Plain & Reinforced Concrete-1

Sixth Term
Civil Engineering

CE-314

Lecture # 27

Simple Columns

BASICS OF SHORT COLUMN DESIGN

Columns are those structural members that are subjected to axial compressive loads as the main force.

However, these may also be subjected to simultaneous bending moments.

The cross-sectional dimensions of a column are generally considerably less than its height.

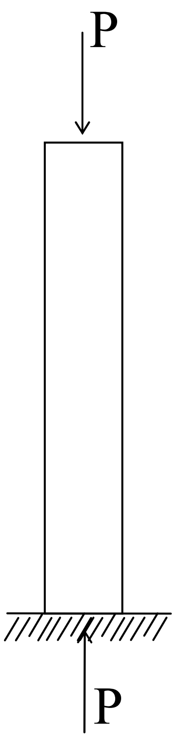


Fig. 7.1.A Simple Compression Member.

Columns support vertical loads from the floors and roof and transfer these loads to the foundations.

Depending upon the presence or absence of bending moment, columns may be of the following two types:

Concentrically loaded columns are those columns that are subjected to only axial load with only some accidental eccentricity of load producing considerably smaller bending moment.

Eccentrically loaded columns are those columns that are subjected to load at an eccentricity resulting in a combined action of axial compression and bending moment.

The column behavior mainly depends on its slenderness ratio, denoted by $K\ell_u/r$, where K is the effective length factor depending on the presence or absence of side-sway and the end conditions, ℓ_u is the unsupported length of column and r is the radius of gyration of the column cross-section along the axis of bending.

Side-sway is the lateral movement between the top and the bottom ends of the column in the deformed shape.

Depending upon the slenderness ratio, the columns may be classified into two categories as under:

1. Short Columns
2. Slender Columns

Short Columns: The slenderness ratio of such columns is so low that the instability, chances of buckling and second order effects is eliminated.

Second order effects usually means the magnification of first order moments produced as a result of product of the first order column deflections and the corresponding axial forces.

The strength of these columns depends only on the material strengths and the cross-sectional dimensions.

Slender Columns: The strength of such columns may be significantly lesser than the short columns depending upon their slenderness ratio and the first order lateral deflection.

Moment magnification may take place due to second order effects.

The effects of slenderness ratio may be ignored and the column may be considered to be behave as a short column if:

$$\frac{K\ell_u}{r} < 34 - 12\frac{M_1}{M_2}$$

where,

M_1 = magnitude of smaller end moment

M_2 = magnitude of larger end moment

M_1 / M_2 = zero if moment is absent, positive for single curvature and negative for reverse curvature.

For a concentrically loaded rectangular column with shorter side equal to *h* having *K* = 0.75, the limiting slenderness ratio is:

$$\frac{K\ell_u}{r} = \frac{0.75 \times \ell_u}{0.3h} = 34 \qquad \frac{\ell_u}{h} \cong 13.5$$

This means that if the unsupported height for a partially fixed braced column is less than 13.5 times its least lateral dimension, the column may be considered as a short column.

In other words, if the center-to-center height for a partially fixed braced column is less than 15 times its least lateral dimension, it is a short column.

Similarly, for a concentrically loaded rectangular column with least lateral dimension equal to h with $K = 1$, $\ell_u / h \cong 10$ and for a concentrically loaded circular column $\ell_u / d = 8.5$.

TYPES OF COLUMN REINFORCEMENT

The main steel in columns is provided along the length of the members in the corners or closer to the outer periphery, called ***longitudinal steel***.

The functions of this main column reinforcement are as under:

1. To reduce creep and shrinkage in the columns.
2. To provide certain minimum ductility in the columns.

3. To prevent reduction in column stiffness against lateral movement by reducing the crack widths.
4. To provide resistance against bending moments.
5. To provide resistance against bending moments.

The longitudinal bars are held together by the ***transverse reinforcement***.

This transverse reinforcement may be in the form of closely spaced ***ties*** or ***spirals***.

The functions performed by the ties are as under:

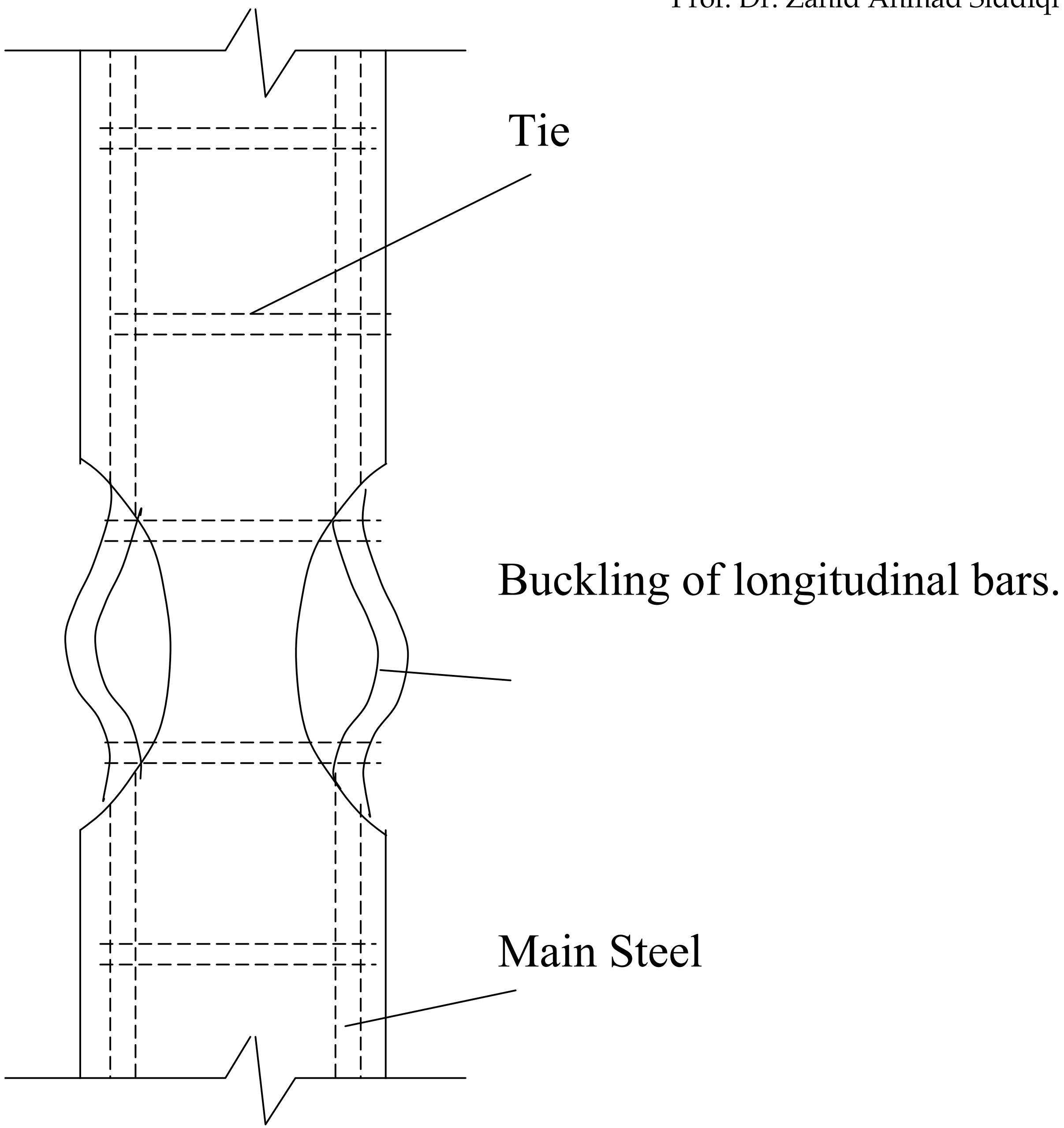
1. To prevent the outward buckling of the longitudinal bars that may occur by breaking of the concrete cover.
2. To provide some confinement to the inner concrete, preventing its sudden collapse and hence increasing ductility.
3. To provide resistance against shear and shrinkage cracking.
4. To hold the column steel at its proper position during casting of concrete.

5. Due to their relatively larger spacing, lateral ties do not provide the level of confinement to concrete as that provided by the spirals.

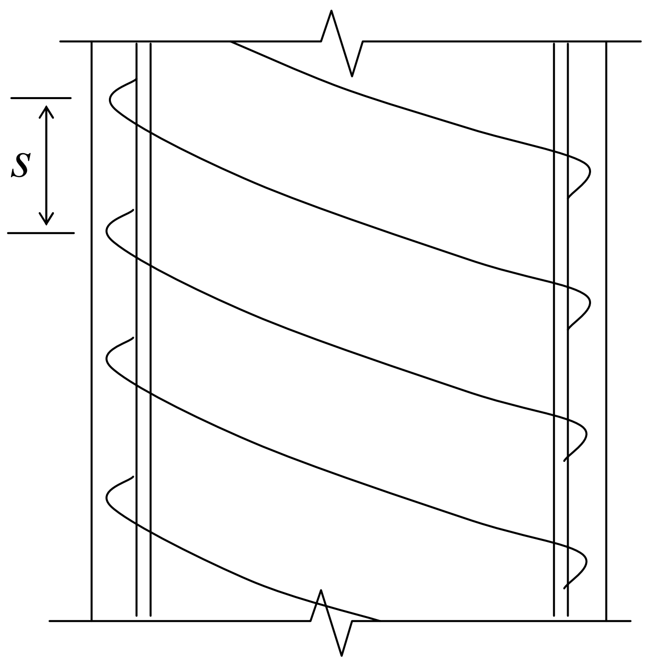
This gives relatively less ductility.

