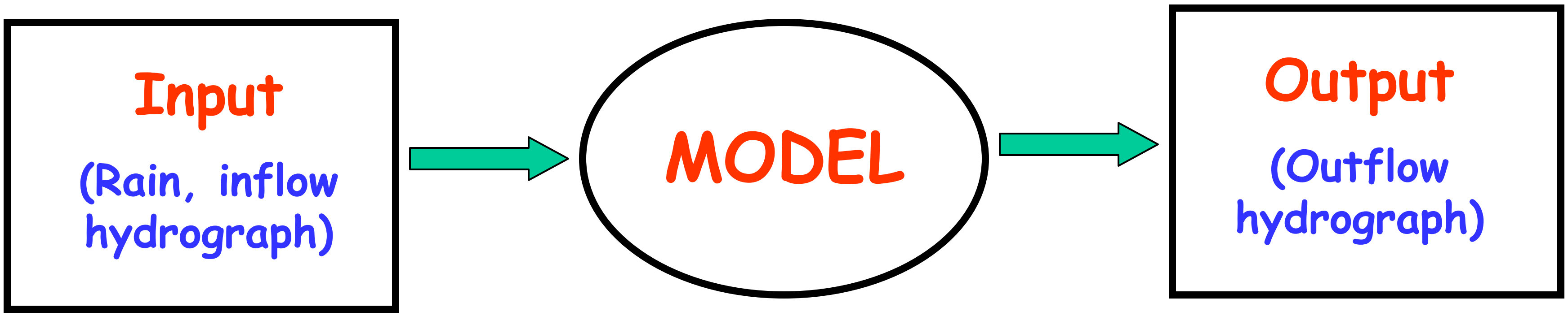
CHAPTER-7

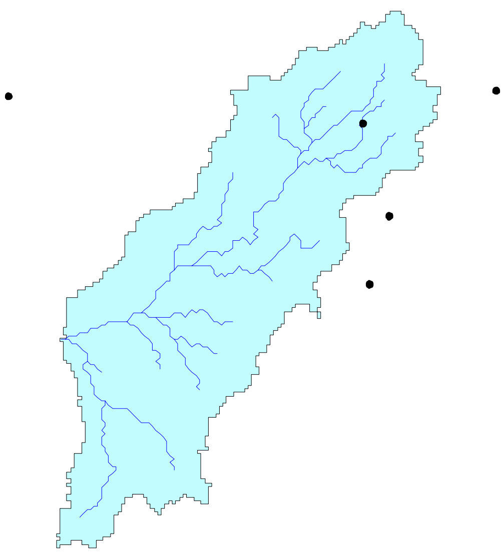
Hydrological Modelling

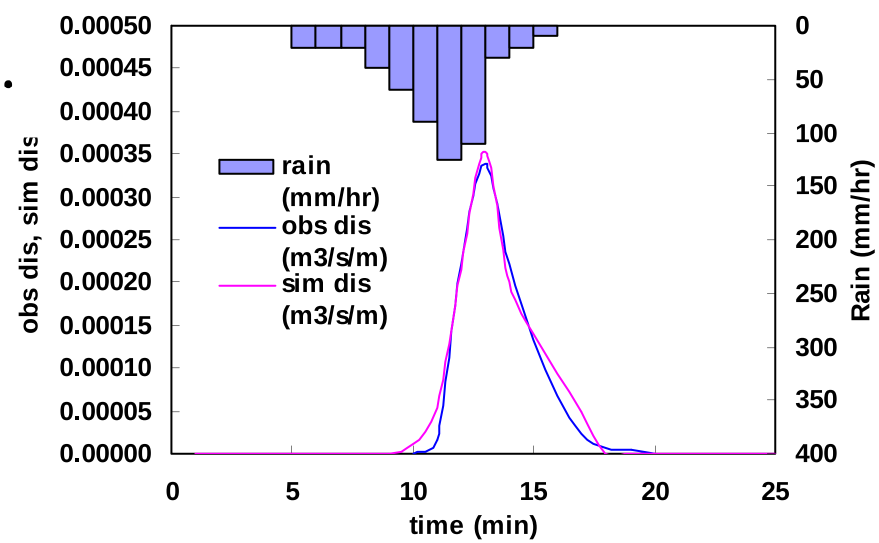
Hydrological Model

It is a set of mathematical expressions that simulate the behavior of the catchment water cycle resulting from rainfall input.

(All hydrological models are approximations of reality).







Hydrological Processes

(1) Precipitation

(2) Canopy Interception

(3) Evaporation

(4) Transpiration

(5) Depression Storage

(6) Infiltration

(7) Sub-surface Flow

(8) Groundwater Flow

(9) Surface Runoff/Overland Flow

(10) River Flow

Surface Runoff will occur when either/or:

(1) Rainfall Rate > Infiltration Rate (even for short rainstorms)

(2) Rainfall Rate < Infiltration Rate and W.T. rises up to G.L. (for longer rainstorms)

Types of Hydrological Models

Deterministic Model:

Determines hydrologic process on the basis of some technical logic. It does not consider randomness, make forecasts, variability is small in randomness, more appropriate

Stochastic Model:

Statistical/probabilistic approach. Requires long past data. It has outputs that are at least partially random, makes predictions more suitable for large random variation (ppt. Models)

Lumped Models:

The system is spatially averaged or regarded as single point in space.

Distributed Models:

Watershed is divide into number of small elements. Considers the hydrological processes taking place at various points in space, defines the model variables as functions of the space dimensions.

Physically based model:

Based on the physics of the phenomina

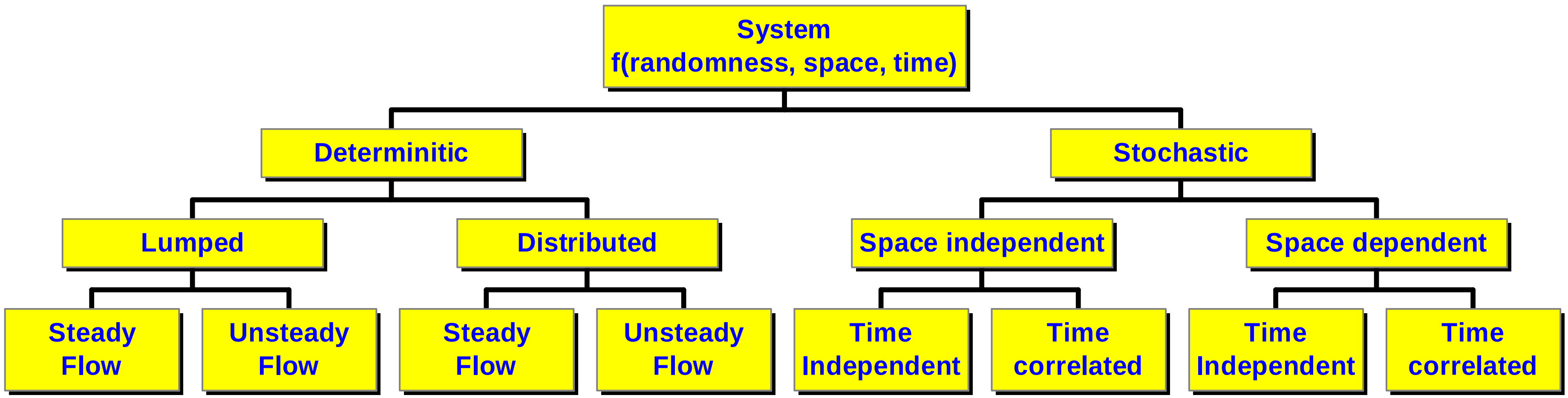
Time independent:

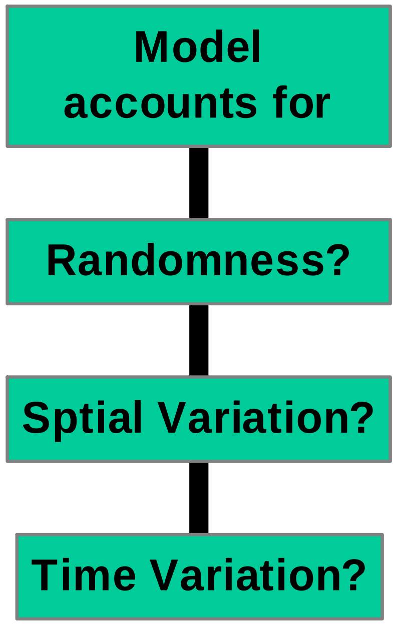
Represents a sequence of hydrologic events that do not influence each other

Time correlated:

Model represents a sequence in which the next event is partially influenced by the current one.

