Hydraulics Engineering Lec #2 : Surface Profiles and Backwater Curves in Channels of Uniform sections

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Steady Flow in Open Channels

- Specific Energy and Critical Depth
- Surface Profiles and Backwater Curves in Channels of Uniform sections
- Hydraulics jump and its practical applications.
- Flow over Humps and through Constrictions
- Broad Crested Weirs and Venturi Flumes

Types of Bed Slopes

- Mild Slope (M)

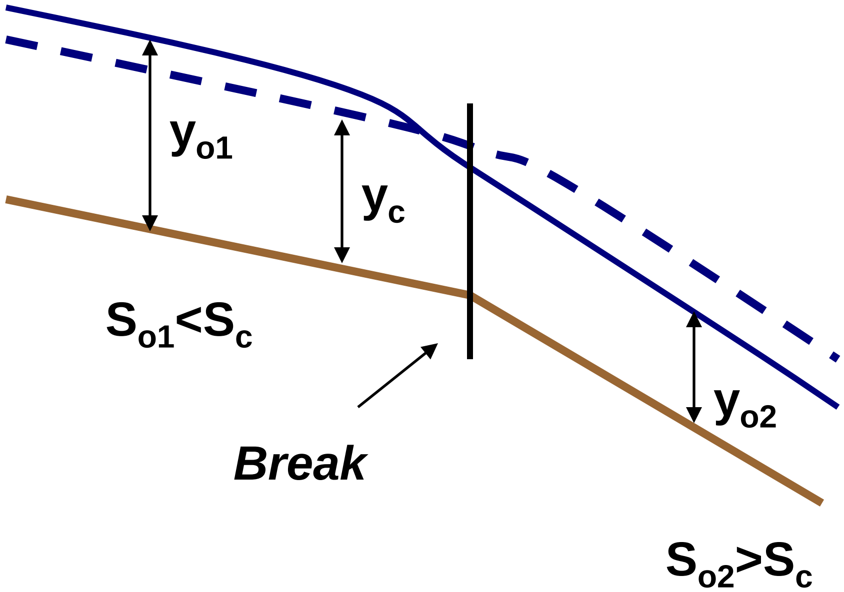
  $y_o > y_c$
  $S_o < S_c$

- Critical Slope (C)

  $y_o = y_c$
  $S_o = S_c$

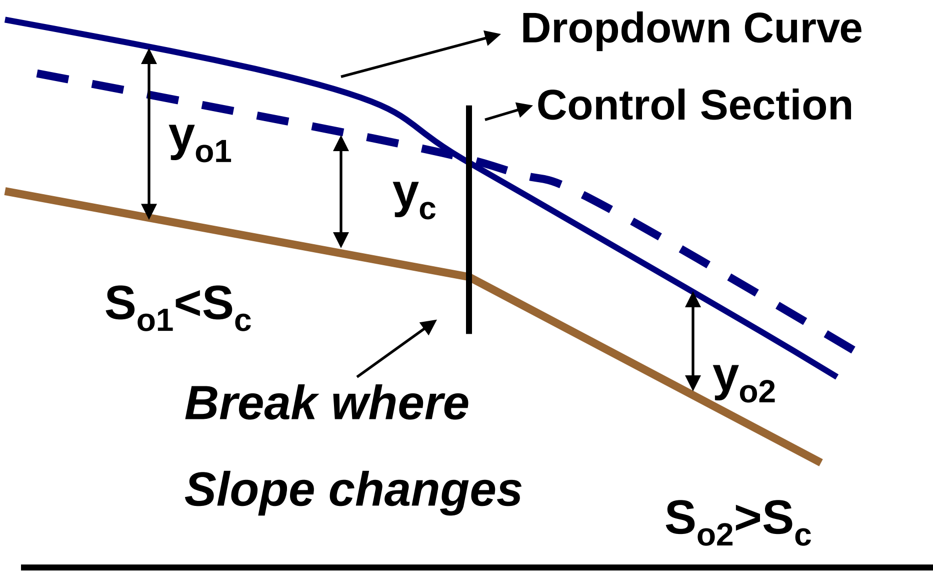
- Steep Slope (S)

  $y_o < y_c$
  $S_o > S_c$

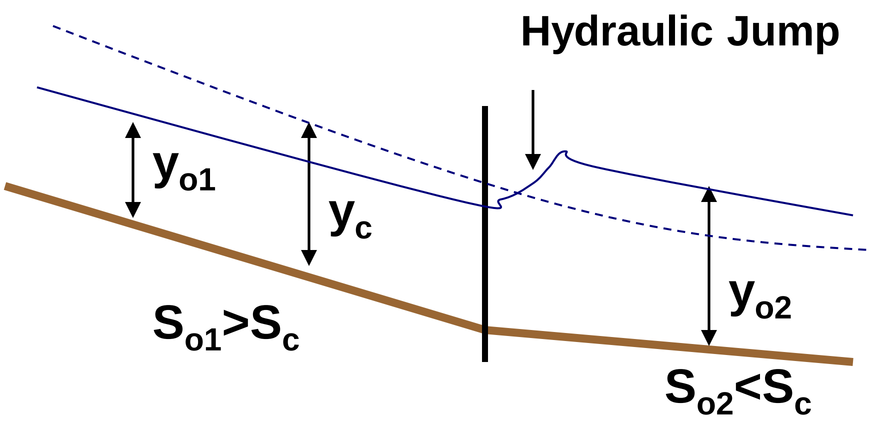


Occurrence of Critical Depth

- Change in Bed Slope
  - **Sub-critical to Super-Critical**
    - Control Section