The failure of a tied column is abrupt and complete.

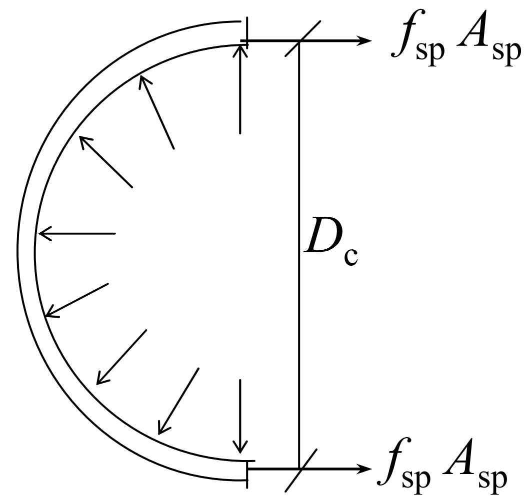
A tied column fails at the ultimate load when concrete fails by crushing and shears outward along the inclined planes, and the longitudinal steel buckles outward between ties as is diagrammatically shown in Fig. 7.2.


Tie
Buckling of longitudinal bars.
Main Steel

The spiral reinforcement consists of a continuous or lapped wire wrapped around the longitudinal steel in the form of spiral at a closer pitch (s), as shown in Fig. 7.3.



s = pitch of the spiral



Free Body of Half Turn of Spiral

The spiral reinforcement provides the following functions:

1. It more effectively brace the longitudinal bars against their outward buckling compared with the ties.

2. It provides much more confinement to the inner core of concrete.

   Closer to the ultimate loads, the concrete cover spalls down.

   However, the spiral keeps the longitudinal steel and the inner concrete core intact.

The load carrying capacity of confined concrete becomes sufficiently larger and may offset the decrease in strength due to loss of cover.

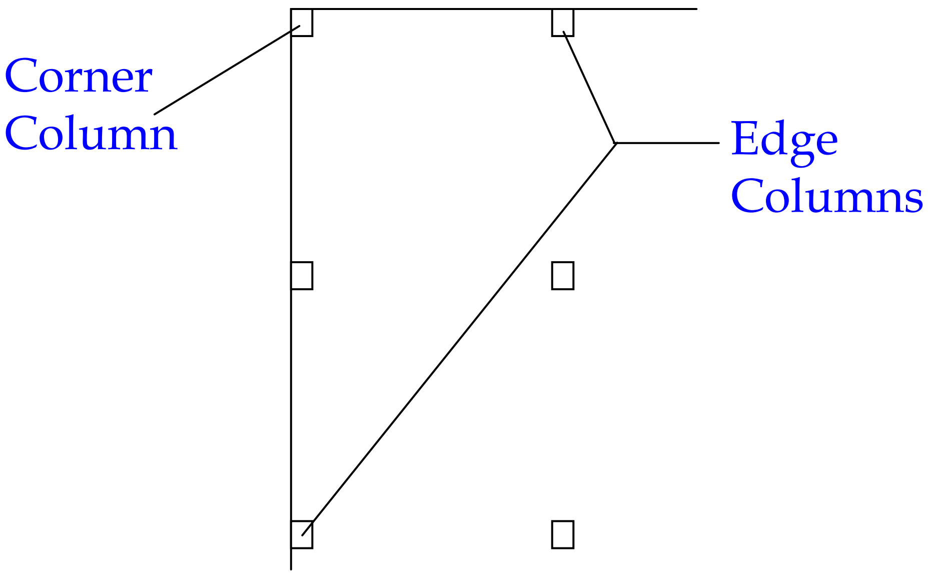
The loads are sustained for much longer times without final collapse, which provides lot of ductility.

3. It can hold the steel firmly during casting.
4. Hoop tension is developed in the spirals due to internal pressure of the concrete.

SOURCES OF MOMENT IN COLUMNS

1. ***Unequal Loading On Both Sides Of Column***

   When load on both sides of a column is unequal, the resultant load will have eccentricity with respect to the column centroid and hence moment will be generated.



2. *Unequal Spans On Both Sides Of Column*

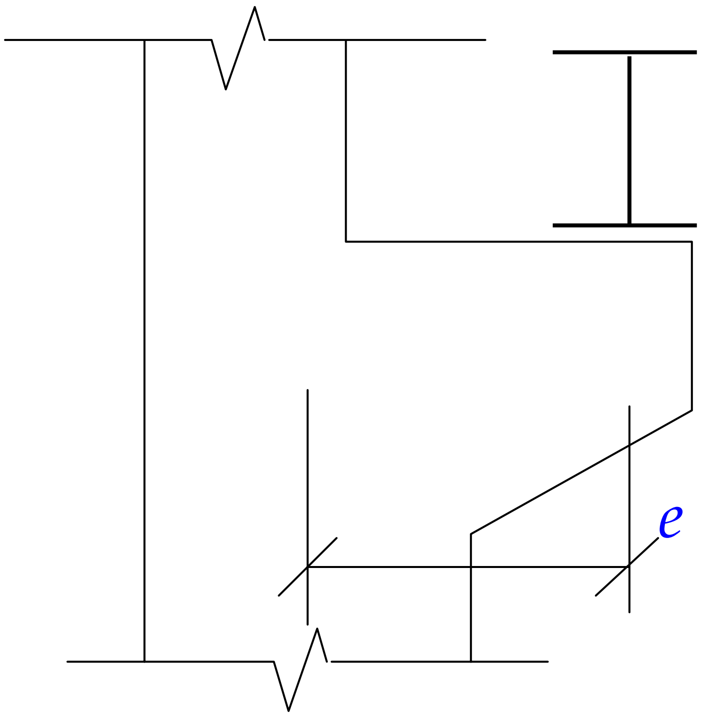
Presence of unequal spans on sides of a column also produces resultant eccentricity and moment.

3. *Rigid Frame Action*

If the joint between the concrete beams and the column is monolithic, it behaves as a rigid joint and the end moment of the beam is transferred to the column.

4. Eccentricity Of Load

Sometimes the load is applied on a column at an eccentricity through brackets or other mechanisms as in case of crane girders producing considerably larger moments.



5. ***Out – Of – Plumbness Of Column***

If the column is not exactly vertical, the load at the top produces some eccentricity at the bottom.

6. ***Out – Of – Straightness Of Column***

In case the column is not perfectly straight, some eccentricity and hence moment is generated.

NOT CONSIDERING THE SERVICE LOAD BEHAVIOR DUE TO SHORTAGE OF TIME

STEEL YIELDS EARLIER THAN CRUSHING OF CONCRETE IN COLUMNS

When the column load is gradually increased, the question arises that whether the steel will yield first or the concrete will crush first.

To answer this question, consider a column consisting of concrete strength $f_c' = 20$ MPa and steel of strength of 300 or 420 MPa.

$$E_c = 4700\sqrt{f_c'} = 21019 \text{ MPa}$$

$$E_s = 200{,}000 \text{ MPa}$$

$\varepsilon_c = 0.003$

$\varepsilon_y = 300 / 200{,}000 = 0.0015$ for grade 300 steel

$\varepsilon_y = 420 / 200{,}000 = 0.0021$ for grade 420 steel

This means that the steel will yield at a lesser strain or earlier than the crushing of concrete.

When the load is less than that causing yielding of steel, the steel takes *n*-times more load than the concrete per unit area.

After yielding of steel, the additional load is transferred to the concrete.

CAPACITY REDUCTION FACTORS FOR COLUMNS

The capacity reduction factors (ϕ) for columns are lower than the beams because of more relative importance of columns and relatively less ductility giving sudden failure of columns.

A column failure may cause the collapse of a complete structure, whereas, a beam may only affect a local portion.

The reduction factor is 0.65 for tied compression controlled columns and is 0.70 for spirally reinforced compression controlled columns.

Additional reduction factor is also applied on nominal strength of concentrically loaded columns to take care of accidental eccentricities, which is 0.80 for tied columns and 0.85 for spirally reinforced columns.

These values approximate the axial load strengths at e/h ratios of 0.05 and 0.10, specified in the earlier codes.

ULTIMATE STRENGTH OF CONCENTRICALLY LOADED SHORT COLUMNS

- For a slow rate of loading, much lesser than prescribed for a cylinder test, the maximum reliable strength of concrete is approximately 0.85 f'_c.
- As the loading on a column is increased, the steel yields first and its contribution to strength becomes constant at the yield stress level.
- Hence, the total nominal strength of a column becomes the yield strength of steel multiplied by its area and 0.85 f'_c times the concrete area.

$$P_{no} = 0.85\, f'_c\, A_c + f_y\, A_{st}$$

$$= 0.85\, f'_c\, A_g + (f_y - 0.85\, f'_c)\, A_{st}$$

Axially Loaded Spiral Short Columns

$$\phi P_{no} = 0.85 \times 0.75\,[0.85\, f'_c\,(A_g - A_{st}) + f_y\, A_{st}]$$

Axially Loaded Tied Short Columns

$$\phi P_{no} = 0.80 \times 0.65\,[0.85\, f'_c\,(A_g - A_{st}) + f_y\, A_{st}]$$

Example 7.2: A reinforced concrete concentrically loaded short column has a cross-sectional size of 450 × 450 mm and is reinforced by 8 # 19 (US) bars of grade 420. Calculate the maximum design axial load capacity of the tied column.

Solution:

$f_c' = 20$ MPa : $f_y = 420$ MPa

$A_{st} = 2280$ mm^2 : $A_g = 450 \times 450$ mm^2

$$\phi P_{no} = 0.80 \times 0.65\,[0.85\, f'_c\,(A_g - A_{st}) + f_y\, A_{st}]$$

$$= 0.52 \times [0.85 \times 20\,(450 \times 450 - 2280) + 420 \times 2280] / 1000$$

$$= 2268 \text{ kN}$$

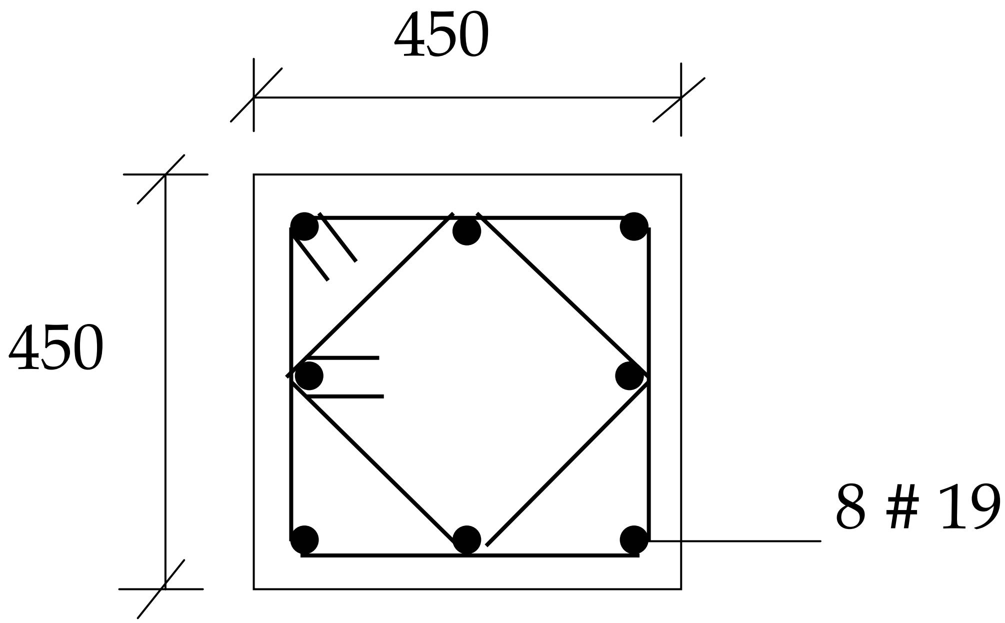


Fig. 7.8. Column Cross-Section for Example 7.2.

MINIMUM CONCRETE COMPRESSIVE STRENGTH

In order to keep creep and shrinkage of columns within acceptable limits, the concrete compressive strength should be kept not less than 20MPa for important and multistory buildings.

MINIMUM AND MAXIMUM STEEL RATIOS IN COLUMNS

The steel ratio in columns is defined as the total amount of longitudinal steel in the cross-section divided by the gross area of the section.

$$\rho = \frac{A_{st}}{b\,h}$$

The minimum steel ratio in columns is 1%, which is provided to strengthen concrete against excessive creep.

If steel is provided less than this amount, creep occurs in concrete and strain increases without increase of stress.

This transfers load to the steel at the service load stage up to which the steel may not have yielded.

This phenomenon continues until steel yields and both materials undergo large strains.

The lower limit is also necessary to ensure resistance to bending moments not accounted for in the analysis.

The maximum steel ratio given by the ACI Code is 8% to avoid congestion of steel, particularly where it must be spliced.

The practical values of steel ratio range from 3 to 6% as the upper limit for easy placement of concrete.

The most economical tied column section generally involves ρ_{st} from 1 to 2%.

According to ACI 10.9.2, the minimum number of longitudinal bars in compression members is to be 4 for bars within rectangular or circular ties, 3 for bars within triangular ties, and 6 for bars enclosed by spirals.

Almost universally, an even number of bars is used in a rectangular column so that the column is symmetrical about the axis of bending and usually all the bars are of the same size.

MINIMUM CLEAR COVER AND SPLICING OF BARS

The clear cover for sheltered columns not in contact with the ground should be at least 40mm.

For unsheltered columns having bars greater than No. 16 (US), clear cover is to be increased to 50mm.

For columns in contact with earth, the minimum clear cover should be 75mm.

When detailed calculations are not made for splicing in compression, the overlap of longitudinal bars at the base and at each floor level should be more than 35 times the diameter of the bar, as shown in Fig. 7.9.

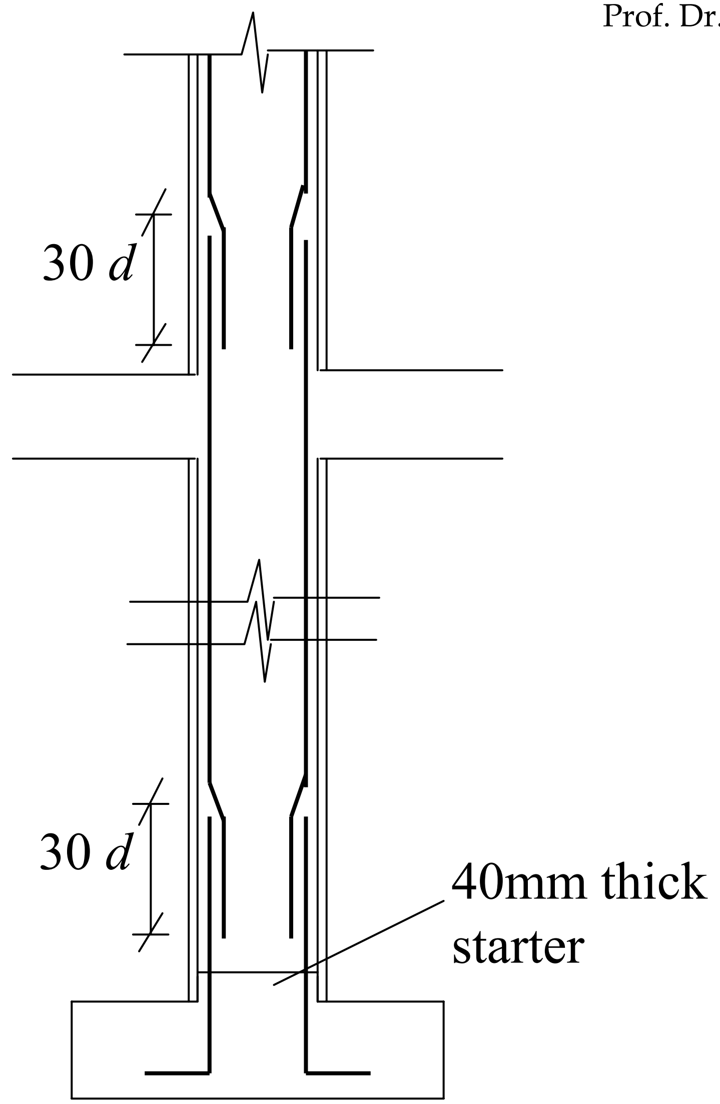


Fig. 7.9.Bar Splicing In Columns.

AMOUNT OF LATERAL TIES

For the design of lateral ties, three parameters are to be decided, which are: (a) diameter of ties (b) shape of ties and (c) spacing of ties.

The diameter of ties should be at least No. 10 (US and SI) for longitudinal bars up to No. 32 US (No. 30 SI) and at least No. 13 US (No. 15 SI) for larger diameter longitudinal bars.

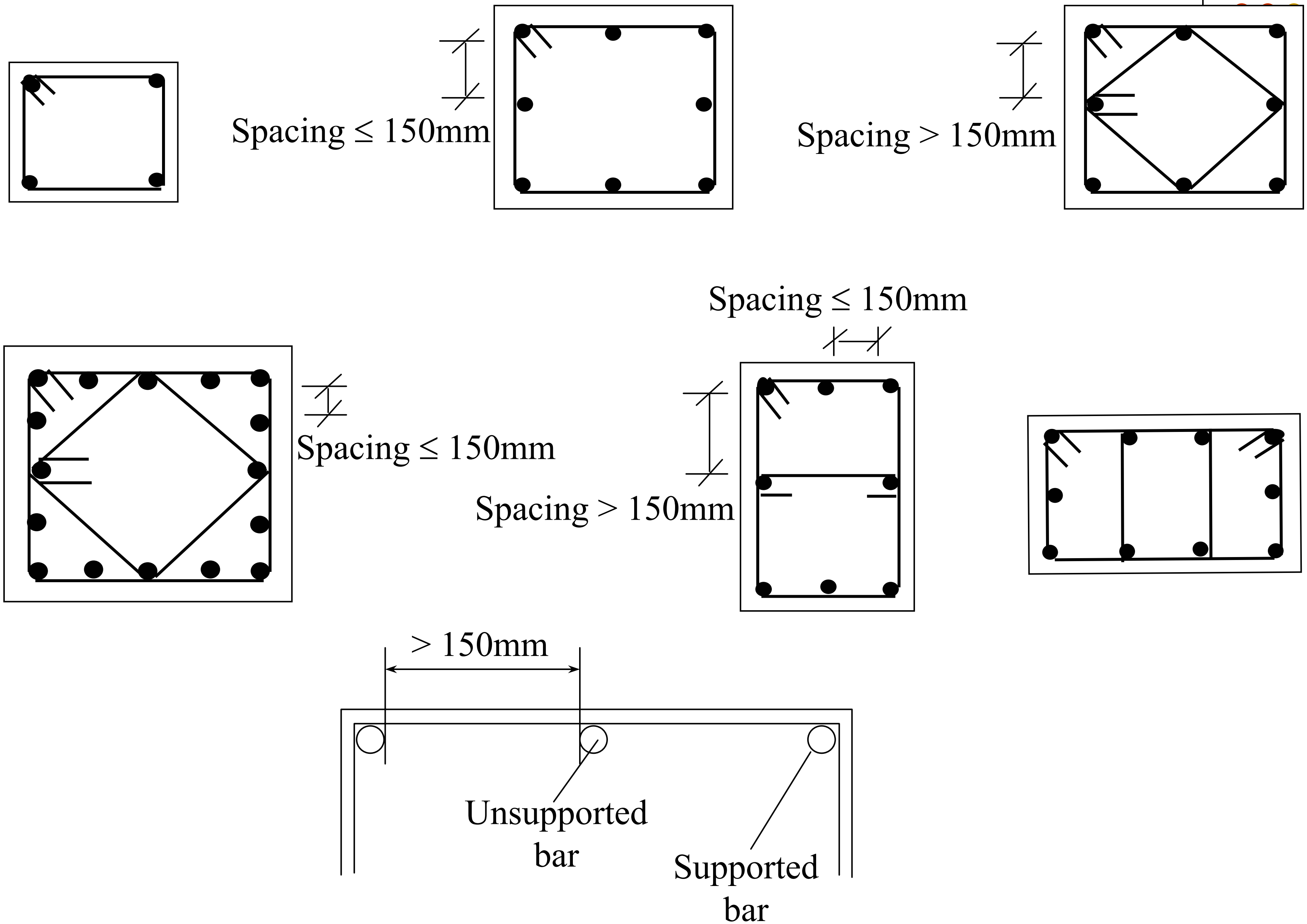
For the shape of ties three different criteria are to be satisfied:

(1) The ties shall be placed such that every corner longitudinal bar, every alternate bar and bars having clear spacing greater than 150mm (6 in) should be enclosed by a tie having an included angle of not more than 135°.

(2) The lateral ties should have proper hooks at the ends.

(3) The ties should not have any bend where there is no longitudinal bar.

According to these criteria, some typical tie-shapes are shown in Fig. 7.10.


Spacing ≤ 150mm
Spacing > 150mm
Spacing ≤ 150mm
Spacing ≤ 150mm
Spacing > 150mm
> 150mm
Unsupported bar
Supported bar

The spacing of the ties should be smaller of the following four quantities:

a) 16 times the diameter of the longitudinal bars.

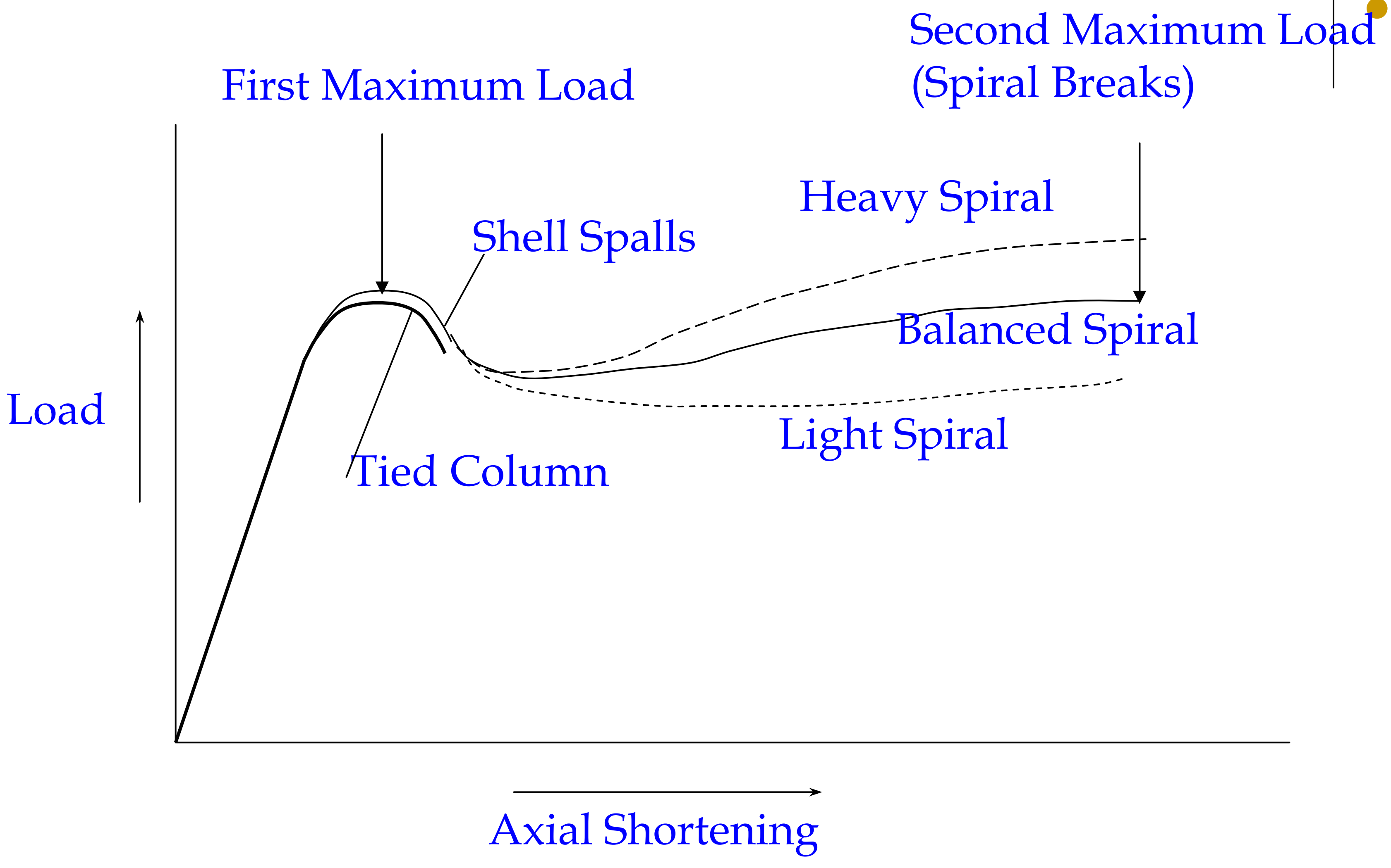
b) 48 times the diameter of the tie bars.

c) Least lateral dimension of the column, not more than 75mm in case of circular columns (as included angle of ties is not less than 135°).

d) 300 mm (not an ACI requirement).

AMOUNT OF SPIRAL REINFORCEMENT

- Figure 7.11 shows the behavior of concentrically loaded short columns as the load is gradually increased.
- The first maximum load is reached after the nominal capacity of the column is exhausted.
- The concrete shell around the longitudinal bars and ties or spiral spalls and the load carrying capacity is reduced due to reduction in the concrete area.
- A tied column collapses only after a little strain after the above stage.


Second Maximum Load
(Spiral Breaks)
First Maximum Load
Heavy Spiral
Shell Spalls
Balanced Spiral
Load
Light Spiral
Tied Column
Axial Shortening

- In case of spirally reinforced column, the inner core concrete starts taking more loads due to lateral confinement.
- The longitudinal steel and the concrete within the core are prevented form outward failing by the spiral.
- The concrete cover in the outer shell does fall down.
- At this stage, the confining action of the spiral has a significant effect, and the failure is caused by the yielding or fracture of spiral steel at a collapse load much larger than that at which the shell spalled off.
- Furthermore, the axial strain at the stage of failure of column will be much greater than a tied column; meaning that the toughness of the column has been significantly increased.

- Core-concrete under compressive load will have a tendency to expand laterally and will bear against the spiral.
- The spiral will thus be under tension and it exerts confining action on the core.
- This increases the load carrying capacity of the core in spite of the loss of shell concrete.

