Hydraulics Engineering Lec # 7:

Unsteady Flow through pipe lines, Water hammer.
Instantaneous and slow closure of valves

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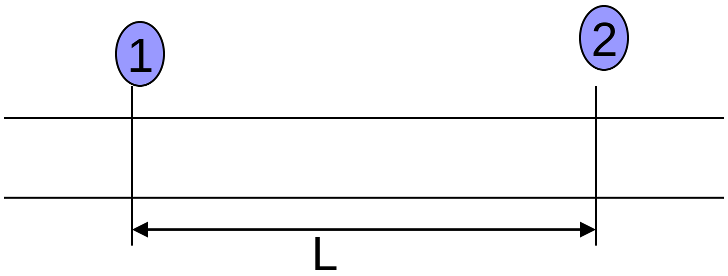
Unsteady Flow

- Discharge through orifices and over weirs under varying heads.
- Unsteady Flow through pipe lines, Water hammer. Instantaneous and slow closure of valves
- Surge wave in open channels.

Unsteady Flow through pipe lines

- Let there be an incompressible accelerating flow and dV is the change in velocity in time dt. The energy equation for unsteady flow (accelerating flow) can be written as



$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2+H_L+H_i$$

Where : H_i = Head consumed in accelerating the flow

b/w section 1 &2

$$\text{Mass of fluid} = \frac{\gamma}{\mathrm{g}}AL$$

$$\text{Force required to accelerate the flow =F=ma=}\frac{\gamma}{\mathrm{g}}AL\frac{dV}{dt}$$

$$\text{Pressure required for accelerating =P=F/A=}\frac{\gamma}{\mathrm{g}}L\ \frac{dV}{dt}$$

$$\text{Hence Head required to accelerate flow=hi=}\frac{\mathrm{P}}{\gamma}=\frac{L}{\mathrm{g}}\frac{dV}{dt}$$

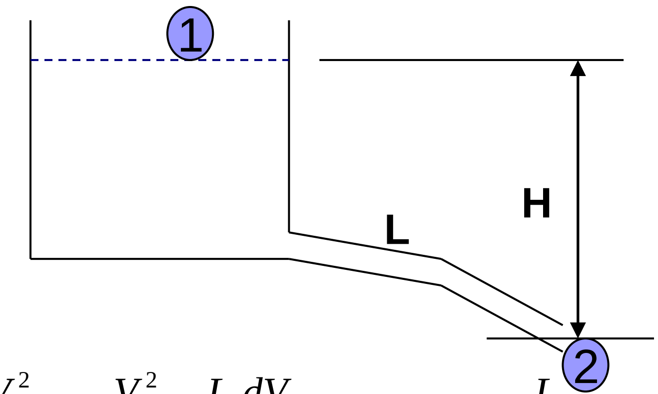
$$\therefore\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2+H_L\pm\frac{L}{\mathrm{g}}\frac{dV}{dt}\Rightarrow Eq.1$$

The last term in eq. 1 will be –ve if the flow is retarding flow and vice versa.

If the valve at section 2 is suddenly/very rapidly closed, the pressure head at this section will suddenly rise up y an amount of *LdV/gdt* and vice versa

Establishment of Steady Flow:

- Determining the time for the flow to become steady in a pipeline when a valve is suddenly opened at end of the pipe can be accomplished through application of Eq. 1.
- Immediate after valve is opened, the head H is available to accelerate the flow. Thus flow commences, but as the velocity increases the accelerating head is reduced by fluid friction and minor losses.
- Let us assume the total head h_L can be expressed as $KV^2/2g$.
- Applying eq. 1 between section 1 & 2.



$$H = \frac{V^2}{2g} + K\frac{V^2}{2g} + \frac{L}{g}\frac{dV}{dt}; \quad K = K + f\frac{L}{D}$$

Let us define the steady flow velocity by V_o.

Noting that for steady flow dV/dt=0, we get

$$V_o = \sqrt{\frac{2gH}{(1+K)}}$$

substituting H from this eq. into above eq.