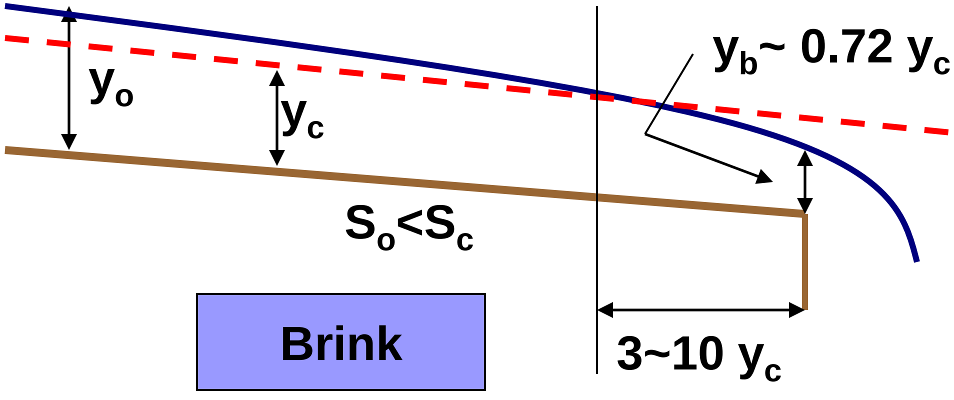


  - **Super-Critical to Sub-Critical**
    - Hydraulics Jump

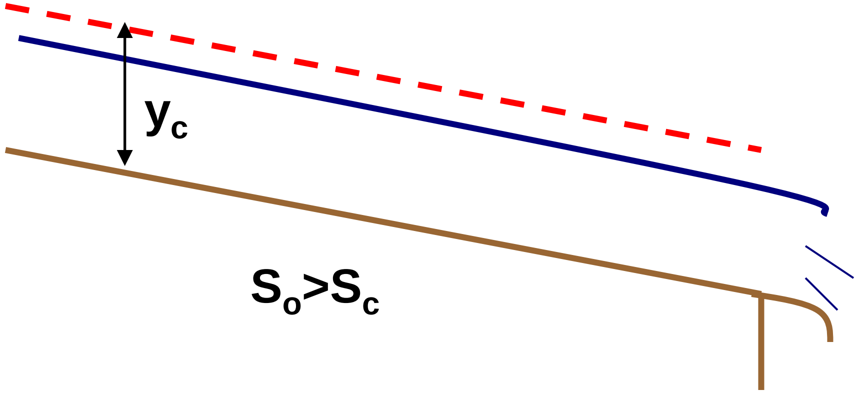


Occurrence of Critical Depth

- Change in Bed Slope
  - Free outfall
    - Mild Slope

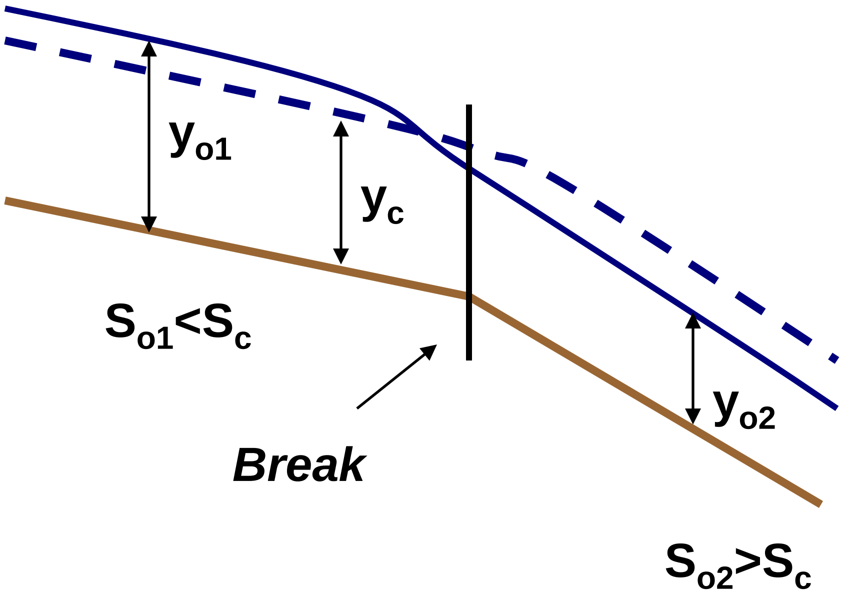


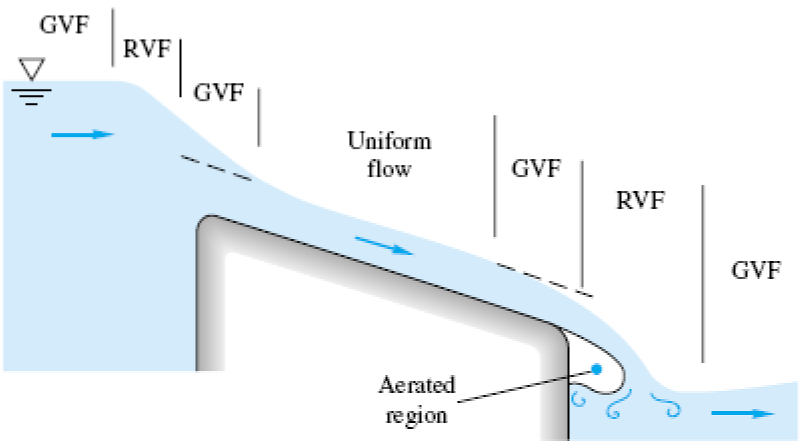
  - Free Outfall
    - Steep Slope



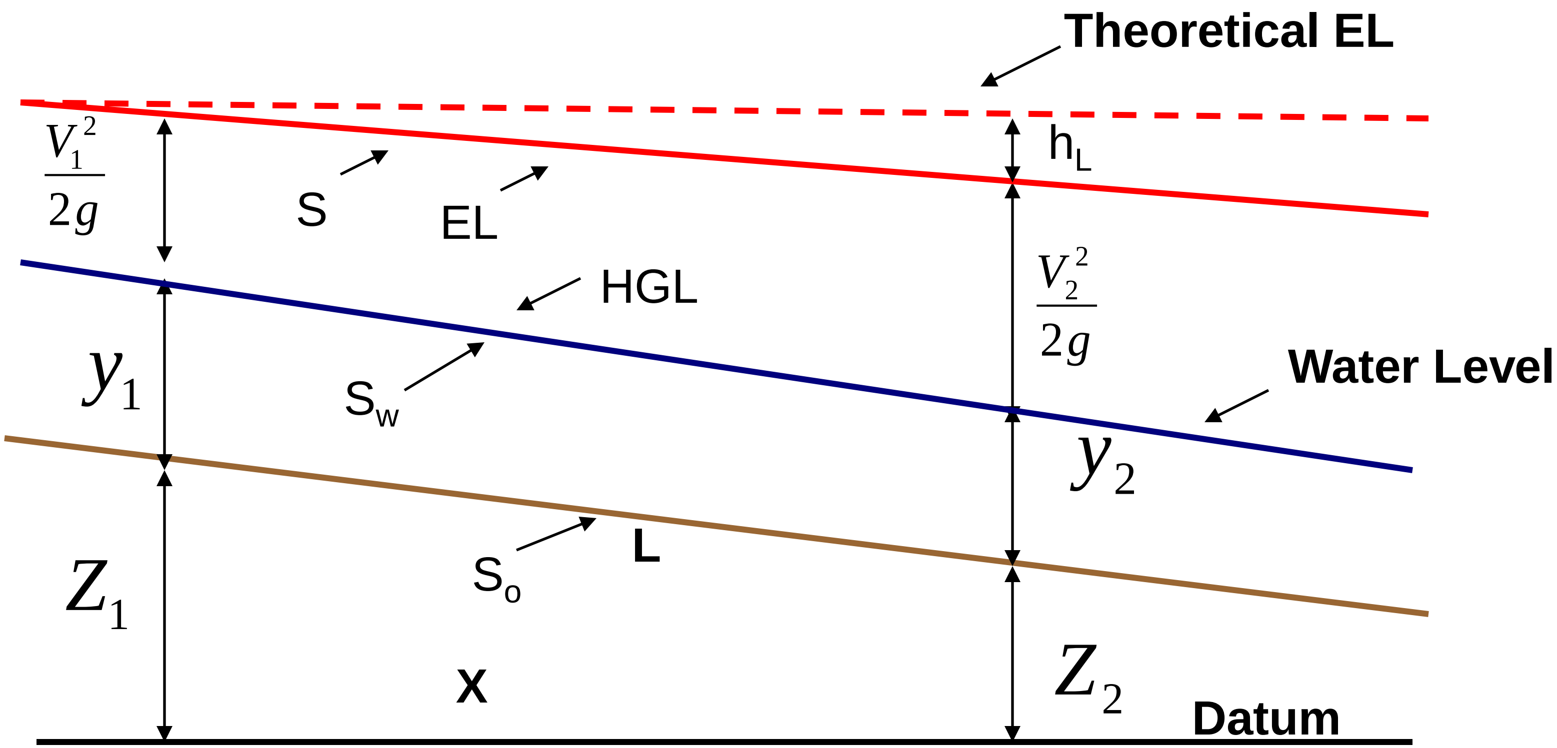
Non Uniform Flow or Varied Flow.

- For uniform flow through open channel, *dy/dl* is equal to zero. However for non uniform flow the gravity force and frictional resistance are not in balance. Thus *dy/dl* is not equal to zero which results in non-uniform flow.
- There are two types of non uniform flows. In one the changing condition extends over a long distance and this is called gradually varied flow. In the other the change may occur over very abruptly and the transition is thus confined to a short distance. This may be designated as a local non uniform flow phenomenon or rapidly varied flow.





Energy Equation for Gradually Varied Flow.



$$Z_1 + y_1 + \frac{V_1^2}{2g} = Z_2 + y_2 + \frac{V_2^2}{2g} + h_l$$

Energy Equation for Gradually Varied Flow.

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} - (Z_1 - Z_2) + h_L$$

$$S = \frac{h_L}{\Delta L}, \qquad S_o = \frac{(Z_1 - Z_2)}{\Delta X} \approx \frac{(Z_1 - Z_2)}{\Delta L} \ for \ \theta < 6^o$$

$$Now$$

$$E_1 = E_2 - S_o \Delta L + S \Delta L$$

$$\Delta L = \frac{E_1 - E_2}{S - S_o} \qquad (1)$$

$$Where \ \Delta L = length \ of \ water \ surface \ profile$$

An approximate analysis of gradually varied, non uniform flow can be achieved by considering a length of stream consisting of a number of successive reaches, in each of which uniform occurs. *Greater accuracy results from smaller depth variation in each reach.*

Energy Equation for Gradually Varied Flow.