Let,

f_c^* = confined core concrete strength

f_2 = lateral confinement stress produced by the spiral

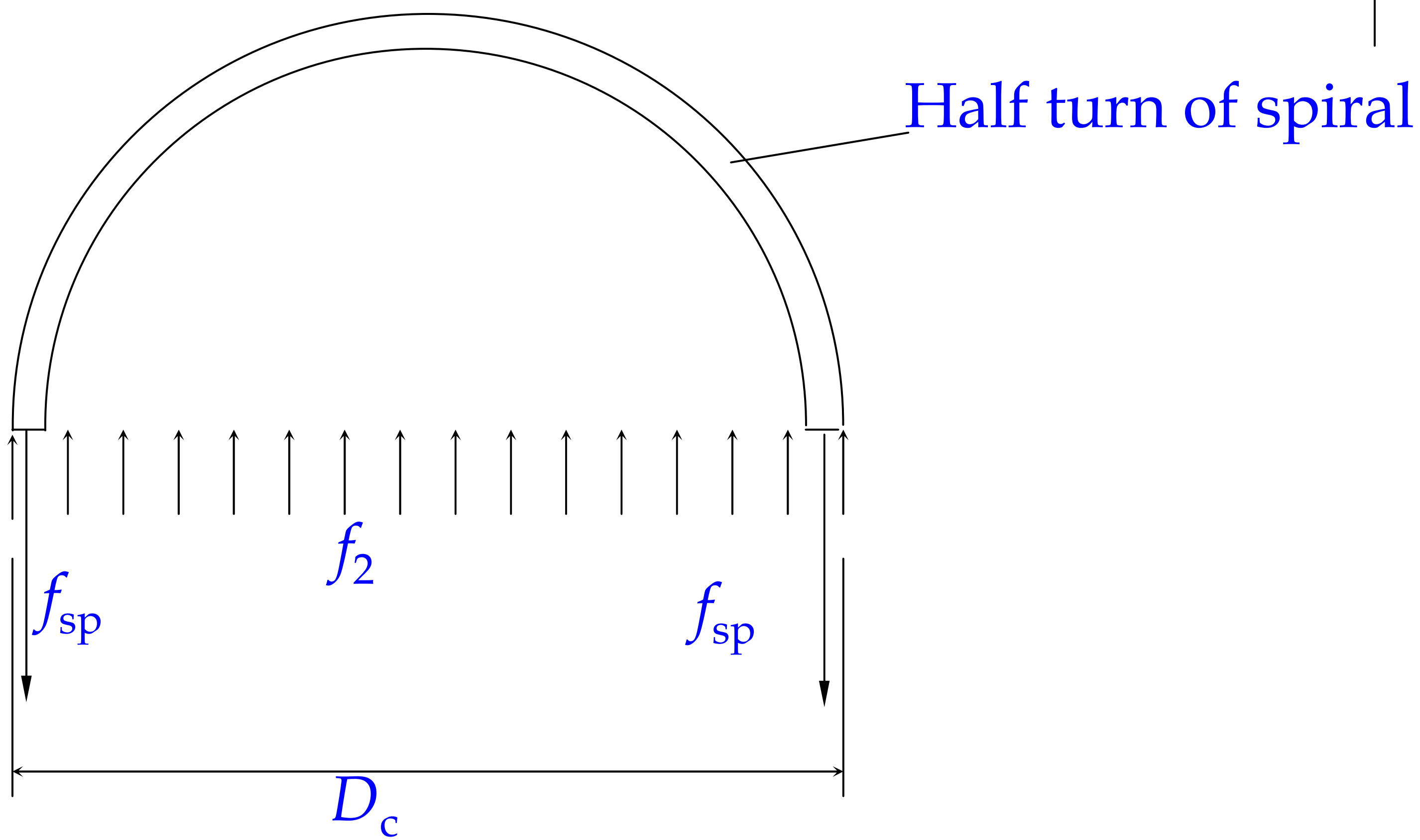
f_2' = the maximum value of f_2 when the spiral is yielding

f_c' = concrete cylinder strength

f_{yt} = specified yield strength of transverse reinforcement

d_{sp} = diameter of spiral

A_{sp} = area of one spiral bar, equal to $(\pi/4)d^2_{sp}$



Half turn of spiral
f_2
f_{sp}
f_{sp}
D_c

ρ_s = volumetric spiral reinforcement ratio

D_c = diameter of core, out-to-out of the spirals

L_{sp} = length of one turn of spiral $\cong \pi D_{ch}$ for smaller pitch

A_c = area of cross section of core, equal to $(\pi/4)D^2_{ch}$

f_{sp} = lateral confinement stress in core concrete produced by spiral

and

s = pitch of the spiral.

- The confined core concrete strength, as increased by the lateral confinement stress, is given by the following empirical equation:

$$f_c^* = 4.0\, f_2' + 0.85\, f_c'$$

- To estimate the confined strength, the lateral confinement pressure must be evaluated.
- For the force equilibrium of this free body diagram along the direction of the shown stresses,

$$2\, A_{sp}\, f_{sp} = f_2\, D_c\, s$$

$$\therefore \quad f_2 = \frac{2 A_{sp} f_{sp}}{D_c\, s}$$

- The volumetric spiral reinforcement ratio (ρ_s) is defined as the ratio of volume of the spiral reinforcement to volume of the core measured out-to-out of the spirals as under:

$$\rho_s = \frac{A_{sp} L_{sp}}{A_c \, s}$$

$$= \frac{A_{sp} \, \pi \, D_{ch}}{(\pi D_{ch}^2 / 4) \, s} = \frac{4 A_{sp}}{D_{ch} \, s}$$

From Eqns. I and II: $f_2 = \dfrac{\rho_s f_{sp}}{2}$

When $f_{sp} = f_{yt}$, the maximum confinement is obtained. That is, $f_2 = f_2'$

$$f_2' = \frac{\rho_s f_{yt}}{2}$$

- The extra strength provided by the spiral is equal to $f_c^* - 0.85f_c'$, and the ultimate strength is given by the following expression:
- Ultimate strength for spirally reinforced column $= (f_c^* - 0.85f_c')\, Ach$

  $= 4.0\, f_2'\, A_{ch}$

  $= 2\, \rho_{sp}\, f_{yt}\, A_{ch}$ (VI)
- Strength contribution of the outer shell before spalling $= 0.85\, f_c'\, (A_g - A_{ch})$

- The amount of spiral reinforcement may be so adjusted that the increased strength due to the spiral becomes exactly equal to the loss of strength due to spalling of shell concrete.
- This gives a second maximum load in Fig. 7.11 exactly equal to the first maximum load.
- This type of spiral is called a ***balanced spiral***. The following may be written to derive formula for the balanced spiral:

$$2\,\rho_s\,f_{yt}\,A_{ch} = 0.85\,f_c'\,(A_g - A_{ch})$$

$$or \qquad \rho_s = 0.425\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}}$$

- The ACI 10.9.3 recommends a minimum spiral steel ratio ρ_s a little more than the above value, as under:

$$(\rho_s)_{\min} = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}}$$

- An ***ACI spiral*** has a minimum spiral reinforcement to provide axial load capacity slightly larger than that of the shell concrete.

- A spiral that provides indirect axial strength more than the loss of shell concrete strength (spiral steel is more than given by above Eq.) is called ***heavy spiral***
- A spiral giving indirect axial strength less than the shell concrete is called ***light spiral***.
- The behavior of light, balanced and heavy spiral is shown in Fig. 7.11.
- The advantage of spiral reinforcement is only significant in axially loaded columns like piles, bridge piers and concentrically loaded columns but is much lesser when bending moment is also present.

- Spirals should consist of a continuous or properly spliced bar of at least 10mm diameter.
- If exact calculations are not performed, the lap should be at least 40 times the diameter with the hooks anchored into the core concrete.
- The minimum ratio of spiral steel is specified so that the structural performance of the column is improved with respect to both the ultimate load and the type of failure.
- To keep the spirals firmly in place and true to line, one-and-a-half extra turns of spiral are provided at each end of spiral.

Rules used to select the pitch of spiral reinforcement

1. The maximum center-to-center pitch should be as under:

$$\rho_s = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}}$$

$$\frac{4\,A_{sp}}{D_{ch}\,s} = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}}$$

$$\frac{\pi\,d_{sp}^2}{D_{ch}\,s} = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}}$$

$$s_{\max} = \frac{\pi\,d_{sp}^2\,f_{yt}}{0.45\,D_{ch}\,f_c'\,(A_g / A_{ch} - 1)}$$

2. To bind the core effectively, the maximum clear pitch should be 75 mm.
3. To avoid problems in placing of concrete, the minimum clear pitch should be larger of 1.5 times the size of coarse aggregate and 25 mm.

TRIAL COLUMN SIZE

- In case the factored column load (P_u) and moments (M_{ux} and M_{uy} in kN-m) are known, the trial column size may be estimated for an assumed steel ratio of 1.5% as follows:

$$A_g\ (trial) \geq \frac{P_u + 2M_{ux} + 2M_{uy}}{0.43\,f_c' + 0.008\,f_y} \text{ for tied column}$$

$$A_g\ (trial) \geq \frac{P_u + 2M_{ux} + 2M_{uy}}{0.5\,f_c' + 0.01\,f_y} \text{ for spiral columns}$$

APPROXIMATE AMOUNT OF STEEL IN COLUMNS

The total amount of steel in columns is calculated from the bar bending schedule. However, as a rough estimate, the amount of steel in columns may be estimated as 175 kgs/m^3.

Example 7.3: An axially loaded short column has a length of 3.0 m and has a factored load of 1500 kN. $f_c' = 20$ MPa, maximum aggregate size = 19mm and $f_y = 300$ MPa. Calculate the suitable dimensions and reinforcement for the following two options:

A) Square tied column, and

B) Circular spirally reinforced column.

Solution:

- f_c' = 20 MPa
- f_y = 300 MPa
- P_u = 1500 kN
- L = 3.0 m

Part-A

$$A_g\ (trial) = \frac{P_u + 2M_{ux} + 2M_{uy}}{0.43 f_c' + 0.008 f_y}$$

$$= \frac{1500 \times 1000}{0.43 \times 20 + 0.008 \times 300} = 136{,}364 mm^2 = 375 \times 375\, mm$$

Length / least dimension = 3000 / 375

= 8.0 (really a short column)

$P_u = \phi P_{no}$

$= 0.80 \times 0.65\ [0.85\, f'_c\, (A_g - A_{st}) + f_y\, A_{st}]$

$$\frac{P_u}{0.80 \times 0.65} = 0.85 f_c' A_g + (f_y - 0.85 f_c') A_{st}$$

$$A_{st} = \frac{P_u / 0.52 - 0.85\, f_c'\, A_g}{f_y - 0.85\, f_c'}$$

$$= \frac{1500{,}000 / 0.52 - 0.85 \times 20 \times 375^2}{300 - 0.85 \times 20} = 1746\ mm^2$$

$A_{st,min}$ = 0.01×3752 = 1406 mm^2

Use 4 – #25 (2040 mm^2) or 8 – #19 (2272 mm^2) or 4 – #19 + 4 – #16 (1932 mm^2)

Note: Compared with four bars, eight bars may require more ties and hence the saving in steel by using eight bars may be somewhat compensated.

- Diameter of ties for #19 (US) bars = #10 (US)
- Spacing of ties = least of

1) $16 \times 16 = 256$ mm
2) $48 \times 10 = 480$ mm
3) 375 mm
4) 300

= 256 mm

$$\text{Clear spacing between the bars} = \frac{375 - 2\times 40 - 2\times 10 - 2\times 19 - 16}{2} = 110mm < 150mm$$

$\therefore$ Additional ring of ties is not required.

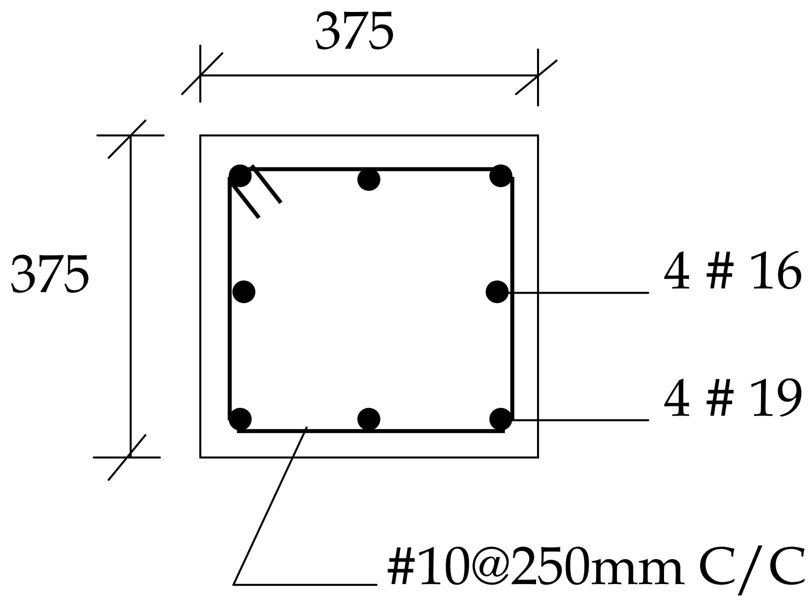


Fig. 7.13. Column Cross-Section for Example 7.3.

Part-B

$$A_g\ (trial) = \frac{P_u + 2M_{ux} + 2M_{uy}}{0.5 f'_c + 0.01 f_y}$$

$$= \frac{1500 \times 1000}{0.5 \times 20 + 0.01 \times 300} = 115{,}385 mm^2 \approx 375 mm \text{ diameter}$$

- Length / least dimension = 3000 / 375

  = 8.0 (really a short column)

- $P_u = \phi P_{no}$

  $= 0.85 \times 0.75\ [0.85\ f'_c\ (A_g - A_{st}) + f_y\ A_{st}]$

$$A_{st} = \frac{P_u / 0.6375 - 0.85 f'_c A_g}{f_y - 0.85 f'_c}$$

$$A_{st} = \frac{1500{,}000 / 0.6375 - 0.85 \times 20 \times \frac{\pi}{4} \times 375^2}{300 - 0.85 \times 20} = 1680 mm^2$$

- $A_{st,min}$ = 0.01 × π/4 × 375^2 = 1105 mm^2
- Use 6 – #19 (1704 mm^2) as shown in Fig. 7.14.
- Diameter of the spiral = 10 mm
- Clear cover = 40 mm
- D_{ch} = 375 – 2 × 40 = 295 mm
- A_g = π/4 × 375^2 = 110,447 mm^2
- A_{gh} = π/4 × 295^2 = 68,349 mm^2

$$s_{\max} = \frac{\pi\, d_{sp}^2\, f_{yt}}{0.45\, D_{ch}\, f_c' (A_g / A_{ch} - 1)}$$

$$s_{\max} = \frac{\pi \times 10^2 \times 300}{0.45 \times 295 \times 20\ (110{,}447 / 68{,}349 - 1)} \cong 57mm$$

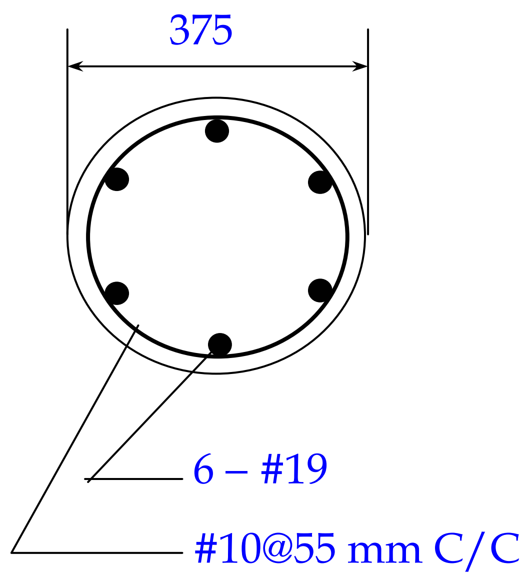


Fig. 7.14. Spiral Column for Example 7.3.

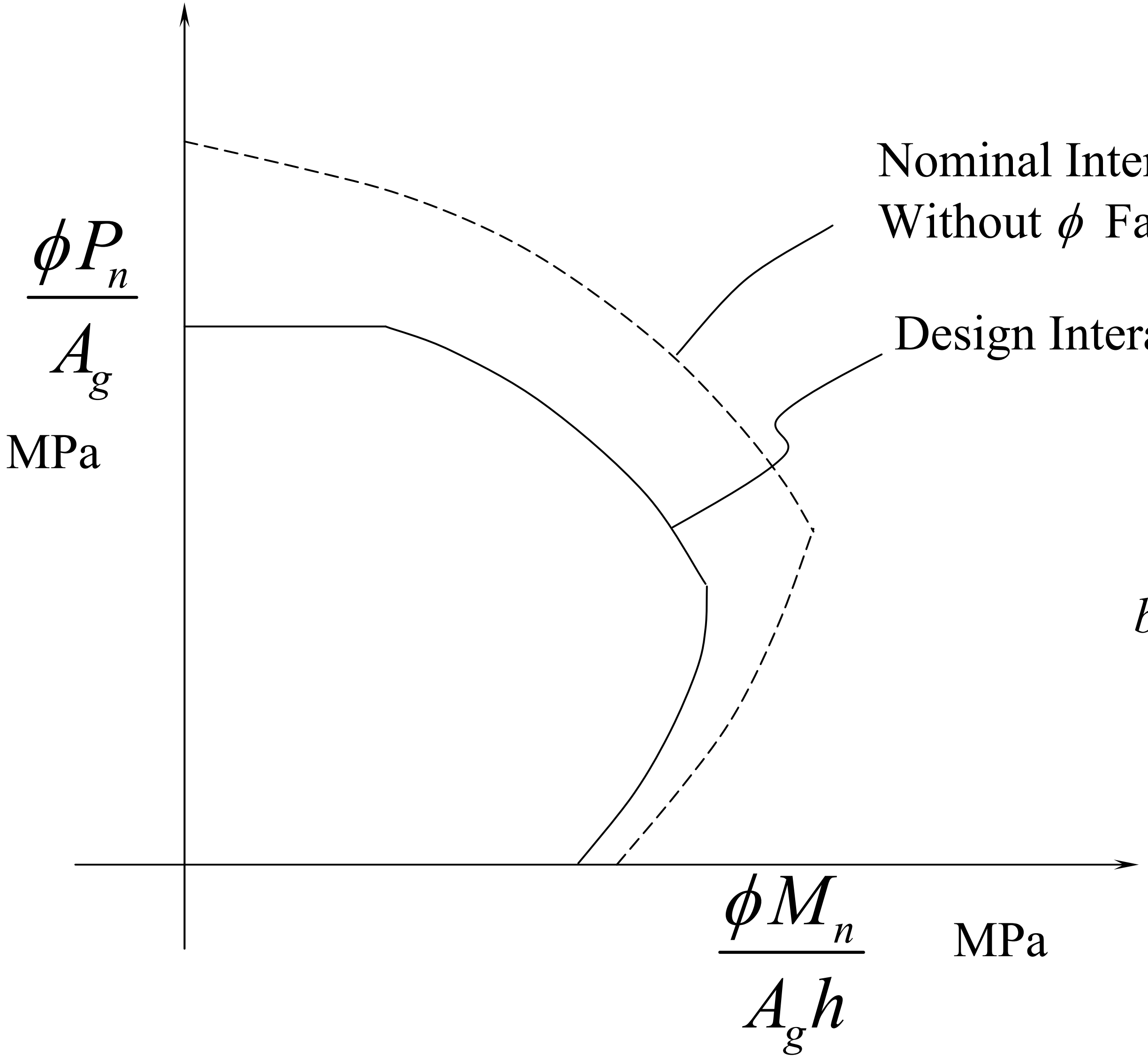
- Provide spiral at a pitch of 55 mm, giving a clear spacing of 45 mm that is larger than 25mm and 1.5×19 = 29mm and less than 75mm.