Types of Hydrological Models





Distributed Flow Routing

- Flow of Water is distributed process because flow rate, velocity and depth vary in space and time throughout the watershed.

- This type of model is based on partial differential equations (Saint Venant's) that allow the flow rate & W.L. to be computed as functions of space and time.

- Distributed models computes the Q and W.L. simultaneously.

- So more closely approximate the actual unsteady, non uniform nature of flow.

- Flood W.Ls. are useful for designing height of bridges and levees.

- Flood Flows are useful in designing the reservoirs.

Distributed Routing Models Can be used

- Transformation of rainfall into runoff.
- W.Ls. At important locations.
- Inundation area, extent of flooding.
- Simulates backwater effects u/s of dam.
- Backwater effect due to sea tides in estuaries.
- For routing low flows (irrigation purpose)
- Can simulate Q & Y anywhere in the catchment.

Saint Venant's Equations

Saint Venant's equations (1871) describe one dimensional unsteady open channel flow.

These equations are:
(1) Continuity equation
(2) Momentum equation

Assumptions.

(1) Flow is one dimensional.

(2) Flow changes gradually along the channel.

(3) Channel is straight.

(4) The bed slope is small.

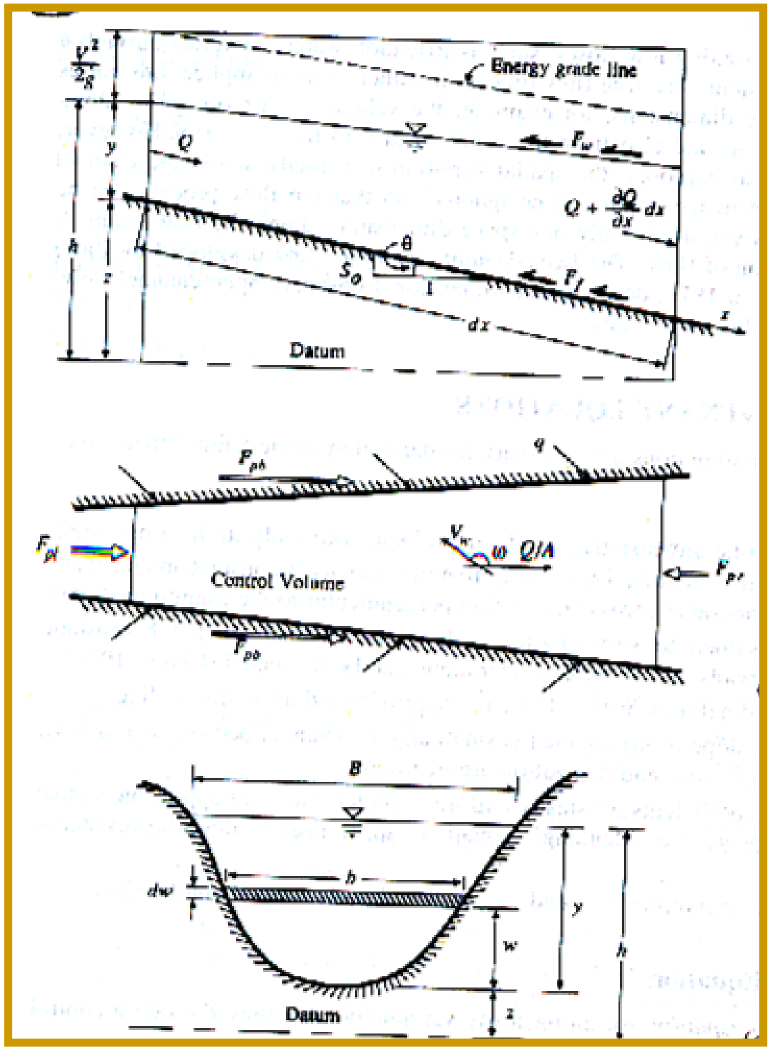
(5) Channel bed is fixed (no erosion, no deposition).

(6) Manning's equation is applicable.

(7) Fluid is incompressible.

(1) Continuity Equation

Consider an elemental control volume of length dx in a channel.



Mass inflow rate to control volume
= mass entering to control volume from u/s + lateral inflow

$$= \rho \left(Q + q \ dx \right) \qquad (1)$$

Mass outflow from the control volume

$$= \rho \left(Q + \frac{\partial Q}{\partial x} \ dx \right) \qquad (2)$$

$\frac{\partial Q}{\partial x}$ = rate of change of channel flow with distance

$A \ dx$ = Volume of channel element

A = Cross sectional area

Rate of change of mass within cont. vol.

$$= \frac{\partial \left(\rho \ A \ dx \right)}{\partial t} \qquad (3)$$

(1) Continuity Equation (cond.)

Mass inflow rate = Mass out flow rate + R.O.C.O.M. within cont. vol.

$$\rho\ (Q + q\ dx) = \rho \left(Q + \frac{\partial Q}{\partial x}\ dx \right) + \frac{\partial (\rho\ A\ dx)}{\partial t} \qquad (4)$$

Assuming density constant, dividing by $\rho\ dx$ throughout

$$\boxed{\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q}$$

(5) Continuity equation in Conservation form

Non conservation form:

V is a dependent variable instead of Q

For unit flow width $A = (y)\ (1) = y$

$$Q = (V)\ (A) = V\ y$$

$$\boxed{\frac{\partial (V\ y)}{\partial x} + \frac{\partial y}{\partial t} = 0}$$

(6) Continuity equation in Non Conservation form

(2) Momentum Equation

From Newton's second law:

Algebraic sum of all external forces acting on the control volume of the fluid is equal to the rate of change of momentum.

$$\sum F = F_g + F_f + F_e + F_w + F_p$$

Where, F_g = gravity force

F_f = Friction force

F_e = Contraction/expansion force (abrupt changes)

F_w = Wind shear force

F_p = Unbalanced pressure force

Substituting values of all forces and simplifying:

$$\frac{\partial Q}{\partial t} + \frac{\partial \left(BQ^2 / A \right)}{\partial x} + g\ A \left(\frac{\partial h}{\partial x} - S_o + S_f + S_e \right) - \beta\ q\ v_x + W_f\ B = 0$$ (7) Momentum Equation

Where, S_f = friction slope, S_e = eddy loss slope, B = width of flow (top),
v_x = velocity in x direction, β = momentum coef., W_f = wind shear factor

Continuity and Momentum Equations

One dimensional distributed routing model:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q$$

(1) Continuity Equation

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g\left(S_o - S_f\right) = 0$$

(2) Momentum Equation (conservation form)

- Local Acc. term — Acc. Due to unsteady ness of the flow
- convective Acc. term — Acc. in steady flow. Acc. Due to change in position
- Pressure force term
- Gravity force term
- friction force term

$$\frac{\partial V}{\partial t} + V\left(\frac{\partial V}{\partial x}\right) + g\left(\frac{\partial y}{\partial x} - S_o + S_f\right) = 0$$

(3) Momentum Equation (Non conservation form)

Classification of distributed flow routing models

Various one dimensional distributed flow routing models:

(1) Kinematic Wave Model
(2) Diffusive Wave Model
(3) Dynamic Wave Model

In all three surface routing models, continuity equation remains same,
Only Momentum equations are different

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q$$

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g\left(S_o - S_f\right) = 0$$

Kinematic wave model

Diffusive wave model

Dynamic wave model

(1) Kinematic Wave Model:

- Simplest distributed model.
- It neglects the local acceleration, convective acceleration and pressure terms in momentum equation.
- Friction and gravity forces balance each other.
- (For modelling Uniform flow, canal flow).

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q$$

(1) Continuity Equation

$$\left(S_o - S_f\right) = 0$$

$$S_o = S_f$$

(2) Momentum Equation

For steady flow $\frac{\partial A}{\partial t} = 0$

For uniform flow $q = 0$

So, Continuity eqn for steady uniform flow $\frac{\partial Q}{\partial x} = 0$

(2) Diffusive Wave Model:

- It neglects the local acceleration and convective acceleration terms in the momentum equation.
- Incorporates the pressure term.
- (For modelling gradually varied flow, backwater effects, daming action).

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q$$

(1) Continuity Equation

$$g \frac{\partial y}{\partial x} - g \left(S_o - S_f\right) = 0$$

(2) Momentum Equation

$$\frac{\partial y}{\partial x} = \left(S_o - S_f\right)$$

$$S_f = \left(S_o - \frac{\partial y}{\partial x}\right)$$

(3) Dynamic Wave Model:

- It considers all the acceleration and pressure terms in momentum eqn.
- When inertial and pressure forces are important
- (For modelling rapidly varied flow, hydraulic jump, flood wave).