The Manning's formula is applied to average conditions in each reach to provide an estimate of the value of S for that reach as follows;

$$V_m = \frac{1}{n} R_m^{2/3} S^{1/2}$$

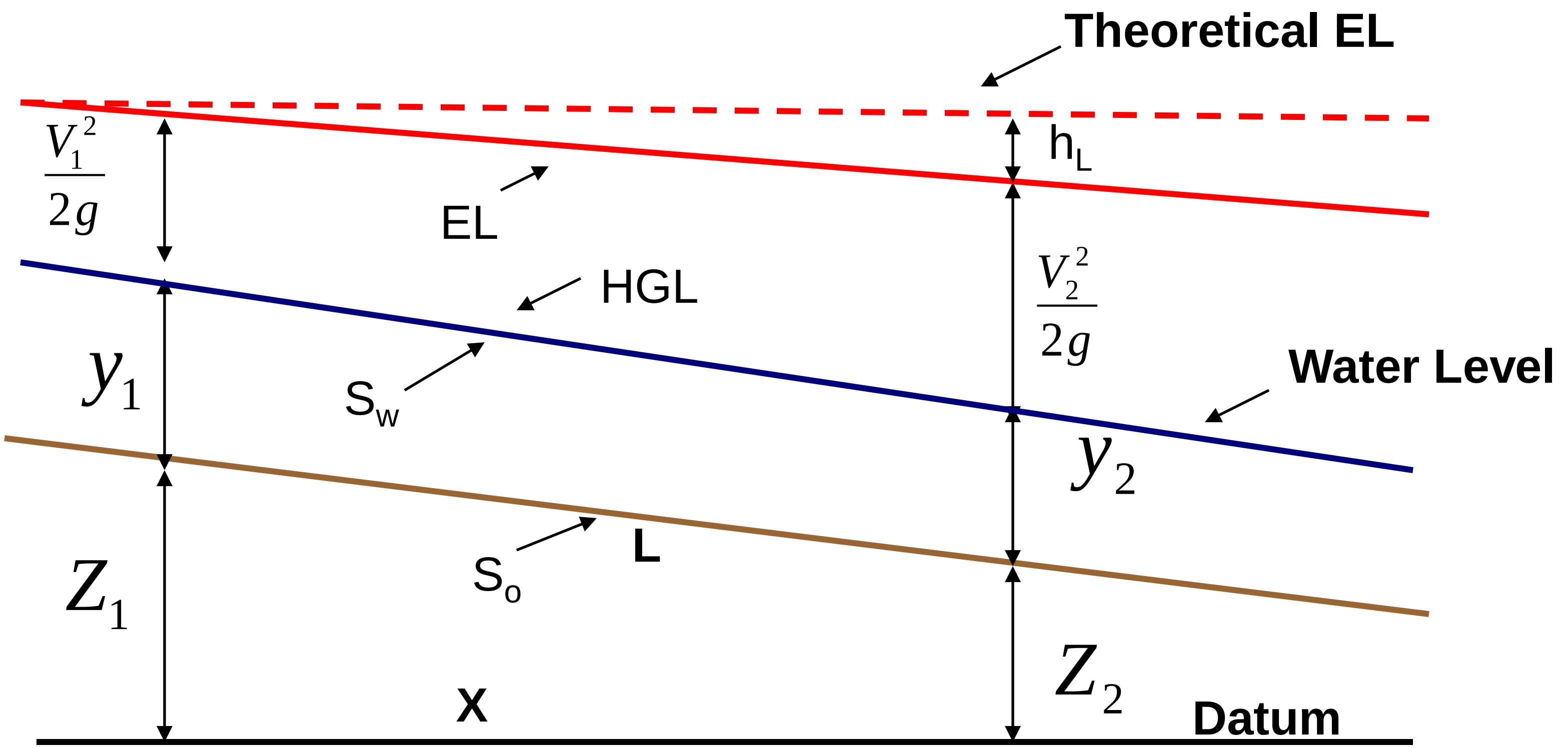
$$V_m = \frac{V_1 + V_2}{2}$$

$$S = \frac{V_m^2 n^2}{R_m^{4/3}}$$

$$R_m = \frac{R_1 + R_2}{2}$$

In practical depth range of the interest is divided into small increments, usually equal, which define the reaches whose lengths can be found by equation (1)

Water Surface Profiles in Gradually Varied Flow.



$$Total \quad Head = Z + y + \frac{V^2}{2g}$$

Water Surface Profiles in Gradually Varied Flow.

Differentiating the total head H w.r.t distance in horizontal direction x.

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{q^2}{2gy^2}\right)$$

Considering cross – section as rectangular

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dy}{dx}\left(1 - \frac{q^2}{gy^3}\right)$$

$$-S = -S_o + \frac{dy}{dx}\left(1 - F_N^{\ 2}\right) \qquad \because F_N = \sqrt{\frac{q^2}{gy^3}}$$

– ve sign shows that total head along direction of flow is decreasing.

$$or \quad \frac{dy}{dx} = \frac{So - S}{1 - F_N^{\ 2}} \qquad (2)$$

Equation (2) is dynamic Equation for gradually varied flow for constant value of *q and n*

For uniform flow $\frac{dy}{dx} = 0$

$$\therefore \frac{So - S}{1 - F_N^{\ 2}} = 0$$

If dy/dx is +ve the depth of flow increases in the direction of flow and vice versa

Water Surface Profiles in Gradually Varied Flow.

For a wide rectangular channel

$$R \approx y$$

$$V = \frac{1}{n} y^{2/3} S^{1/2} \qquad or$$

$$q = \frac{1}{n} y^{5/3} S^{1/2} \qquad or$$

$$S = \frac{n^2 q^2}{y^{10/3}}$$

For uniform flow in rectangular channel

$$S_o = \frac{n^2 q^2}{y_o^{10/3}}$$

$$\therefore \frac{S}{S_o} = \left(\frac{y_o}{y}\right)^{10/3}$$

$$\frac{dy}{dx} = \frac{S_o - S}{1 - F^2}$$

- *Consequently, for constant q and n,* when $y>y_o$, $S<S_o$, and the numerator is +ve. Conversely, when $y<y_o$, $S>S_o$, and the numerator is –*ve*.