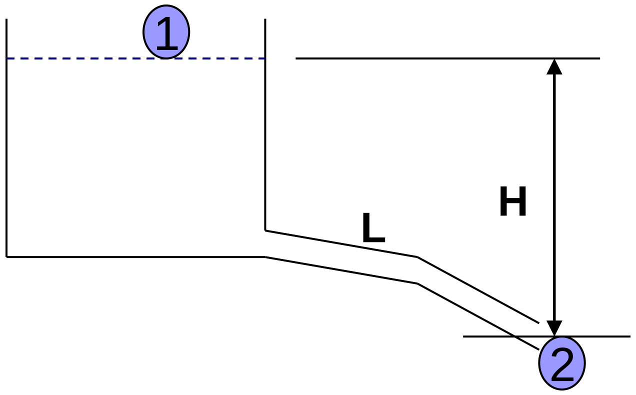
$$dt = \left(\frac{2L}{1+k}\right)\frac{dV}{V_o^2 - V^2}$$

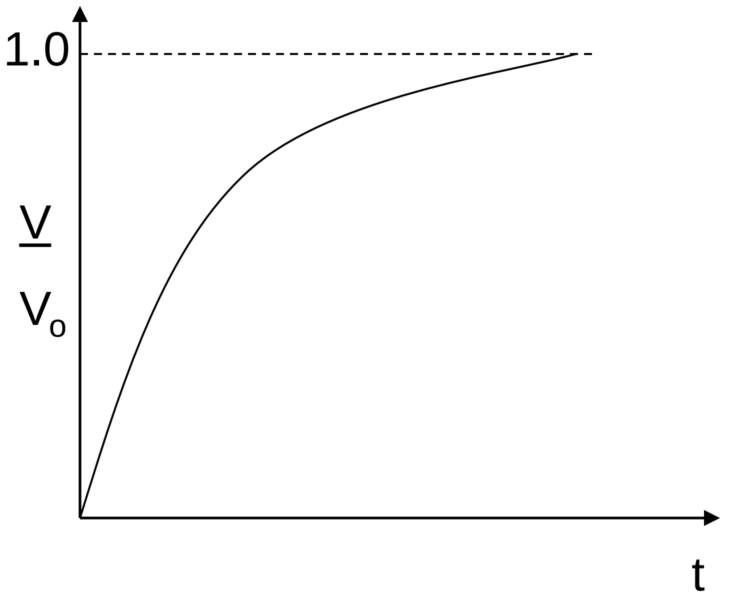
Integrating and noting that constant of integration is zero, since V=0 at t=0 and lnVo/Vo=0,

$$t = \frac{L}{(1+K)V_o}\ln\frac{V_o + V}{V_o - V} \quad \Rightarrow Eq.2$$

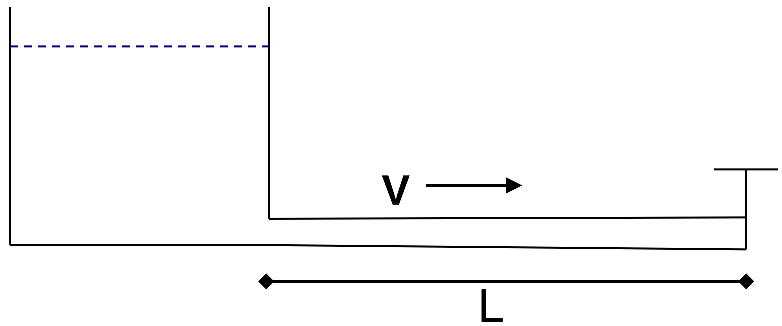
Establishment of Steady Flow:

- Eq. 2 indicated that V approaches Vo asymptotically and that the equilibrium will be attained only after an infinite time, but it must be remembered that this is an idealized case. In reality there will be elastic wave and damping, so that true equilibrium will be reached in a finite time.