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q$$

(1) Continuity Equation

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g\left(S_o - S_f\right) = 0$$

(2) Momentum Equation

Kinematic & Dynamic wave models

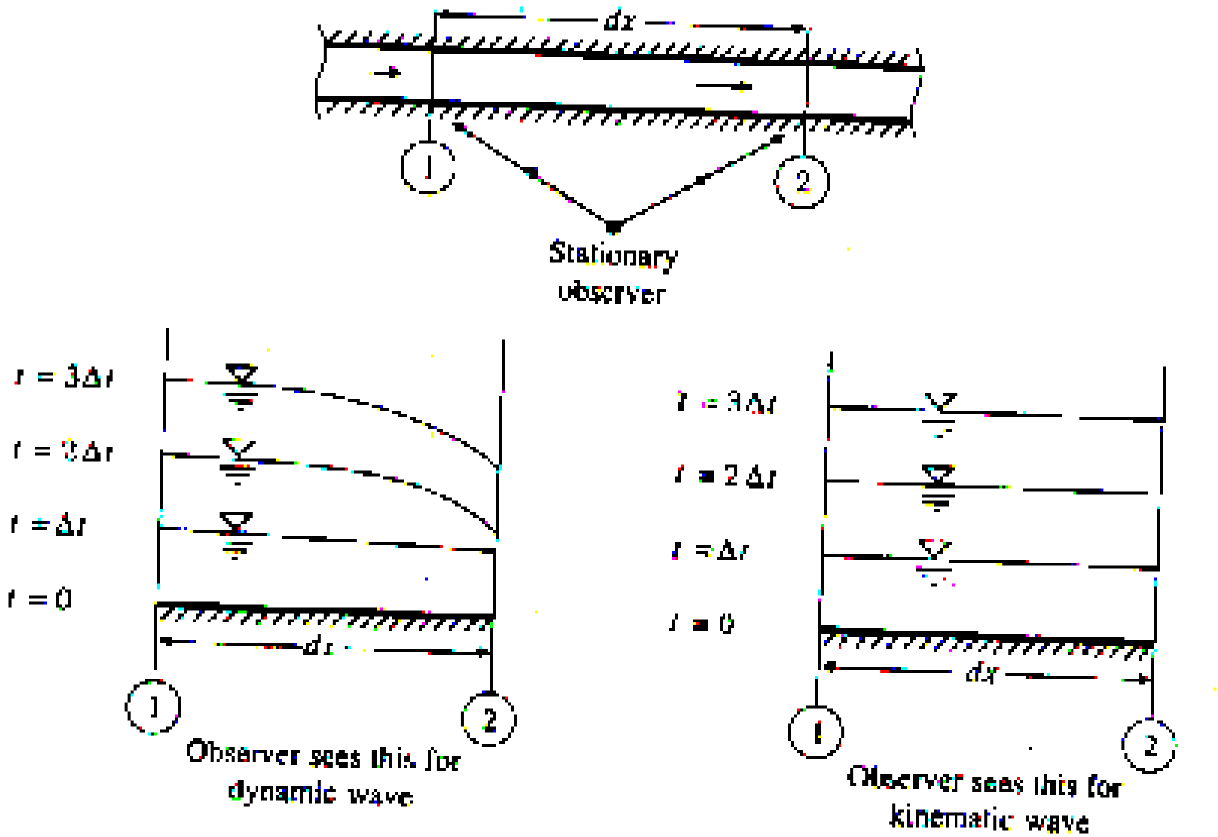


FIGURE 9.3.1
Kinematic and dynamic waves in a short reach of channel as seen by a stationary observer.

Kinematic Wave Celerity

- Wave is a variation in flow (h).
- Wave celerity is the velocity with which this variation travels along the channel.
- Kinematics refers to the study of motion exclusive of the influence of mass and force.

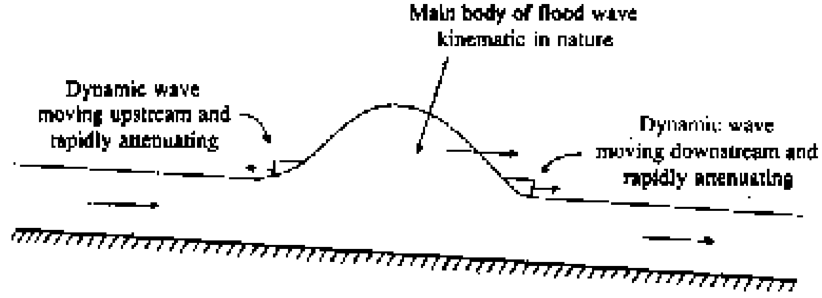


FIGURE 9.3.2
Motion of a flood wave.

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q$$

(1) Continuity Equation (contains 2 variables, A & Q)

$$S_o = S_f$$

(2) Momentum Equation

Using Manning's equation

$$Q = A\,V = A\frac{1.49}{n}R^{2/3}\,S^{1/2} = \frac{1.49\;A^{5/3}\;S^{1/2}}{n\;P^{2/3}} \qquad (3)$$

Which can be solved for A

$$A = \left(\frac{n\;P^{2/3}}{1.49\sqrt{S_o}}\right)^{3/5} Q^{3/5} \qquad (4)$$

$$A = \alpha\ Q^{\beta}$$ where $$\alpha = \left(\frac{n\ P^{2/3}}{1.49\sqrt{S_o}}\right)^{0.6}$$ and $$\beta = 0.6$$

Differentiating w.r.t t

$$\frac{\partial A}{\partial t} = \alpha\ \beta\ Q^{\beta-1}\left(\frac{\partial Q}{\partial t}\right) \qquad (5)$$

Substituting in equation (1)

$$\frac{\partial Q}{\partial x} + \alpha\ \beta\ Q^{\beta-1}\left(\frac{\partial Q}{\partial t}\right) = q$$

Contains one variable, Q (6)

An increment in flow, dQ can be written as

$$dQ = \frac{\partial Q}{\partial x}dx + \frac{\partial Q}{\partial t}dt \qquad (7)$$

Dividing by dx and rearranging

$$\frac{\partial Q}{\partial x} + \frac{dt}{dx}\frac{\partial Q}{\partial t} = \frac{dQ}{dx} \qquad (8)$$

Eqn (6) and (8) are identical if $\frac{dQ}{dx} = q$

$$\alpha\ \beta\ Q^{\beta-1}\left(\frac{\partial Q}{\partial t}\right) = \frac{dt}{dx}\left(\frac{\partial Q}{\partial t}\right) \qquad (9)$$

$$\frac{dx}{dt} = \frac{1}{\alpha \ \beta \ Q^{\beta-1}} \quad (10)$$

differentiating and rearranging gives

$$\frac{dQ}{dA} = \frac{1}{\alpha \ \beta \ Q^{\beta-1}} \quad (11)$$

By comparing (10) & (11)

$$\frac{dx}{dt} = \frac{dQ}{dA} \quad (12)$$

$$C_k = \frac{dx}{dt} = \frac{dQ}{dA}$$

Kinematic wave celerity is due to lateral inflow (13)

Ck can also be expressed interms of depth y

$$C_k = \frac{1}{B}\frac{d}{dy}(Q) \quad \text{as} \quad dA = B \ dy \quad (14)$$

Dynamic wave celerity

$$C_d = \sqrt{g \ y} \quad (15)$$

Analytical Solution of the Kinematic Wave

- It specifies the distribution of the flow as a function of distance x along the channel and time t.
- This solution may be obtained numerically by using **Finite Difference Approximations.**

Assumption

Lateral flow is negligible.

The solution requires:

Initial Condition:

$Q\ (x,0)$ = The value of the flow along the channel at the begining of the calculations.

Boundary Condition:

$Q\ (0,t)$ = Inflow hydrograph at the u/s end of the channel.

$$\frac{dQ}{dx} = q = 0 \qquad or \;\; Q = a$$

Objective:

$Q\ (L,t)$ = To get the outflow hydrograph at the d/s end of the channel

If lateral flow is neglected.

Analytical Solution of the Kinematic Wave (cond.)

- If the flow rate is known at a point in time and space, this flaw can be propagated along the channel at the kinematic wave celerity.

$$C_k = \frac{dx}{dt} = \frac{dQ}{dA}$$

- The equation for these lines are found by

$$\int_0^x dx = \int_{t_o} C_k \; dt$$

$$x = C_k \left(t - t_o\right)$$

So the discharge Q entering a channel of length L at time t0 will appear at the outlet.

$$t = t_0 + \frac{L}{C_k}$$

The slope of the characteristic line is $C_k = \frac{dQ}{dA}$ for the particular value of the flow rate being considered.