- To investigate the denominator we observe that, if *F=1*, *dy/dx*=infinity; if *F>1*, the denominator is *-ve;* and if *F<1*, the denominator is +*ve.*

Classification of Surface Profiles

- Mild Slope (M)

  $y_o > y_c$
  $S_o < S_c$

- Critical Slope (C)

  $y_o = y_c$
  $S_o = S_c$

- Steep Slope (S)

  $y_o < y_c$
  $S_o > S_c$

- Horizontal (H)

  $S_o = 0$

- Adverse (A)

  $S_o = -ve$

- **Type 1:** if the stream surface lies above both the normal and critical depth of flow. (M_1, S_1)

- **Type 2:** if the stream surface lies between normal and critical depth of flow. (M_2, S_2)

- **Type 3:** if the stream surface lies below both the normal and critical depth of flow. (M_3, S_3)

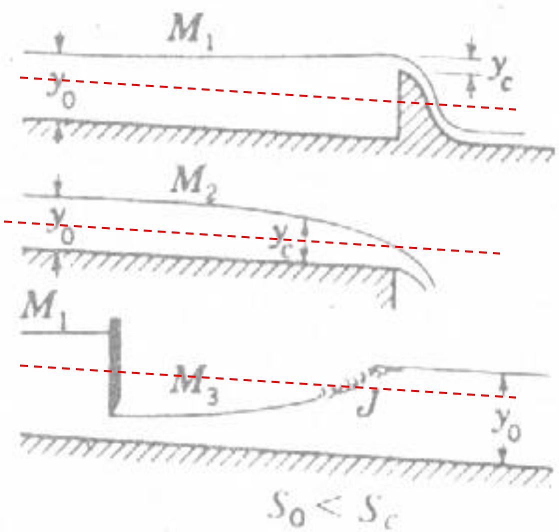
Water Surface Profiles

Mild Slope (M)

1: $$\frac{dy}{dx} = \frac{S_o - S}{1 - F_N} = \quad \Rightarrow$$

2: $$\frac{dy}{dx} = \frac{S_o - S}{1 - F_N} = \quad \Rightarrow$$

3: $$\frac{dy}{dx} = \frac{S_o - S}{1 - F_N} = \quad \Rightarrow$$



<u>Note:</u>

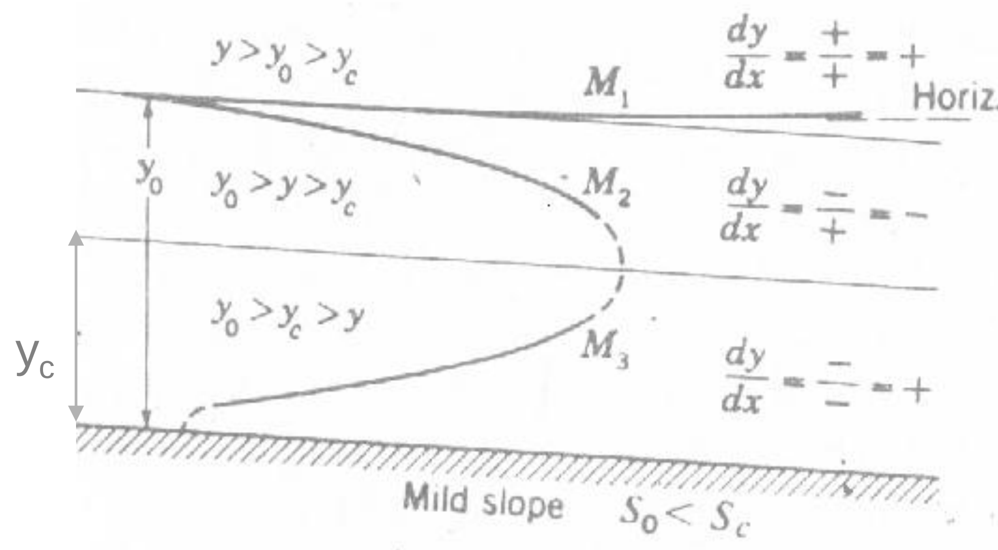
For Sign of Numerator computer

y_o & y

For sign of denominator compare

y_c & y

If $y>y_o$ then $S<S_o$ and Vice Versa



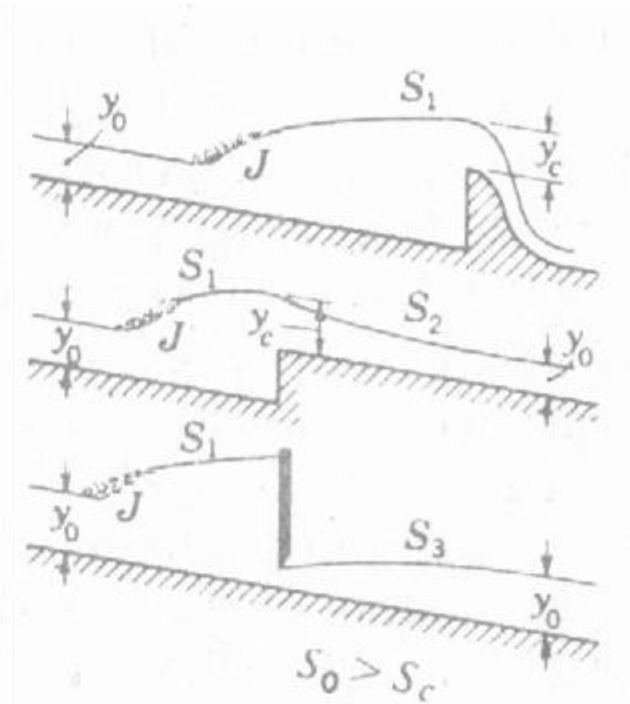
Water Surface Profiles

Steep Slope (S)

$$1: y > y_c > y_o \qquad \frac{dy}{dx} = \frac{S_o - S}{1 - F_N} = \frac{+Ve}{+Ve} = +Ve \quad \Rightarrow S_1$$

$$2: y_c > y > y_o \qquad \frac{dy}{dx} = \frac{S_o - S}{1 - F_N} = \frac{+Ve}{-Ve} = -Ve \quad \Rightarrow S_2$$

$$3: y_c > y_o > y \qquad \frac{dy}{dx} = \frac{S_o - S}{1 - F_N} = \frac{-Ve}{-Ve} = +Ve \quad \Rightarrow S_3$$



