PDHonline Course C155 (2 PDH)

Earth Pressure and Retaining Wall Basics for Non-Geotechnical Engineers

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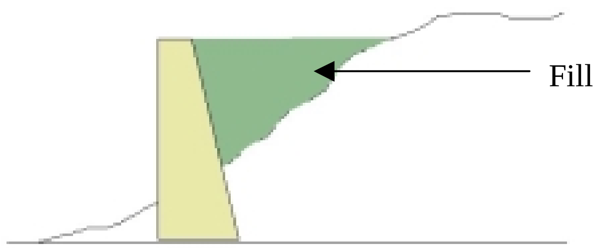
Richard P. Weber

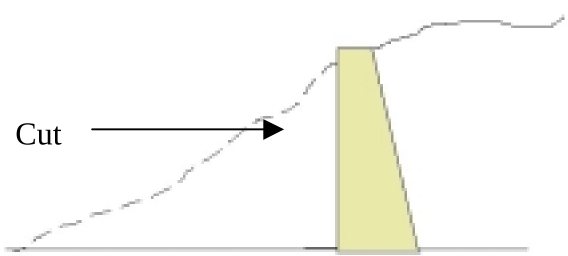
Course Content

Content Section 1

Retaining walls are structures that support backfill and allow for a change of grade. For instance a retaining wall can be used to retain fill along a slope or it can be used to support a cut into a slope as illustrated in Figure 1.



Retaining Wall to Support a Fill.



Retaining Wall to Support a Cut.

Figure 1 – Example of Retaining Walls

Retaining wall structures can be gravity type structures, semi-gravity type structures, cantilever type structures, and counterfort type structures. Walls might be constructed from materials such as fieldstone, reinforced concrete, gabions, reinforced earth, steel and timber. Each of these walls must be designed to resist the external forces applied to the wall from earth pressure, surcharge load, water, earthquake etc. Prior to completing any retaining wall design, it is first necessary to calculate the forces acting on the wall.

This course is not intended to be exhaustive nor does it discuss a wide range of surcharge loads or other lateral forces that might also act on a wall such as earthquake. There are many textbooks and publications that explain loading conditions in depth including:

- Foundations and Earth Structures, NAVFAC, Design Manual 7.2
- Retaining and Flood Walls, Technical Engineering and Design Guides As Adapted from The US Army Corps Of Engineers, No. 4, ASCE
- Standard Specifications for Highway Bridges, AASHTO

In the following sections, we will first discuss basic considerations necessary for calculating lateral earth pressure and then how to apply these pressures in developing the force. We will illustrate how the lateral forces are combined with vertical forces to calculate the factor of safety with respect to sliding, overturning and bearing capacity. These three components are important elements in retaining wall design. Structural design of a retaining wall is beyond the scope of this course.

Content Section 2

Categories of Lateral Earth Pressure

There are three categories of lateral earth pressure and each depends upon the movement experienced by the vertical wall on which the pressure is acting as shown in Figure 2 (Page 4). In this course, we will use the word wall to mean the vertical plane on which the earth pressure is acting. The wall could be a basement wall, retaining wall, earth support system such as sheet piling or soldier pile and lagging etc.

The three categories are:

- At rest earth pressure
- Active earth pressure
- Passive earth pressure

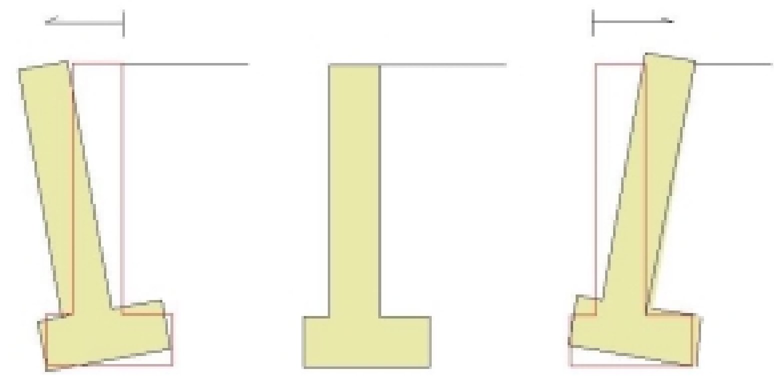
The at rest pressure develops when the wall experiences no lateral movement. This typically occurs when the wall is restrained from movement such as along a basement wall that is restrained at the bottom by a slab and at the top by a floor framing system prior to placing soil backfill against the wall.

The active pressure develops when the wall is free to move outward such as a typical retaining wall and the soil mass stretches sufficiently to mobilize its shear strength.