Lines are straight because q =0, Q is constant.

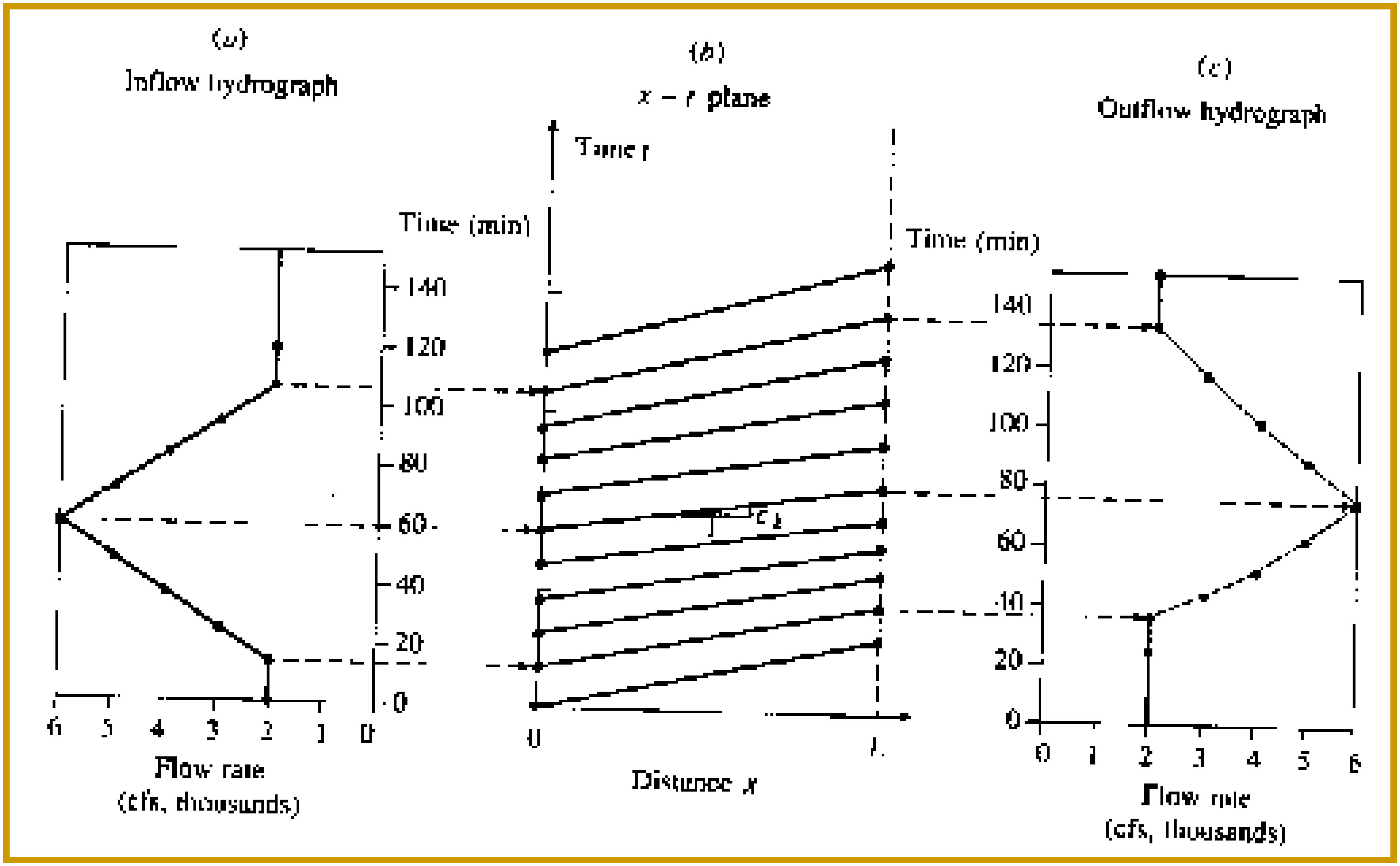
Example 9.4.1

TABLE 9.4.1
Routing of a flow hydrograph by analytical solution of the kinematic wave (Example 9.4.1).

Column:	1 Inflow Time (min)	2 Inflow Rate (cfs)	3 Kinematic wave celerity (ft/s)	4 Travel time (min)	5 Outflow* time (min)
	0	2000	10.0	25.1	25.1
	12	2000	10.0	25.1	37.1
	24	3000	11.7	21.3	45.3
	36	4000	13.2	19.0	55.0
	48	5000	14.4	17.4	65.4
	60	6000	15.5	16.1	76.1
	72	5000	14.4	17.4	89.4
	84	4000	13.2	19.0	103.0
	96	3000	11.7	21.3	117.3
	108	2000	10.0	25.1	133.1
	120	2000	10.0	25.1	145.1

*Outflow time = Inflow time + Travel time.

Example 9.4.1



(a)
Inflow hydrograph
Time (min)
140
120
100
80
60
40
20
0
6 5 4 3 2 1 0
Flow rate
(cfs, thousands)
(b)
x – t plane
Time t
0
L
Distance x
(c)
Outflow hydrograph
Time (min)
140
120
100
80
60
40
20
0
0 1 2 3 4 5 6
Flow rate
(cfs, thousands)

Merits & Demerits of Physically based distributed Models

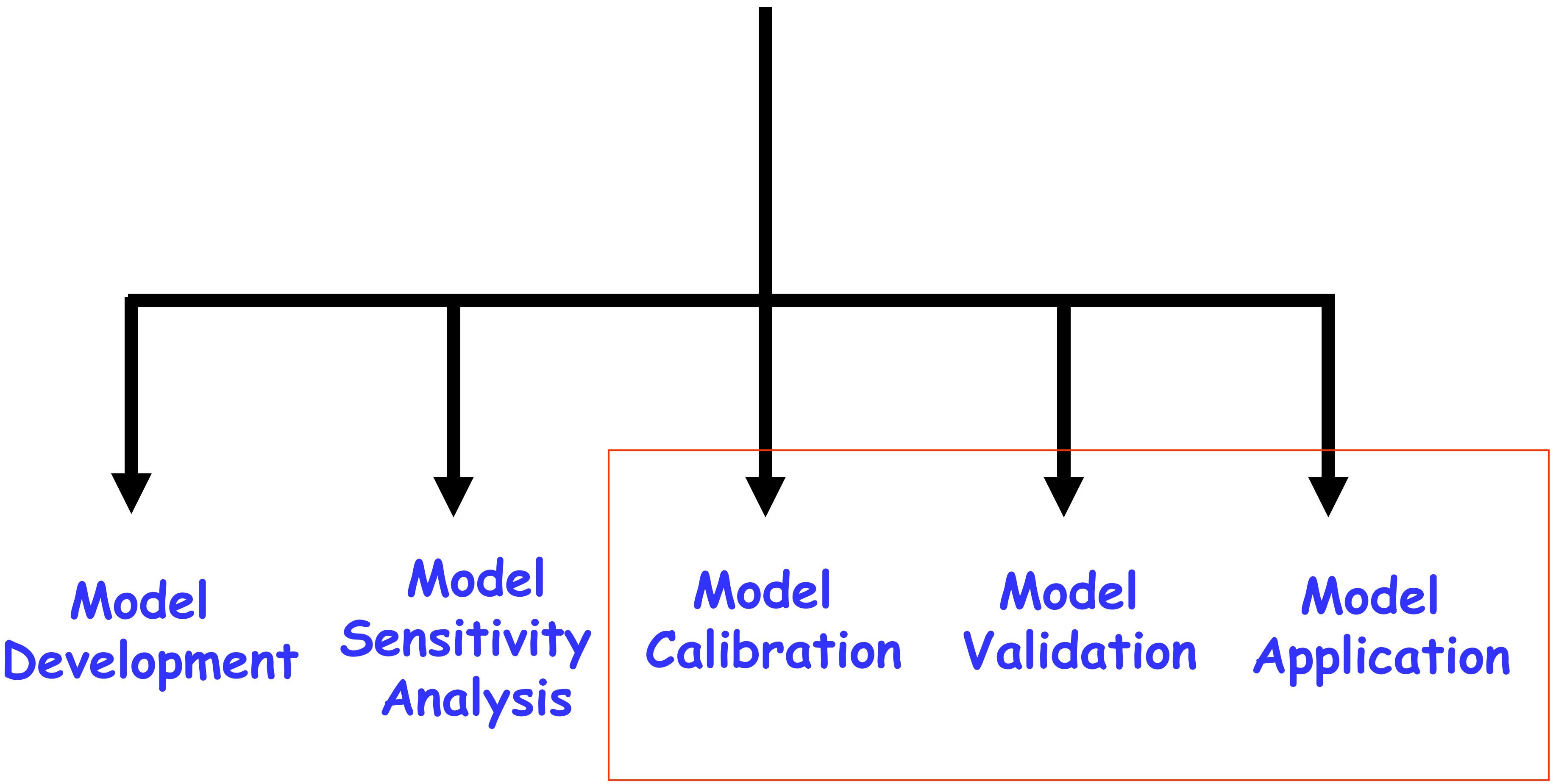
Merits

- Models are based on the physics of the phenomena.
- Governing equations are non linear partial differential equations.
- Considers transfer of mass and energy within the watershed.
- Can simulate hydrology in space and time both.
- Can forecast spatial patterns
- Can forecast temporal and spatial distribution of floods
- Depth and flow of water can be obtained anywhere in the watershed.
- Can model urban hydrology
- Parameters have physical meaning.
- Can be applied out side the calibration range
- Can be used to model un-gauged catchments (Flow data can be generated).
- Climate change scenario can be modelled
- Land-use change scenario can be modelled.
- Forecast the movement of pollutants and sediments