ECCENTRICALLY LOADED COLUMNS

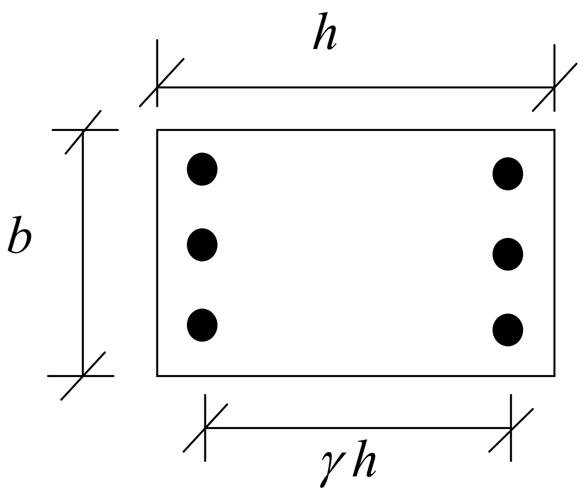
When load on a column is applied at an eccentricity or when axial load (P_u) and bending moment ($M_u = P_u \times e$) are simultaneously applied on a column, the stress distributions due to axial load and bending moment interact with each other.

This produces more compression on one side and less compression or a small tension on the other side.

In short, the capacity of such a column may be represented by a curve between $\frac{\phi P_n}{A_g}$ and $\frac{\phi M_n}{A_g h}$, as shown in Fig. 7.16. Any point corresponding to the applied combination of P_u and M_u that is lying on or inside the curve is safe and any point outside of the curve is unsafe.



$\frac{\phi P_n}{A_g}$
MPa
Nominal Interaction Curve
Without ϕ Factor
Design Interaction Curve
$\frac{\phi M_n}{A_g h}$
MPa

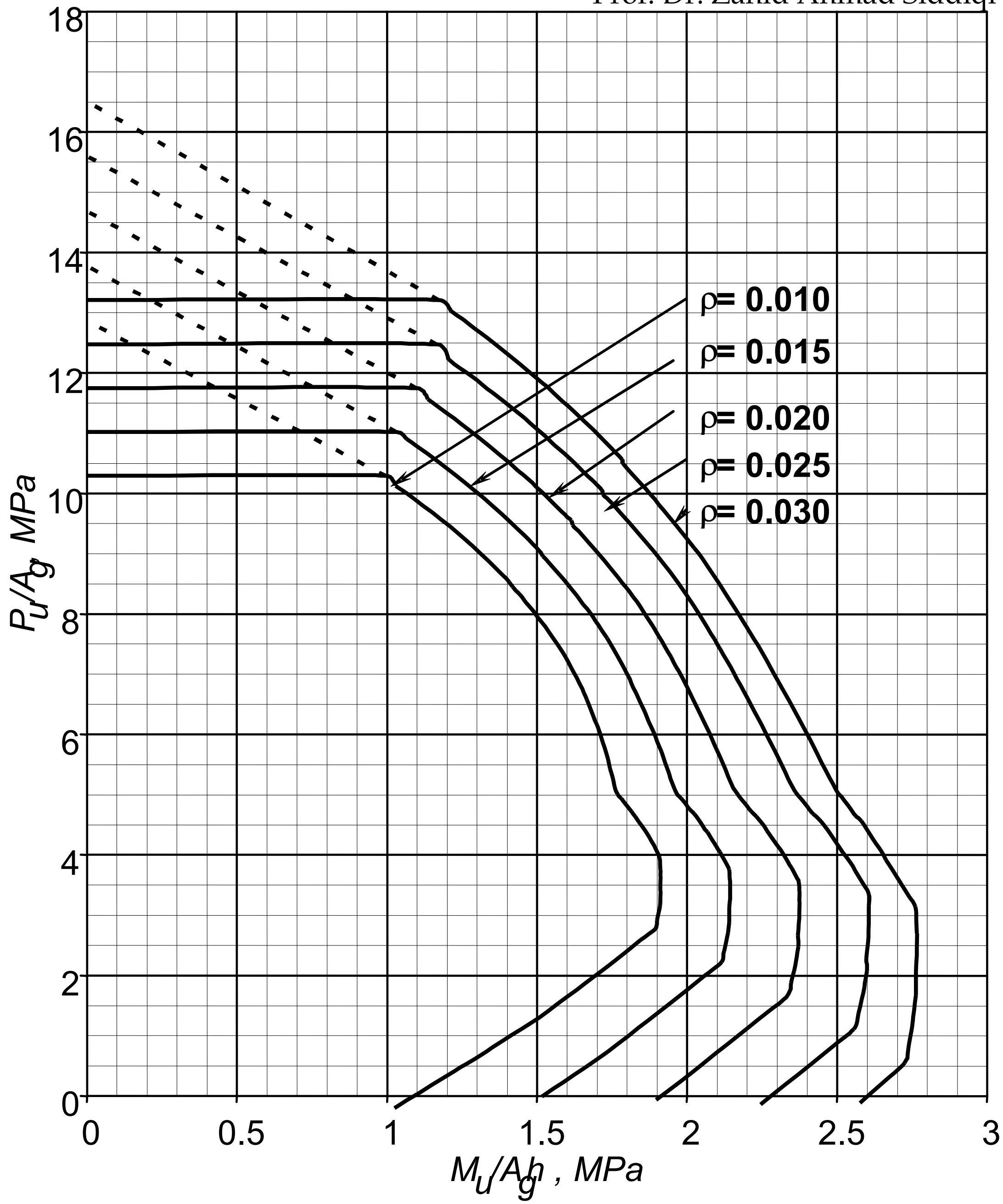


h
b
γh

$\gamma = 0.6$

$f_c' = 20$ MPa

$f_y = 300$ MPa



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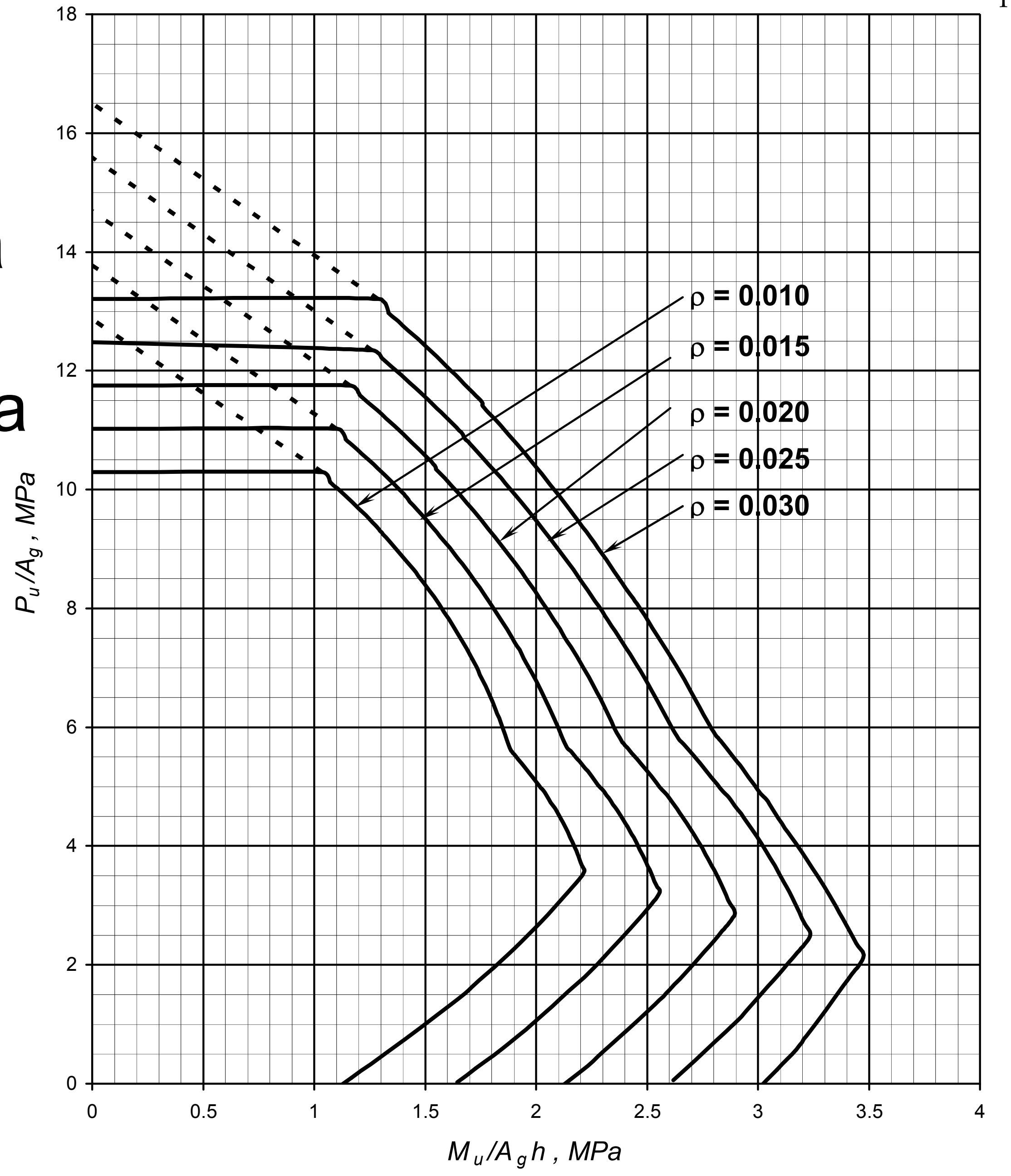
$\gamma = 0.75$

$f_c' = 20$ MPa

$f_y = 300$ MPa



PROCEDURE FOR DESIGN OF ECCENTRICALLY LOADED COLUMNS

1. Select trial cross-sectional dimensions *b* and *h*, considering the load and the approximate effect of the bending moment, as discussed for the axially loaded column.