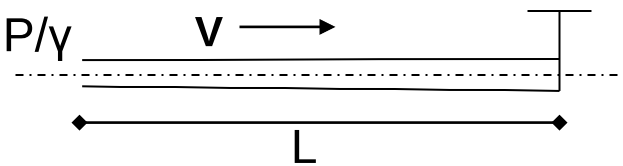
Water Hammer (Hammer Blow)



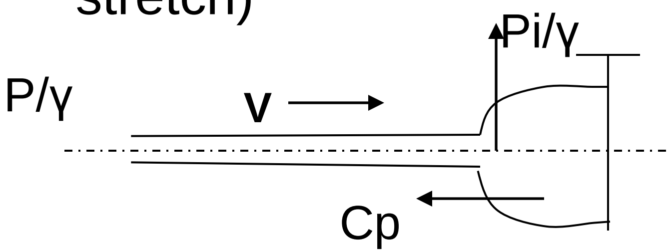
- Whenever velocity in a pipe line is reduced instantaneously or in a very short time a sudden increase in pressure takes place. This sudden rise in pressure in a pipe line due to stoppage of flow is known as water hammer or hammer blow.
- The increase in pressure is so high that the pipe may burst if it is not strong enough to withstand that pressure.
- Water hammer is used for all type of fluids, its not specific for water.

Water Hammer (Hammer Blow)

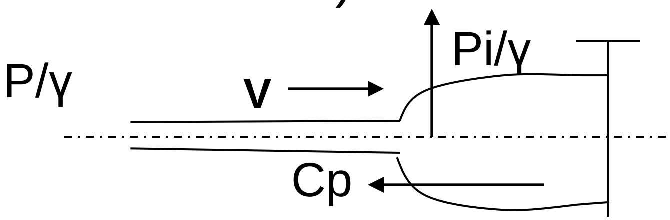
- (a). Normal steady flow taking place: (Pressure head throughout is P/γ)



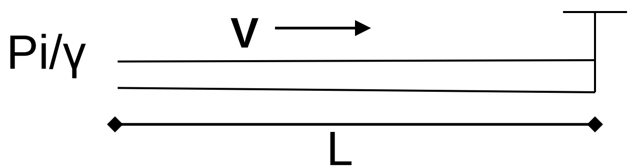
- (b). Valve closed instantaneously: (Pressure wave travels u/s with a velocity Cp. The pressure rises to Pi/γ and the wall stretch)



- (c). Valve closed instantaneously : (Pressure wave travels further and pipe wall stretches)



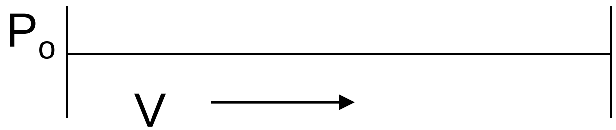
- (c). After Time L/Cp: (The entire pipe is under very high pressure)



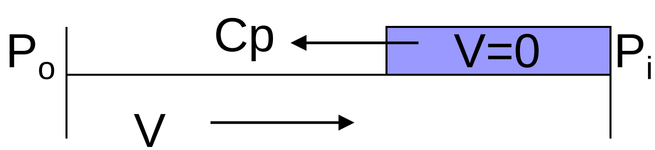
Cp=Celerity=Velocity of pressure wave

Movement of Pressure Wave

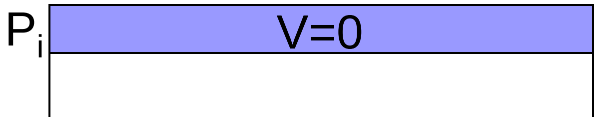
- 1. Initial Condition



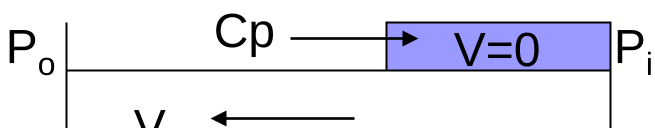
- 2. Transmission of original wave (Valve closed at t=0)