Note:

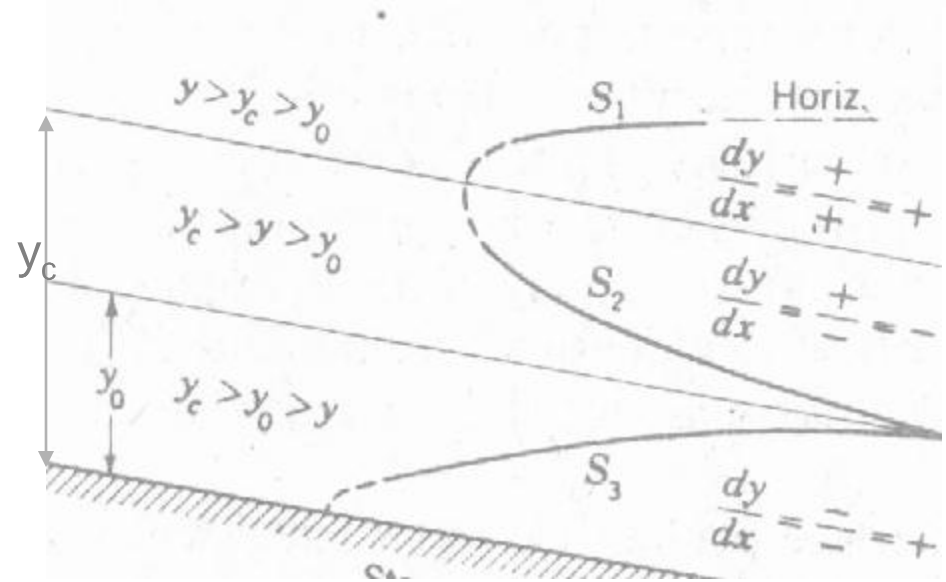
For Sign of Numerator computer

y_o & y

For sign of denominator compare

y_c & y

If $y > y_o$ then $S < S_o$ and Vice Versa

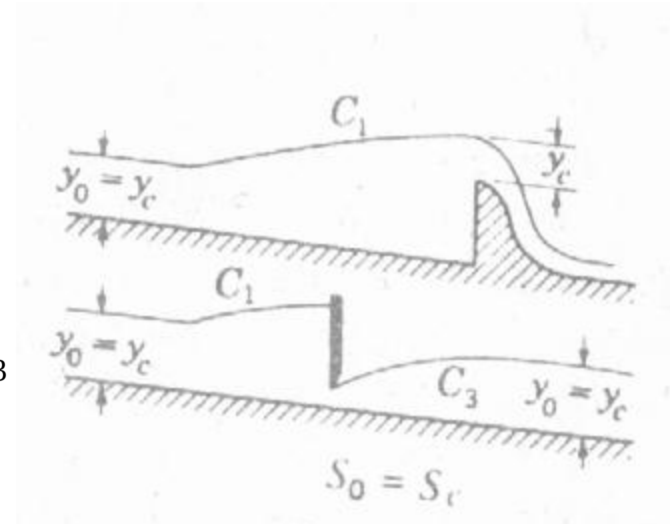


Water Surface Profiles

Critical (C)

$$1: y > y_o = y_c \qquad \frac{dy}{dx} = \frac{S_o - S}{1 - F_N} = \frac{+Ve}{+Ve} = +Ve \quad \Rightarrow C_1$$

$$2: y_o = y_c > y \qquad \frac{dy}{dx} = \frac{S_o - S}{1 - F_N} = \frac{-Ve}{-Ve} = +Ve \quad \Rightarrow C_3$$