Demerits

- Experiments are required for Parameter estimation
- Computer resources are required
- Data preparation is required.
- Governing equations cannot be solved analytically.
- Numerical solution is required for partial differential equations.
- More complicated computer programming is required.
- More qualified persons are required.
- It may take 3-6 months for the training of these models.
- More funds are required.

How to use a Physically based distributed Model?



Types of Hydrological Models based on Spatial information discritization

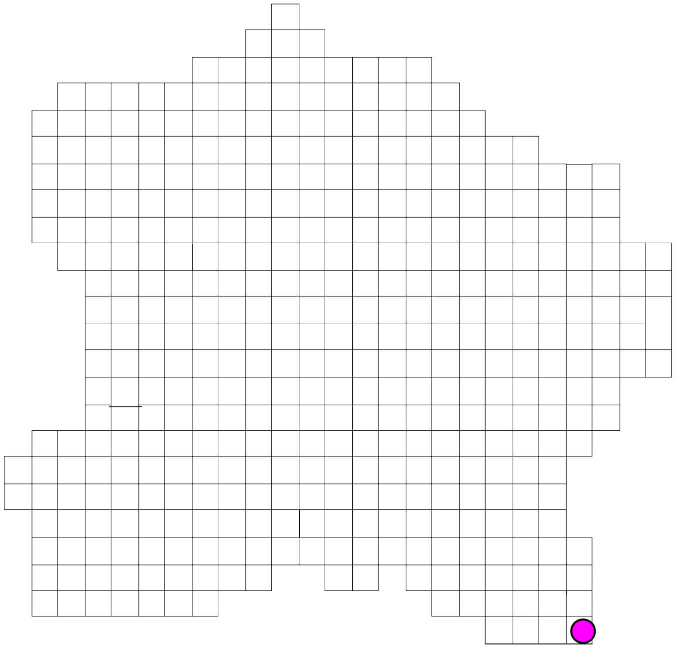
(1) Grid based Hydrological Model

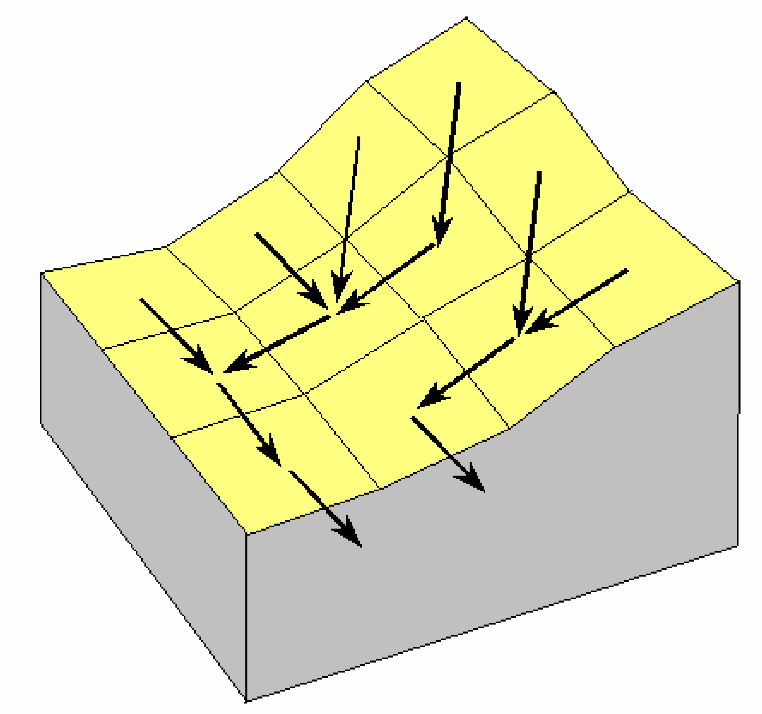
(2) Slope based Hydrological Model

(1)Grid based Hydrological Model

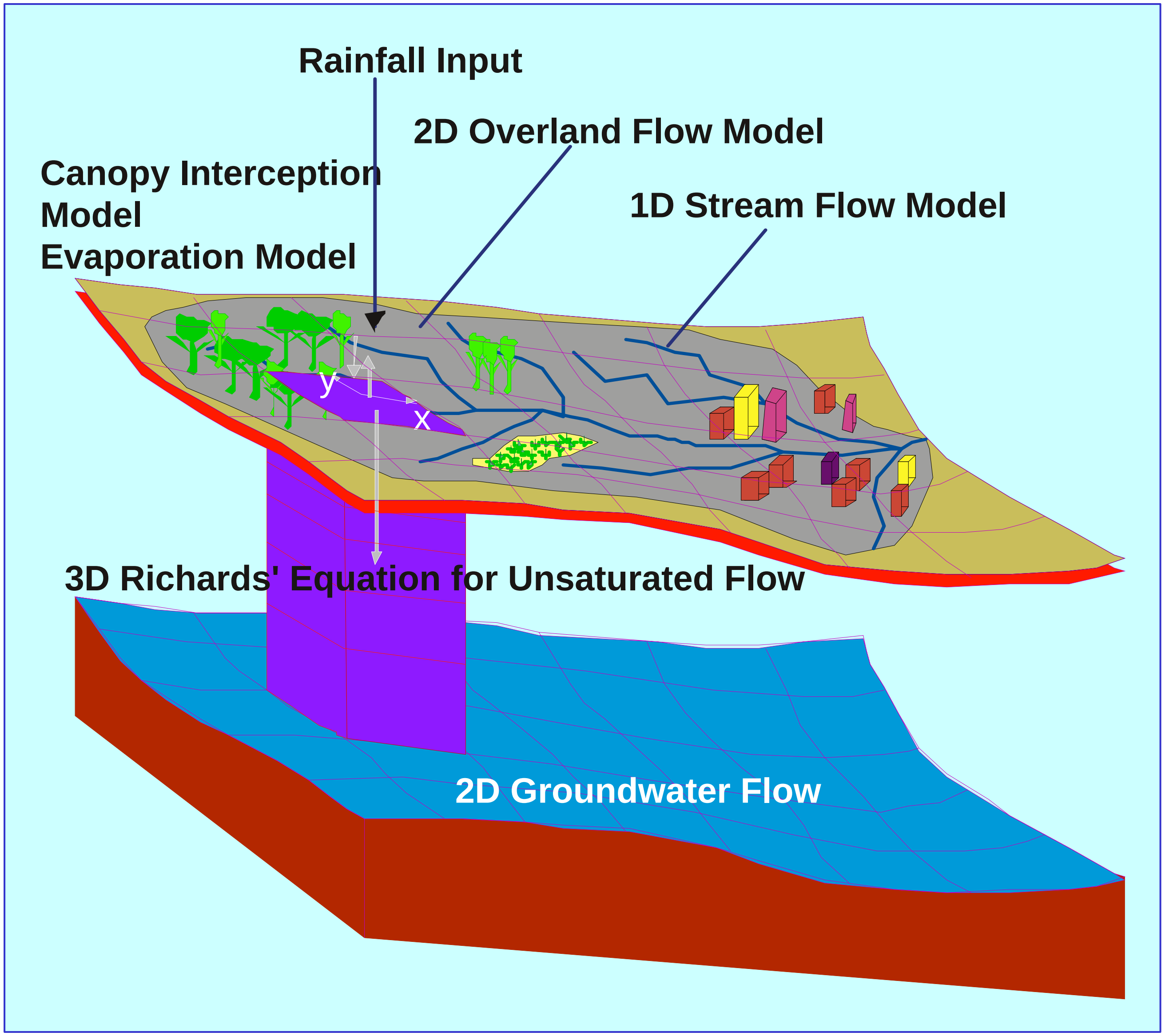
Watershed is divided into number of elements of square shape and of uniform size.