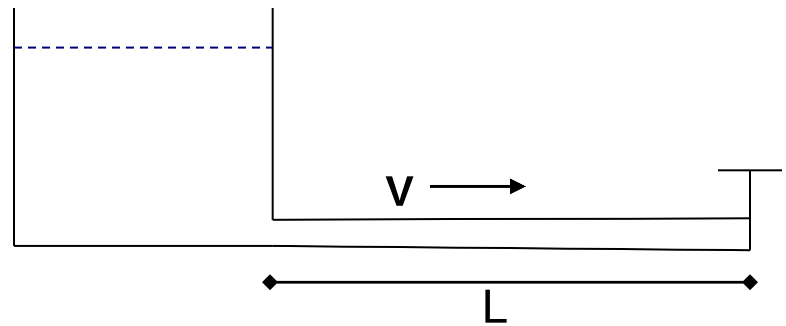


- 3. Wave reaches reservoir

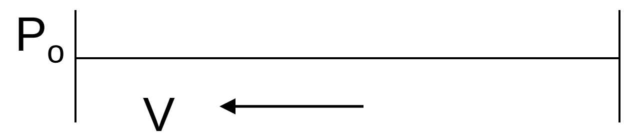


- 4. Reflected wave canceling original pressure moving back towards the valve

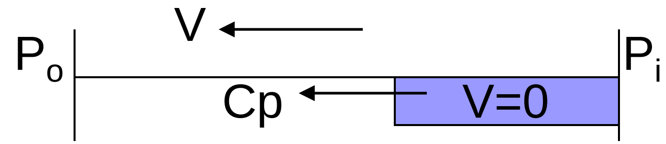




- 5. Reflected wave reaches the valve

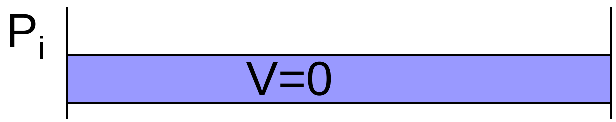


- 6. Reflection of negative wave from closed end

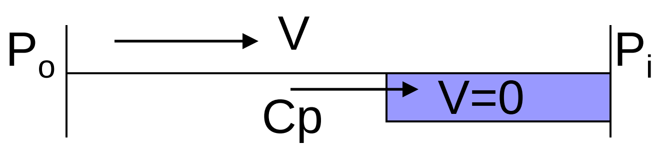


Movement of Pressure Wave

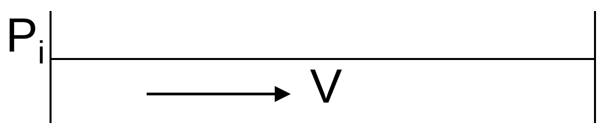
- 7. Negative wave reaches the reservoir end.

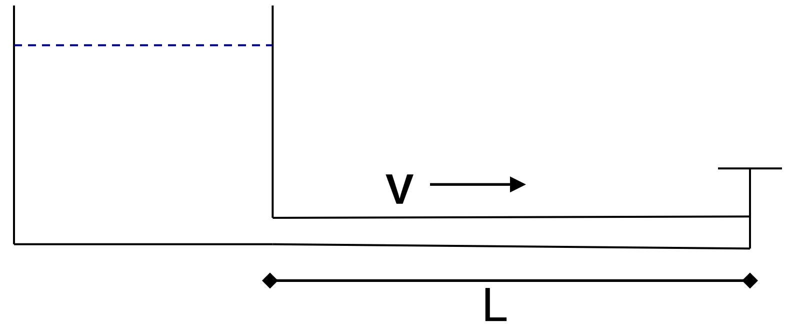


- 8. Reflection of negative wave from reservoir as +ve wave canceling the pressure drop