On the other hand, if the wall moves into the soil, then the soil mass is compressed, which also mobilizes its shear strength and the passive pressure develops. This situation might occur along the section of wall that is below grade and on the opposite side of the retained section of fill. Some engineers might use the passive pressure that develops along this buried face as additional restraint to lateral movement, but often it is ignored.

In order to develop the full active pressure or the full passive pressure, the wall must move. If the wall does not move a sufficient amount, then the full active or full passive pressure will not develop. If the full active pressure does not develop, then the pressure will be higher than the expected active pressure. Likewise, significant movement is necessary to mobilize the full passive pressure.

How movement affects development of the active and passive earth pressure is illustrated in Figure 3 shown on Page 4. Note that the "at rest" condition is shown where the wall rotation is equal to 0, which is the condition of zero lateral strain.



Active Case

(Wall moves away from soil)

At Rest Case

(No movement)

Passive Case

(Wall moves into soil)

Figure 2 - Wall Movement

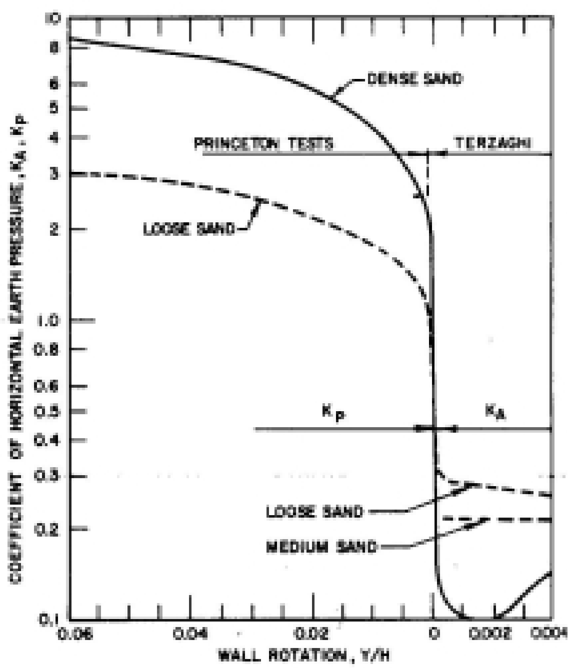


Figure 3 - Effect of Wall Movement on Wall Pressure **[Ref: NAVFAC DM-7]**

From Figure 3 it is evident that:

- As the wall moves away from the soil backfill (left side of Figure 2), the active condition develops and the lateral pressure against the wall decreases with wall movement until the minimum active earth pressure force (Pa) is reached.

- As the wall moves towards (into) the soil backfill (right side of Figure 2), the passive condition develops and the lateral pressure against the wall increases with wall movement until the maximum passive earth pressure (Pp) is reached.

Thus the intensity of the active / passive horizontal pressure, which is a function of the applicable earth pressure coefficient, depends upon the degree of wall movement since movement controls the degree of shear strength mobilized in the surrounding soil.

Calculating Lateral Earth Pressure Coefficients

Lateral earth pressure is related to the vertical earth pressure by a coefficient termed the:

- At Rest Earth Pressure Coefficient (Ko)
- Active Earth Pressure Coefficient (Ka)
- Passive Earth Pressure Coefficient (Kp)

The lateral earth pressure is equal to vertical earth pressure times the appropriate earth pressure coefficient. There are published relationships, tables and charts for calculating or selecting the appropriate earth pressure coefficient.

Since soil backfill is typically granular material such as sand, silty sand, sand with gravel, this course assumes that the backfill material against the wall is coarse-grained, non-cohesive material. Thus, cohesive soil such as clay is not discussed. However, there are many textbooks and other publications where this topic is fully discussed.

At Rest Coefficient

Depending upon whether the soil is loose sand, dense sand, normally consolidated clay or over consolidated clay, there are published relationships that depend upon the soil's engineering values for calculating the at rest earth pressure coefficient. One common earth pressure coefficient for the "at rest" condition in granular soil is:

$$Ko = 1 - \sin(\phi) \qquad (1.0)$$

Where: Ko is the "at rest" earth pressure coefficient and ϕ is the soil friction value.

Active and Passive Earth Pressure Coefficients

When discussing active and passive lateral earth pressure, there are two relatively simple classical theories (among others) that are widely used:

- Rankine Earth Pressure Theory
- Coulomb Earth Pressure Theory

The Rankine Theory assumes:

- There is no adhesion or friction between the wall and soil
- Lateral pressure is limited to vertical walls
- Failure (in the backfill) occurs as a sliding wedge along an assumed failure plane defined by ϕ.
- Lateral pressure varies linearly with depth and the resultant pressure is located one-third of the height (H) above the base of the wall.
- The resultant force is parallel to the backfill surface.