Elements/grids/pixels/cells.

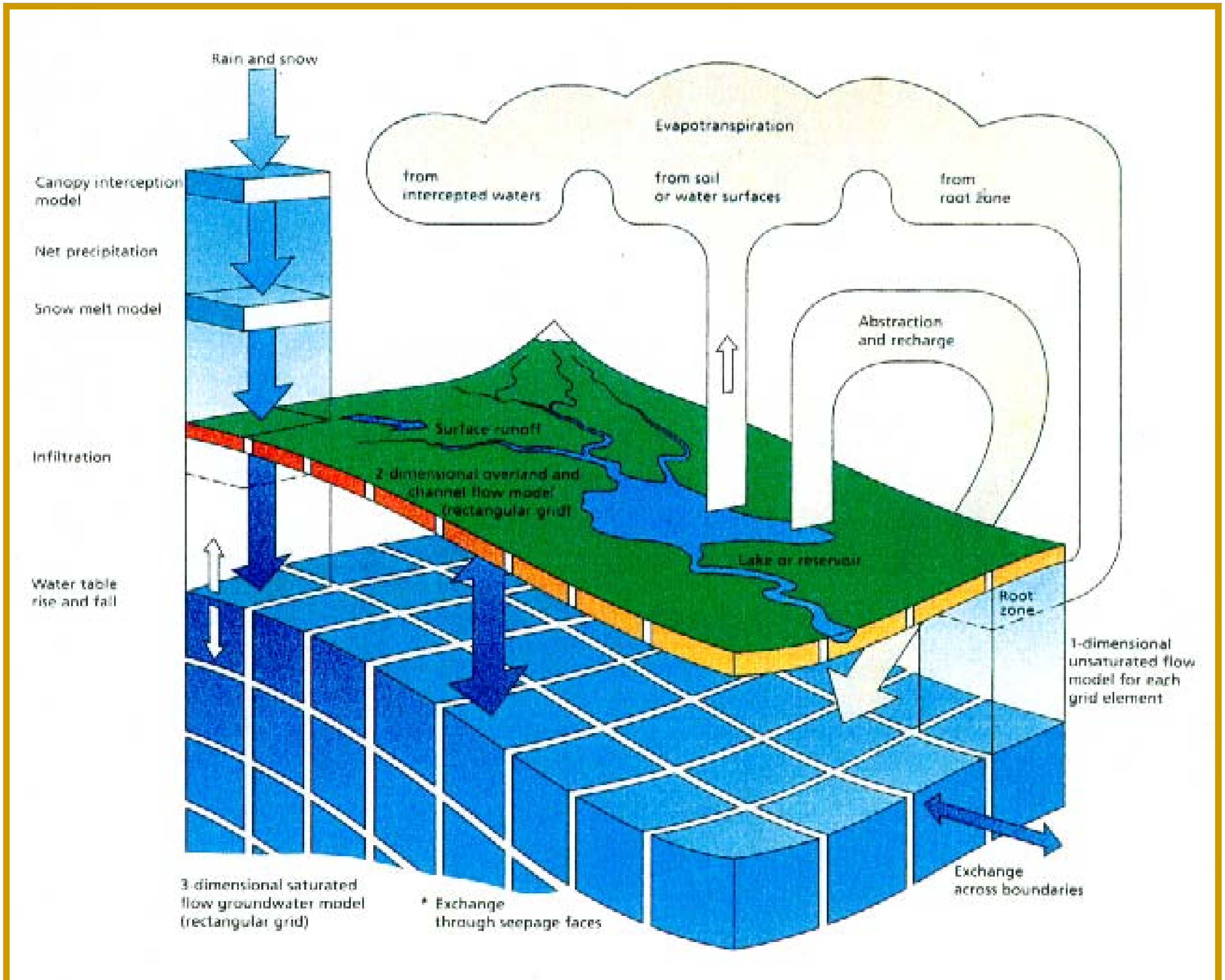




(1)Grid based Hydrological Model for Tropical areas (cond.)



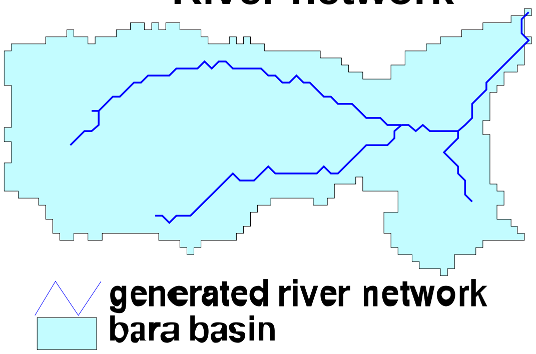
(1)Grid based Hydrological Model (cond.)



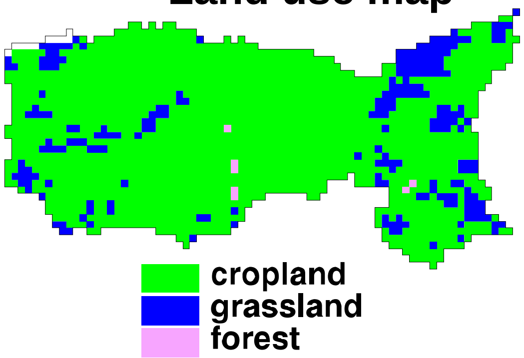
APPLICATION TO BARA RIVER BASIN AT JHANSI-POST

(catch area = 1841 km^2)

