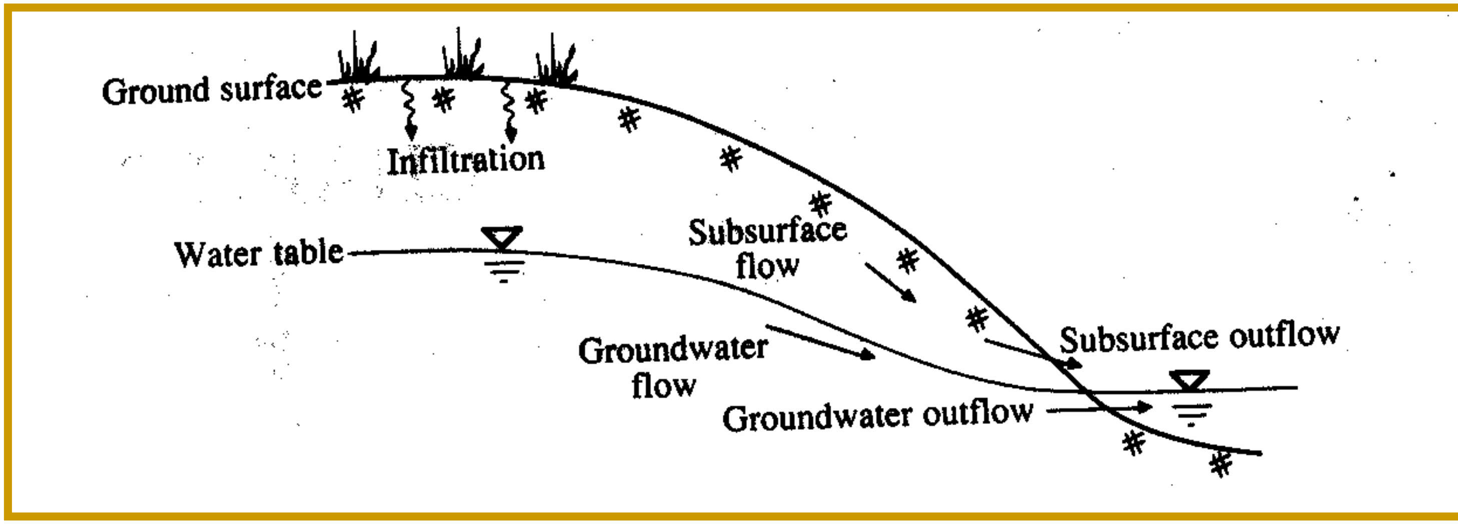
CHAPTER-6

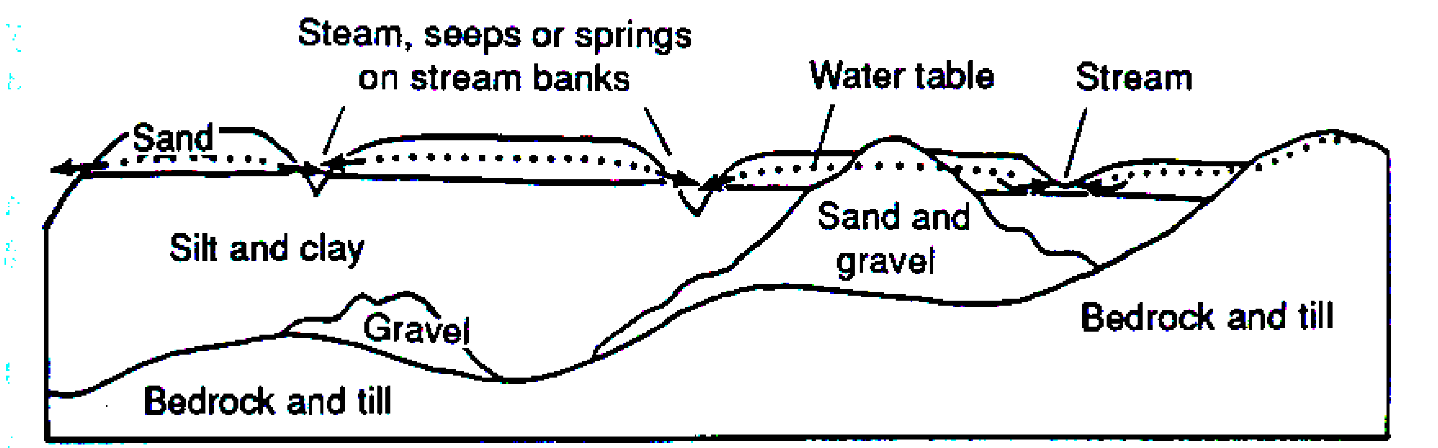
GROUNDWATER FLOW

Groundwater Hydrology

- It may be defined as the science of occurrence, distribution and movement of water below the surface of earth.
- About 30% of the world's freshwater resources exist in the form of GW.
- In USA, about 25% of total fresh water is GW.
- After rainfall and surface water, GW is the next most important water source for irrigation.
- GW is free of pollution
- GW has low temperatures, so suitable for cooling.
- In Pakistan, Mean Annual River discharge = 133 MAF
- Mean Annual discharge into the Sea = 32 MAF
- Total GW Reserve available = 64 MAF
- %age of total GW of the total fresh water = 32 %

Groundwater Hydrology





Lake or marine-dominated lowland

Sources of GW Recharge in Pakistan

S.No.	Source	Volume (MAF)
1	Rainfall	2.50
2	Rivers	3.10
3	Canal Seepage	24.2
4	Water Course Seepage	4.40
5	Field Infiltration	12.0
6	SUB-Total	46.2
7	Non useable GW	17.8
	Grand Total	**64**

Source: Water Resources of Pakistan by Dr. Nazir Ahmad

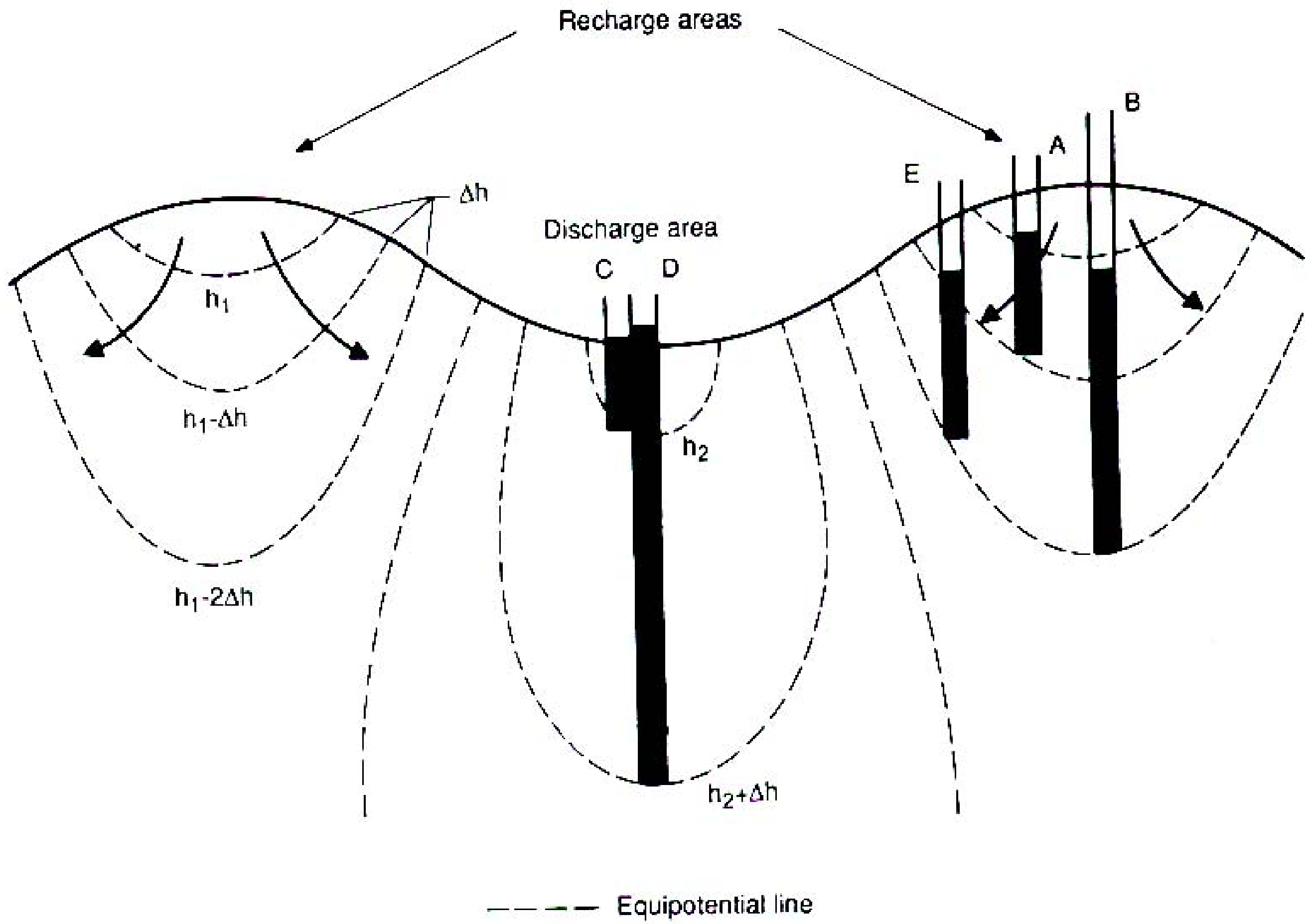


FIGURE 6.3.5 Conceptual model of a groundwater flow system. The dashed lines are lines of constant hydraulic head; head decreases with depth in recharge areas and increases with depth in discharge areas. The piezometers have small screens at the bottom of the borehole.

Terms Related to GW Hydrology

Aquifer (contains water and transmits it, contain interconnected openings, coarse sands)

Aquitard (partly permeable, sandy clays)

Aquiclude (contains water but does not transmit it, clays)

Aquifuge (neither porous nor permeable, rocks)

Unconfined Aquifer

Confined Aquifer/Artesian aquifer (may be leaky)

Artesian Well (Artois, a former province in France)

Leaky Aquifer

Water Table (fluctuates)

Springs (when W.T. = G.S.)

Perched W.T.

