Hydraulics Engineering Lec # 6:

Discharge through orifices and over weirs under varying heads.

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Unsteady Flow

- Discharge through orifices and over weirs under varying heads.
- Unsteady Flow through pipe lines, Water hammer. Instantaneous and slow closure of valves
- Surge wave in open channels.

Unsteady Flow

For unsteady flow

$Q_1 \neq Q_2$

let dh be the change in head in

small time period dt

$(Q_1 - Q_2)\,dt = Adh$

or

$$dt = \frac{Adh}{Q_1 - Q_2} \Rightarrow Eq.(1)$$

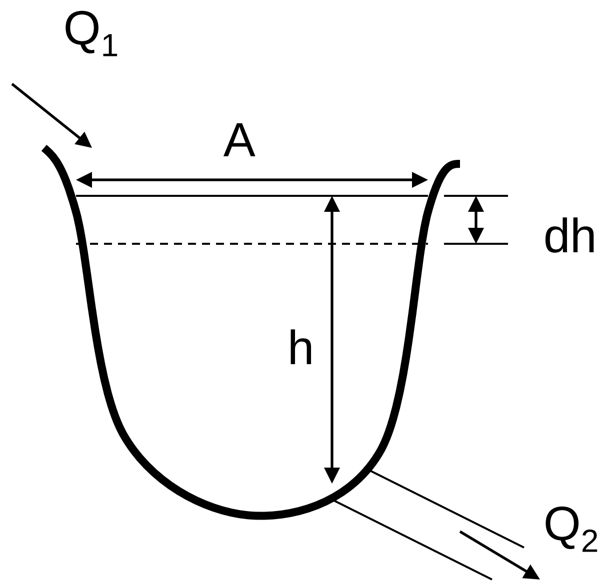
$Eq.(1)$ is called as continuity equation

for unsteady flow

Integrating above Eq.(1)

$$\int_0^t dt = \int_{h_1}^{h_2} \frac{Adh}{Q_1 - Q_2}$$

$$\therefore t = \int_{h_1}^{h_2} \frac{Adh}{Q_1 - Q_2}$$



For Steady Flow

$Q_1 = Q_2$

$i.e. \Rightarrow h = constt$

$\therefore For \quad Orifice$

$$Q_{act} = cd.a.\sqrt{2gh}$$

Discharge Between reservoirs under Varying Heads

- Figure shows two reservoirs, of constant area, interconnected by a pipeline, water transfer occurring under gravity. Reservoir *A* receives an inflow *I* while a discharge Q_2 is withdrawn from *B*; *I* and Q_2 may be time variant.

Let

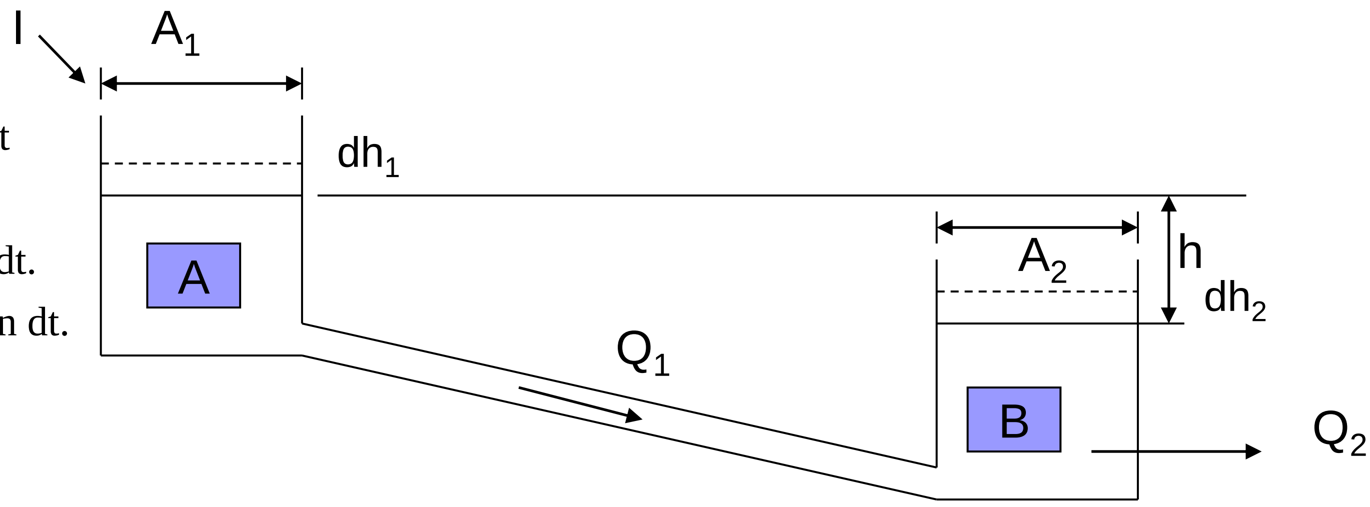
h = Gross head at any instant

dh_1 = Change in level in A during a small time interval dt.

dh_2 = Change in level in B in dt.

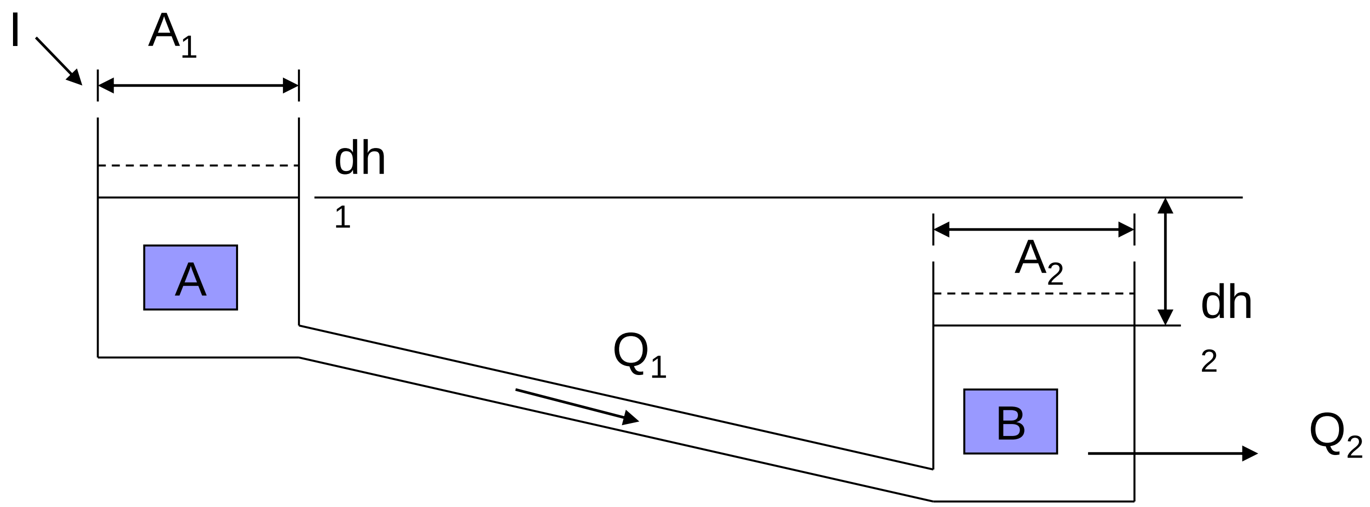
dh= Change in total head

$= dh_1 - dh_2$



- In general case the head *h*, and hence the transfer discharge Q_1, will vary with time and object is to obtain a differential equation describing the rate of variation of head with time, *dh/dt*.

Discharge Between reservoirs under Varying Heads



Continuity Equation for A: $I - Q_1 = A_1 \dfrac{dh_1}{dt}$

Continuity Equation for B: $Q_1 - Q_2 = A_2 \dfrac{dh_2}{dt}$

Note that Q_1 is the discharge in the pipeline and hence is the inflow rate into B

Hence

$$dh = \left(\frac{I - Q_1}{A_1} - \frac{Q_1 - Q_2}{A_2} \right) dt$$

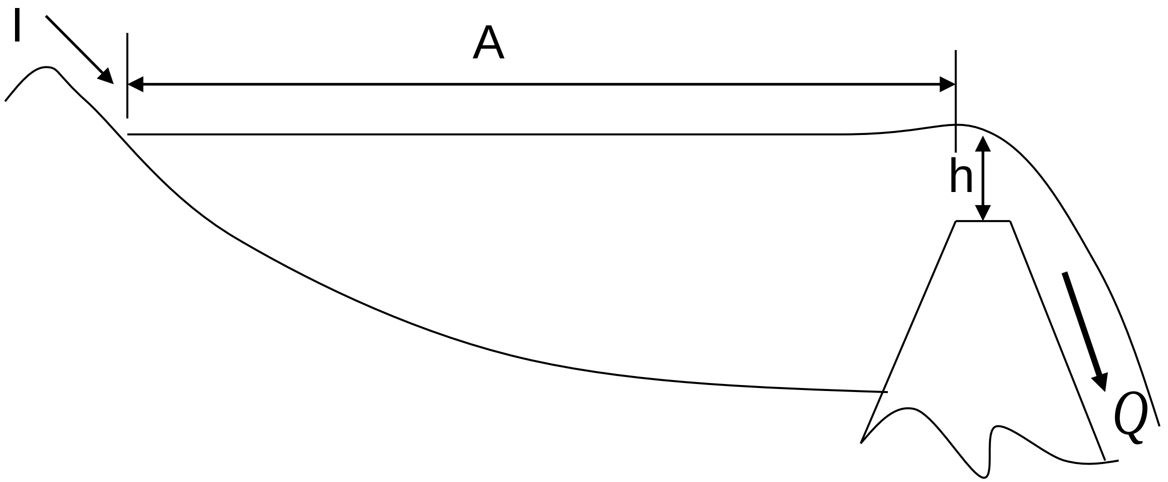
Discharge in pipleline

$$h_L = \left(K_m + \frac{fL}{D} \right) \frac{Q^2}{2gA_p{}^2}$$

$$\therefore Q = K\sqrt{h} \text{ where } K = A_p \sqrt{\frac{2g}{\left(K_m + \frac{fL}{D} \right)}}$$

Unsteady Flow over Weir/Spillway

- Figure shows spillway, of varying area with *h*. Reservoir receives an inflow *I* while a discharge *Q* is withdrawn from it.



- The Computation of time variant in reservoir elevation and spillway discharge during a flood inflow to the reservoir is essential in design of the spillway to ensure safety of the impounding structure.

Unsteady Flow over Weir/Spillway

Now Continuity Equation becomes;

$$I - Q = \frac{dS}{dt};\text{where } \frac{\mathrm{dS}}{\mathrm{dt}} \text{ is the rate of change of storage, or volume.}$$

$$\frac{\mathrm{dS}}{\mathrm{dt}} = A\frac{\mathrm{dh}}{\mathrm{dt}}; \textit{Where} \text{ A is intantenous plan area of reservoir and}$$

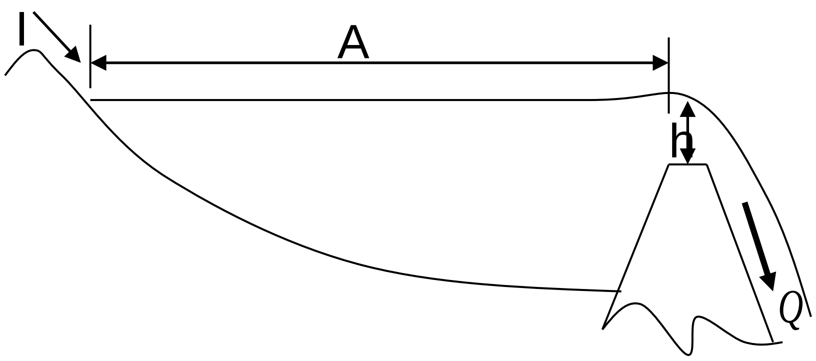
$$\frac{\mathrm{dh}}{\mathrm{dt}} \text{ is intantenous rate of change of depth.}$$

Since Q for Spillway/Weir is

$$Q = \frac{2}{3}C_d\sqrt{2g}BH^{3/2} = KH^{3/2}$$

Therefore equation becomes

$$I - KH^{3/2} = \frac{\mathrm{dS}}{\mathrm{dt}} = A\frac{dh}{dt} \Rightarrow Eq.(1)$$



- *I* is the known time variant inflow rate. Except in the special case where *A* does not vary with depth and *I* is constant, eq.1 is not directly integrable and in general must be evaluated numerically.

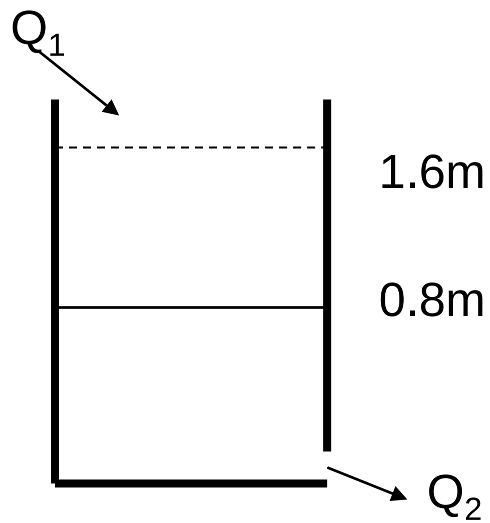
Problem

- **A cylindrical tank with its axis vertical is filled with water and discharges through an orifice 2.5 cm diameter at the bottom with a Cd=0.623. If the diameter of the tank is 0.6m; find the time required for water level to drop from 1.8m to 0.6m above the orifice when supply is cut off.**

Given that

diameter of orifice = d= 0.25 cm

Continuity Eq. for unsteady flow is



$$\therefore t = \int_{h_1}^{h_2} \frac{Adh}{Q_1 - Q_2}$$

$$Here:\ \mathrm{Q}_1 = 0\ \&\ \mathrm{Q}_2 = \mathrm{Cd.a.}\sqrt{\mathrm{2gh}}$$

$$t = \int_{h_1}^{h_2} \frac{Adh}{-C_d.a.\sqrt{2gh}} = \frac{-A}{C_d.a\sqrt{2g}} \int_{h_1}^{h_2} h^{1/2} dh$$

$$t = \frac{2A}{C_d.a\sqrt{2g}} \left(\sqrt{h_1} - \sqrt{h_2}\right)$$

$$\therefore = \frac{2\left(\frac{\pi}{4} 0.6^2\right)}{0.623\left(\frac{\pi}{4} 0.025^2\right)\sqrt{2x9.81}} \left(\sqrt{1.8} - \sqrt{0.6}\right) = 236.718\,\mathrm{sec}$$

Assignment Problems

- Q1: A vertical cylinder tank 1.8 m diameter has at the bottom a 5cm diameter sharp edge orifice for which C_d=0.6
  - (a) If the water enters the tank at a constant rate of 0.009 cumec. Find the depth of water above the orifice when the flow becomes steady.
  - (b) Find the time for the water level to fall from 2.4m to 0.6m above orifice if there is no inflow.
  - (c) If the water now runs into the tank at a constant rate of 0.02 cumec, the orifice remaining open, find the rate of rise of water level in cm/min when this level has reached 1.5m above orifice.
- Q2: A Reservoir of water surface area 900,000 m^2 is provided with a spillway 30m long which may be treated as rectangular notch with Cd=0.72
  - Find the time in hours for head over the spillway to fall from 0.6m to 0.15m. Neglect inflow.
- Q3: A reservoir is circular in plan and when full the diameter of water surface is 60m. When the water level falls 1.2m the diameter of the surface is 48m. Discharge takes place through a 0.6m diameter outlet 3m below the high water level which can be treated as orifice with a coefficient of discharge 0.8.
  - Determine the time required to lower the water level by 1.2m if the reservoir is full at the start.

Assignment Problems

- Q4: A cylindrical tank 1.0 m diameter empty through a 0.5m diameter pipe 3.5m long for which the friction factor, *f=0.04.*
  - Find the time taken for head over the outlet to fall from 2.5m to 1.2m. There is no inflow.
- Q5: A tank 3m long and 1.5m wide is divided into two parts by a partition so that the area of one part is 3 times the area of other. The partition contains a square orifice 7.5cm sides through which the flow may flow from one side to other.
  - If the water level in smaller dimension is 3m above that of larger. Find the time taken to reduce the difference of water level to 0.6m. Take Cd=0.62
- Q6: A pipe discharges into a measuring tank 2m long and 1m wide. One wall of the tank has a V notch through which the water escapes freely and a gauge is provided to measure the head of water above the bottom of the notch. When the pipe discharges steadily at the rate of $0.02 m^3/sec$, the gauge shows 22.5cm head.
  - If the supply of water is suddenly cutoff, how long will it take for the head to fall to 7.5cm. Take Cd=0.62

Questions