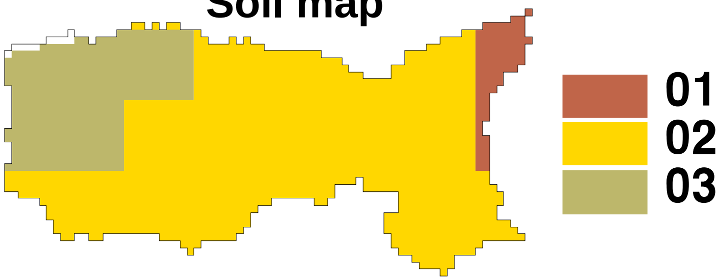
River network



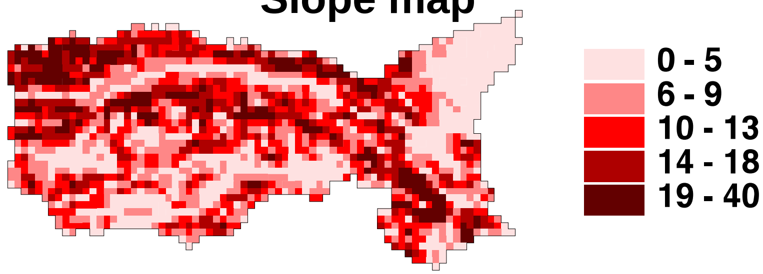
Land use map



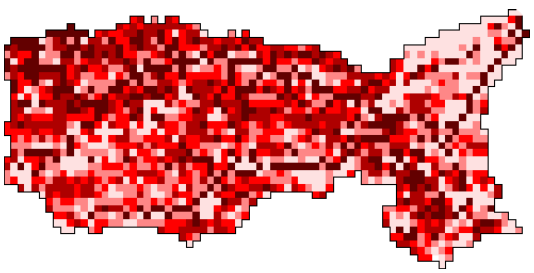
Soil map



Slope map



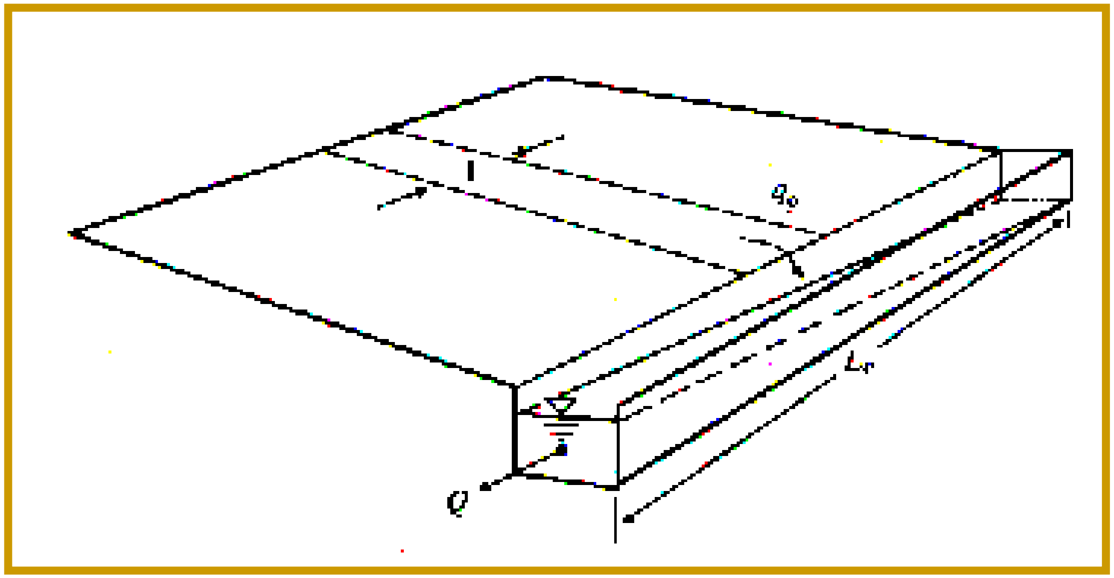
Surface runoff map

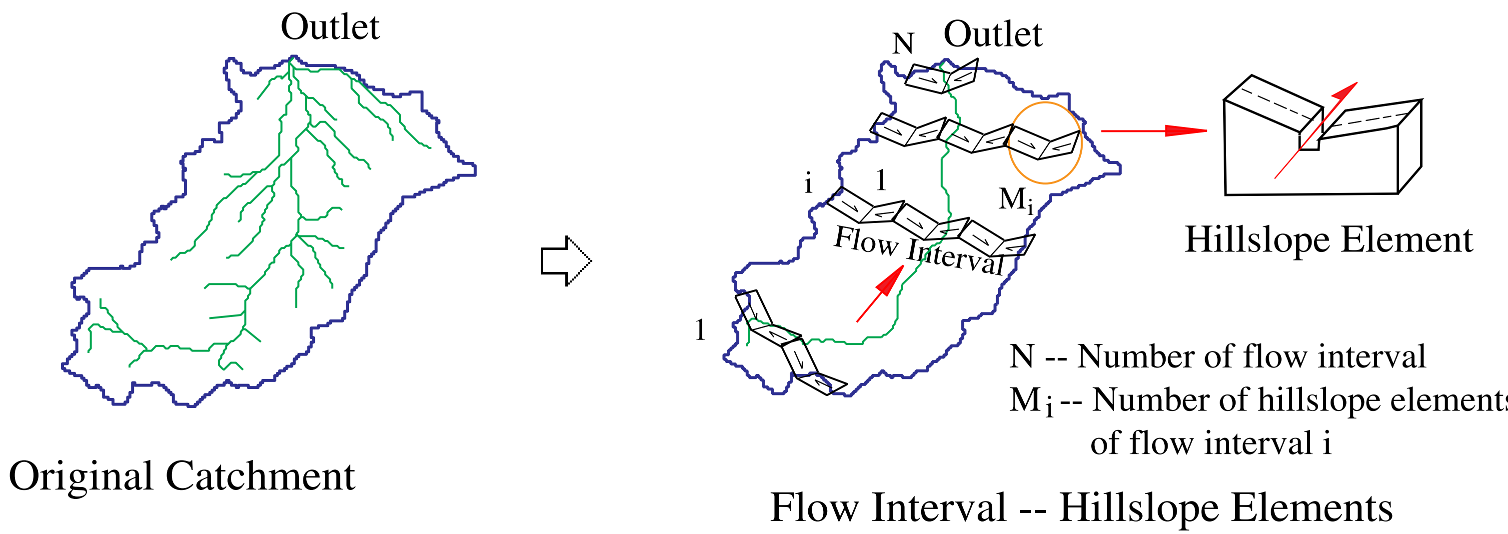


(2) Slope based Hydrological Model

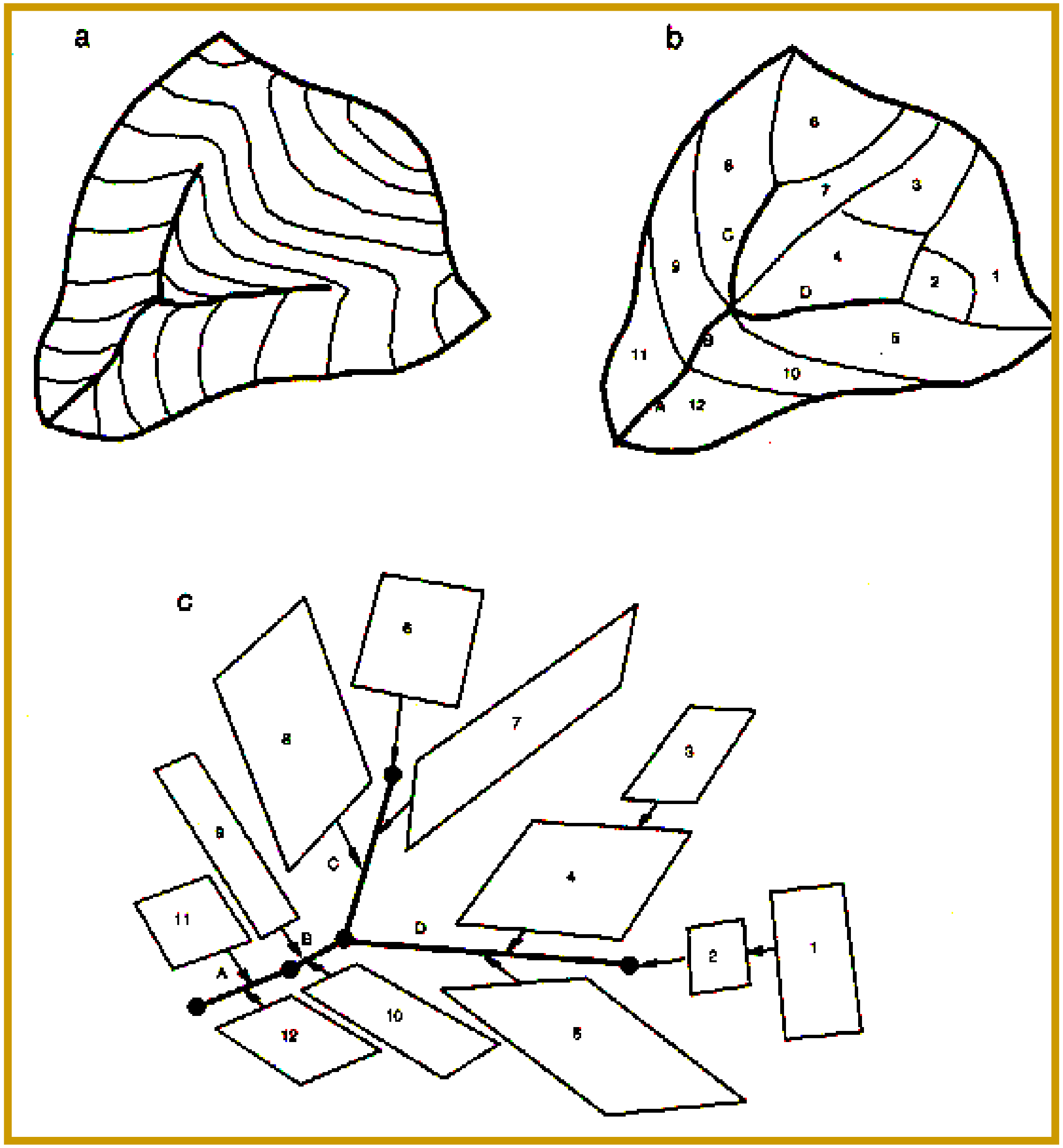
Watershed is divided into number of elements of various shapes and sizes.

Elements are called hillslopes. They are assumed of uniform shape.





(2) Slope based Hydrological Model (contd.)