2. Calculate the ratio γ using the clear cover and the assumed diameter of the ties and the main reinforcement. The value may be rounded to any lower value of γ for which the curve is available.

3. Curve is selected for the required values of γ, f_c', f_y and shape of cross-section.

4. The parameters $\frac{P_u}{A_g}$ and $\frac{M_u}{A_g h}$ are calculated for each combination of P_u and M_u such as (P_{u1} and M_{u1}) and (P_{u2} and M_{u2}), etc.

5. The point corresponding to each $\frac{P_u}{A_g}$ and $\frac{M_u}{A_g h}$ is located on the diagram and the required steel ratio is approximately interpolated between the two adjacent interaction curves.

6. The maximum steel ratio (ρ_g) is found for all the pairs of values.

7. The total steel area A_{st} is calculated as follows:

$$A_{st} = \rho_g \, bh$$

8. The column size may be revised if the reinforcement is excessive.

9. The lateral ties are designed as for the concentrically loaded columns.

Example 7.5 (SI): A 3.0 m long column, which is short and tied, is to be used for the following combinations of loads:

1st combination: P_D = 300 kN, P_L = 450 kN,
M_D = 58 kN-m, M_L = 85 kN-m

2nd combination: P_D = 300 kN, P_L = 225 kN,
M_D = 58 kN-m, M_L = 85 kN-m

3rd combination: P_D = 300 kN, P_L = 600 kN,
M_D = 58 kN-m, M_L = 40 kN-m

The column is to be of rectangular section with at least one side not larger than 375 mm. f_c' = 20 MPa, maximum aggregate size = 19mm and f_y = 300 MPa. Calculate the suitable dimensions and reinforcement.

Solution:

$$f_c' = 20 \text{ MPa}$$

$$f_y = 300 \text{ MPa}$$

1st combination:

$$P_u = 1.2 \times 300 + 1.6 \times 450 = 1080 \text{ kN}$$

$$M_u = 1.2 \times 58 + 1.6 \times 85 = 205.6 \text{ kN-m}$$

2nd combination:

$P_u = 1.2 \times 300 + 1.6 \times 225 = 720$ kN

$M_u = 1.2 \times 58 + 1.6 \times 85 = 205.6$ kN-m

3rd combination:

$P_u = 1.2 \times 300 + 1.6 \times 600 = 1320$ kN

$M_u = 1.2 \times 58 + 1.6 \times 40 = 133.6$ kN-m

The trial size of the column is decided as on the next slide:

$$A_g \text{ (trial)} = \frac{P_u + 2M_{ux} + 2M_{uy}}{0.43 f_c' + 0.008 f_y}$$

$$= \frac{(1320 + 2 \times 133.6) \times 1000}{0.43 \times 20 + 0.008 \times 300}$$

$$= 144{,}291 \text{ mm}^2$$

$$\approx 375 \times 375 \text{ mm}$$

For 10 mm diameter ties and 25 mm diameter longitudinal steel,

$$\gamma = \frac{375 - 2 \times 40 - 2 \times 10 - 25}{375} = 0.67 \approx 0.60$$

Check For 1st Combination:

$$\frac{P_u}{A_g} = \frac{1080 \times 1000}{375^2} = 7.68 \text{ MPa}$$

$$\frac{M_u}{A_g h} = \frac{205.6 \times 10^6}{375^3} = 3.90 \text{ MPa}$$

Using the relevant interaction curve for $\gamma = 0.60$, $f_c' = 20$ MPa and $f_y = 300$ MPa, the required steel ratio becomes greater than 3%. It is better to revise the column size to 375 × 450 mm.

$$\gamma = \frac{450 - 2 \times 40 - 2 \times 10 - 25}{450} = 0.72 \approx 0.75$$

Check For 1st Combination:

$$\frac{P_u}{A_g} = \frac{1080 \times 1000}{375 \times 450} = 6.40 \text{ MPa}$$

$$\frac{M_u}{A_g h} = \frac{205.6 \times 10^6}{375 \times 450^2} = 2.71 \text{ MPa}$$

Using the interaction curve, $\rho_g \cong$ *0.03.*

Check For 2nd Combination:

$$\frac{P_u}{A_g} = \frac{720 \times 1000}{375 \times 450} = 4.27 \text{ MPa}$$

$$\frac{M_u}{A_g h} = \frac{205.6 \times 10^6}{375 \times 450^2} = 2.71 \text{ MPa}$$

Using the interaction curve, $\rho_g < 0.03$ *(OK).*

Check For 3rd Combination:

$$\frac{P_u}{A_g} = \frac{1320 \times 1000}{375 \times 450} = 7.82 \text{ MPa}$$

$$\frac{M_u}{A_g h} = \frac{133.6 \times 10^6}{375 \times 450^2} = 1.76 \text{ MPa}$$

Using the interaction curve, ρ_g < *0.03* *(OK).*

$$A_{st} = \rho_g \times bh = 0.03 \times 375 \times 450$$

$$= 5063 \text{ mm2} \quad (10 – \#25)$$

Diameter of ties for #25 (US) bars = #10 (US)

Spacing of ties = least of

1) $16 \times 25 = 400$ mm

2) $48 \times 10 = 480$ mm

3) 375 mm

4) 300 mm

= 300 mm

$$s_1 = \frac{375 - 2 \times 40 - 2 \times 10 - 25}{3}$$

$= 83.3$ mm $<$ 150 mm

$$s_2 = \frac{450 - 2 \times 40 - 2 \times 10 - 25}{2}$$

$= 162.5$ mm $>$ 150 mm

∴ Additional ring of ties is as provided.

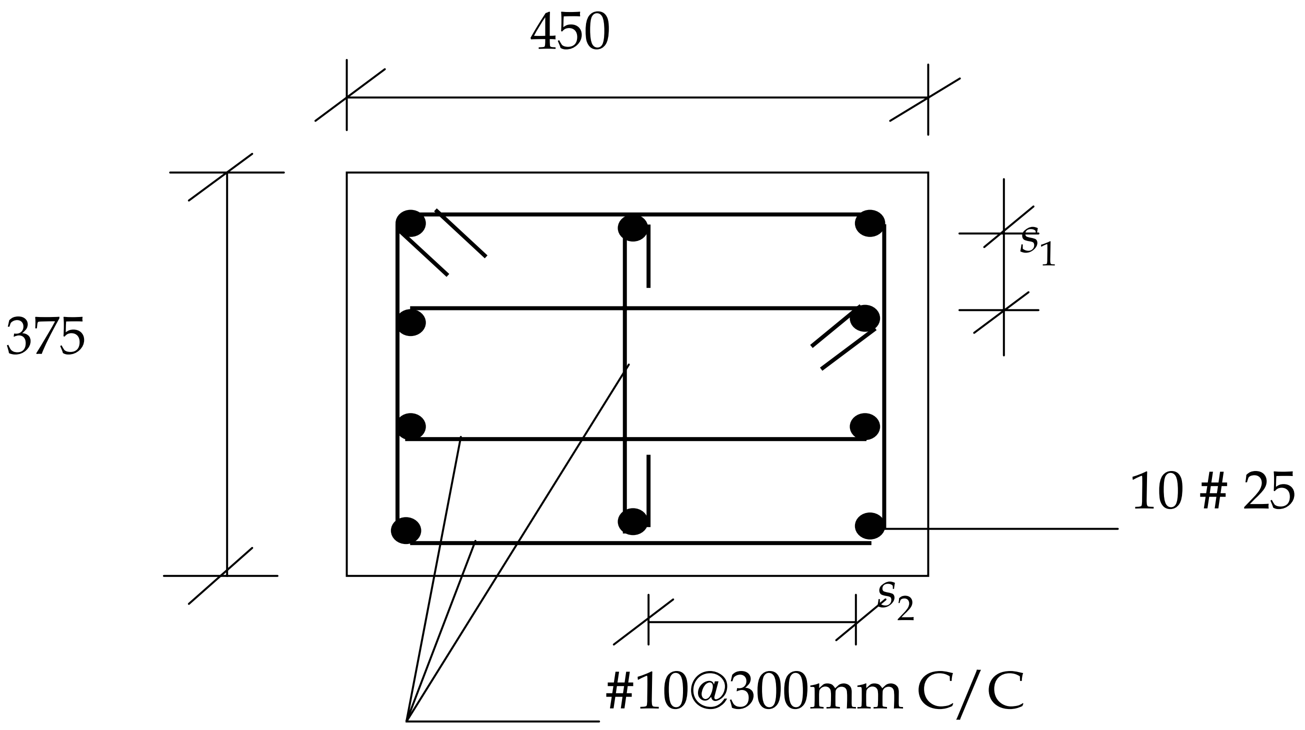


Fig. 7.17. Column Cross-Section for Example 7.5.

Assignment – Chapter 7