Confined & Unconfined Aquifers

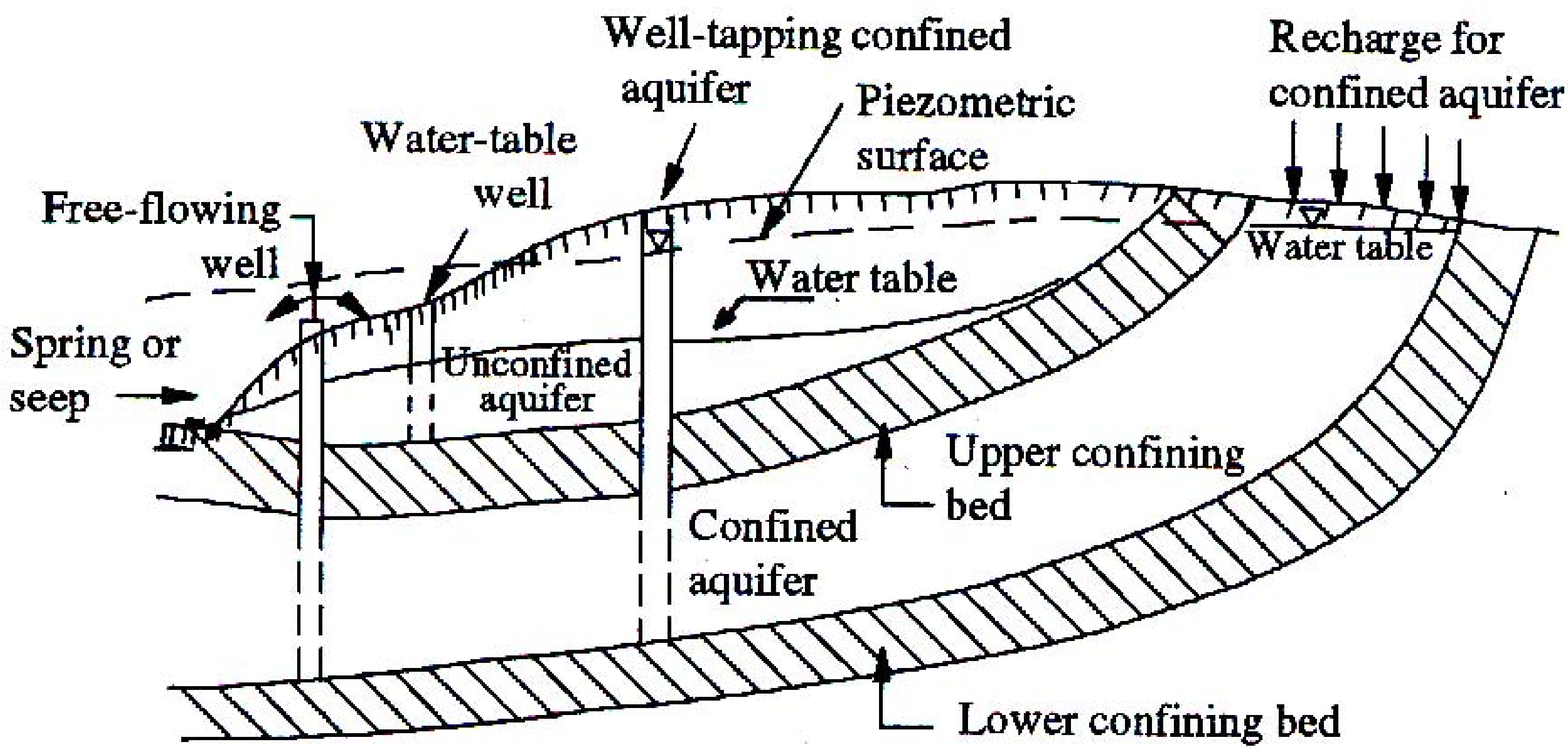


Fig. 9.2 Confined and unconfined acquifers

Perched Water Table

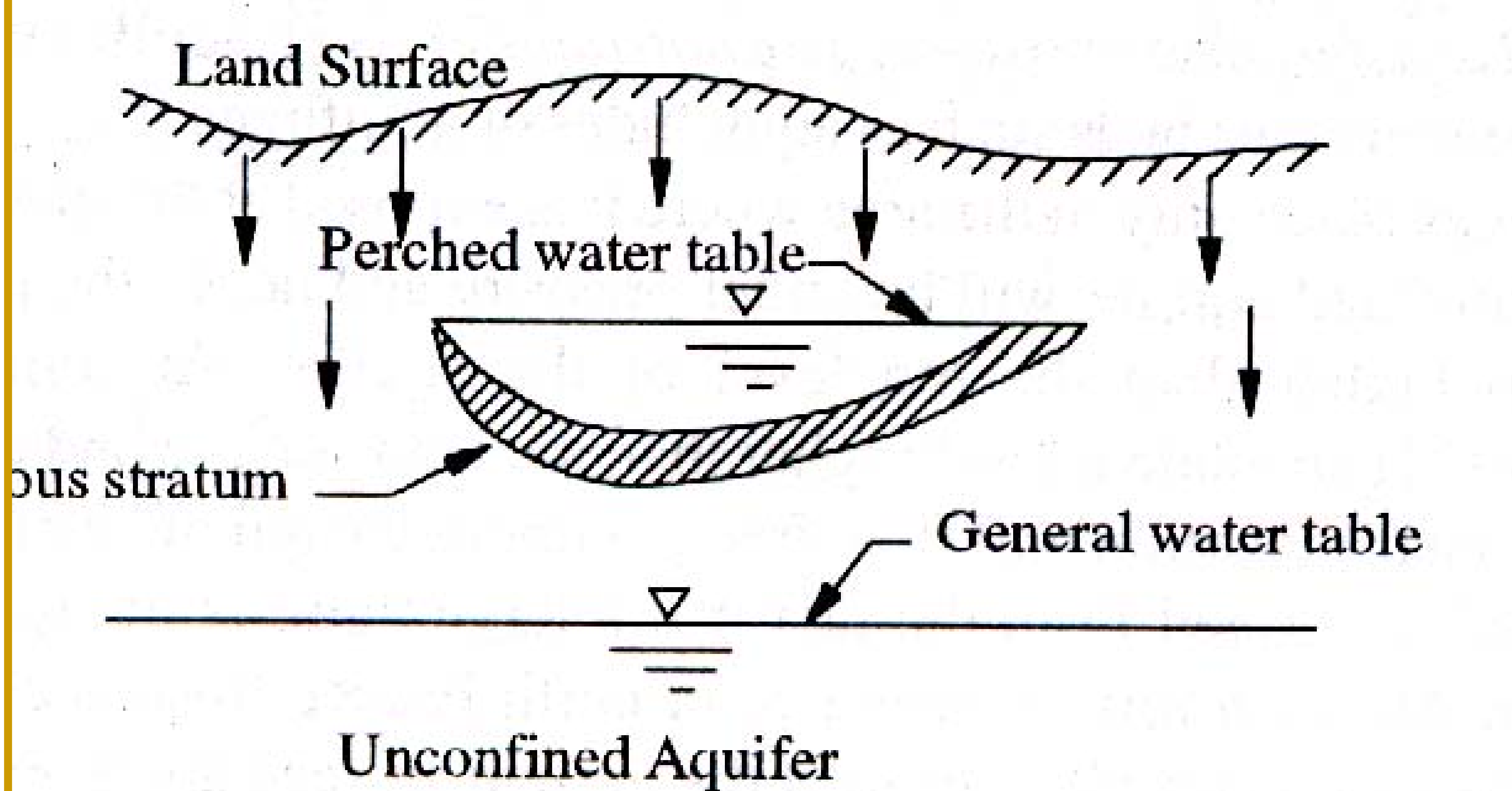


Fig. 9.3 Perched water table

Effluent and Influent Streams

Effluent Streams / Perennial Streams

If Bed of the river is below W.T. during low flows

Influent Streams / Intermittent Streams

If W.T. is below the bed of the river

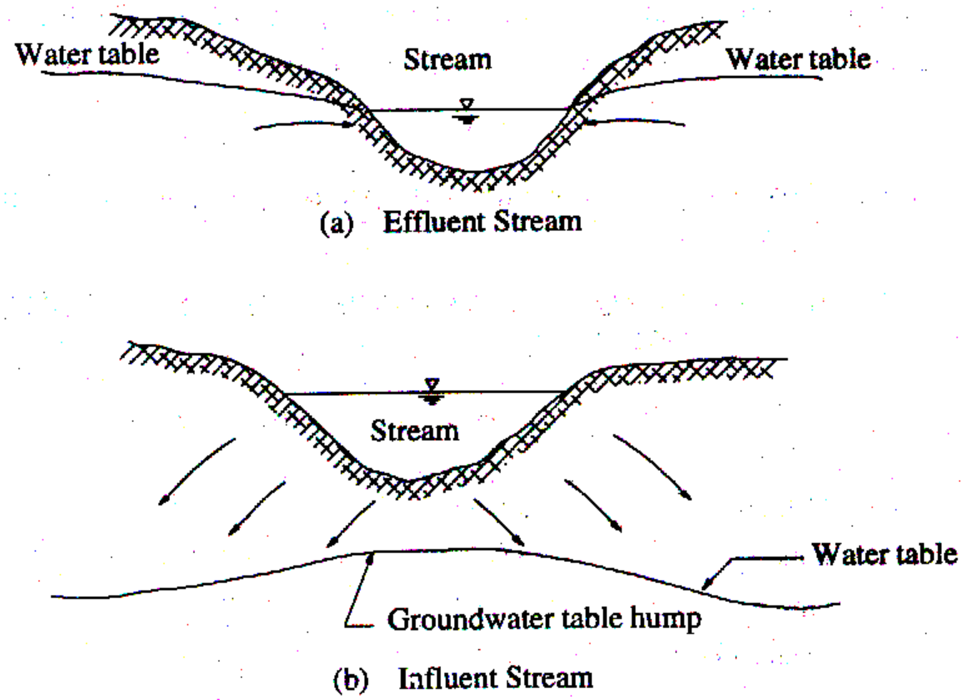


Fig. 9.4 Effluent and influent streams

Aquifer Properties

(1) Porosity

(2) Specific Yield

(3) Specific retention

(4) Hydraulic Conductivity / Coefficient of Permeability

(5) Transmissivity

(6) Specific Storage

(7) Storativity or Storage Constant

Porosity

In soils, depends on shape, size distribution & packing of particles.

$\eta = 0.25 - 0.75$ (> 20% is large, 5-20% medium, <5% small)

Specific Yield

The actual volume of water that can be extracted by the force of gravity from a unit volume of aquifer material is known as Specific Yield.

Specific Retention

The fraction of water held back by in the aquifer material is know as specific retention.

$$\eta = S_y + S_r$$

Aquifer Properties (cond.)

TABLE 9.1 POROSITY AND SPECIFIC YIELD OF SELECTED FORMATIONS

Formation	Porosity %	Specific yield %
Clay	45–55	1–10
Sand	35–40	10–30
Gravel	30–40	15–30
Sand stone	10–20	5–15
Shale	1–10	0.5–5
Lime stone	1–10	0.5–5

Source: Subramanya

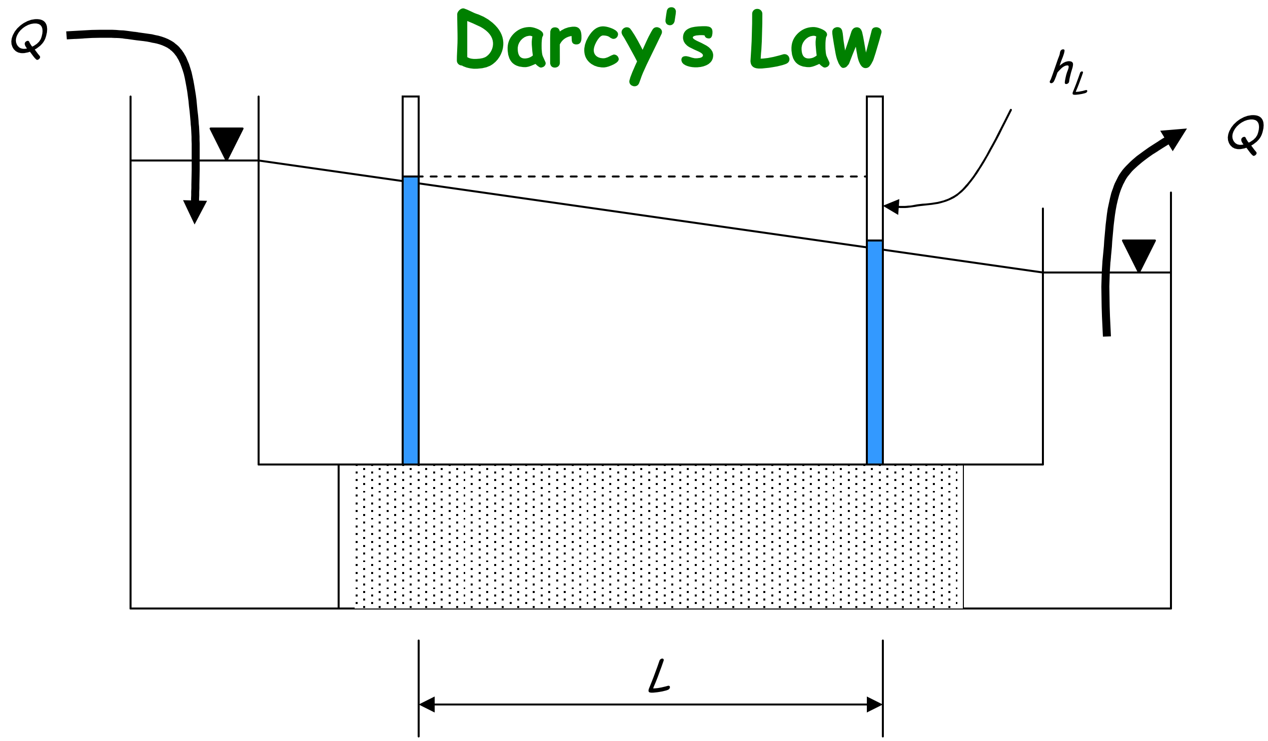
Aquifer Properties (cond.)