The Coulomb Theory is similar to Rankine except that:

- There is friction between the wall and soil and takes this into account by using a soil-wall friction angle of δ. Note that δ ranges from $\phi/2$ to $2\phi/3$ and $\delta = 2\phi/3$ is commonly used.
- Lateral pressure is not limited to vertical walls
- The resultant force is not necessarily parallel to the backfill surface because of the soil-wall friction value δ.

The general cases for calculating the earth pressure coefficients can also be found in published expressions, tables and charts for the various conditions such as wall friction and sloping backfill. The reader should obtain these coefficients from published sources for conditions other than those discussed herein.

The Rankine active and passive earth pressure coefficient for the specific condition of a horizontal backfill surface is calculated as follows:

- (Active) $Ka = (1 - \sin(\phi)) / (1 + \sin(\phi))$ (2.0)
- (Passive) $Kp = (1 + \sin(\phi)) / (1 - \sin(\phi))$ (3.0)

Some tabulated values base on Expressions (2.0) and (3.0) are shown in Table 1.

Table 1 - Rankine Earth Pressure Coefficients

ϕ (deg)	Rankine Ka	Rankine Kp
28	.361	2.77
30	.333	3.00
32	.307	3.26

The Coulomb active and passive earth pressure coefficient is derived from a more complicated expression that depends on the angle of the back of the wall, the soil-wall friction value and the angle of backfill. Although this expression is not shown, these values are readily obtained in textbook tables or by programmed computers and calculators. Table 2 and Table 3 show some examples of the Coulomb active and passive earth pressure coefficient for the specific case of a vertical back of wall angle and horizontal backfill surface. The Tables illustrate increasing soil-wall friction angles (δ).

Table 2 - Coulomb Active Pressure Coefficient

	δ (deg)				
ϕ (deg)	0	5	10	15	20
28	.3610	.3448	.3330	.3251	.3203
30	.3333	.3189	.3085	.3014	.2973
32	.3073	.2945	.2853	.2791	.2755

Table 3 - Coulomb Passive Pressure Coefficient

	δ (deg)				
ϕ (deg)	0	5	10	15	20
30	3.000	3.506	4.143	4.977	6.105
35	3.690	4.390	5.310	6.854	8.324

Some points to consider are:

- For the Coulomb case shown above with no soil-wall friction (i.e. δ = 0) and a horizontal backfill surface, both the Coulomb and Rankine methods yield equal results.

- As the soil friction angle (ϕ) increases (i.e. soil becomes stronger), the active pressure coefficient decreases, resulting in a decrease in the active force while the passive pressure coefficient increases, resulting in an increase in the passive force.

Calculating the Vertical Effective Overburden Pressure

The vertical effective overburden pressure is the effective weight of soil above the point under consideration. The term "effective" means that the submerged unit weight of soil is used when calculating the pressure below the groundwater level. For instance, assume that a soil has a total unit weight (γ) of 120 pcf and the groundwater level is 5 feet below the ground surface. The vertical effective overburden pressure ($\sigma v'$) at a depth of 10 feet below the ground surface (i.e. 5 feet below the groundwater depth) is:

$$\sigma v' = 5(\gamma) + 5(\gamma')$$

Where γ is the total unit weight of the soil and γ' is the effective (or submerged) unit weight of the soil which equals the total unit weight of soil minus the unit weight of water (i.e. 62.4 pcf). Thus:

$$\sigma v' = 5(120) + 5(120\text{-}62.4) = 888 \text{ psf}$$

Calculating the Lateral Earth Pressure

There is a relationship between the vertical effective overburden pressure and the lateral earth pressure. The lateral earth pressure (σ) at a point below ground surface is:

- $\sigma a = Ka\ (\sigma v')$ Active lateral earth pressure (4.0)
- $\sigma p = Kp\ (\sigma v')$ Passive lateral earth pressure (5.0)

Where ($\sigma v'$) is the vertical effective overburden pressure. The symbols σa and σp denote active and passive earth pressure respectively.

If water pressure is allowed to accumulate behind a retaining wall, then the total pressure and the resulting total force along the back of the wall is increased considerably. Therefore, it is common for walls to be designed with adequate drainage to prevent water from accumulating behind the wall and introducing a separate water pressure force. Thus, weepholes, lateral drains or blanket drains along with granular soil (freely draining backfill) are commonly used behind retaining walls. In the case of a drained condition, the total unit weight of soil (γ) is used behind the full height of the wall and there is no contribution from hydrostatic water pressure.

An example of an earth pressure calculation using the Rankine active earth pressure coefficient is shown later in Example (1.0). A similar calculation can be performed for the Coulomb case by using the applicable Coulomb earth pressure coefficient.

Calculating the Total Lateral Earth Pressure Force

The total lateral earth pressure force is the area of the pressure diagram along the wall. In the example shown later in this course, the area of the earth pressure diagram is the lateral earth pressure at the bottom of the wall $Ka\gamma H$ (note that γH is the vertical effective overburden pressure in this example) times the height of the wall (H) times one-half (1/2) since the pressure distribution increases linearly with depth creating a triangular shape. Thus the total active earth pressure force (Pa) acting along the back of the wall is the area of the pressure diagram expressed as:

- $Pa = ½\ Ka\ \gamma\ H^2$ (6.1)

The total passive earth pressure force is:

- $Pp = ½\ Kp\ \gamma\ H^2$ (6.2)

The total force acts along the back of the wall at a height of H/3 from the base of the wall.

In more complicated cases, the earth pressure distribution diagram is drawn and the total force is calculated by determining the separate areas of the pressure diagram. The location of the resultant force is also determined. The direction of the force is based upon the angle of backfill and in the Coulomb case, it is also based upon the soil-wall friction value.

Other Forces Acting on the Wall

Aside from the earth pressure force acting on the wall, other forces might also act on the wall and these are superimposed onto the earth pressure force. For example, these forces might include:

- Surcharge load
- Earthquake load
- Water Pressure

Surcharge Load

A surcharge load results from forces that are applied along the surface of the backfill behind the wall. These forces apply an additional lateral force along the back of the wall. Surcharge pressures result from loads such as a line load, strip load, embankment load, traffic (such as a parking lot), floor loads and temporary loads such as construction traffic and stockpiles of material. Generally, elastic theory is used to determine the lateral pressure due to the surcharge and solutions are available in published references.

In the case of a uniform surcharge pressure (q) taken over a wide area behind the wall, the lateral pressure due to the uniform surcharge:

- K_0q (7.0)

Where K_0 is the applicable at rest, active or passive pressure coefficient. The pressure diagram behind the wall for a uniform surcharge is rectangular and acts at a height of H/2 above the base of the wall. Thus, the additional lateral force (Ps) acting behind the wall resulting from a uniform surcharge is the area of the rectangle, or:

- $Ps = K_0qH$ (8.0)

Whether the total surcharge force is calculated from elastic theory or as shown in Expression (8.0), the pressure (force) is superimposed onto the calculated lateral earth pressure (force).

Earthquake Force

Additional lateral loads resulting from an earthquake are also superimposed onto the lateral earth pressure where required. Publications such as AASHTO Standard Specifications for Highway Bridges and other textbooks provide methods for calculating the earthquake force.

Water Pressure

Walls are typically designed to prevent hydrostatic pressure from developing behind the wall. Therefore the loads applied to most walls will not include water pressure. In cases where hydrostatic water pressure might develop behind an undrained wall, the additional force resulting from the water pressure must be superimposed onto the lateral earth pressure. Since water pressure is equal in all directions (i.e. coefficient (K) = 1), the water pressure distribution increases linearly with depth at a rate of $\gamma_w z$ where γ_w is the unit weight of water (62.4 pcf) and z is the depth below the groundwater level. If the surface of water behind a 10-foot high wall (H) were located 5-feet (d) below the backfill surface, then the superimposed total lateral force resulting from groundwater pressure would be:

- $W = ½ (\gamma_w)(H\text{-}d)^2 = 780$ pounds (which is the area of the linearly increasing pressure distribution).

W acts at a height of (H-d)/3 (or 1.67-ft) above the base of the wall.

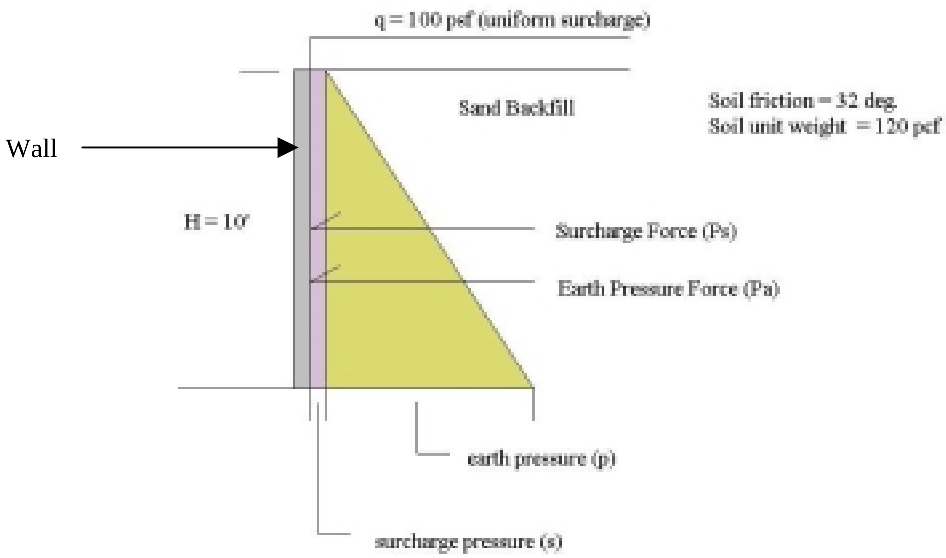
If seepage occurs, then the water pressure must be derived from seepage analysis, which is beyond the scope of this course.