Input Data in Hydrological Models

INPUT

Spatial Data

1) Digital Elevation Model (DEM)
2) Slope Map
3) Flow Direction Map
4) Flow Accumulation MAP
5) River network
6) Watershed Boundary
7) Land-use MAP
8) Soil Map
9) Canopy cover
10) Leaf Area Index Map
11) Root Depth
12) Rainfall Distribution Map
13) Evaporation distribution Map
14) Water Table Map

Temporal Data

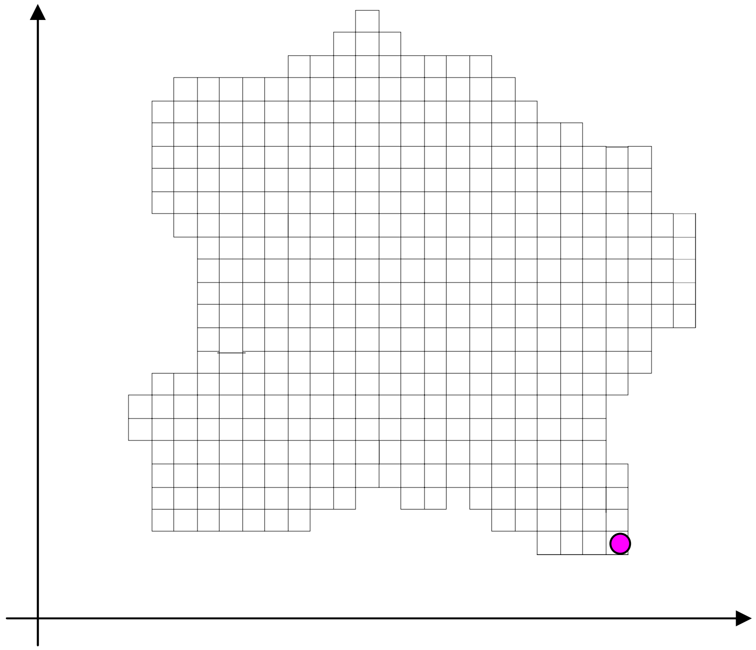
1) Hourly / Daily Rainfall data
2) Potential Evaporation distribution
3) Leaf Area Index
4) Irrigation Supplies

Digital Elevation Model

A GIS file having z coordinate as elevation/R.L.

How it can be generated?

- Digitization from Contour Maps
- National (Local) Digital Data Sets
- Global Data Sets
  - GLOBE project, 99% of the earth 30-arc-second resolution National Imagery and Mapping Agency Digital Terrain Elevation Data. CD ROM
  - GTOPO30, developed by USGS, over a 3 year period covers the globe at 30 arc seconds and is available for download at www



Creating Digital Elevation Model (DEM/DTM)

- Select an Area
- Capture the contour from Topo Map
  - Digitized the contour using digitizer software, assign the contour height as attribute
  - Scan the Topo map by A0 scanner and use some software to convert raster data into vector data (Vtrak,..) and contour height as attribute
- Input other spot elevation if any available
- Create TIN (Triangular Irregular Network)
- Create DEM at Desired GRID Resolution

78	72	69	71	58	49
74	67	56	49	46	55
69	53	44	37	38	49
64	58	55	22	31	24
68	61	47	21	16	19
74	53	34	12	11	12

Flow Direction

Is the value assigned to the pixel in which direction water has to flow. These directions are 1, 2, 4, 8, 16, 32, 64 and 128. It is computed on the basis of steepest descent direction.

78	72	69	71	58	49
74	67	56	49	46	55
69	53	44	37	38	49
64	58	55	22	31	24
68	61	47	21	16	19
74	53	34	12	11	12

32	64	128
16		1
8	4	2

Slope = change in z value / distance

2	2	2	4	4	8
2	2	2	4	4	8
1	1	2	4	8	4
128	128	1	2	4	8
2	2	1	4	4	4
1	1	1	1	4	16

Flow Accumulation

No. of cells accumulating in a cell.

For out let of the watershed, flow accumulation = Total no. of cells - 1

2	2	2	4	4	8
2	2	2	4	4	8
1	1	2	4	8	4
128	128	1	2	4	8
2	2	1	4	4	4
1	1	1	1	4	16

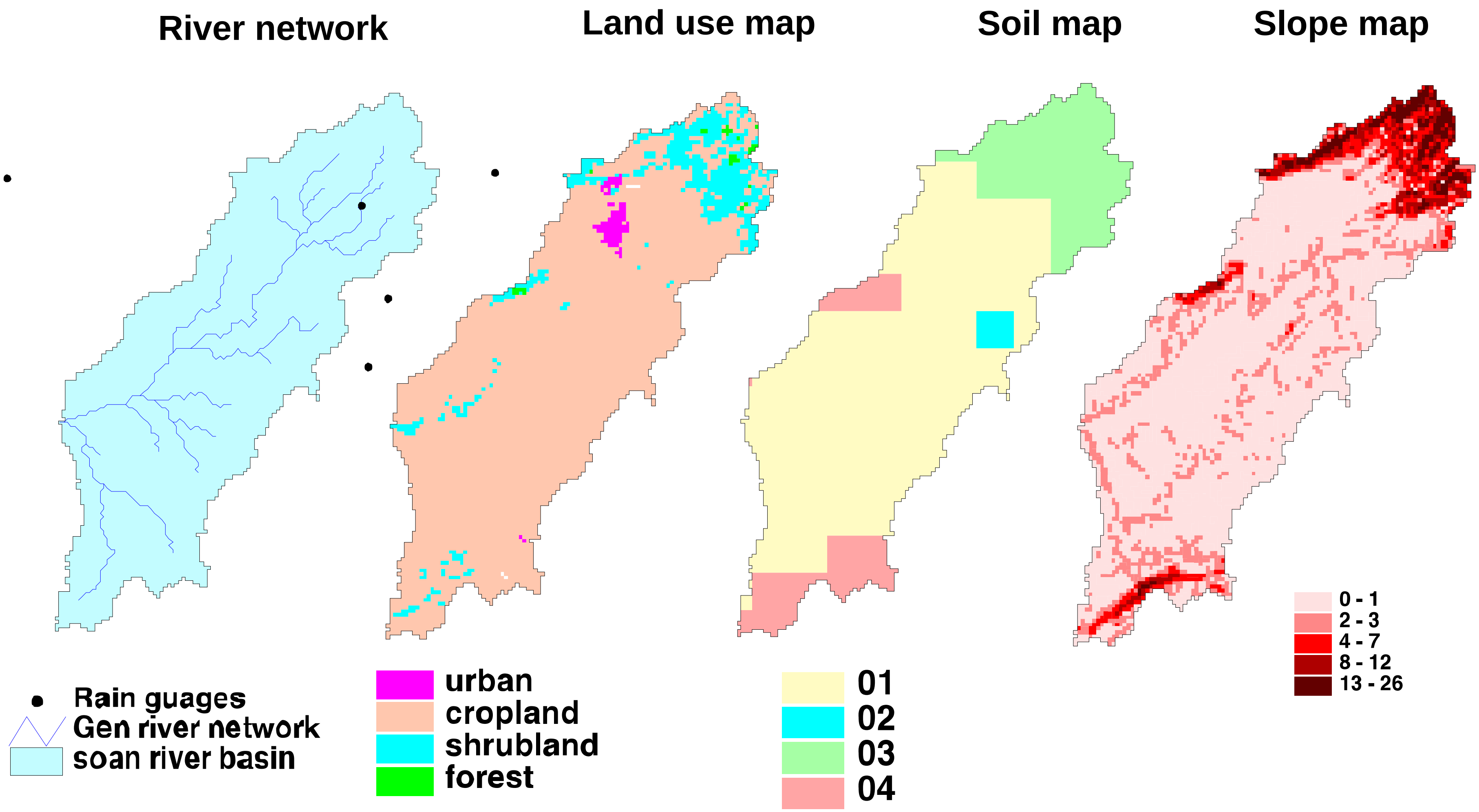
0	0	0	0	0	0
0	1	1	2	2	0
0	3	7	5	4	0
0	0	0	20	0	1
0	0	0	1	24	0
0	2	4	7	35	2

1. Cells with higher flow Accumulation are areas of concentrated flow and will be used as stream channels

2. Cells with 0 FA are local topographic heighs and used to identify ridges.

SOAN RIVER BASIN

(catch area = 6549 km^2)



Well Known Physically based distributed Hydrologic Models

(1) SHE Model

- Systeme Hydrologique Europeaon
- Denmark, France and Britain
- 1-d channel flow
- 2-d overland flow
- 1-d vertical unsaturated flow
- 2-d saturated zone component

(2) IHDM Model

- Institute of Hydrology Distribution Model
- 1-d channel flow
- 1-d overland flow component
- 2-d unsaturaterd flow
- 2-d saturated flow

(3) SWAM Model

- Small Watershed Model
- USDA Agricultural Research Service

(4) IISDHM

- Institute Of Industrial Science Distributed Hydrology Model
- University of Tolyo, Japan

(5) GBHM

- Geomorphological based Hydrological Model
- Slope based model
- University of Tokyo, Japan

Choice of distributed Models?
It is not simple!

Factors

- Economic Constrants
- Personal Experience
- Highly Qualified Persons (Ph.Ds.)
- Hydrological Conditions
- Data Preparation
- Data availability (Fine resolutions Spatial and Temporal data)
- Simulation time
- Accuracy required