TABLE 6.3.1 Representative Values of Porosity

Sediment or rock type	Porosity
Clays	0.40–0.60
Silts	0.35–0.50
Fine sands	0.20–0.45
Coarse sands	0.15–0.35
Shales (near-surface, weathered)	0.30–0.50
Shales (at depth)	0.01–0.10
Sandstones	0.05–0.35
Limestones	See Table 6.5.3
Bedded salt	0.001–0.005
Unfractured igneous rocks	0.0001–0.01
Fractured igneous rocks	0.01–0.10
Basalts	0.01–0.25

Source: Handbook of Hydrology by Maidment

Darcy's Law



In 1856 Henry Darcy, a French Hydraulic Engineer, on the basis of experiments, shows that

$$V \, \alpha \, i \qquad V = K \, i \qquad i = -\frac{h_L}{L} \qquad Q = K \, i \, A$$

Darcy law is valid only for laminar flow (Re <= 1) $R_e = \frac{V \, d_a}{\upsilon} = 1$

Where, $d_a = d_{10}$

V is not the actual velocity of flow through the pores. $V_a = \frac{V}{\eta}$

V_a = Bulk pore velocity

Hydraulic Conductivity

It is the flow rate through a porous medium for unit cross sectional area under a unit hydraulic gradient.

$$K = \frac{Q}{i\ A}$$

According to Hagen Poiseuille flow: $K = C\ d_m^2\ \frac{\gamma}{\mu}$

d_m = mean particle size of the porous medium

C = shape factor (depends on porosity, packing, shape of grains, grain size distribution).

Factors affecting Hydraulic Conductivity (K)

- Specific weight of fluid
- Viscosity of fluid
- Temperature
- Porosity of Porous medium
- Packing
- Shape of grains
- Grain size distribution

The standard value of K is taken as that of for pure water at a standard temperature of 20 °C.

The value of Kt at any temperature t can be converted to Ks:

$$K_s = K_t \left(\frac{\upsilon_t}{\upsilon_s} \right)$$

Hydraulic Conductivity (cond.)

TABLE 9.2 REPRESENTATIVE VALUES OF THE PERMEABILITY COEFFICIENT

No.	Material	K (cm/s)	K_0 (darcys)
A.	*Granular material*		
1.	Clean gravel	1–100	$10^3 - 10^5$
2.	Clean coarse sand	0.010–1.00	$10 - 10^3$
3.	Mixed sand	0.005–0.01	$5 - 10$
4.	Fine sand	0.001–0.05	1–50
5.	Silty sand	$1 \times 10^{-4} - 2 \times 10^{-3}$	0.1–2
6..	Silt	$1 \times 10^{-5} - 5 \times 10^{-4}$	0.01–0.5
7.	Clay	$< 10^{-6}$	$< 10^{-3}$
B.	*Consolidated material*		
1.	Sandstone	$10^{-6} - 10^{-3}$	$10^{-3} - 1.0$
2.	Carbonate rock with secondary porosity	$10^{-5} - 10^{-3}$	$10^{-2} - 1.0$
3.	Shale	10^{-10}	10^{-7}
4.	Fractured and weathered rock (aquifers)	$10^{-6} - 10^{-3}$	$10^{-3} - 10$

At 20° C, for water, $\nu = 0.01\ cm^2/s$ and substituting in Eq. (9.8)

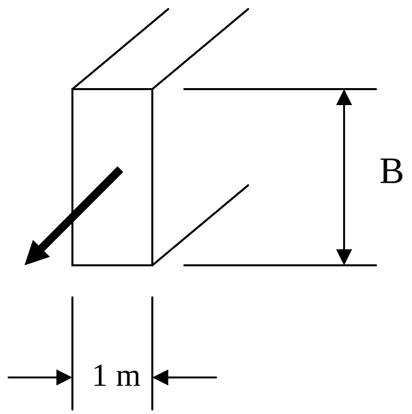
$$K_0 \text{ [darcys]} = 10^3\ K \text{ [cm/s] at } 20°\ C$$

Transmissivity

It is flow rate through a porous medium of unit width and thickness of aquifer B under a hydraulic gradient of unity.

Product of Hydraulic Conductivity and thickness of aquifer.

$$T = K\ B = \frac{Q}{i\ A} B$$



Measurement of Hydraulic Conductivity

(1) Laboratory Tests

- (i) Constant Head Test (for granular soils)
- (ii) Falling Head Test (for clayey soils)

(2) Field Tests

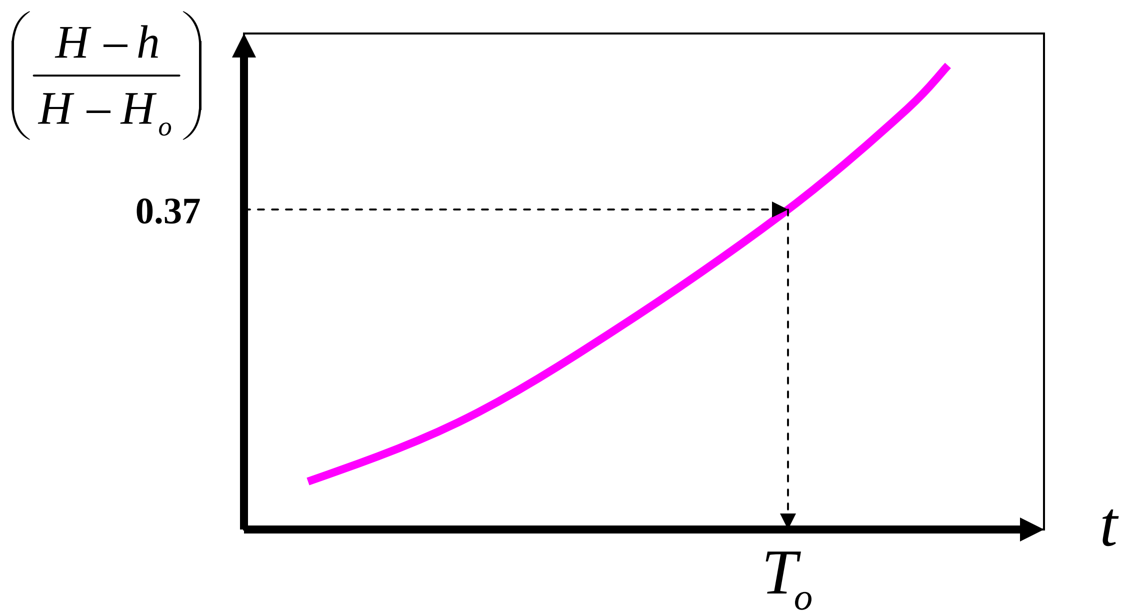
- (i) Single Piezometer Test/Slug test/Bail test
- (ii) Packer Test
- (iii) Pumping Test
  - (a) Steady State Test (Theim formula)/Equilibrium well formula.
  - (b) Unsteady State Test (Theis Formula)/Non equilibrium well formula

(3) Using Empirical Equations

- (i) Hazen's equation
- (ii) Kozeny Karman equation

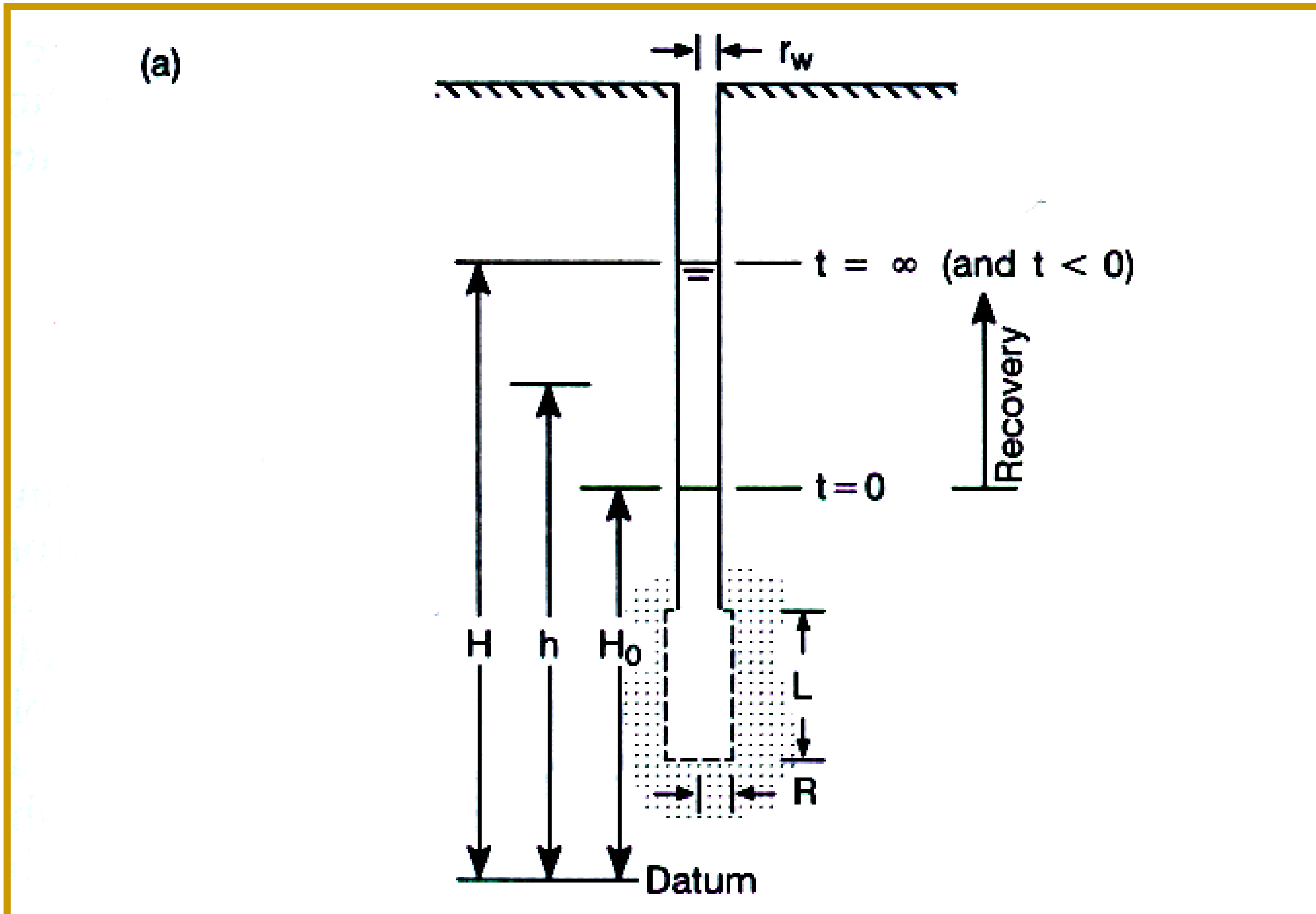
Single Piezometer Test/Slug/Bail Test

- Test is based on rate of recovery of W.L. within a Piezometer.
- Water is added to the standing water column to raise the W.L. from the piezometer.
- Cost effective
- Quick
- Plot graph between $\left(\frac{H-h}{H-H_o}\right) \sim t$
- Pick T_o corresponding to 0.37 value