Compaction

If heavy rollers are used to compact soil adjacent to walls, then high residual pressures can develop against the wall increasing the total pressure. Although a reasonable amount of backfill compaction is necessary, excess compaction must be avoided.

Example (1.0)

Use the Rankine method to calculate the **total active lateral force** and location of the forces behind a 10-foot high vertical wall. Assume that the soil has a total unit weight of 120 pcf and a friction value of 32 degrees. Assume that there is a uniform surcharge of 100 psf located along the surface behind the wall. Groundwater is well below the depth of the foundation so that groundwater pressure does not develop behind the wall.



Ka = 1 – sin (32) / 1 + sin (32) = 0.307 is the Active Earth pressure Coefficient

At bottom of wall (surcharge pressure) s = Ka (q) = 0.307(100) = 30.7 psf

At bottom of wall (active lateral earth pressure) p_a = Ka (γ) H = 0.307(120)(10) = 368.4 psf

Total Surcharge Force: Ps = Ka(q)H = 307 pounds and acts at a height of H/2 from the base of the wall.

Total Earth Pressure Force: Pa = ½ Ka (γ) H^2 = ½ (0.307) (120) $(10)^2$ = 1842 pounds and act at a height of H/3 from the base of the wall.

Total Active Force = 1842 + 307 = **2149 pounds**

Content Section 3

Earth Pressure Force

Although there are three categories of lateral earth pressure as discussed in Section 2, this section discusses the active earth pressure because it is the active pressure that produces the destabilizing earth force behind retaining walls. Although passive pressures might develop along the toe of the wall and provide resistance, it is commonly ignored and therefore it is not discussed in this section. Other lateral forces such as those resulting from surcharge, earthquake, etc., also produce additional destabilizing forces. The reader is advised to review the concepts developed in Section 2 before continuing in this Section.

Since soil backfill is typically granular material such as sand, silty sand, sand with gravel, this course assumes that the backfill material against the wall is coarse-grained, non-cohesive material. For this reason, cohesive soil such as clay is not discussed.

It is important to note that the full active earth pressure condition will only develop if the wall is allowed to move a sufficient distance. The lateral outward movement required to develop the full active pressure condition ranges from:

- Granular soil: 0.001H to 0.004H
- Cohesive soil: 0.01H to 0.04H

Where H is the height of the wall.

The Rankine active lateral earth pressure in the following discussions will be developed using Expression (2.0), which is for the specific case of a horizontal backfill surface. The expression is modified for sloping backfill surfaces and the reader can find these modified expressions in published references.

The Coulomb active earth pressure coefficient is a more complicated expression that depends on the angle of the back of the wall, the soil-wall friction and the angle of backfill slope. Although the expression is not shown, these values are readily obtained in textbook tables or by programmed computers and calculators. Table 4 shows an example of the Coulomb active earth pressure coefficient for the specific case of a wall with a back of wall angle (β) of 80 degrees and a horizontal backfill surface. In this Table, the soil-wall friction value (δ) has been taken as $(2/3)\phi$.

Table 4 - Coulomb Active Pressure Coefficient
[Note: $\delta = (2/3)\phi$]

ϕ (deg)	β = 80 deg
28	.4007
30	.3769
32	.3545

Calculating the Total Active Earth Pressure Force

The total lateral force is the area of the pressure diagram acting on the wall surface. The examples in this course assume drained conditions and a homogeneous granular soil backfill behind the wall, which results in a simple triangular distribution. Although this is a common case, the pressure diagram can become more complicated depending upon actual soil conditions that might have different values.

With the Coulomb method, the active force acts directly on the wall and friction develops between the soil and wall. With the Rankine method however, wall friction is ignored and the active force acts directly on a vertical face extending through the heel of the wall. If the back of the wall were vertical, then the force acts on the wall. On the other hand, if the back of the wall were sloping, then the force acts on the vertical soil plane as illustrated in Figure 4.

In the Example (2.0), the area of the earth pressure diagram is the earth pressure at the bottom of the wall ($K_a \gamma H$) times the height of the wall (H) times one-half (1/2) since the pressure distribution increases linearly with depth creating a triangular shape. Thus the total active earth pressure force (Pa) acting along the back of the wall is the area of the pressure diagram expressed as:

- $Pa = ½\ Ka\ \gamma\ H^2$ (6.1)

The total force acts along the back of the wall at a height of H/3 from the base of the wall. So far we have not stated whether this is the Rankine or Coulomb Case. The calculation for the active earth pressure force (Pa) is the same provided that the appropriate earth pressure coefficient (Ka) is used. Selecting whether the Rankine method or Coulomb method will be used is usually a matter of choice or convention.

The example shown in Figure 4 relates specifically to a wall supporting a horizontal backfill. Thus the active earth pressure coefficient (Ka) can be derived directly from Expression (2.0) or Table 2. For the case of a sloping backfill and other wall geometries, the reader should refer to the published references.

The example shown in Figure 4 (Page 15) assumes that a 9-foot high gravity type retaining structure supports soil backfill having a total unit weight of 125 pcf.

Groundwater is well below the structure and the backfill material is freely draining. The backfill soil has an angle of internal friction (ϕ) of 32 degrees and the backfill surface behind the wall is horizontal. Both the Rankine and Coulomb earth pressure force is shown.

Note that the location and direction of the active forces follows the assumptions stated above for the Rankine and Coulomb Theory. Although the back of the wall has an angle of 80 degrees, The Rankine force acts along a vertical plane beginning at the heel of the wall while the Coulomb force acts directly along the back of the wall. Since the Rankine Theory assumes that there is no soil – wall friction, the force (Pa) is parallel to the backfill surface. On the other hand, since the Coulomb Theory takes the soil – wall friction into consideration, the force (Pa) acts at an angle of δ from the perpendicular to the wall. This results in both a vertical and horizontal component of the force (Pa), which is expressed as Pa_h and Pa_v. The Rankine method will also produce a vertical and horizontal component of the force (Pa) if the backfill surface is sloped.

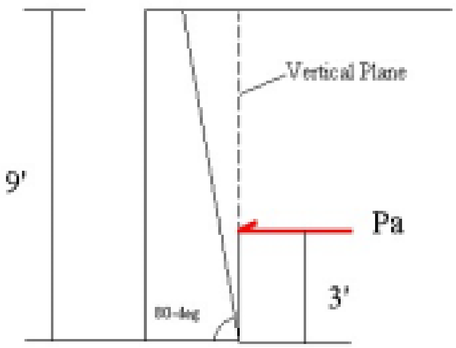
In each case, the resultant force Pa acts at a height of H/3 from the base of the wall where H is the height of the wall for the simple case illustrated herein. If the pressure diagram were more complicated due to differing soil conditions for instance, then the location of the force (Pa) will change. In all cases however, the resultant of the force (Pa) is located at the centroid of the combined mass area.

Other Forces Acting on the Wall

Aside from the earth pressure force acting on wall, other forces might also act on the wall as discussed in Section 2 such as.

- Surcharge load
- Earthquake load
- Water Pressure

These additional forces would be superimposed onto the earth pressure force to yield the total lateral force in a similar way as shown of the uniform surcharge in Example (1.0).

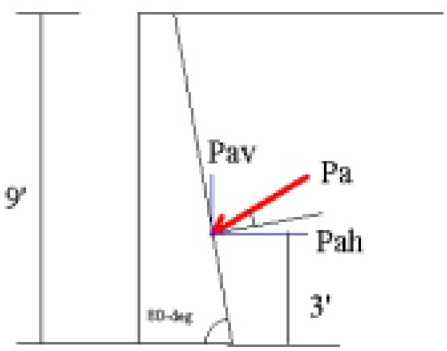


Rankine Active Earth Pressure Force

$Ka = (1 - \sin(\phi)) / (1 + \sin(\phi)) = 0.307$

$Pa = ½ \; Ka \, \gamma \, H^2 = (0.5)(0.307)(125)(9^2)$

Pa = 1554.2 pounds



Coulomb Active Earth Pressure Force

Ka = 0.3545 from Table 1 for conditions stated

$Pa = ½ \; Ka \, \gamma \, H^2 = (0.5)(0.354)(125)(9^2)$

Pa = 1792.1 pounds

Calculate horizontal and vertical components of Pa where Pa acts 31.3 deg from the horizontal.

Pa_h = Pa cos (31.3) = 1531.3 pounds

Pa_v = Pa sin (31.3) = 931.0 pounds

Figure 4 - Calculation of Earth Pressure Force for a Homogeneous Cohesionless Backfill

Factors of Safety

Retaining wall design is an iterative process. An initial geometry is usually assigned to the wall and the appropriate forces are calculated. The actual forces are then checked using acceptable factors of safety and the geometry is revised until satisfactory factors of safety are reached. There are however, common dimensions for walls that are available that can be used as a first cut.

Proportioning Walls

In order to achieve stability, retaining walls are usually proportioned so that the width of the base (B) is equal to approximately 0.5 to 0.7 times the height of the wall (H). This is illustrated in Example (2.0). Thus, the 9-foot high wall would have a base approximately 4.5 feet to 6.3 feet wide, which provides a convenient starting point.

Sliding

A retaining structure has a tendency to move away from the backfill surface because of the horizontal driving forces resulting from the soil backfill and other forces such as surcharge. Generally, the wall resists sliding by the frictional resistance developed between the foundation of the wall and foundation soil. Although other horizontal forces act opposite to the driving force such as passive soil pressure in the fill in front of the wall, it is often ignored.

The factor of safety with respect to sliding equals the resisting force divided by the driving force and is shown in Expression (9.0). A minimum factor of safety of 1.5 is desirable to resist sliding assuming that passive resistance from any fill in front of the wall is ignored. This is a common assumption and avoids relying on the presence of soil in front of the wall for additional resistance.

$$FS_s = \Sigma V \tan(k\phi_1) / Pa_h \qquad (9.0)$$

ΣV is the total vertical force, Pa_h is the horizontal active earth pressure force and $\tan(k\phi_1)$ is the coefficient of friction between the base of the wall and the soil. The factor "k" ranges from ½ to 2/3 and ϕ_1 is the friction angle of the foundation soil. Friction factors between dissimilar materials can also be found in publications such as NAVFAC Design Manual 7.2. Expression (9.0) assumes that the soil below the wall is a cohesionless material such as sand without any cohesive strength. Therefore there is no additional resistance due to cohesion.

Overturning

A retaining structure also has a tendency to rotate outward around the toe of the wall. The moment resulting from the earth pressure force (as well as other lateral forces such as surcharge) must be resisted by the moments resulting from the vertical forces produced by the wall including any vertical component (Pa_v) of the earth pressure force. Thus, the factor of safety with respect to overturning is the resisting moment divided by the overturning moment as shown in Expression (10.0). A minimum factor of safety of 2 to 3 is desirable to resist overturning.

$$FS_o = \Sigma Mr / \Sigma Mo \qquad (10.0)$$

Where ΣMr is the sum of the resisting moments around the toe of the wall and ΣMo is the sum of the overturning moments around the toe of the wall.

Bearing Capacity

As with any structure, the bearing capacity of the soil must be adequate to safely support the structure. The ultimate bearing capacity of the foundation soil (q_u) is calculated using theoretical bearing capacity methods presented in textbooks and other published resources. The calculation of bearing capacity is not presented in this course.

The resultant of all forces acting along the base of the wall from earth pressure and the weight of the wall result in a non-uniform pressure below the base of the wall with the greatest pressure below the toe of the base and the least pressure below the heel of the base.

The maximum and minimum pressure below the base of the wall (B) is:

$$q_{max} = (\Sigma V / B) (1 + 6e / B) \quad (11.0)$$

$$q_{min} = (\Sigma V / B) (1 - 6e / B) \quad (12.0)$$

Where e = eccentricity; $e = (B / 2) - (\Sigma Mr - \Sigma Mo) / \Sigma V$ (13.0)

The factor of safety with respect to bearing capacity is shown in Expression (14.0). Generally, a factor of safety of 3 is required.

$$FS_{bc} = q_u / q_{max} \quad (14.0)$$

Eccentricity is an important consideration when proportioning the walls. Consider the eccentricity (e) in relationship to the minimum pressure (q_{min}). Substituting for (e) in Expression (12.0):

$$\text{If } e = B / 6 \text{ then } q_{min} = (\Sigma V / B) (1 - 6e / B) = 0 \quad (15.0)$$

$$\text{If } e < B / 6 \text{ then } q_{min} = (\Sigma V / B) (1 - 6e / B) > 0 \quad (16.0)$$

$$\text{If } e > B / 6 \text{ then } q_{min} = (\Sigma V / B) (1 - 6e / B) < 0 \quad (17.0)$$

Expressions (15.0) and (16.0) give acceptable results since the pressure at the heel is zero or greater (positive). Thus the entire base lies in contact with the soil. If Expression (17.0) were true, then the pressure at the heel is negative indicating the heel of the base is tending toward lifting off the soil, which is unacceptable. If this condition occurs, then the wall must be re-proportioned.

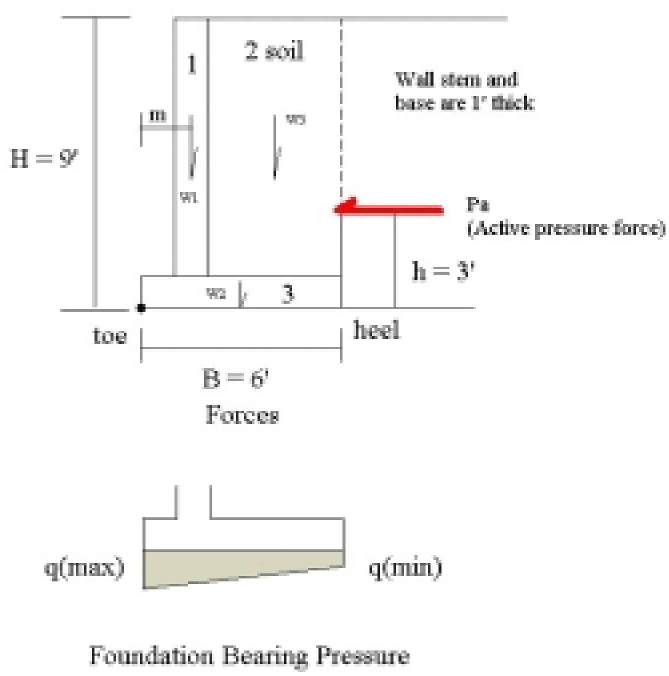
Other Considerations

Before a wall design is complete, the settlement of the wall and the global stability of the entire mass on which the wall is supported must be checked. Settlement must lie within tolerable ranges and global stability such as slope stability calculations must be adequate. These calculations however, are beyond the scope of this course.

Example (2.0)

Using the Rankine method of analysis, calculate the factors of safety with respect to sliding, overturning and bearing capacity. Use the values presented in the following Table and refer to Figure 5. It is inferred that all calculations relate to a unit length of wall.

Friction angle of soil backfill (ϕ)	32 degrees
Soil Backfill Unit Weight (γ)	125 pcf
Friction angle of the foundation soil (ϕ_1)	33 degrees
Rankine active pressure coefficient (Ka)	0.307
Concrete Unit Weight (γ_c)	150 pcf
Dimensions of the concrete wall section 1	1-ft by 8-ft
Dimensions of the soil backfill section 2	4-ft by 8-ft
Dimensions of the concrete wall section 3	6-ft by 1-ft



Note:

Since Pa is horizontal, there is no vertical component of the force. If the backfill surface were sloping then Pa would slope at an angle parallel to the backfill slope. In this case there would be both a vertical and horizontal component of Pa. The lateral thrust would be the horizontal component and the vertical component would be an additional vertical force included in ΣV.

Figure 5

Solution

Calculate the values shown in the following Table. The dimensions for "Area" relate to each of the three sections identified in Figure 3. The unit weight (γ) is provided for the concrete wall and soil backfill over the base of the wall. W is the weight of each section and it acts at the centroid of the mass area as shown in Figure 3. The value "m" is the moment arm measured from the toe to the location of the individual W values. M is the resisting moment for each of the individual areas.

Section	Area (sf)	γ (pcf)	W (lbs)	m (ft)	M (ft-lb)
1	1 x 8	150	1200	1.5	1800
2	4 x 8	125	4000	4	16000
3	6 x 1	150	900	3	2700
			ΣV = **6100**		ΣMr = **20500**

$Pa = ½\ Ka\ \gamma\ H^2$ = (0.5) (0.307) (125) (81) = **1554.2 lbs**

$\Sigma Mo = Pa\ (h)$ = (1554.2) (3) = **4662.6 ft-lbs**

Overturning: $Fs_o = \Sigma Mr / \Sigma Mo$ = 20500 / 4662.6 = **4.4** > 2 **OK**

Sliding: $FS_s = \Sigma V \tan(k\phi_1) / Pa_h$ = (6100) tan (22) / 1554.2 = **1.58** > 1.5 **OK**
Where k = 2/3

Bearing Capacity:

Assume that the ultimate bearing capacity of the foundation soil is 5000 psf.

$e = (B / 2) - (\Sigma Mr - \Sigma Mo) / \Sigma V$ = (6 / 2) – (20500 – 4662.6) / 6100 = **0.4** (i.e. e < B / 6)

$q_{max} = (\Sigma V / B) (1 + 6e / B)$ = (6100 / 6) (1 + 2.4 / 6) = (1016.6) (1.4) = **1423.4 psf**

$q_{min} = (\Sigma V / B) (1 - 6e / B)$ = (6100 / 6) (1 - 2.4 / 6) = (1016.6) (.6) = **610 psf** (i.e. base of wall is in full soil contact)

$FS_{bc} = q_u / q_{max}$ = 5000 / 1423.4 = **3.5** > 3.0 **OK**

Disclaimer

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