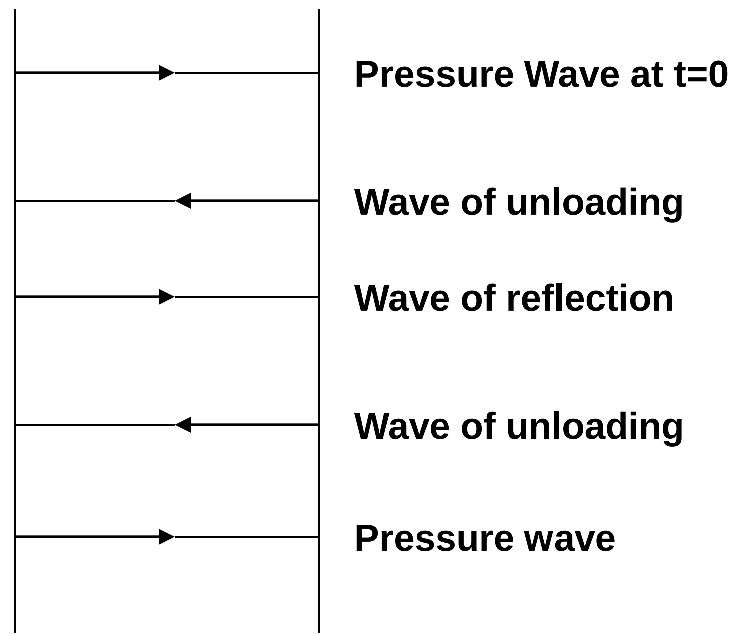


- 9. Initial Condition again





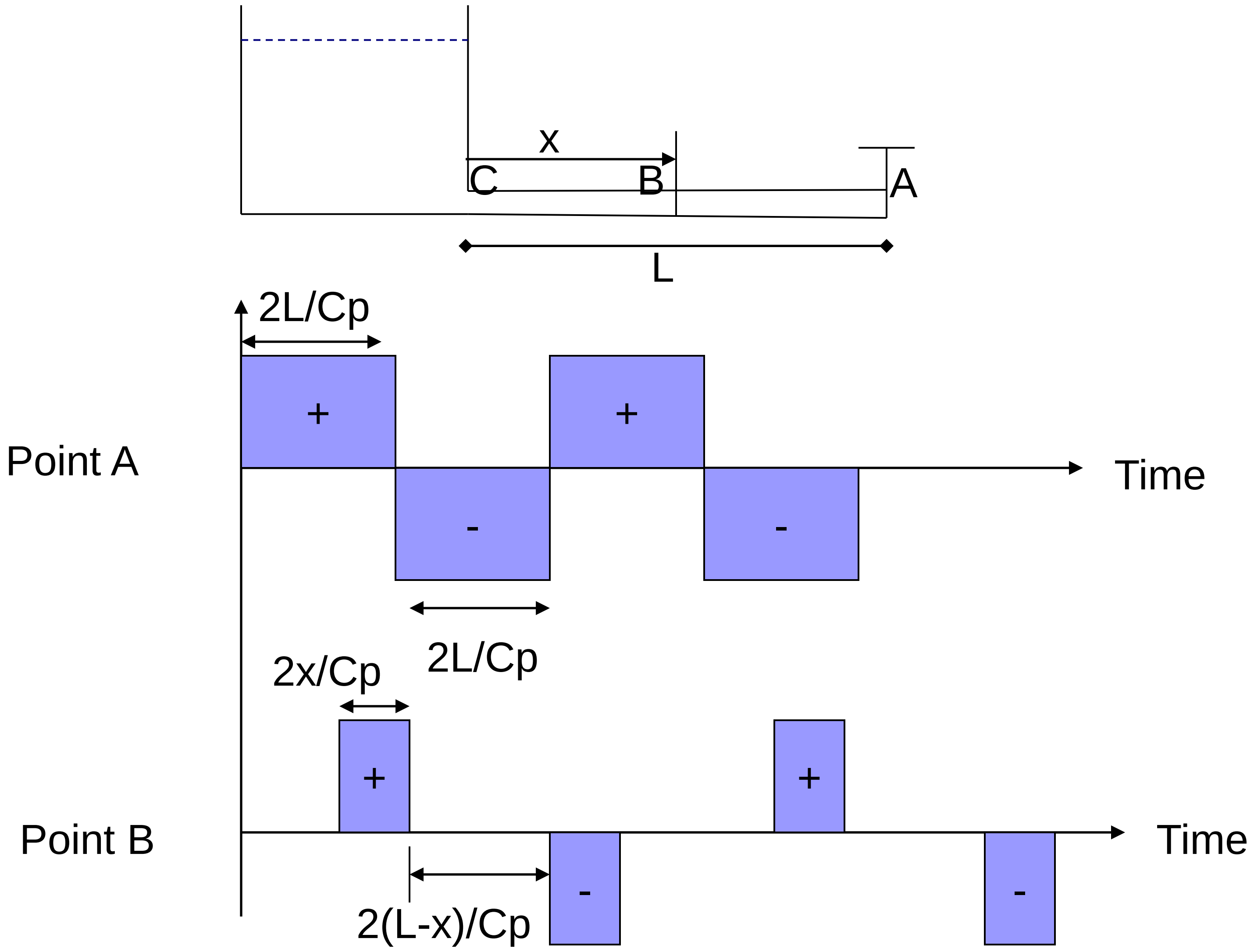
- The Movement of wave is summarized below:



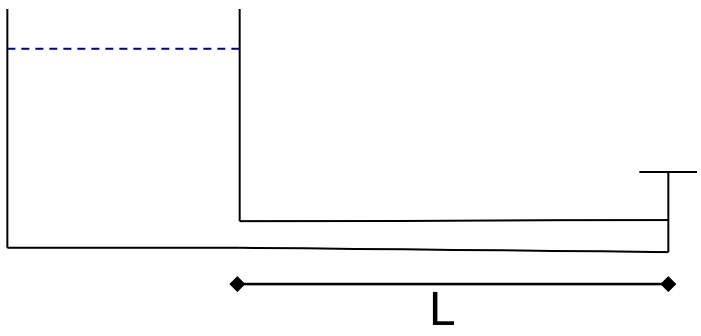
Movement of Pressure Wave

- Ideally, there would be a series of pressure waves traveling back and forth over the length of pipe b/w high and low pressures about the valve for zero flow.
- In actual practice, as a result of friction of the pipe and the different values of elasticity for pipe material and liquid, the amplitude of each reflected wave would be less than the previous wave. This reduction in the pressure of wave is know as damping of waves and the result is that after sometimes the situation becomes normal.
- However, the most critical condition in pipeline would be when valve is instantaneously closed and critical section is the one very close to the valve.
- Due to movement of pressure wave u/s and d/s of pipeline, a sound is sometimes heard which is known a knocking.

Pressure Variation with time at different sections.



Equation for Water Hammer Pressure



- Types of Valve Closures
  - Instantaneous Valve Closure: t=0
  - Rapid Valve Closure: t<2L/Cp
  - Slow Valve Closure: t>2L/Cp
    - Where Cp= Celerity or velocity of pressure wave

Equation for Water Hammer Pressure (Rigid Pipe Theory)

- Let the valve is closed instantaneously. Let the pressure wave cover a distance Cp.dt in time period dt and change in velocity is dv. Volume of water the is compressed against valve and comes to rest in time dt is
  - *Volume =Cp.dt.A*
  - *Mass = ρ.Cp.dt.A*
  - *Force = ρ.Cp.dt.A. dv/dt*
  - *Pressure*=P_h = *ρ.Cp.dv*
- *Where* h stands for water hammer pressure
- ρ = mass density

- For rapid valve closure
  - *Pressure*=P_h = *ρ.Cp.dv*
- For slow valve closure
  - Pressure=P_h =2ρL(V_1-V_2')/t
- For a rigid pipe
  - $Cp=(Ev/\rho)^{1/2}$
- For an elastic pipe
  - $Cp=(Ev/(\rho(1+D/t+Ev/E))^{1/2}$
- Where
  - Ev=Vol. Modulus of Elasticity
  - E=Young's Modulus of Elasticity
  - D/t= Diameter to thickness ratio of pipe

Methods of eliminating/controlling water hammer pressure in pipelines

- 1. Slow Valve Closure
- 2. Pressure Relief Valve
- 3. Surge Tanks

Questions

- P1. water from a reservoir flowing through a rigid pipe 15cm diameter with a velocity 2.4m/sec is completely stopped by a valve situated at 1000m from the reservoir. Determine the maximum rise of pressure when valve closure takes place in
- A) 1 sec B) 5 sec

Assume that pressure increase at uniform rate & there is no damping of pressure wave. Take Cp=1433 m/sec

A): $2L/Cp = 2x1000/1433 = 1.4\sec$

$\therefore$ tc <1.4 sec $\Rightarrow$ Rapid Valve Closure

$$Ph = \rho Cpdv = \rho Cp(V-0)$$

$$= 1000x1433x2.4$$

$$= 3439200 N/m^2$$

B): $2L/Cp = 2x1000/1433 > 1.4\sec$

$\therefore$ tc>1.4 $\Rightarrow$ Slow Valve Closure

$$Ph = 2\rho L\left(\frac{V_1 - V_2'}{t}\right) = 2\rho L\left(\frac{V_1 - 0}{t}\right)$$

$$= 2x1000x1000(2.5/5)$$

$$= 1000000 N/m^2$$

Questions

- P2. A cast iron pipe 15cm diameter and 1.5 cm thick is conveying water when outlet valve is suddenly closed. Calculate the maximum permissible discharge if pressure rise is not to exceed 1700KN/m². Take Ev for water as 2.06 E6 KN/m² and E for cast iron as 117 E6 KN/m²

$$Cp = \sqrt{\frac{Ev}{\rho\left(1 + \frac{D}{t} + \frac{Ev}{E}\right)}}$$

$$Cp = \sqrt{\frac{2.06x10^9}{1000\left(1 + \frac{15}{1.5} + \frac{2.06x10^9}{117x10^9}\right)}}$$

$$= 1323.48 m/\sec$$

$$Ph = \rho Cpdv$$

$$1700x10^3 = 1000x1323.48xV$$

$$V =$$

$$Q = \frac{\pi}{4} D^2 xV$$

$$Q =$$

Questions

- P3. A pipeline 60cm diameter is 1500m long & is carrying a discharge of 0.5 m^3/sec. The valve at the outlet end is closed in 10 secs in such a way that the time of retardation at any instant is proportional to the time elapsed since the commencement of closure.
- A: calculate the rise in pressure
- B: Calculate the rise of pressure if the closure were practically instantaneous
- Take Ev=1895 E6 N/m^2

$$-\frac{dv}{dt} \propto t \Rightarrow -\frac{dv}{dt} = ct$$

$$-dv = ctdt$$

$$\int_{Vo}^{0} dv = -\int_{0}^{10} ctdt$$

$$V\Big|_{Vo}^{0} = -\frac{ct^2}{2}\Bigg|_{1}^{10}$$

$$C = 0.03536$$

$$\therefore \frac{dv}{dt} = -0.3536t$$

$$\frac{P_o}{\gamma} + \frac{V_o^2}{2g} + Z_o + h_f = \frac{P_o}{\gamma} + \frac{V_o^2}{2g} + Z_o + h_f \pm \frac{L}{g}\frac{dv}{dt}$$

$$\frac{P_o - P_1}{\gamma} = \frac{V_o^2}{2g} - \frac{L}{g}\frac{dv}{dt}$$

$$\frac{P_o - P_1}{\gamma} = \frac{V_o^2}{2g} - \frac{L}{g}(-0.3536t)$$

$$\frac{P_o - P_1}{\gamma} = -------m$$

Assignment

- Ex. 13.4
- 13.29, 13.30, 13.32
- Submission Date: ____________________

Questions