$$K = \frac{r_w^2 \ \ln(\frac{L}{R})}{2 \ L \ T_o}$$

Single Piezometer Test/Slug/Bail Test



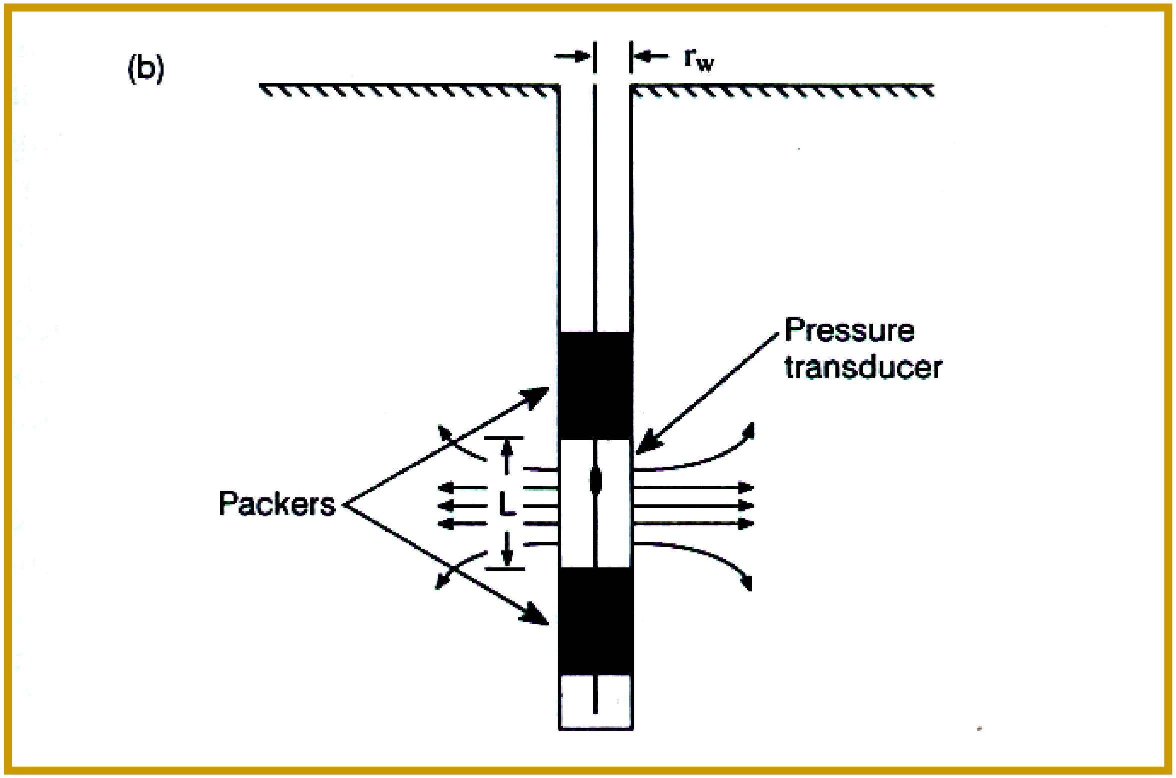
Packer Test

- Length L of borehole is isolated between inflatable packers.
- Fluid is injected into the test interval.
- Injection is continued untill the flow rate stabilizes.

$$K = \frac{Q}{2\ \pi\ L\ (\Delta h_w)} \ln(\frac{L}{r_w}) \qquad L >> 2\ r_w$$

Where

Δh_w = pressure head where flow stablizes.



Pumping Test

(a) Steady State Test (Theim formula)/Equilibrium well formula.

(b) Unsteady State Test (Theis Formula)/Non equilibrium well formula

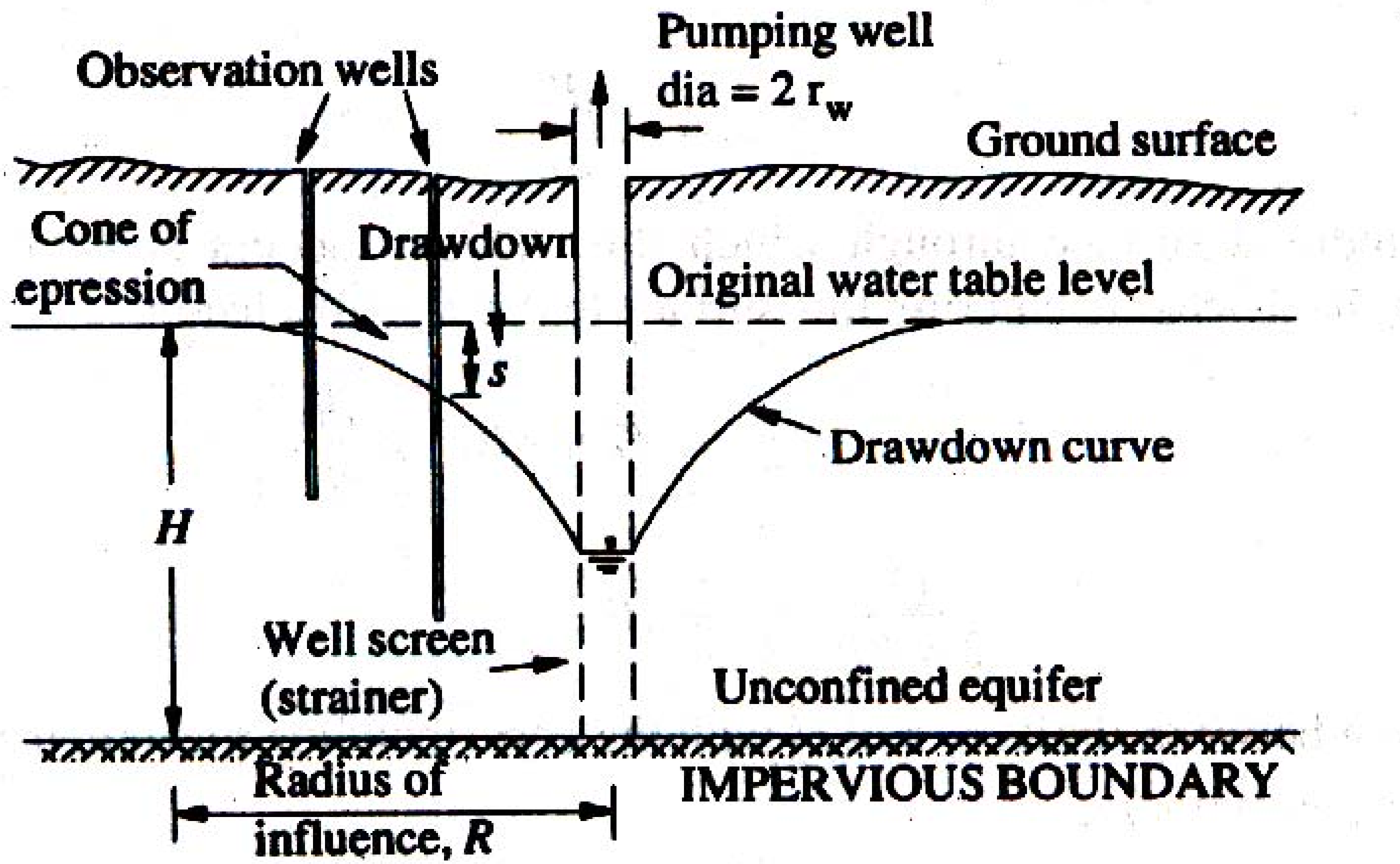


Fig. 9.15 Well operating in an unconfined aquifer, (definition sketch)

Empirical Correlations

(1) Hazen's Equation

$$K = C\ d_{10}^2$$

K (cm/s)

d_{10} (mm)

C = 0.4 - 0.8 (for sands)

= 0.8 - 1.2 (medium to coarse sand)

= 1.2 - 1.5 (well sorted coarse sand)

(2) Kozeny Karman Equation

$$K = \left(\frac{\gamma}{\mu}\right)\left[\frac{\eta^3}{(1-\eta)^2}\right]\left(\frac{d_m^2}{180}\right)$$

dm = mean grain size (mm)

Stratification & Equivalent Hydraulic Conductivity

(1) When flow is parallel to the stratification

$$B = \sum_1^n B_i$$

Equiv. Hyd.Cond. $$K_e = \frac{\sum_1^n (K_i \; B_i)}{\sum_1^n (B_i)}$$

$$T = K_e \sum B_i = \sum_1^n K_i \; B_i$$

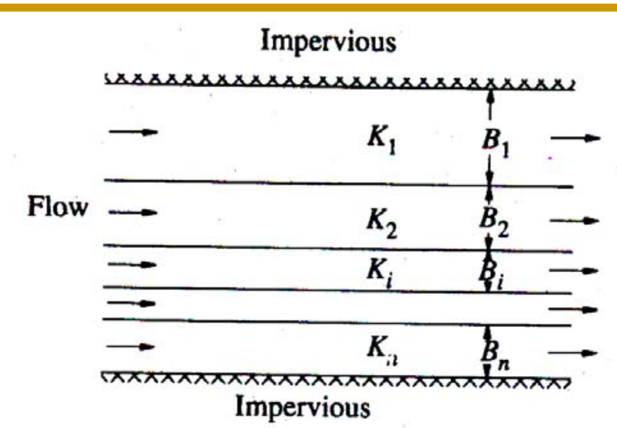


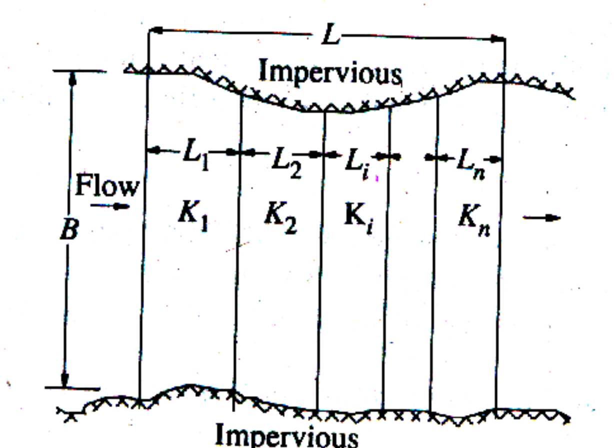
Fig. 9.5 (a) Flow parallel to stratification

(2) When flow is normal to the stratifications

$$L = \sum_i^n L_i$$

$$K_e = \frac{\sum_i^n L_i}{\sum_i^n \left(\frac{L_i}{K_i} \right)}$$

$$T = K_e \; B$$



Geologic Formation of Aquifers & K

Broad Classification

(1) Unconsolidated Deposit

deposits of Sand, Gravel, fluvial alluvial deposits, coastal alluvial and glacial deposits. Having good yields (50-100 m^3/hr).

(2) Consolidated Rocks

Sandstones are good aquifers, joints and fractures improve K.

Yields are lesser (20-50 m^3/hr)

Limestones contain numerous secondary openings.

Basalt aquifers are available both in confined and unconfined state and Yield (> 20 m^3/hr)

Igneous & Metamorphic rocks with weathered and fractured horizons are good aquifers. The yield is fairly low (5-10 m^3/hr).

TABLE 6.2.1 Geologic Time Scale

Era	Period	Epoch	Duration (Millions of years)	Before present (Millions of years)
Cenozoic	Quaternary	Holocene	0.01	
		Pleistocene	3	3
	Tertiary	Pliocene	4	7
		Miocene	19	26
		Oligocene	12	38
		Eocene	16	54
		Paleocene	11	65
Mesozoic	Cretaceous		71	136
	Jurassic		54	190
	Triassic		35	225
Paleozoic	Permian		55	280
	Pennsylvanian		45	325
	Mississippian		20	345
	Devonian		50	395
	Silurian		35	430
	Ordovician		70	500
	Cambrian		70	570
Precambrian				>570

Compressibility of Aquifers

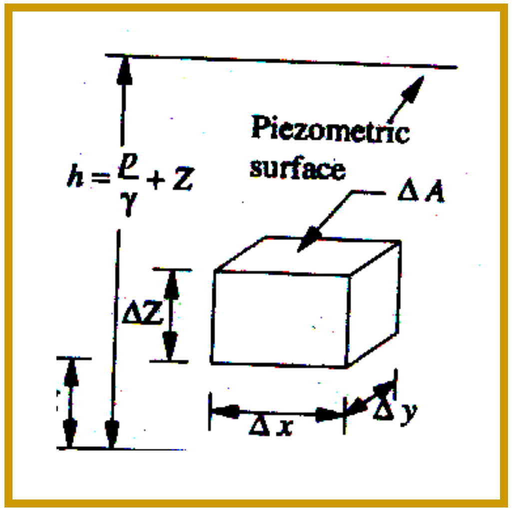
- Total press. = pore press. + intergranular press.
- Assumptions

(1) Delta A remains constant, only length changes in Z-direction.

(2) The pore water is compressible

(3) The solid grains of the aquifer are incompressible.

(4) Pore structure is compressible.



Compressibility of water:

It is the ratio between change in volume of water per unit water volume and change in pressure. Reciprocal of Bulk Modulus of elasticity of water.

$$\beta = -\frac{\left(\frac{d\Delta V_w}{\Delta V_w}\right)}{dp}$$

ΔV_w = Volume of water in element

Compressibility of pores:

Reciprocal of the bulk modulus of elasticity (m^2/N)

$$\alpha = -\frac{\left[\frac{d(\Delta V)}{\Delta V}\right]}{d\sigma_z}$$

ΔV = Volume of element

Specific Storage:

Volume of water that a unit volume of porous medium releases from storage per unit change in piezometric head. Units [L^{-1}].

Numerical values are very small (1×10^{-4} m^{-1}).

$$S_s = \gamma \left(\eta\ \beta + \alpha\right)$$

Storage Coefficient / Storativity:

Volume of water that an aquifer releases (takes into) from storage per unit surface area of aquifer per unit change in head normal to that surface. Product of Spec. Storage and layer thickness B.

Numerical values (5×10^{-5} to 5×10^{-3}).

No units.

$$S = S_s\ B = \gamma \left(\eta\ \beta + \alpha\right) B$$ Confined aquifer

$$S = S_y + \gamma \left(\eta\ \beta + \alpha\right) B_s$$ Unconfined aquifer

$$S \approx S_y$$

B_s = Saturated thickness of aquifer

Barometric Efficiency:

Ratio of the W.L. change in the well to pressure head change.

$$BE = -\left[\frac{\eta\ \beta}{\alpha + \eta\ \beta}\right] = -\left[\frac{\eta\ \beta}{S/\gamma\ B}\right]$$

It affords a means of finding S.

The barometric efficiency ranges (10-75%)

Unconfined aquifers have no barometric efficiency $BE \approx 0$

Examples of W.L. change in artisian wells due to compressibility:

- Tidal action in coastal aquifers.
- Earthquake or underground explosions
- Passing of heavy railway trains

Values of Alpha for some formation Materials

Material	Bulk modulus of elasticity, E_s (N/cm^2)	Compressibility $\alpha = 1/E_s$ (cm^2/N)
Loose clay	$10^2 - 5 \times 10^2$	$10^{-2} - 2 \times 10^{-3}$
Stiff clay	$10^3 - 10^4$	$10^{-3} - 10^{-4}$
Loose sand	$10^3 - 2 \times 10^3$	$10^{-3} - 5 \times 10^{-4}$
Dense sand	$5 \times 10^3 - 8 \times 10^3$	$2 \times 10^{-4} - 1.25 \times 10^{-4}$
Dense sandy gravel	$10^4 - 2 \times 10^4$	$10^{-4} - 5 \times 10^{-5}$
Fissured and jointed rock	$1.5 \times 10^4 - 3 \times 10^5$	$6.7 \times 10^{-5} - 3.3 \times 10^{-6}$

Values of Beta for water at various temperatures

	E_w (N/cm^2)	(cm^2/N)
0	2.04×10^5	4.90×10^{-6}
10	2.11×10^5	4.74×10^{-6}
15	2.14×10^5	4.67×10^{-6}
20	2.20×10^5	4.55×10^{-6}
25	2.22×10^5	4.50×10^{-6}
30	2.23×10^5	4.48×10^{-6}
35	2.24×10^5	4.46×10^{-6}

Soil is more Compressible than water

	Compressibilty		
Material	soil	water	Ratio
loose clay	5.10E-02	4.55E-06	11209
stiff clay	5.05E-03	4.55E-06	1110
loose sand	5.25E-03	4.55E-06	1154
dense sand	1.63E-04	4.55E-06	36
dense sandy gravel	5.25E-04	4.55E-06	115
fissured & jointed rock	3.52E-05	4.55E-06	8

Equation of Motion

(1) Confined Groundwater Flow

(a) Confined Groundwater Flow between two water bodies

(2) Unconfined Flow by Dupit's Assumption

(a) Unconfined Flow with a Recharge

(b) One dimensional Dupit's Flow with Recharge

(i) The general Case

(ii) Flow without Recharge

(iii) The Drain Problem

(1) Confined Groundwater Flow

$$\frac{\partial(\Delta M)}{\partial t} = -\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right]$$

Continuity equation (9.22)

$$\frac{\partial(\Delta M)}{\partial t} = S_s\ \rho\ \left(\frac{dh}{dt}\right)$$

From equation (9.18), (9.18a)

For isotropic aquifer, Darcy's equations in x, y and z directions

$$u = -K\frac{\partial h}{\partial x},\ \ v = -K\frac{\partial h}{\partial y},\ \ and\ \ w = -K\frac{\partial h}{\partial z} \qquad (9.25)$$

where $h = \frac{p}{\gamma} + z$

Equation (9.22) can be written as

$$\frac{\partial(\rho u)}{\partial x} = \rho\frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x} = -K\rho\frac{\partial^2 h}{\partial x^2} - K\rho^2\beta\ g\ \left(\frac{\partial h}{\partial x}\right)^2$$

$$\frac{\partial(\rho v)}{\partial y} = \rho\frac{\partial v}{\partial y} + v\frac{\partial \rho}{\partial y} = -K\rho\frac{\partial^2 h}{\partial y^2} - K\rho^2\beta\ g\ \left(\frac{\partial h}{\partial y}\right)^2$$

$$\frac{\partial(\rho w)}{\partial z} = \rho\frac{\partial w}{\partial z} + w\frac{\partial \rho}{\partial z} = -K\rho\frac{\partial^2 h}{\partial z^2} - K\rho^2\beta\ g\ \left(\frac{\partial h}{\partial z}\right)^2$$

Assembling these, Equation (9.22) can be written as

$$K\ \rho\left[\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}\right]+K\ \rho^2\ \beta\ g\left[\left(\frac{\partial h}{\partial x}\right)^2-\left(\frac{\partial h}{\partial y}\right)^2-\left(\frac{\partial h}{\partial z}\right)^2\right]=\rho\ S_s\frac{\partial h}{\partial t}$$

The second term on left side can be neglected as small $\frac{\partial h}{\partial x} << 1$

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=\frac{S_s}{K}\frac{\partial h}{\partial t}$$

defining

$$S_S\ B=S, \quad KB=T, \ and \ \nabla^2 h=\left[\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}\right]$$

$$\nabla^2 h=\frac{S}{T}\frac{\partial h}{\partial t}$$

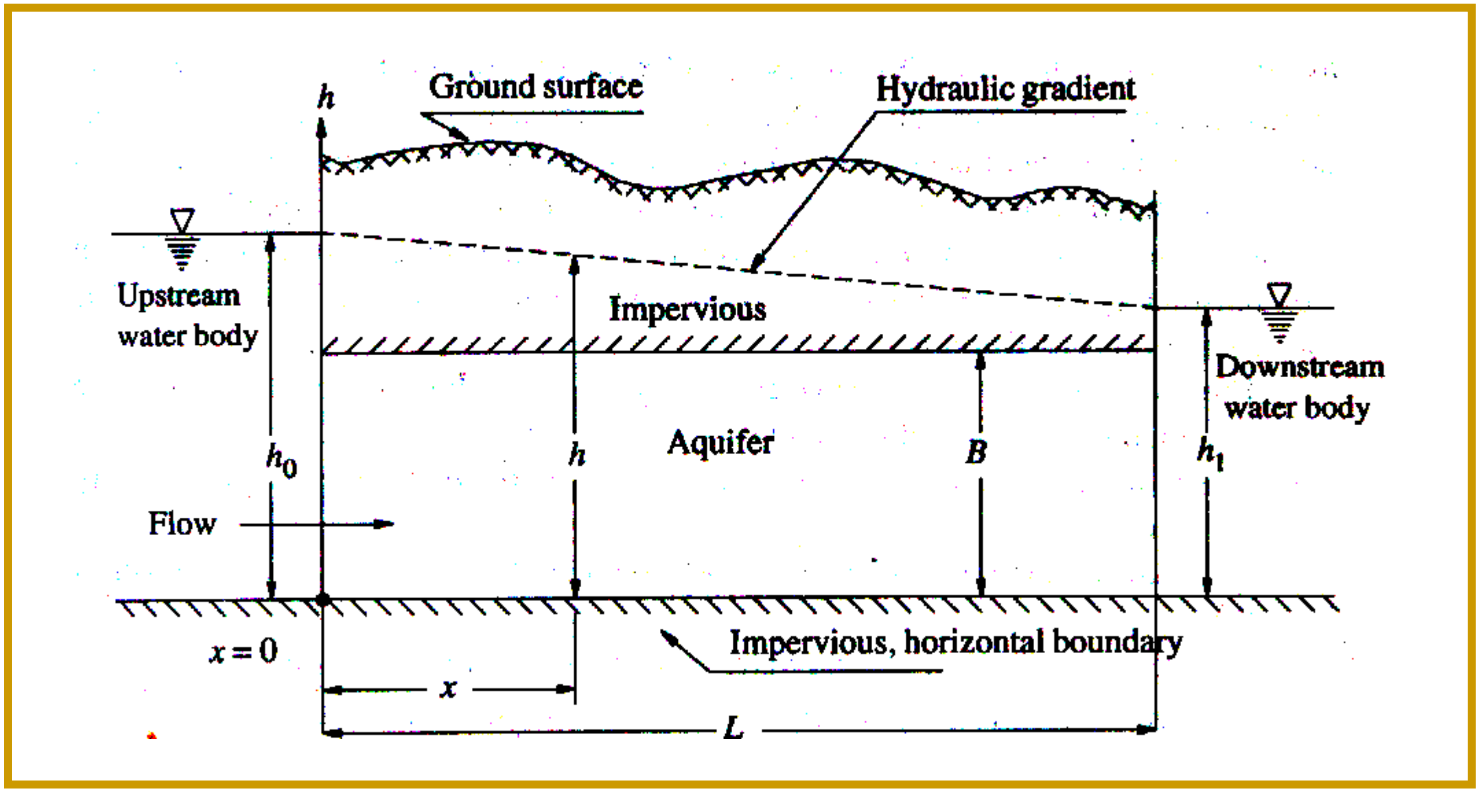
Basic differential equation for unsteady groundwater flow, in a homogenous isotropic confined aquifer (9.26), also known as diffusion equation.

If the flow is steady, $\frac{\partial h}{\partial t}$ term does not exist.

$$\nabla^2 h=0$$

Laplace equation for all potential steady flow problems (9.27)

(1) Confined Groundwater Flow between two Water bodies



$h_o =$ Piezometric head at the u/s end

$h =$ Piezometric head at a distance x from the u/s end

As the flow is in x-direction only, equation (9.27) becomes

$$\frac{\partial^2 h}{\partial x^2} = 0$$

On integrating twice

$$h = c_1\ x + c_2$$

Applying boundary conditions $h = h_o\ at\ x = 0$ $\qquad c_2 = h_o$

$$h = c_1\ x + h_0$$

Also at $x = L,\ h = h_1$ $\qquad c_1 = -\left[\frac{h_o - h}{L}\right]$

$$h = h_o - \left[\frac{h_o - h_1}{L}\right] x$$

Equation of hydraulic grade line (linear) (9.29)

By Darcy's law, the discharge per unit width of aquifer

$$q = -K\ B\ \frac{dh}{dx}$$

$$= -K\ B\left[\frac{-(h_o - h_1)}{L}\right]$$

$$q = \left[\frac{(h_o - h_1)}{L}\right] K\ B$$

Equation of Darcy flux / unit width of aquifer(9.30)

(2) Unconfined Flow by Dupit's Assumption

Unconfined aquifer have boundary condition, free surface pressure = constt.
Line representing the W.T. is a streamline
These boundary conditions cause considerable difficulties in analytical solution.

Dupit's (1863) Assumptions are:

(1) Free surface curvature is very small, streamlines can be assumed horizontal at all sections.
(2) The hydraulic grade line is equal to the free surface slope and does not vary with depth.

Consider an elementary prism of fluid bounded by water table

V_x = gross velocity of ground water entering the element in x direction

V_y = = gross velocity of ground water entering the element in y direction

According to Dupit's assumption

$$\frac{\partial V_x}{\partial z} = 0 \quad \text{and} \quad \frac{\partial V_y}{\partial z} = 0$$

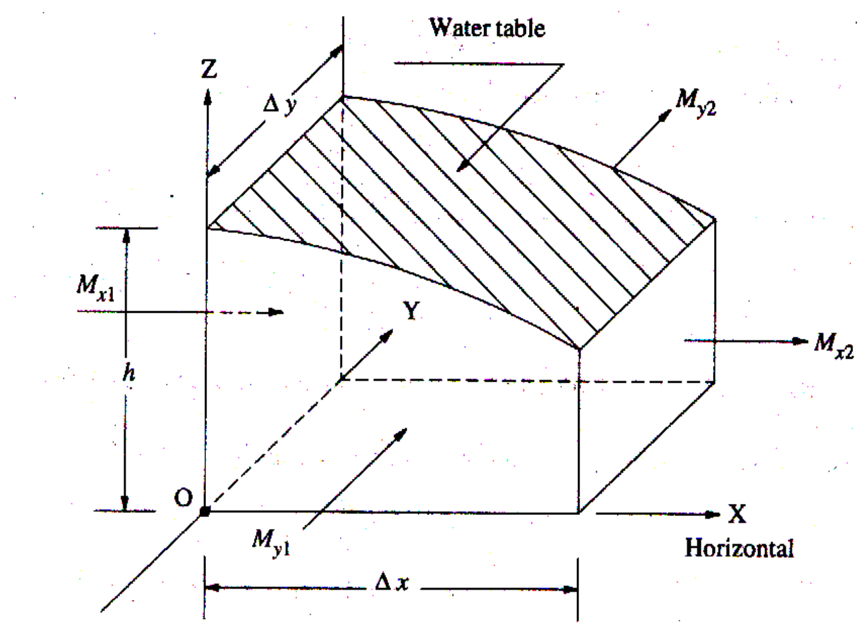


Fig. 9.9 (a) Definition sketch—unconfined groundwater flow without recharge

The mass flux entering the element $M_{x1} = \rho\, V_x\, h\, \Delta_y$

The mass flux leaving the element $M_{x2} = \rho\, V_x\, h\, \Delta_y + \frac{\partial}{\partial x}\left(\rho\, V_x\, h\, \Delta_y\right) \Delta x$

The net mass efflux from the element in x-direction

$$M_{x1} - M_{x2} = \Delta M_x = -\frac{\partial}{\partial x}\left(\rho\, V_x\, h\, \Delta_y\right) \Delta_x$$

The net mass efflux from the element in y-direction

$$M_{y1} - M_{y2} = \Delta M_y = -\frac{\partial}{\partial y}\left(\rho\, V_y\, h\, \Delta_x\right) \Delta_y$$

No inflow or outflow in Z-direction. Thus for steady incompressible flow, by continuity:

$$\Delta M_x + \Delta M_y = 0 \qquad (9.31)$$

Substituting values and simplifying

$$\frac{\partial}{\partial x}\left(V_x\, h\right) + \frac{\partial}{\partial y}\left(V_y\, h\right) = 0 \qquad (9.32)$$

By Darcy Law $V_x = -K\frac{\partial h}{\partial x}$ *and* $V_y = -K\frac{\partial h}{\partial y}$

Substituting values $\frac{\partial}{\partial x}\left(-K\, h\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(-K\, h\frac{\partial h}{\partial y}\right) = 0$ or $\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} = 0$

or $\boxed{\nabla^2 h^2 = 0}$ (9.33)

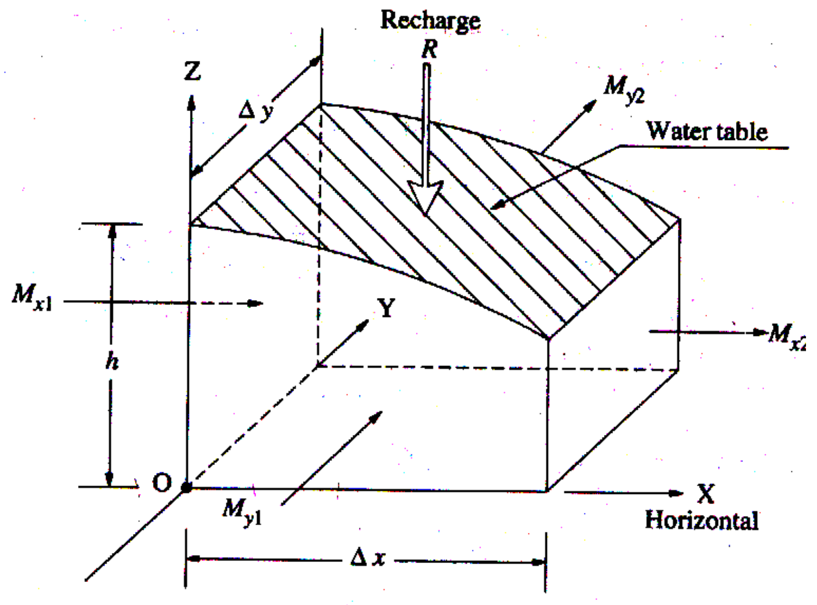
Thus, Unconfined groundwater flow with Dupit's assumption is governed by Laplace equation in h^2

(a) Unconfined Flow with a Recharge

Recharge => Infiltration of water
R is the recharge rate ($m^3/s\ m^2$)

Modifying the continuity equation (9.31) to account the recharge.

Consider the element of an unconfined aquifer on hz impervious bed.



Net inflow into the element in Z direction:

$$\Delta M_z = \rho\ R\ \Delta_x\ \Delta_y$$

For steady, incompressible flow, the continuity equation for the element

$$\Delta M_x + \Delta M_y + \Delta M_z = 0$$

$$-\frac{\partial}{\partial x}\left(\rho\ V_x\ h\ \Delta_x\ \Delta_y\right) - \frac{\partial}{\partial y}\left(\rho\ V_y\ h\ \Delta_x\ \Delta_y\right) + \rho\ R\ \Delta_x\ \Delta_y = 0$$

Substituting $V_x = -K\frac{\partial h}{\partial x}$ *and* $V_y = -K\frac{\partial h}{\partial y}$ and simplyfying

$$\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} = -\frac{2R}{K}$$

Basic differential equation with Dupit's assumption for unconfined GWF with recharge (9.34)

Or

$$\nabla^2 h^2 = -\frac{2R}{K}$$

Eq (9.33) is a special case of eq. (9.34) with R = 0.

Applications of eq. (9.34) for W.T. profiles in unconfined aquifers

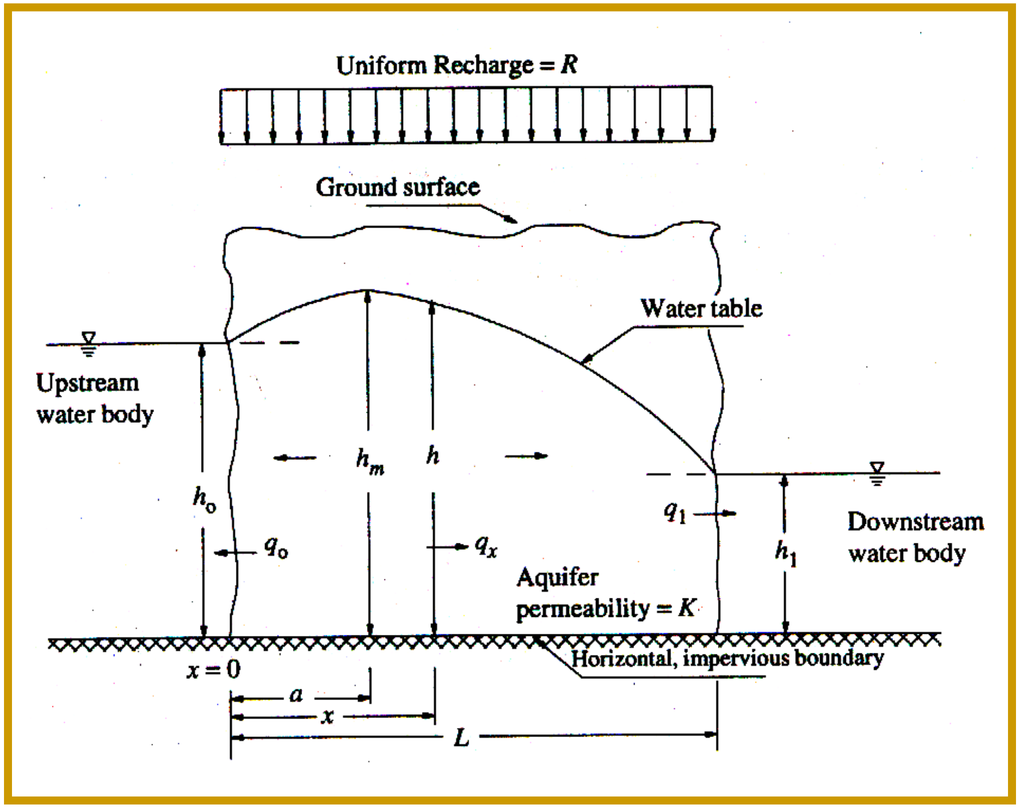
(i) An unconfined aquifer separating two water bodies (canal, river).
(ii) Various recharge situations
(iii) Drainage problems
(iv) Infiltration galleries.

One dimensional Dupit's Flow with Recharge

(i) The general Case

The aquifer is of infinite length and hence one dimensional method of analysis is adopted.

From eq. (9.34) $$\frac{\partial^2 h^2}{\partial x^2} = -\frac{2R}{K}$$



Integrating w.r.t x twice

$$h^2 = \frac{R}{K} x^2 + c_1 \; x + c_2 \quad (9.36)$$

Where C1 and C2 are constants of integration.

The boundary conditions are:

$$(i) \;\; at \;\; x = 0, \;\; h = h_o \;\; hence \;\;\; c_2 = h_o^2$$

$$(ii) \;\; at \;\; x = L, \;\; h = h_1 \;\; hence, \;\;\; h_1^2 - h_o^2 = \frac{R}{K} L^2 + c_1 L$$

$$c_1 = -\frac{\left[h_o^2 - h_1^2 - \frac{RL^2}{K}\right]}{L}$$

Thus eq. (9.36) becomes

$$h^2 = -\frac{R\ x^2}{K} - \frac{\left(h_o^2 - h_1^2 - \frac{RL^2}{K}\right)x}{L} + h_o^2$$

(9.37), So W.T. is ellipse

The value of a is obtained by equation $\frac{dh}{dx} = 0$ and is given by

$$a = \frac{L}{2} - \frac{K}{R}\left(\frac{h_o^2 - h_1^2}{2L}\right)$$

(9.38), Water divide, flow directions will be different to the left and right

The discharge per unit width of aquifer at any location x is

$$q_x = -K\ h\ \frac{\partial h}{\partial x}$$

$$= -K\left[-\frac{R\ x}{K} - \frac{\left(h_o^2 - h_1^2 - \frac{RL^2}{K}\right)}{2L}\right]$$

$$q_x = R\left(x - \frac{L}{2}\right) + \frac{K}{2L}\left(h_o^2 - h_1^2\right)$$

(9.39), q_x varies with x

At u/s water body, x=0

$$q_o = q_{x=0} = -\frac{RL}{2} + \frac{K}{2L}\left(h_o^2 - h_1^2\right)$$

(9.40)

At d/s water body, x=L

$$q_1 = q_{x=L} = +\frac{RL}{2} + \frac{K}{2L}\left(h_o^2 - h_1^2\right) = RL + q_o$$

(9.40a)

(ii) Flow without Recharge

If there is no recharge, R=0

Steady one dimensional flow in unconfined aquifer.

Putting R=0 in eqn (9.37)

$$\left(h^2 - h_o^2\right) = \left[\frac{h_1^2 - h_o^2}{L}\right] x \qquad (9.41)$$

This represents Dupit's Parabola.

Differentiating w.r.t. x

$$2h\frac{dh}{dx} = \frac{\left(h_1^2 - h_o^2\right)}{L}$$

The discharge per unit width of aquifer is

$$q = -K\ h\ \frac{dh}{dx} = K\frac{\left(h_o^2 - h_1^2\right)}{2L} \qquad (9.42)$$

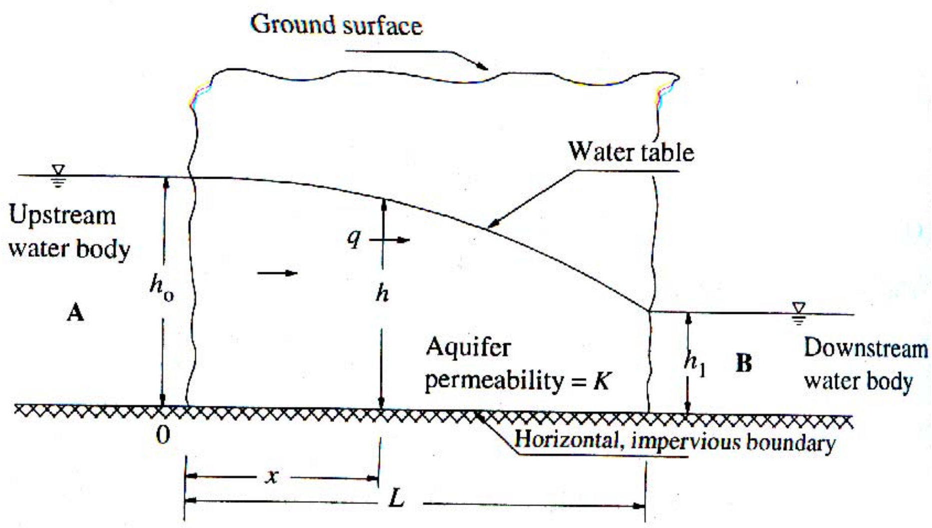


Fig. 9.11 One dimensional unconfined flow without recharge

(iii) The Drain Problem

To drain water logged areas

To reduce level of W.T.

Constant recharge rate R at top

Using eqn.(9.37) for aprroximate Solution

Neglecting depth of water in the drains, i.e. $h_o = h_1 = 0$

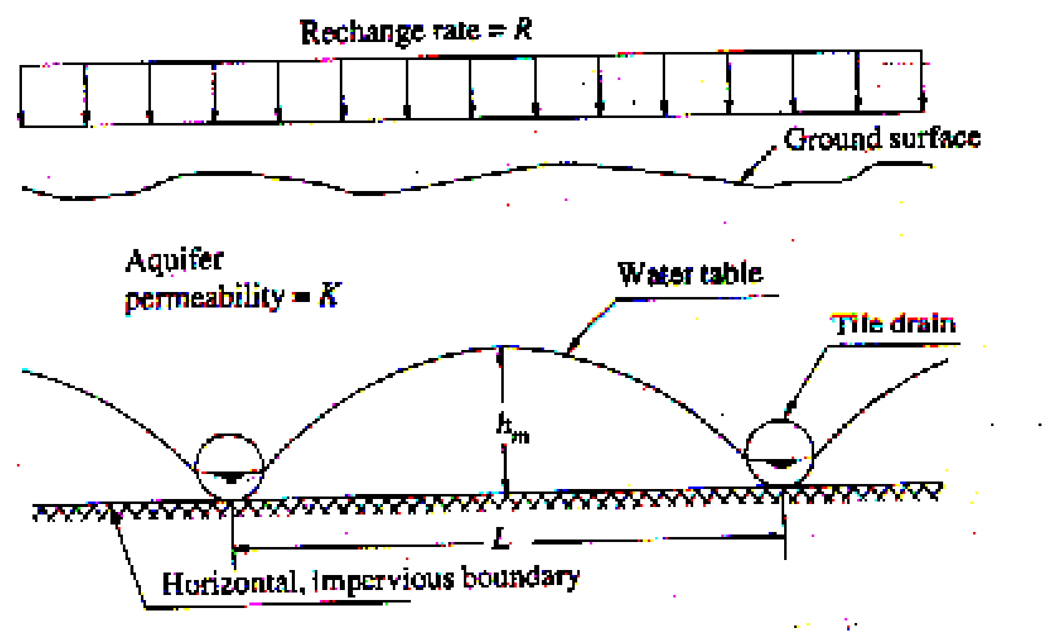


Fig. 9.12 Tile Drains under a constant Recharge rate

The W.T. profile will then be

$$h^2 = \frac{R}{K}(L-x)\ x \quad (9.43)$$

The maximum height of W.T. occurs at a=L/2

$$h_m = \frac{L}{2}\sqrt{\frac{R}{K}} \quad (9.44)$$

The discharge entering a drain per unit length of the drain

$$q = 2\left[\frac{RL}{2}\right] = RL \quad (9.45)$$

Sea Water intrusion in coastal area and island

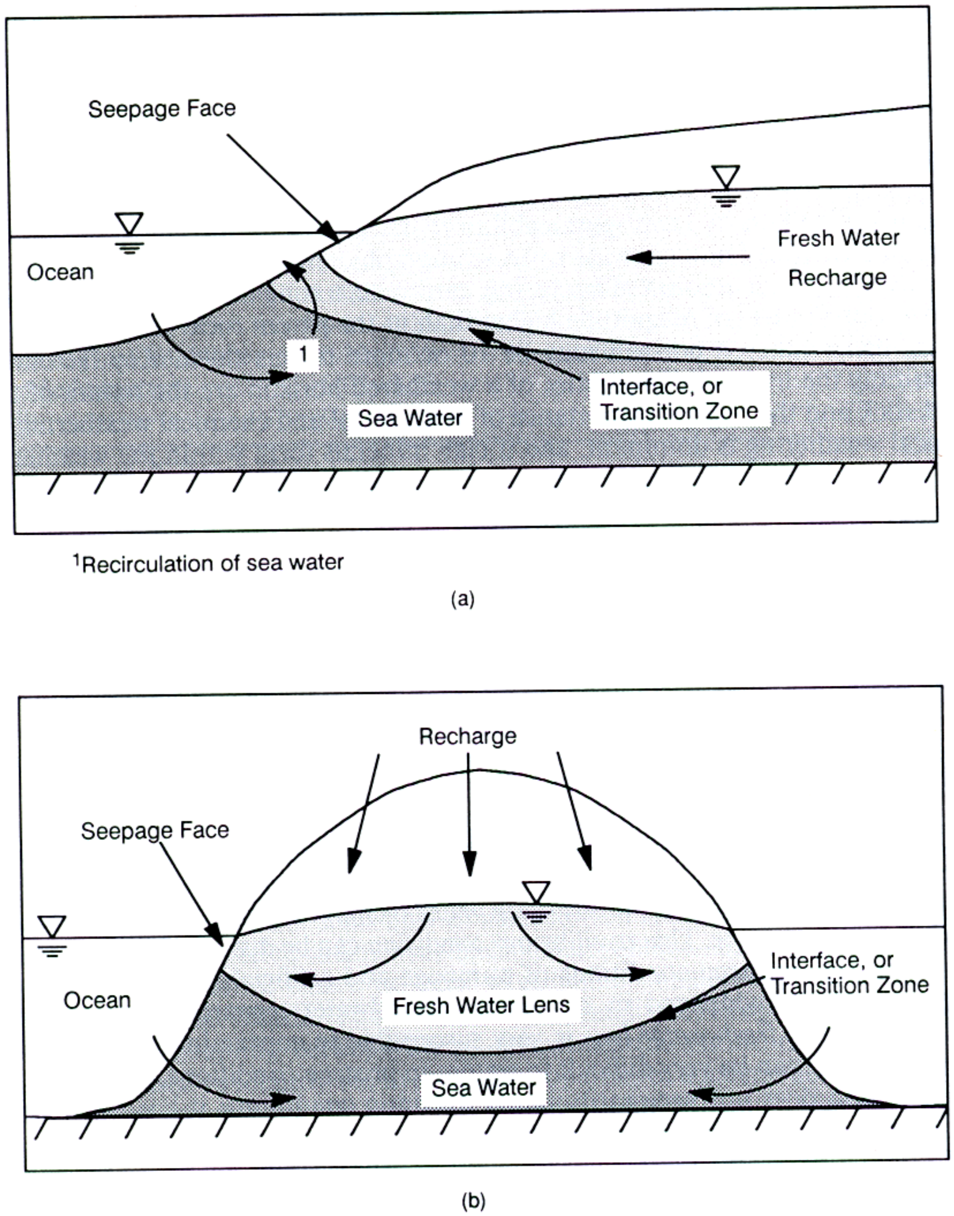
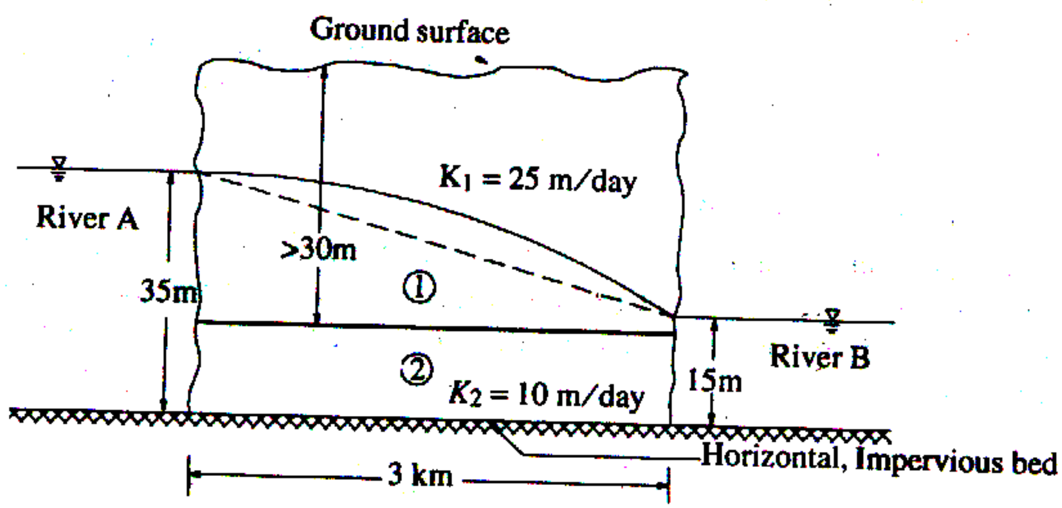


FIGURE 6.6.1 Occurrence of seawater intrusion in *(a)* coastal aquifers, *(b)* island aquifers.

Example of Groundwater

Example of GWF



Ground surface
River A
35m
>30m
K1 = 25 m/day
①
②
K2 = 10 m/day
15m
River B
Horizontal, Impervious bed
3 km

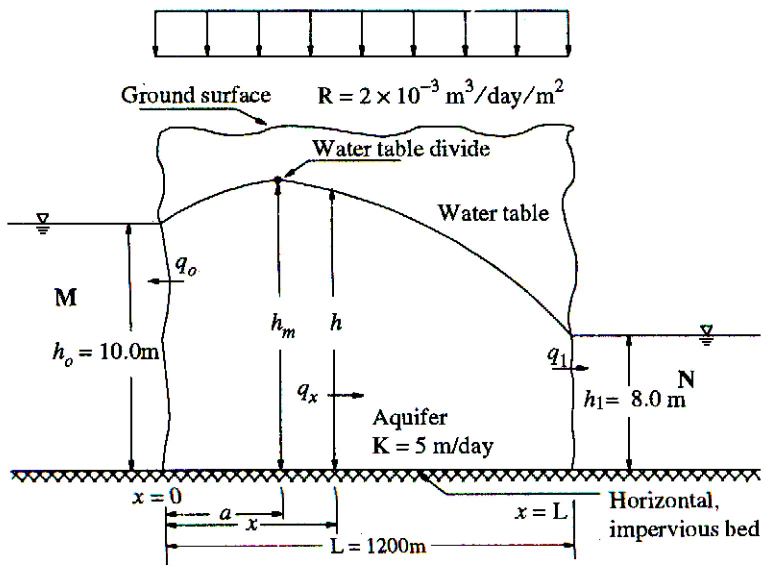


Fig. 9.14 Schematic Layout of Example 9.5

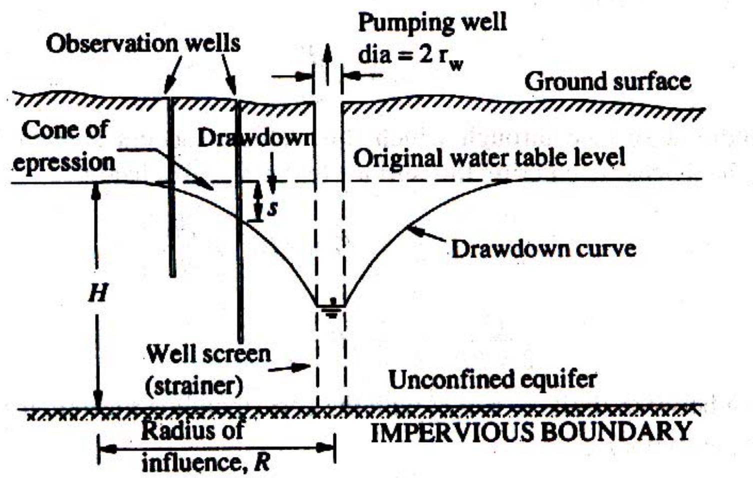
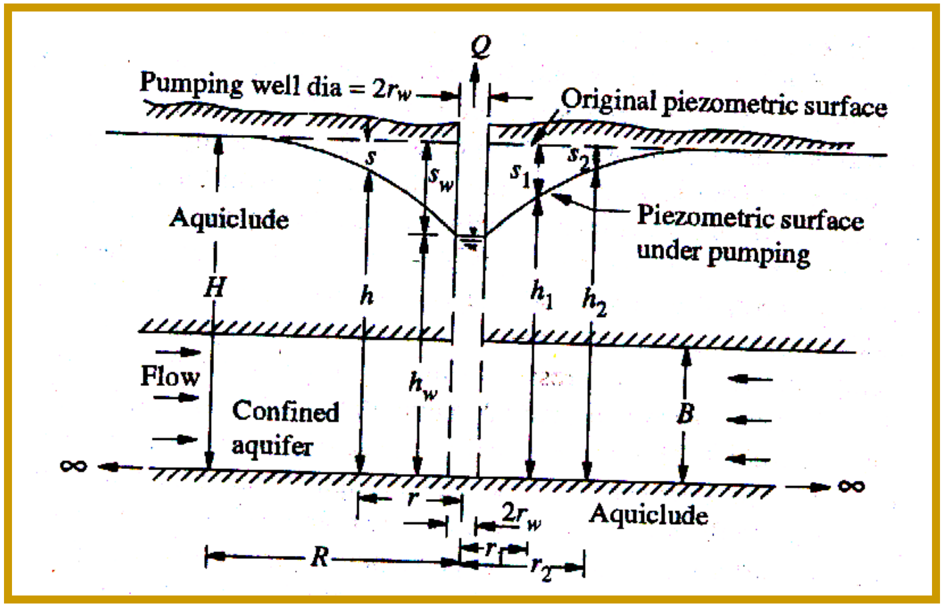
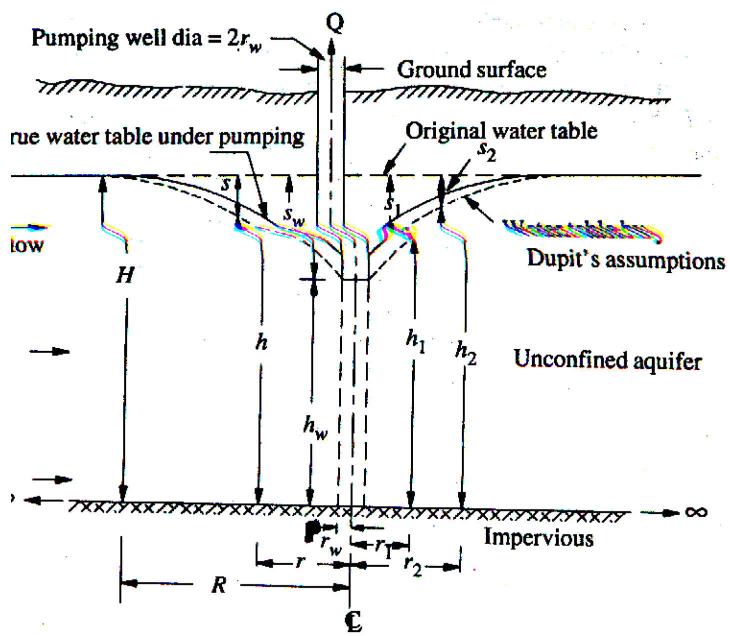


Fig. 9.15 Well operating in an unconfined aquifer, (definition sketch)



Q
Pumping well dia = $2r_w$
Original piezometric surface
s
s_w
s_1
s_2
Aquiclude
Piezometric surface under pumping
H
h
h_1
h_2
Flow
h_w
Confined aquifer
B
∞
∞
r
$2r_w$
Aquiclude
R
r_1
r_2


Pumping well dia = $2r_w$
Q
Ground surface
rue water table under pumping
Original water table
s_2
s
s_w
s_1
low
Dupit's assumptions
H
h
h_1
h_2
Unconfined aquifer
h_w
∞
r_w
r_1
Impervious
r
r_2
R
℄

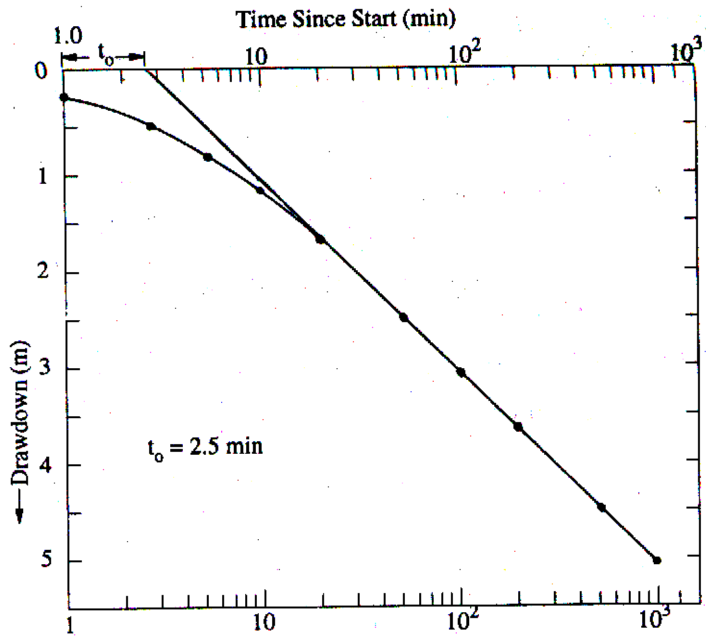
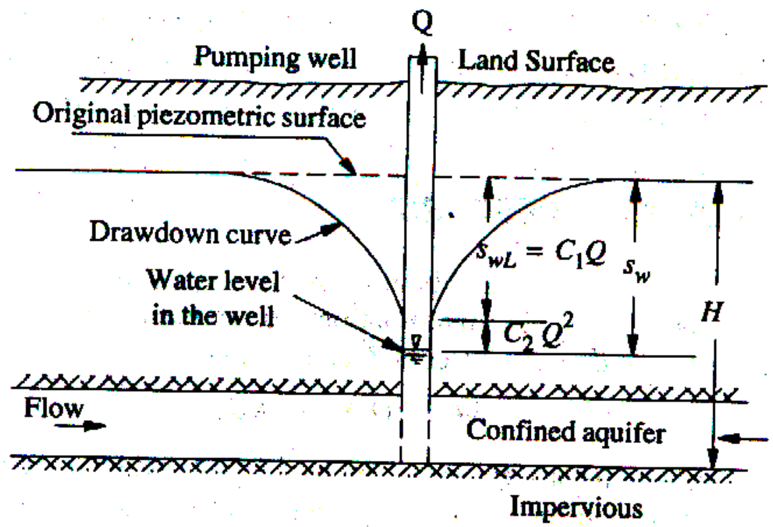


Fig. 9.18 Time-drawdown plot — Example 9.9


Q
Pumping well
Land Surface
Original piezometric surface
Drawdown curve
Water level
in the well
$s_{wL} = C_1Q$
s_w
H
C_2Q^2
Flow
Confined aquifer
Impervious