C_2 is not possible

<u>Note:</u>

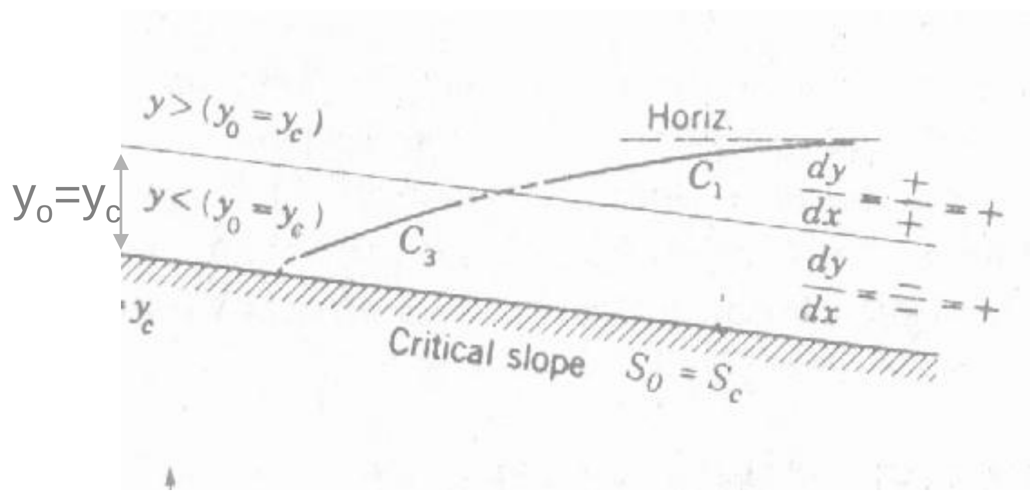
For Sign of Numerator computer

$y_o \quad \& \quad y$

For sign of denominator compare

$y_c \quad \& \quad y$

If $y>y_o$ then $S<S_o$ and Vice Versa

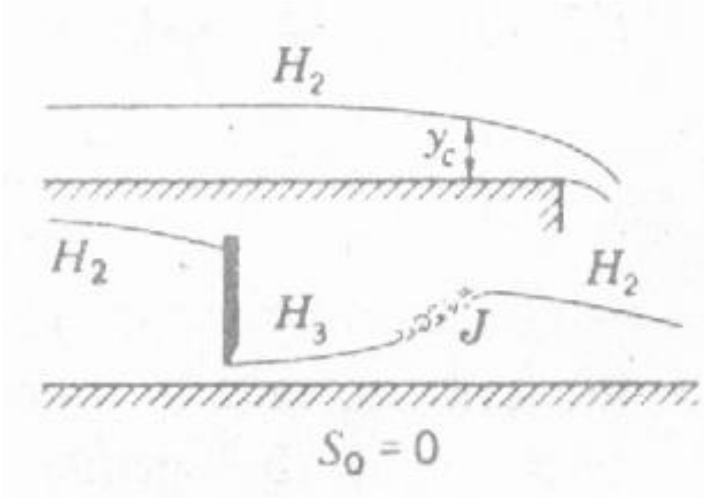


Water Surface Profiles

Horizontal (H)

$$1: y_{o(\infty)} > y > y_c \qquad \frac{dy}{dx} = \frac{S - S_o}{1 - F_N} = \frac{-Ve}{+Ve} = -Ve \quad \Rightarrow H_2$$

$$2: y_{o(\infty)} > y_c > y \qquad \frac{dy}{dx} = \frac{S - S_o}{1 - F_N} = \frac{-Ve}{-Ve} = +Ve \quad \Rightarrow H_3$$



H_1 is not possible bcz water has to lower down

Note:

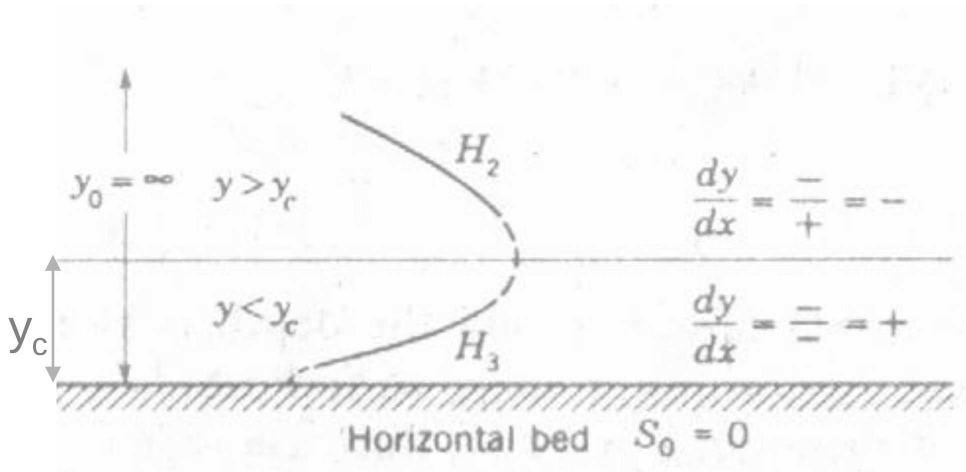
For Sign of Numerator computer

$$y_o \quad \& \quad y$$

For sign of denominator compare

$$y_c \quad \& \quad y$$

If $y>y_o$ then $S<S_o$ and Vice Versa

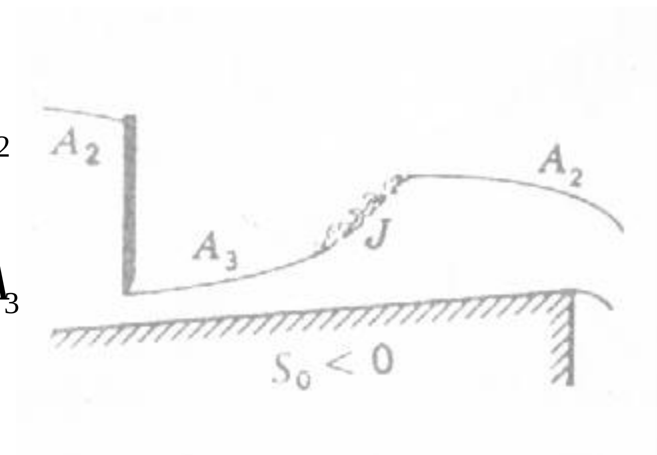


Water Surface Profiles

Adverse (A)

$$1: y_{o(\infty)} > y > y_c \qquad \frac{dy}{dx} = \frac{S - S_o}{1 - F_N} = \frac{-Ve}{+Ve} = -Ve \quad \Rightarrow A_2$$

$$2: y_{o(\infty)} > y_c > y \qquad \frac{dy}{dx} = \frac{S - S_o}{1 - F_N} = \frac{-Ve}{-Ve} = +Ve \quad \Rightarrow A_3$$



A_1 is not possible bcz water has to lower down

<u>Note:</u>

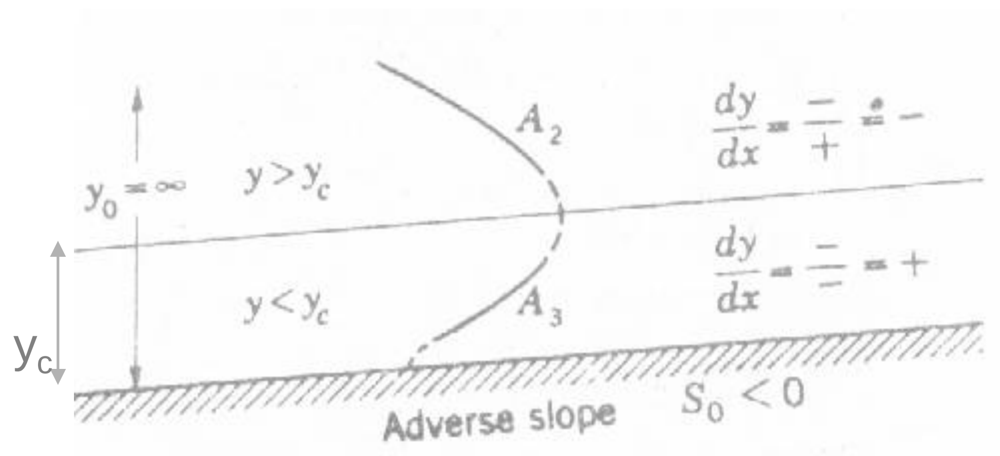
For Sign of Numerator computer

$y_o \quad \& \quad y$

For sign of denominator compare

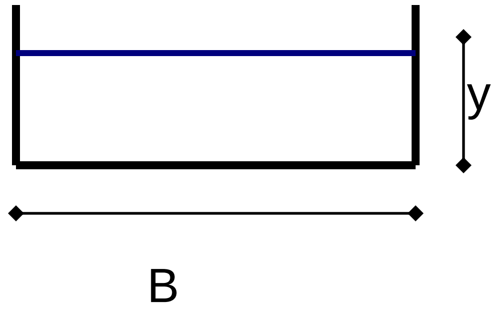
$y_c \quad \& \quad y$

If $y>y_o$ then $S<S_o$ and Vice Versa



Problem 11.59

- A rectangular flume of planer timber (n=0.012) is 1.5 m wide and carries 1.7m^3/sec of water. The bed slope is 0.0006, and at a certain section the depth is 0.9m. Find the distance (in one reach) to the section where depth is 0.75m. Is the distance upstream or downstream?



Rectangular Channel

$$n = 0.012$$

$$B = 1.5m$$

$$Q = 1.7m^3 / \sec$$

$$S_o = 0.0006$$

$$y_1 = 0.9m$$

$$y_2 = 0.75$$

Problem 11.59 Solution

Since

$$\Delta L = \frac{E_1 - E_2}{S - S_o}$$

$$\& \qquad S = \frac{V_m^2 n^2}{R_m^{4/3}}$$

$$A_1 = 1.5x0.9 = 1.35m^2$$

$$A_2 = 1.5x0.75 = 1.125m^2$$

$$P_1 = 1.5 + 2x0.9 = 3.3m$$

$$P_2 = 1.5 + 2x0.75 = 3m$$

$$R_1 = A_1 / P_1 = 0.41$$

$$R_2 = A_2 / P_2 = 0.375$$

$$V_1 = Q / A_1 = 1.26m / \sec$$

$$V_2 = Q / A_2 = 1.51m / \sec$$

$$R_m = 0.3925m$$

$$V_m = 1.385m$$

$$\& \qquad S = \frac{V_m^2 n^2}{R_m^{4/3}} = 0.000961$$

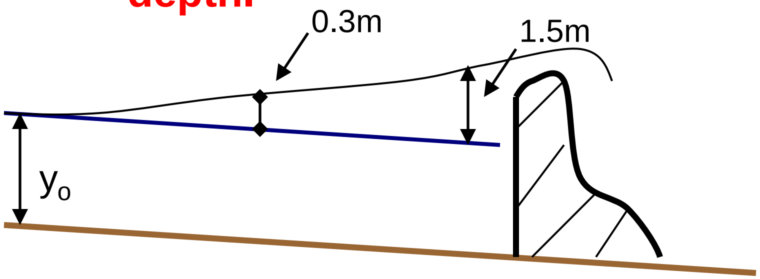
$$Now \quad \Delta L = \frac{E_1 - E_2}{S - S_o}$$

$$\Delta L = \frac{\left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)}{S - S_o}$$

$$= 317.73m \quad Downstream$$

Problem 11.66

- **The slope of a stream of a rectangular cross section is S_o=0.0002, the width is 50m, and the value of Chezy C is 43.2 $m^{1/2}$/sec. Find the depth for uniform flow of 8.25 m^3/sec/m of the stream. If a dam raises the water level so that at a certain distance upstream the increase is 1.5m, how far from this latter section will the increase be only 30cm? Use reaches with 30cm depth.**



- Given That

$$S_o = 0.0002$$

$$B = 50m$$

$$C = 43.2m^{1/2} / \sec$$

$$q = 8.25m^3 / \sec/ m$$

$$q = y_o C \sqrt{\frac{A_o}{P_o} S_o}$$

$$8.25 = y_o 43.2 \sqrt{\frac{50 y_o}{50 + 2 y_o} 0.0002}$$

$$y_o = 6.1m$$

Problem 11.66

$V=C(RS)^{1/2}$

y	A	P	R	V	E	E1-E2	Vm	Rm	S	$S-S_o$	ΔL	ΣΔL
m	m^2	m	m	m/s	m	m	m/s	m	m/m	m/m	m	m
7.6	380	65.2	5.82	1.09	7.66	0.295	1.11	5.74	0.000115	-0.000085	-3454.33	-3454.33
7.3												
7.0												
6.7												
6.4												

Assignment

- Problems:

  11.60, 11.63, 11.64, 11.65, 11.72, 11.73, 11.74, 11.75

- Date of Submission: