Hydraulics Engineering
Lec #1 : Specific Energy and Critical Depth

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Books

- Fluid Mechanics with Engineering applications
  - By: Roberts L. Duagherty, Joseph B. Franzini, E. John Finnemore
- Open Channel Flow
  - By: Ven te Chow
- Civil Engineering Hydraulics
  - By: R.E. Featherstone & C. Nalluri

Steady Flow in Open Channels

- Specific Energy and Critical Depth
- Surface Profiles and Backwater Curves in Channels of Uniform sections
- Hydraulics jump and its practical applications.
- Flow over Humps and through Constrictions
- Broad Crested Weirs and Venturi Flumes

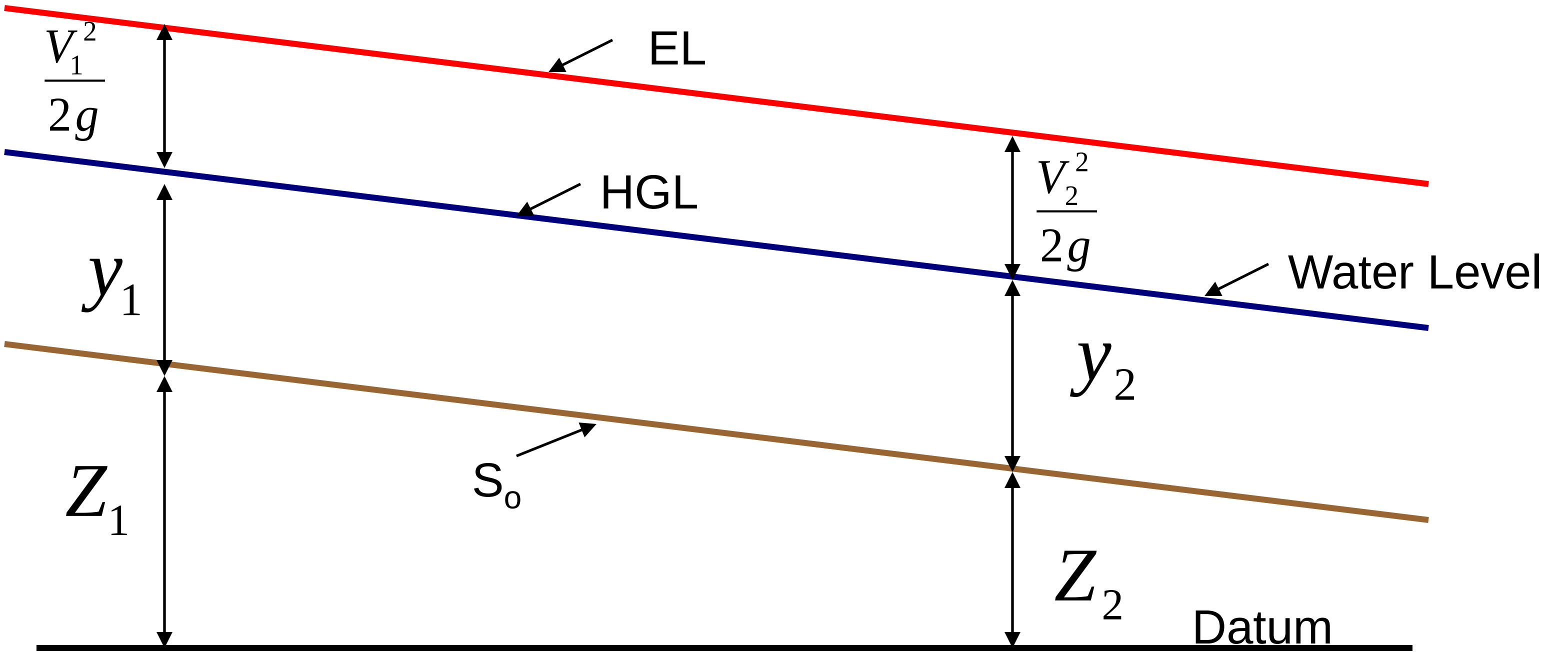
Specific Energy and Critical Depth
Basic Definitions

- Head
  - Energy per unit weight
- Energy Line
  - Line joining the total head at different positions.
- Hydraulics Grade Line
  - Line joining the pressure head at different positions.

Specific Energy and Critical Depth

Basic Definitions

- Open Channel Flow

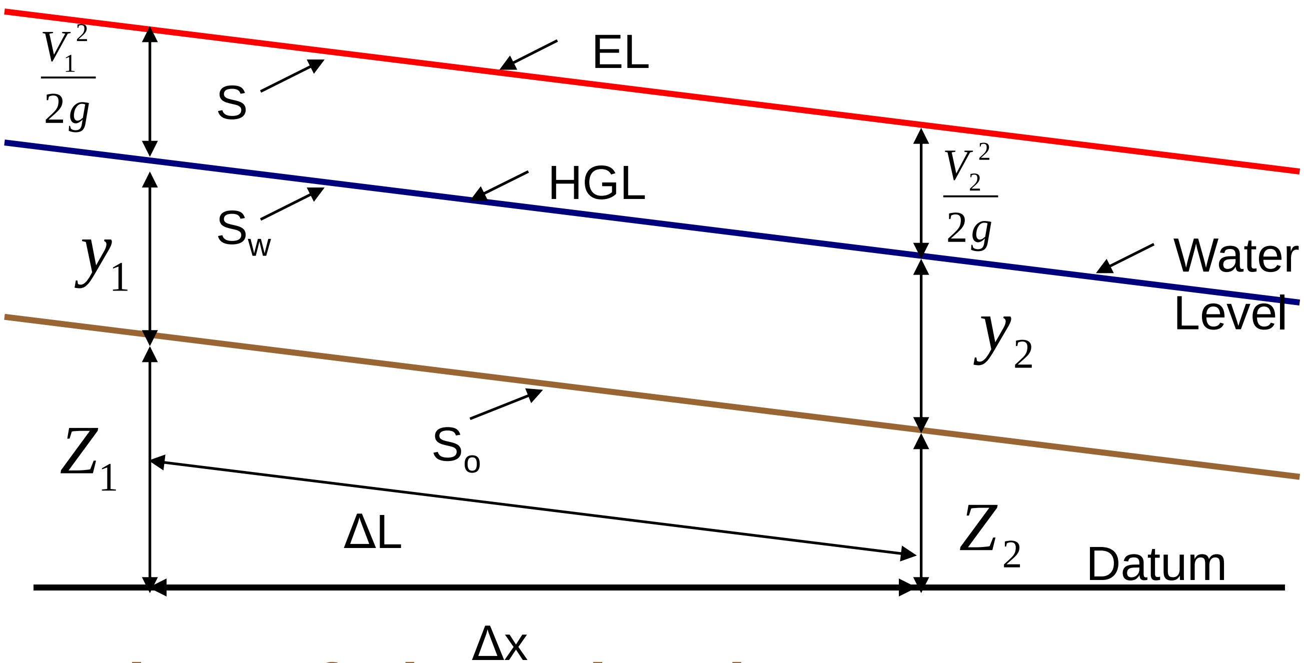


$$Z_1 + y_1 + \frac{V_1^2}{2g} = Z_2 + y_2 + \frac{V_2^2}{2g} + h_l$$

Specific Energy and Critical Depth

Basic Definitions

- Slopes in Open Channel Flow



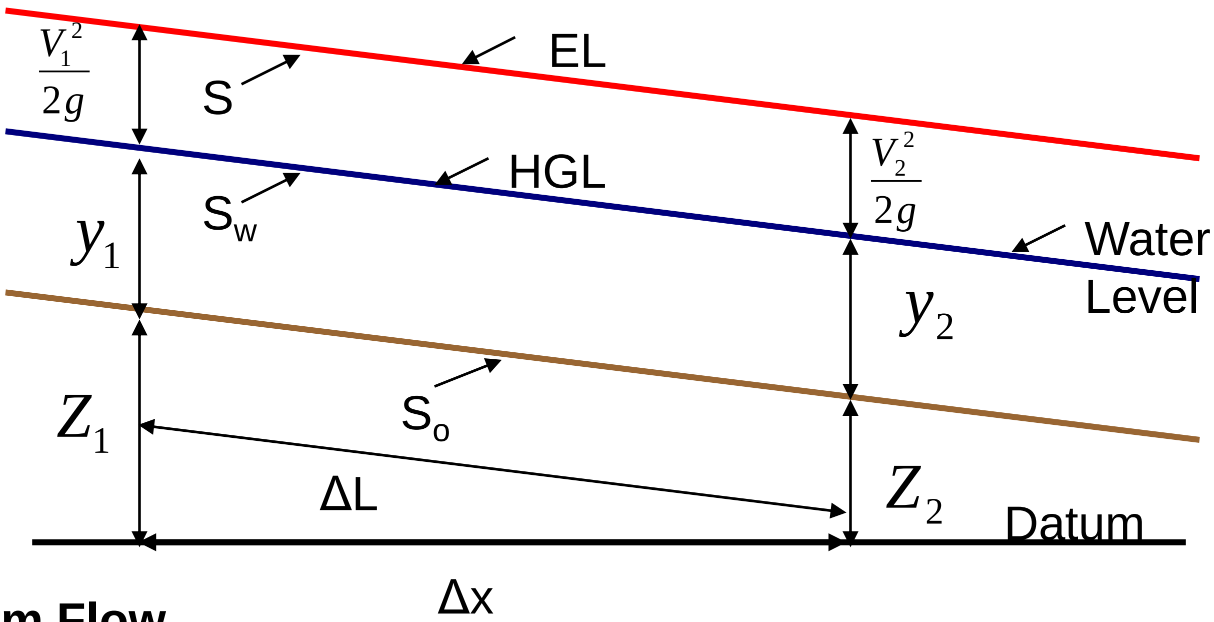
- So= **Slope of Channel Bed** $= (Z_1-Z_2)/(\Delta x) = -\Delta Z/\Delta x$
- S_w= **Slope of Water Surface** $= [(Z_1+y_1)-(Z_2+y_2)]/\Delta x$
- S= **Slope of Energy Line** $= [(Z_1+y_1+V_1^2/2g)-(Z_2+y_2+V_2^2/2g)]/\Delta x$

  $= hl/\Delta L$

Specific Energy and Critical Depth

Basic Definitions

- Slopes in Open Channel Flow



For Uniform Flow

$y_1 = y_2$ **and** $V_1^2/2g = V_2^2/2g$

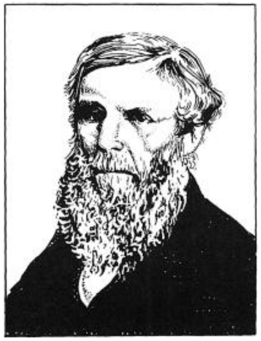
Hence the line indicating the bed of the channel, water surface profile and energy line are parallel to each other.

For θ being very small (say less than 5 degree) i.e Δx=ΔL

So=Sw=S

Specific Energy and Critical Depth

Basic Definitions



- Froude's Number (F_N)
  - It is the ratio of inertial forces to gravitational forces.
  - For a rectangular channel it may be written as

$$F_N = \frac{V}{\sqrt{gy}}$$

  - F_N= 1 Critical Flow

    > 1 Super-Critical Flow

    < 1 Sub-Critical Flow

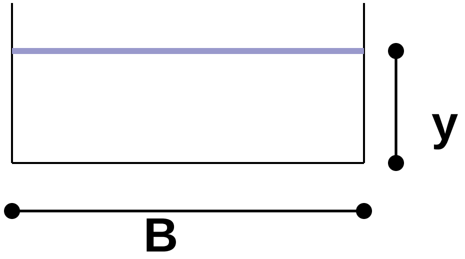
William Froude (1810-79)

Born in England and engaged in shipbuilding. In his sixties started the study of ship resistance, building a boat testing pool (approximately *75* m long) near his home. After his death, this study was continued by his son, Robert Edmund Froude (1846-1924). For similarity under conditions of inertial and gravitational forces, the non-dimensional number used carries his name.

Specific Energy and Critical Depth (Rectangular Channels)

- Specific Energy
  - Specific Energy at a section in an open channel is the energy with reference to the bed of the channel. Mathematically;

    Specific Energy = E = y+V²/2g



For a rectangular Channel

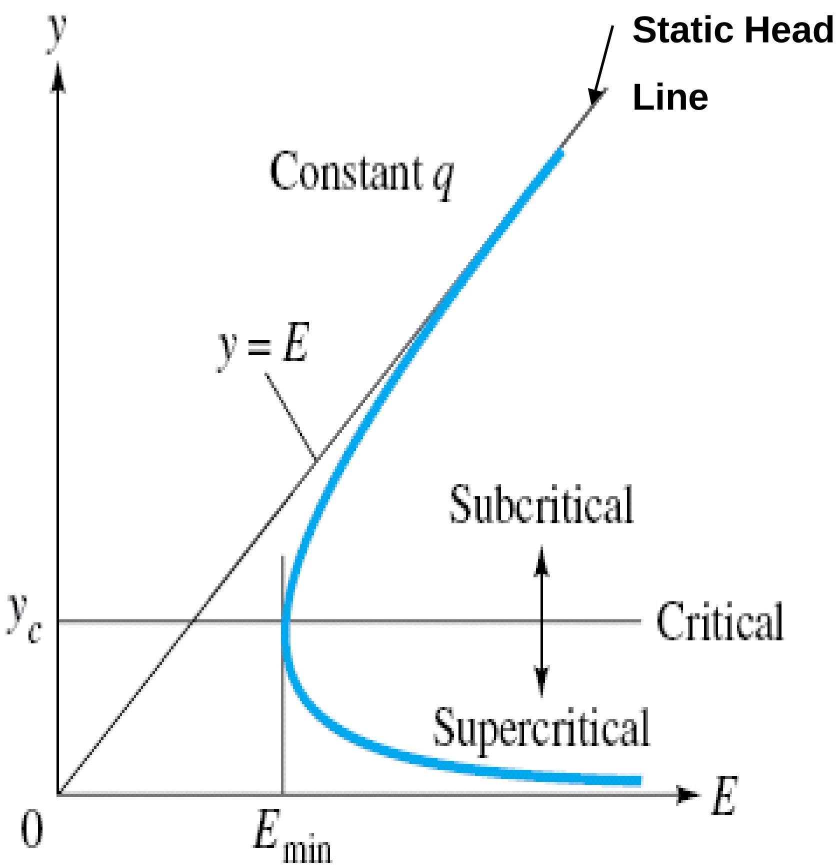
$$E = y + \frac{V^2}{2g}$$

$$E = y + \frac{q^2}{2g\,y^2} \quad where \quad q = Q/B, q = yb$$

q= Discharge per unit width m³/sec per m

Specific Energy and Critical Depth

- E~y Diagram or E-Diagram



- As it is clear from E~y diagram drawn for constant discharge for any given value of E, there would be two possible depths, say y_1 and y_2. These two depths are called Alternate depths.
- However for point C corresponding to minimum specific energy E_{min}, there would be only one possible depth yc. The depth y_c is know as critical depth.
- The critical Depth may be defined as *depth corresponding to minimum specific energy discharge remaining Constant.*

Specific Energy and Critical Depth

- For $y>y_c$, $V<V_c$ Deep Channel
  - Sub-Critical Flow, Tranquil Flow, Slow Flow.
- For $y<y_c$, $V>V_c$ Shallow Channel
  - Super-Critical Flow, Shooting Flow, Rapid Flow and Fast Flow.

Specific Energy and Critical Depth

Relationship Between Critical Depth and Specific Energy

$$E = y + \frac{q2}{2gy^2} \qquad (1)$$

$$\frac{dE}{dy} = 1 - \frac{2q2}{2gy3}$$

$$\frac{dE}{dy} = 1 - \frac{q2}{gy3} = 0$$

$$y^3 = \left(\frac{q2}{g}\right)$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad (2)$$

$$y_c = \left(\frac{V_c y_c}{g}\right)^{1/3} \qquad q = V_c y_c$$

$$\frac{Vc^2}{2g} = \frac{y_c}{2} \qquad (3)$$

$$\frac{V_c}{\sqrt{gy_c}} = 1 \qquad (4)$$

Substituting $\frac{V_c}{2g} = \frac{y_c}{2}$ in eq. (1)

$$E_{\min} = Ec = y_c + \frac{y_c}{2}$$

$$E_c = \frac{3}{2} y_c \qquad (5)$$

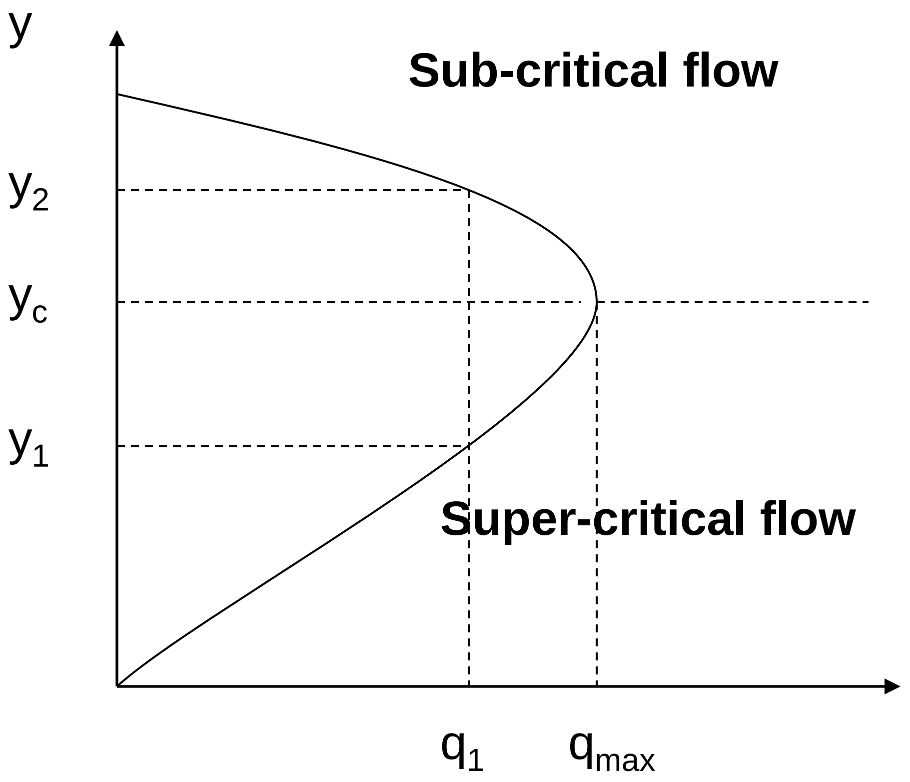
Specific Energy and Critical Depth

- Since the equation (1) can be written as

$$q = y\sqrt{2g(E-y)}$$

- Therefore, **Critical Depth** may also be defined as *the depth corresponding to maximum discharge specific energy remaining constant.*

- **Discharge~Depth Diagram**



Specific Energy and Critical Depth

- *Relationship Between Critical Depth and Specific Energy*

$$\frac{dq}{dy} = \sqrt{2g}\left[\frac{-y}{2\sqrt{(E-y)}} + \sqrt{(E-y)}\right]$$

$$\frac{dq}{dy} = 0 \quad for \quad q_{\max}$$

$$\frac{-y}{2\sqrt{(E-y)}} + \sqrt{(E-y)} = 0$$

$$E = \frac{3}{2}y_c \quad or \quad y_c = \frac{2}{3}E$$

Problem 11.38

- Water is released from a sluice gate in a rectangular channel 1.5m wide such that depth is 0.6 m and velocity is 4.5 m/sec. Find

  (a). Critical Depth for this specific energy

  (b). Critical Depth for this rate of Discharge

  (c). The type of flow and alternate depth by either direct solution or the discharge Curve.

Problem 11.38

(b) $Q = AV = ByV = 4.05 m^3/\sec$

$q = vy = 2.7 m/\sec \; per \; m$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = 0.906 m > y$$

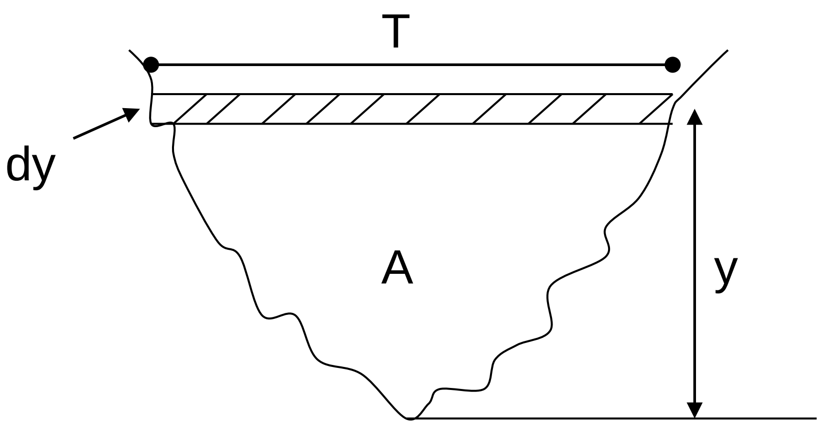
$Therefore\ Flow\ is\ Super-Critical$

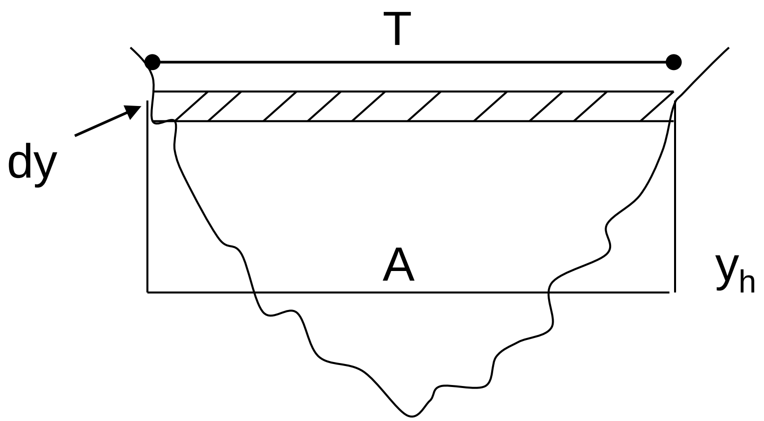
$$F_N = \frac{V}{\sqrt{gy}} = 1.855 > 1$$

$Flow\ is\ Super-Critical$

Specific Energy and Critical Depth (Non Rectangular Channels)

- Hydraulic Depth





- The hydraulic depth, y_h for non rectangular channel is the depth of a rectangular channel having flow area and base width the same as the flow area and top width respectively as for non rectangular channel.

Specific Energy and Critical Depth

Relationship Between Critical Depth and Specific Energy

- Froude's number may be numerically calculate as

$$F_N = \frac{V}{\sqrt{gy_h}}$$

$$F_N^{\,2} = \frac{Q^2}{A^2 g \frac{A}{T}} \qquad y_h = \frac{A}{T}$$

$$F_N = \sqrt{\frac{Q^2 T}{gA^3}}$$

$$Eq.(1) \Rightarrow \quad E = y + \frac{Q^2}{2gA^2}$$

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy}$$

$$Since \qquad dA = Tdy$$

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}T$$

$$for\ Critical\ \ flow \quad \frac{dE}{dy} = 0$$

$$Therefore$$

$$\left(\frac{A^3}{T} = \frac{Q^2}{g}\right)_{y=yc}$$

Problems11.45

- A Trapezoidal canal with side slopes 1:2 has a bottom width of 3 m and carries a flow of 20 m^3/sec.
  - a). Find the Critical Depth and Critical velocity.
  - b). If the canal is lined with Brick (n=0.015), find the critical slope for the same rate of discharge.
- **Solution**

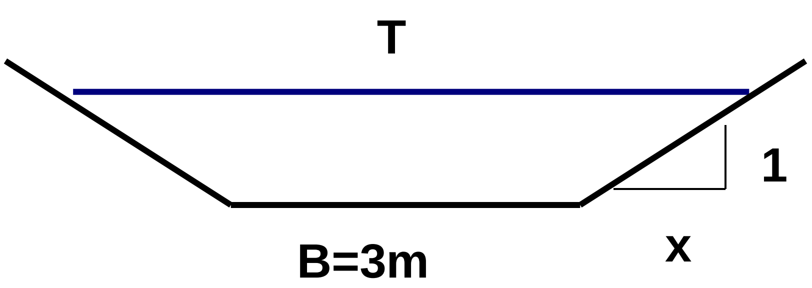
Q=20 m^3/sec

x= 2

A= (B+xy)y

P= $B+2y(1+x^2)^{1/2}$

T= B+2xy



Problem 11.45

- Solution (a)
  - $Q^2/g = A^3/T$

Q^2/g	y	A	T	A^3/T
40.775	1	5	7	17.85
	2	14	11	249.45
	1.2	6.48	7.8	34.88
	1.25	6.883	8.004	40.74
	1.2512	**6.885**	**8.0048**	**40.77**

- (b)

$$Q = \frac{A}{n}\left(\frac{A}{P}\right)^{3/2} S^{1/2}$$

$$S_c = 0.224433$$

Assignment

- Problems: 11.37, 11.39, 11.40, 11.43,
- Problems: 11.44, 11.46, 11.47

- Submission Date: