Plain & Reinforced Concrete-1

Sixth Term
Civil Engineering

CE-314

Lecture # 18

Analysis and Design of Slabs

Example 6.1: Design a slab of 10 m × 3.75 m clear dimensions supported over 342 mm thick walls on all the four sides. This slab is part of a residential house. Use C–18 concrete and Grade 280 steel. Use US Customary bars and prepare bar bending schedule.

Solution:

- The slab is supported on all the four sides and longer to shorter dimensions is 10 / 3.75 $\cong$ 2.67.
- Hence, the slab is one-way along the 3.75 m side.
- For the calculation of slab depth, the span length may be considered equal to the clear dimension plus half brick length bearings on the two sides.

End Conditions	Steel Grades		
	280 or 300	**420**	
Simply Supported	L/25	L/20	
One end continuous	L/30	L/24	
Both ends continuous	L/35	L/28	
Cantilever	L/12	L/10	

- h_{min} = L / 25 = (3750 + 2×114) / 25 = 159 mm (say 160 mm)
- d ≅ h – 27 = 133 mm
- L = clear span + h = 3.91 m

Dead Load

- R. C. slab: 0.160 × 2400 = 384 kgs / m^2
- 75 mm screed of brick ballast: 0.075 × 1800 = 135 kgs / m^2
- P. C. C. + terrazzo: 0.060 × 2300 = 138 kgs / m^2

q_D = 657 kgs / m^2

Live Load

- For residential building: q_L = 200 kgs / m^2

Factored Load

- $q_u = 1.2\ q_D + 1.6\ q_L$

  $= (1.2 \times 657 + 1.6 \times 200) \times 9.81 / 1000$

  $= 10.87$ kN / m^2

  $= 10.87$ kN / m per meter width

Factored Bending Moment

- $M_u = 1 / 8\ q_u\ L_n^2 \cong 1 / 8\ q_u\ L^2$ (approximation on safe side)

  $= 1/8 \times 10.87 \times 3.91^2 = 20.78$ kN-m per meter width

Main Reinforcement

- R = M_u / bd^2 = 20.78×10^6 /(1000×133^2)
  = 1.1747 MPa
- f_c' = 18 MPa ≅ 17.27 MPa : f_y = 280 MPa
- ρ = 0.005
- A_s = $\rho \times b \times d$ = $0.005\times1000\times133$ = 665 mm^2

Diameter And Spacing

- Using #10 bars: #10 @ 100 mm c/c provides
  A_s = 710 mm^2
- Using #13 bars: #13 @ 190 mm c/c provides
  A_s = 679 mm^2

Maximum preferred spacing: least of

i) $2h$ = 320 mm (Code value is $3h$)

ii) 300 mm (Code value is 450 mm)

iii) $159{,}600 / f_y - 2.5c_c$

$= 159{,}600 / 280 - 2.5 \times 20 = 520$ mm

iv) $126{,}000 / f_y = 126{,}000 / 280 = 450$ mm

$s_{max} = 300$ mm

Selected main reinforcement:

#13 @ 200 mm c/c

Temperature Reinforcement

- Temperature steel: $0.002 \times b \times h$
  $= 0.002 \times 1000 \times 160 = 320\ \text{mm}^2$
- Using #10 bars: #10 @ 200 mm c/c
  provides $A_s = 355\ \text{mm}^2$

Maximum preferred spacing: least of

i) $2.5h = 400$ mm (Code value is $5h$)

ii) 375 mm (Code value is 450 mm)

iii) $159{,}600 / f_y - 2.5c_c$
$= 159{,}600 / 280 - 2.5\times20 = 520$ mm

iv) $126{,}000 / f_y = 126{,}000 / 280 = 450$ mm

$s_{max} = 375$ mm

Selected temperature reinforcement: #10 @ 200 mm c/c

Check For Shear

- $V_u = q_u\,(L_n / 2 - d)$

  $= 10.87 \times (3.75/2 - 0.133) = 18.94$ kN

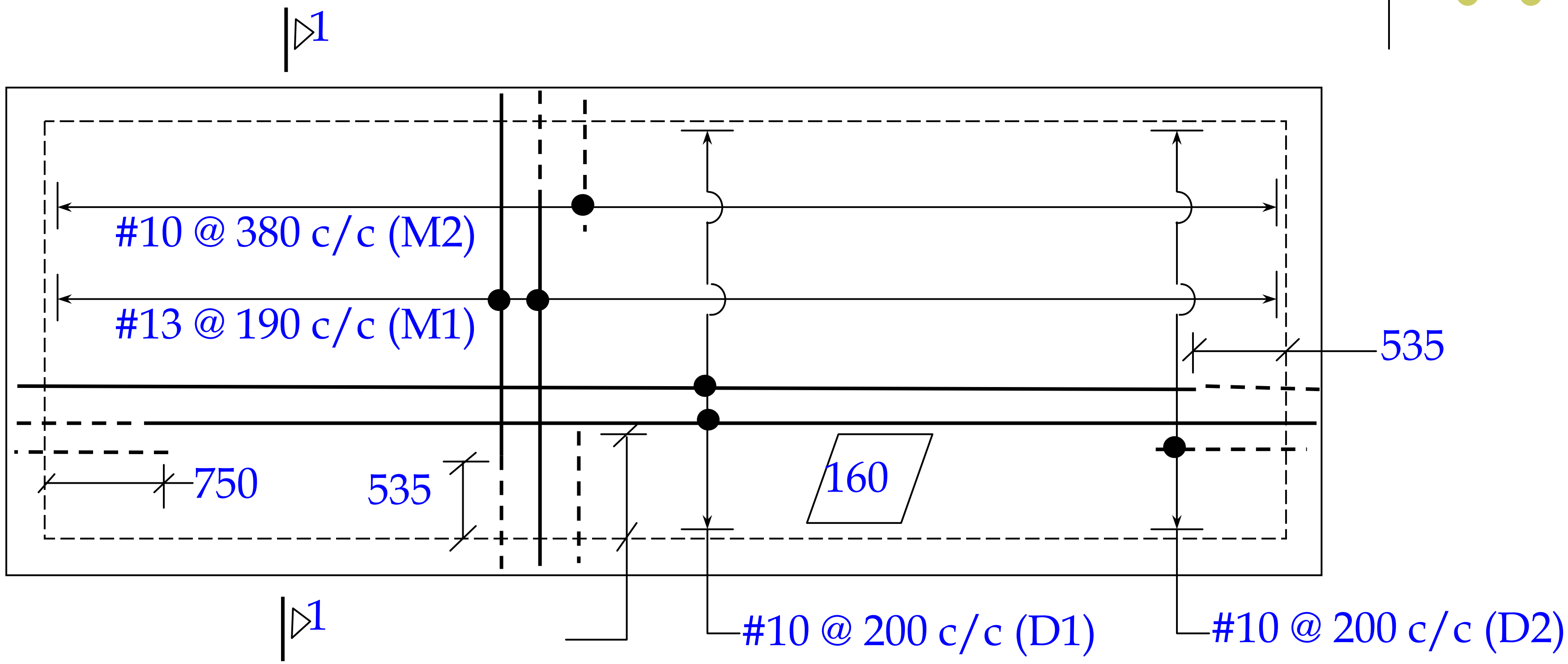
- $\phi_c V_c = 0.75 \times 0.17\lambda\sqrt{f_c'}\; b_w\, d$

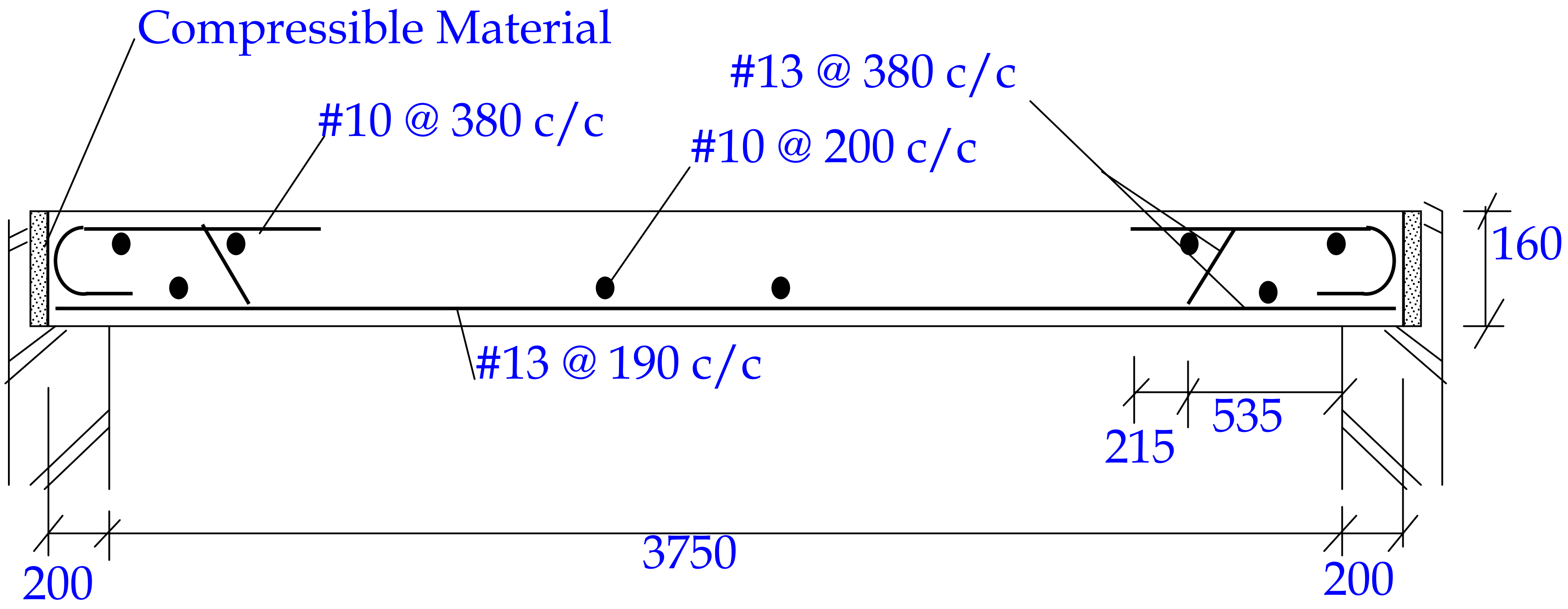
  $= 0.75 \times 0.17 \times 1 \times \sqrt{18} \times 1000 \times 133 / 1000$

  $= 71.94$ kN

- The applied shear force is significantly lesser than even $\phi_c V_c / 2$.

Sketch Of Reinforcement: The reinforcement details are shown in Fig. 6.2.

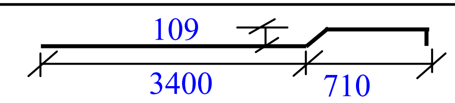
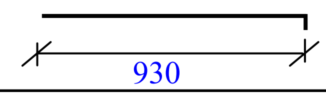
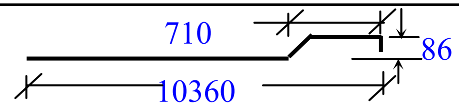
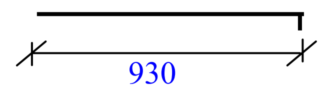
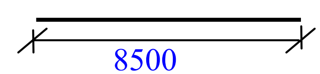
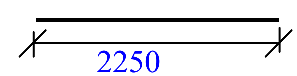

1
#10 @ 380 c/c (M2)
#13 @ 190 c/c (M1)
535
750
535
160
1
#10 @ 200 c/c (D1)
#10 @ 200 c/c (D2)


Compressible Material
#13 @ 380 c/c
#10 @ 380 c/c
#10 @ 200 c/c
160
#13 @ 190 c/c
535
215
3750
200
200

Bar Bending Schedule

- Number of bars for M-1 = 10456 / 190 = (say 55)
- Total length of bars M-1 = 3750 + 2 × 180 = 4110 mm
- Length M-1 for estimation = 3750 + 2 (200 – 20) + 0.414 × 109 + 18 × 13 = 4.389 m
- Number of bars for M-2 = 10456 / 380 × 2 = 55
- Number of bars for D-1 = 3750 / 200 = 19 (say 51)
- Bottom length of bars D-1 = 10000 – 530 + 180 = 9650 mm
- Total length of bars D-1 = 10000 + 2 × 180 + 18 × 10 = 10.540 m
- Length D-1 for estimation = 10360 + 0.414 × 86 = 10.40 m
- Number of bars for D-2 = 3750 / 400 × 2 = 20
- Length of bars D-2 and M-2 = 750 + 180 + 18 × 10 = 1.110 m

Table 6.3. Bar Bending Schedule For Example 6.1. Steel Grade: 300

S. No.	**Bar Desig-nation**	**No. Of Bars**	**Len. Of Bar (m)**	**Dia. Of Bar (mm)**	**Weight Of Bars**				**Shape Of Bar**
					No.10	**No.13**			
1	M-1	55	4.389	#13		240			
2	M-2	55	1.110	#10	35				
3	D-1	19	10.54	#10	113				
4	D-2	20	1.110	#10	13				
5	H-1	4	8.50	#10	19				
6	H-2	4	2.25	#10	5				
				$\sum =$	195	252			

Total Steel Required ≅ 447 kgs

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Example: Design a cantilever projecting out from a room slab extending 1.0m and to be used as balcony (LL = 300 kg/m^2). A brick wall of 250 mm thickness including plaster of 1m height is provided at the end of cantilever.

$f_c' = 18$ MPa $f_y = 280$ MPa

Slab thickness of room = 125 mm. Slab bottom steel in the direction of cantilever is **# 13 @ 190 mm c/c**, alternate bars are bent up.

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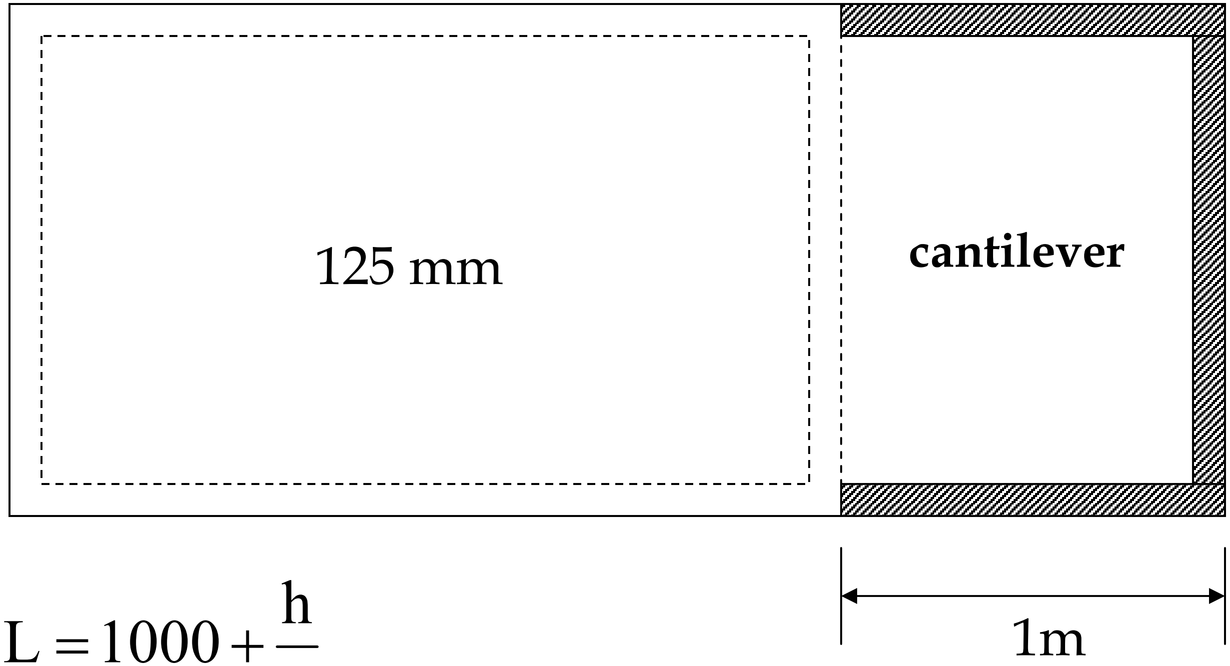
Solution:



$$L = 1000 + \frac{h}{2}$$

$$L = 1000 + \frac{125}{2} = 1063mm \approx 1.07m$$

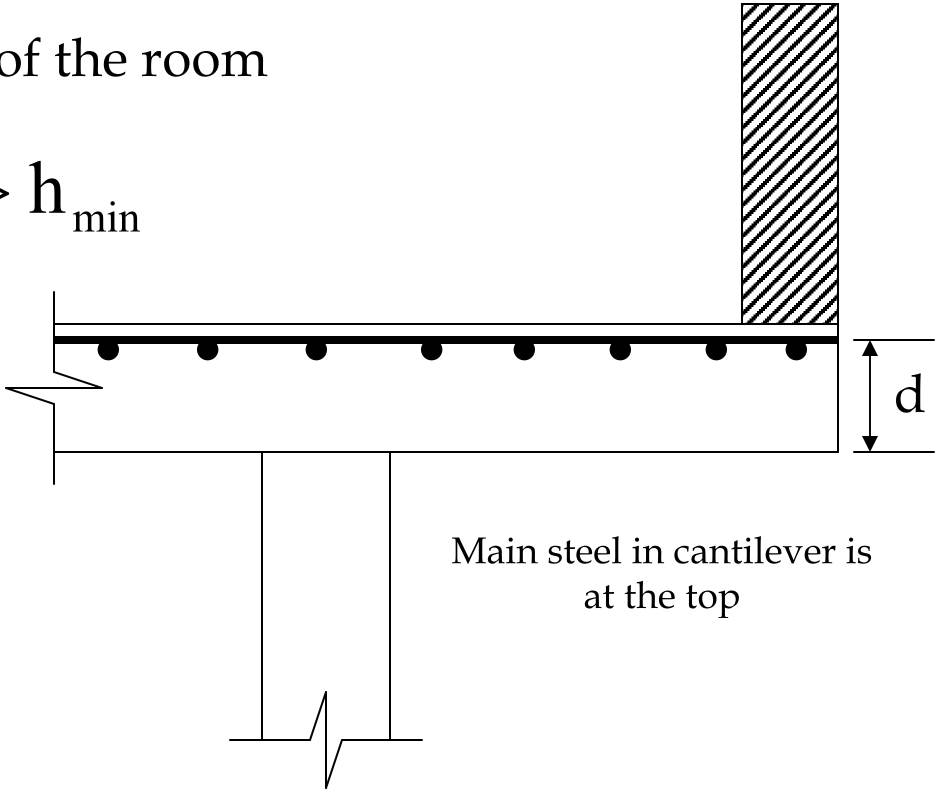
Plain & Reinforced Concrete-1

Solution: (contd…)

$$h_{\min} = \frac{L}{12} = \frac{1063}{12} = 89mm$$

Let us use the same thickness as of the room

$$\mathrm{h} = 125\mathrm{mm} > \mathrm{h}_{\min}$$



$$\mathrm{d} = 125 - 20 - 7 = 98\mathrm{mm}$$

Plain & Reinforced Concrete-1

Solution: (contd…)

Slab Load

Self weight of slab $= \frac{125}{1000} \times 2400 = 300 \text{kg}/\text{m}^2$

75 mm brick ballast/ screed $= \frac{75}{1000} \times 1800 = 135 \text{kg}/\text{m}^2$

60 mm floor finishes $= \frac{60}{1000} \times 2300 = 138 \text{kg}/\text{m}^2$

Total dead load $= 300 + 135 + 138 = 573 \text{kg}/\text{m}^2$

Plain & Reinforced Concrete-1

Solution: (contd…)

Slab Load

$$\text{Live Load} = 300\text{kg}/\text{m}^2$$

$$w_u = (1.2\times573+1.6\times300)\times\frac{9.81}{1000}$$

$$=11.46kN/m^2$$

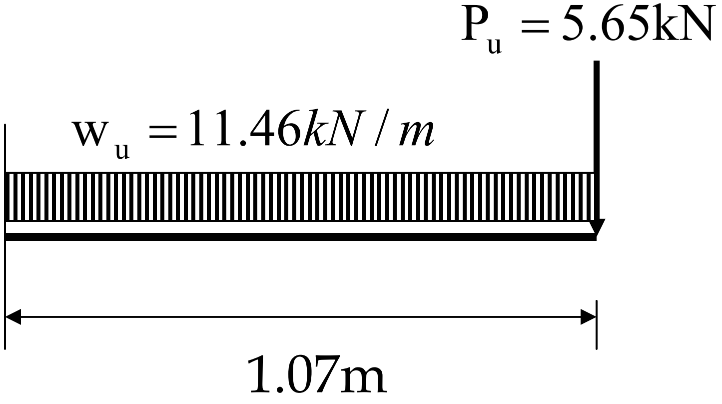
$$=11.46kN/m$$ For a unit strip

$$P_u = 1.2(0.25\times1\times1)\times1920\times\frac{9.81}{1000}$$

$$=5.65kN$$

Plain & Reinforced Concrete-1

Solution: (contd…)



$$M_u = P_u \times L + \frac{w_u L^2}{2}$$

$$= 5.65 \times 1.07 + \frac{11.46 \times 1.07^2}{2}$$

$$= 12.6kN - m$$

Per meter width

$$\frac{M_u}{bd^2} = \frac{12.6 \times 10^6}{1000 \times 98^2} = 1.3120 \qquad \omega = 0.85 \frac{f_c'}{f_y} = 0.0546$$

$$\rho = \omega\left(1 - \sqrt{1 - \frac{2.614R}{f_c'}}\right) = 0.0546\left(1 - \sqrt{1 - \frac{2.614 \times 1.312}{18}}\right) = 0.00548$$

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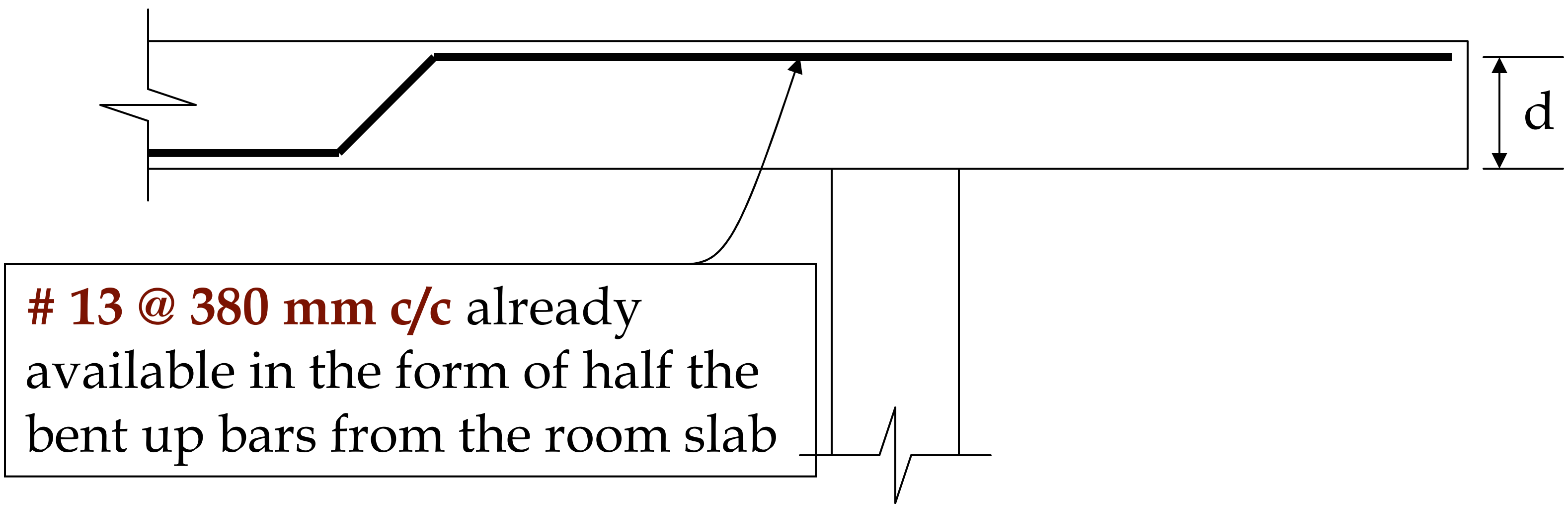
Solution: (contd…)

$$A_s = 0.00548 \times 1000 \times 98 = 538\text{mm}^2$$



$$\#13@380c/c \Rightarrow A_s = 342\text{mm}^2$$

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Solution: (contd…)

Remaining steel required at the top $= 538 - 342 = 196\text{mm}^2$

$$\#10 @ 350 c/c$$

Or $\boxed{\#13 @ 380 c/c}$

Distribution steel $= 0.002 \times 1000 \times 125 = 250\text{mm}^2$

$$\boxed{\#10 @ 275 c/c}$$

Maximum preferred spacing: least of

i) $2.5h$ = 312 mm (Code value is $5h$)

ii) 375 mm (Code value is 450 mm)

iii) $159{,}600 / f_y - 2.5c_c$

$= 159{,}600 / 280 - 2.5 \times 20 = 520$ mm

iv) $126{,}000 / f_y = 126{,}000 / 280 = 450$ mm

$s_{max} = 375$ mm

Selected temperature reinforcement:

#10 @ 275 mm c/c

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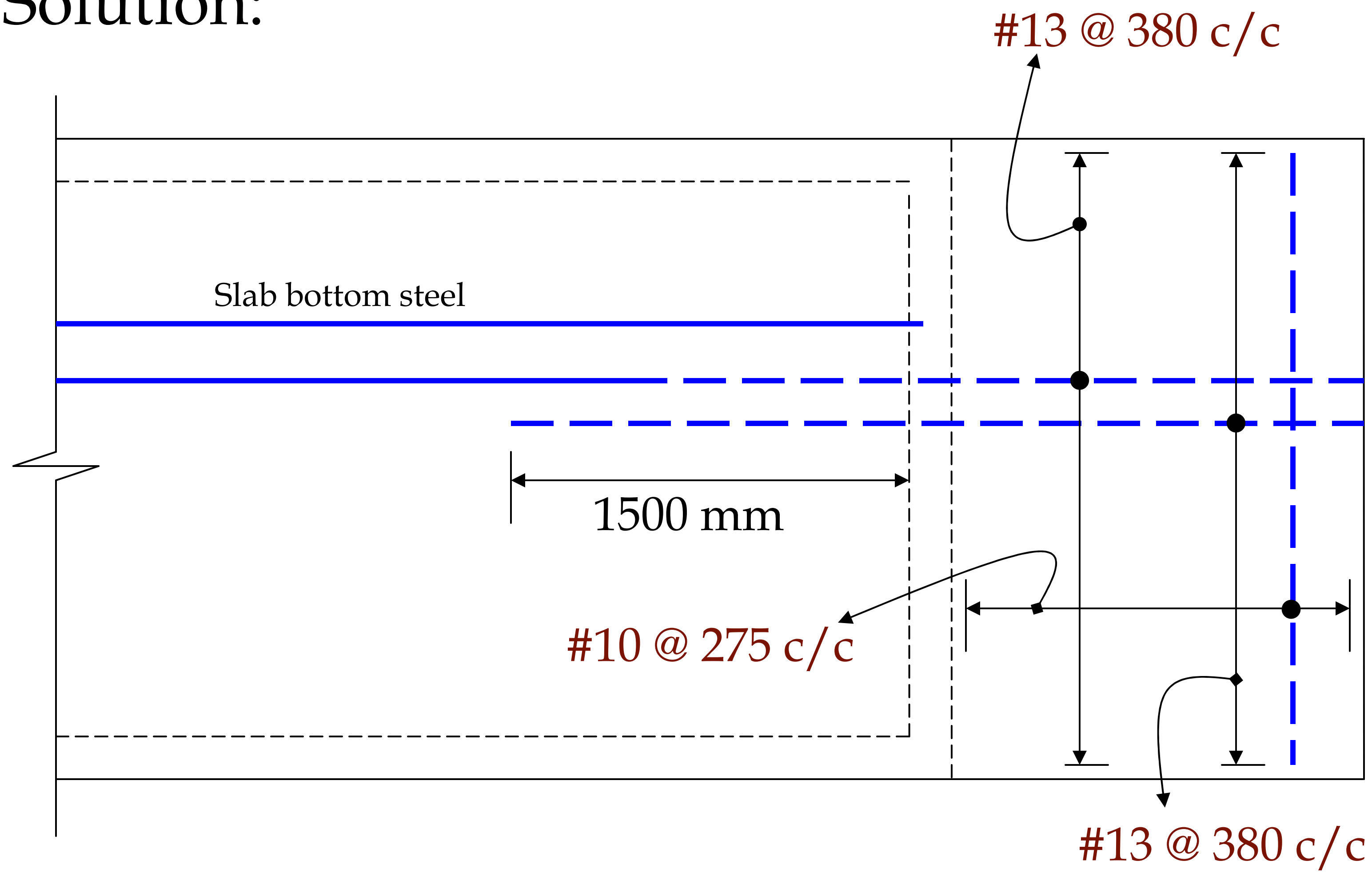
Solution:



Example 6.3: Design a slab consisting of eight panels of 8 m × 3. 5 m clear dimensions, continuous along their longitudinal edges, that are supported on 300 mm wide beams. Office live load is to be used along with a floor finish load of 300 kg/m^2 and 200 kg/m^2 immovable partition load. Use C–20 concrete, Grade 280 steel and US Customary bars.

Solution:

A unit strip of slab, taken along the shorter direction, acts as a continuous beam and is shown in Fig. 6.5.

$L \cong 3500 + 300 = 3.8 \text{ m}$

h_{min} for end panel $= L / 30 = 3800 / 30$

$= 127$ mm(say 130 mm)

$d \cong h - 27 = 103$ mm

Dead Load

- R. C. slab: 0.130×2400 = 312 kgs / m^2
- Floor finish: = 300 kgs / m^2
- Partition load: = 200 kgs / m^2

q_D = 812 kgs / m^2

Live Load

For office building: q_L = 250 kgs / m^2

Factored Load

$$q_u = 1.2\, q_D + 1.6\, q_L$$

$$= (1.2 \times 812 + 1.6 \times 250) \times 9.81 / 1000$$

$$= 13.48 \text{ kN / m}^2$$

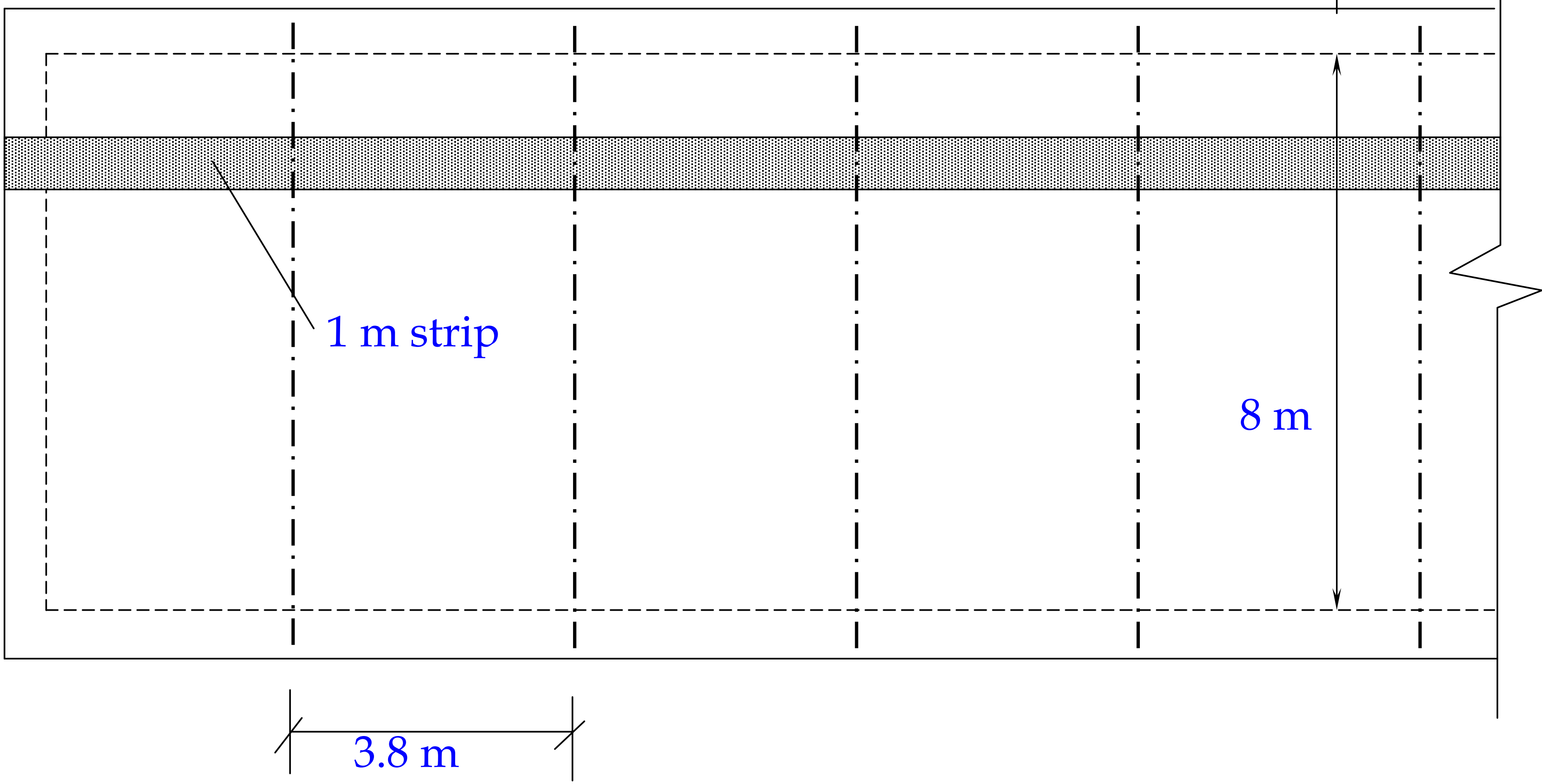

1 m strip
8 m
3.8 m

Table 4.1. Moment Coefficients for Slabs Having Spans Lesser Than 3.0 m ***OR*** Beams Having Ratio of Sum of Column Stiffness to Beam Stiffness More Than 8 at Each End of the Span.

1. Negative moments at all supports, integrally built with beams.	$\frac{1}{12} w_u \ell_n^2$
2. Positive moment in end panel.	$\frac{1}{14} w_u \ell_n^2$
3. Positive moment in central panels.	$\frac{1}{16} w_u \ell_n^2$

Table 4.2. Moment and Shear Values for Beams and Slabs Having Spans Greater Than 3.0 m.

1. Positive Moment	
End spans:	
If discontinuous end is unrestrained	$\frac{1}{11} w_u \ell_n^2$
If discontinuous end is integral with the support	$\frac{1}{14} w_u \ell_n^2$
Interior spans:	$\frac{1}{16} w_u \ell_n^2$
2. Negative moment at exterior face of first interior support	
Two spans:	$\frac{1}{9} w_u \ell_n^2$
More than two spans:	$\frac{1}{10} w_u \ell_n^2$

3. Negative moment at other faces of interior supports	$\frac{1}{11}\, w_u \ell_n^{\,2}$
(ℓ_n in no. 3 is the average of clear spans of the two adjacent panels.)	
4. Negative moment at interior faces of exterior supports for members built Integrally with their supports:	
The support is a spandrel beam or girder:	$\frac{1}{24}\, w_u \ell_n^{\,2}$
The support is a column:	$\frac{1}{16}\, w_u \ell_n^{\,2}$
The support is not monolithic:	Zero
5. Shear in end members at first interior support	$1.15\, \frac{w_u \ell_n}{2}$
6. Shear at all other supports	$\frac{w_u \ell_n}{2}$

Factored Bending Moments

ℓ_n = 3.5 m

- Exterior support M_u^- = $1 / 24\ q_u\ \ell_n^2$

  = $13.48 \times 3.5^2 / 24$ = 6.88 kN-m / m

- Exterior span M_u^+ = $1 / 14\ q_u\ \ell_n^2$

  = $13.48 \times 3.5^2 / 14$ = 11.80 kN-m / m

- First int. support M_u^- = $1 / 10\ q_u\ \ell_n^2$

  = $13.48 \times 3.5^2 / 10$ = 16.51 kN-m / m

- Interior support M_u^- = $1 / 11\ q_u\ \ell_n^2$

  = $13.48 \times 3.5^2 / 11$ = 15.01 kN-m / m

- Interior span M_u^+ = $1 / 16\ q_u\ \ell_n^2$

  = $13.48 \times 3.5^2 / 16$ = 10.32 kN-m / m

Table 6.4. Calculation of Steel for Example 6.3.					
Section	M_u/bd^2	ρ	ρbd	$A_{s,min}$	A_s
Exterior support M_u^-	0.6485	0.0027	278	260	278
Exterior span M_u^+	1.1123	0.0047	484	260	484
First interior support M_u^-	1.5562	0.0066	680	260	680
Interior support M_u^-	1.4148	0.0060	618	260	618
Interior span M_u^+	0.9728	0.0041	423	260	423

- Exterior span M_u^+: A_s = 475 mm²
  #13 @ 250 mm c/c
- Interior span M_u^+: A_s = 420 mm²
  #10 @ 160 mm c/c
- The balance top steel, after considering the area of bent up bars, at the supports is given below:

Section	A_s	***Available Steel***	***Balance*** A_s	***Extra Steel***
Exterior support M_u^-	278	258	20	#10 @ 500 mm c/c
First interior support M_u^-	680	468	212	#10 @ 300 mm c/c
Interior support M_u^-	618	420	198	#10 @ 350 mm c/c

Check For Maximum Spacing Of Main Steel

Maximum preferred spacing: least of

i) $2h$ = 260 mm (Code value is $3h$)

ii) 300 mm (Code value is 450 mm)

iii) $159{,}600 / f_y - 2.5c_c$

$= 159{,}600 / 280 - 2.5c_c = 520$ mm

iv) $126{,}000 / f_y = 126{,}000 / 280 = 450$ mm

$s_{max} = 260$ mm

Temperature Reinforcement

- Temperature steel: $0.002 \times b \times h$
  $= 0.002 \times 1000 \times 130 = 260 \text{ mm}^2$
- #10 @ 250 mm c/c provides $A_s = 284 \text{ mm}^2$

Maximum preferred spacing: least of

i) $2.5h = 400$ mm (Code value is $5h$)

ii) 375 mm (Code value is 450 mm)

iii) $159{,}600 / f_y - 2.5c_c = 159{,}600 / 280 - 2.5c_c$
$= 520$ mm

iv) $126{,}000 / f_y = 126{,}000 / 280 = 450$ mm

$s_{max} = 375$ mm

Selected temperature reinforcement:

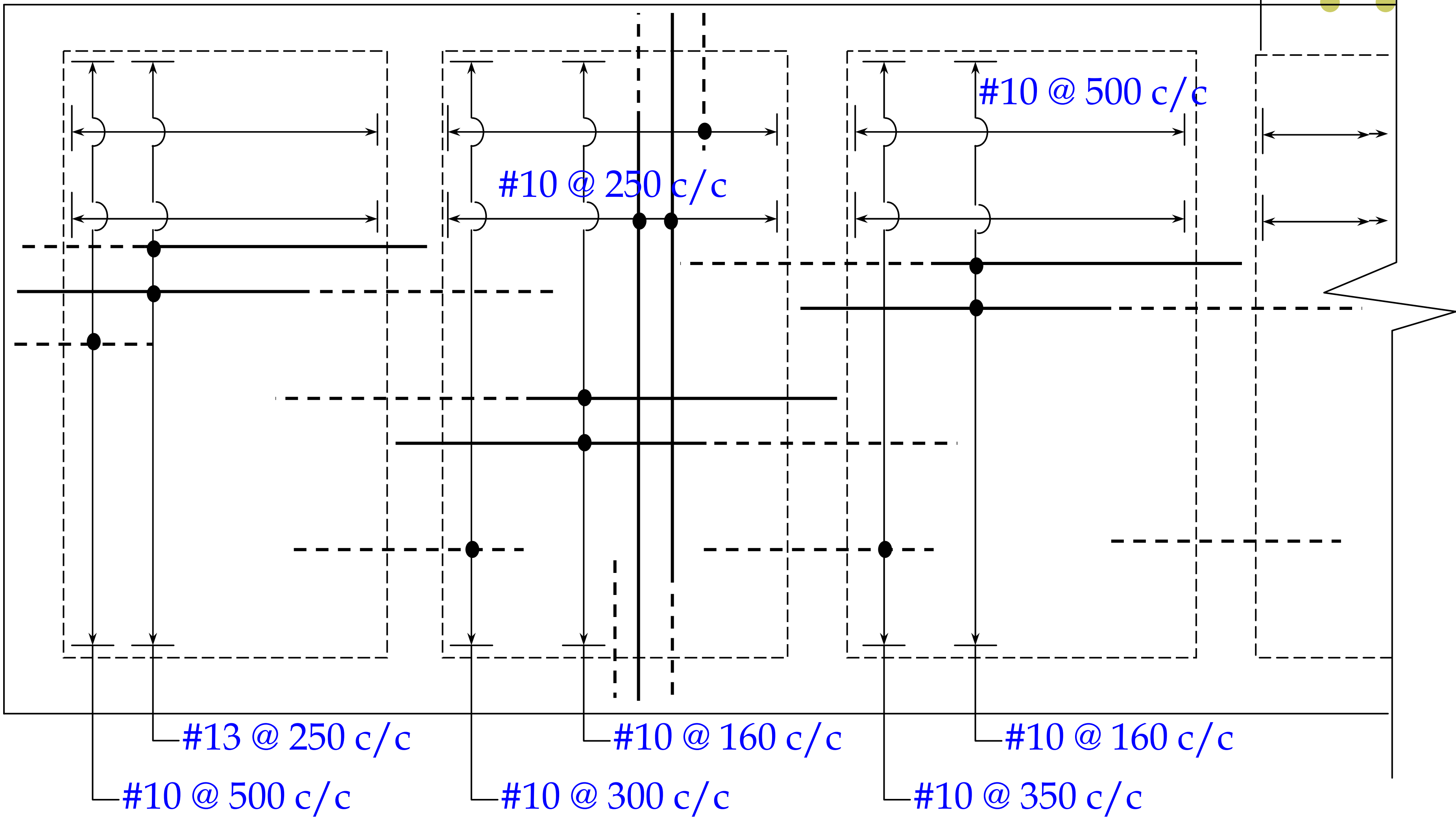
#10 @ 250 mm c/c

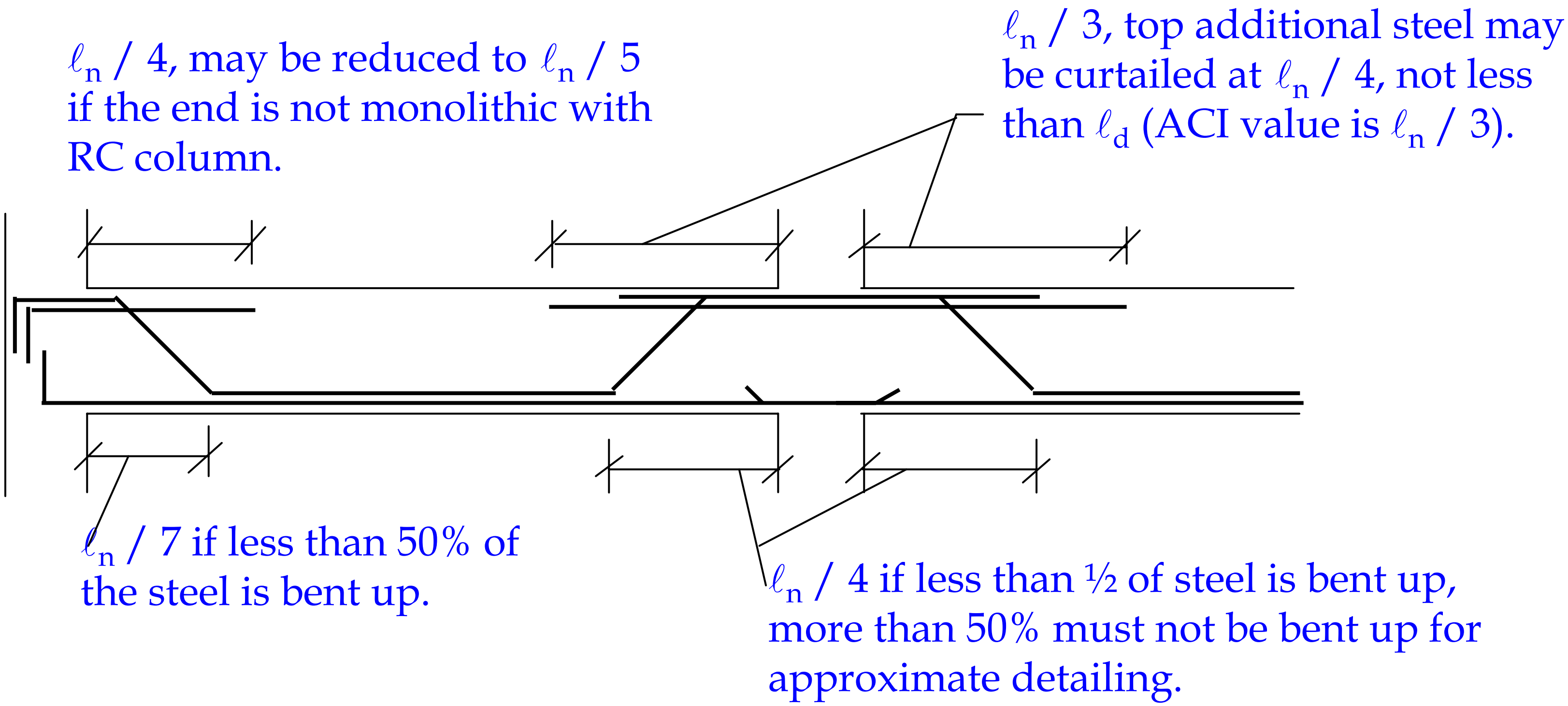
Check For Shear

$$V_u = 1.15 \times q_u \,(L_n / 2 - d)$$
$$= 1.15 \times 13.48 \times (3.5 /2 - 0.103)$$
$$= 25.53 \text{ kN}$$

$$\phi_c V_c = 0.75 \times 0.17\lambda\sqrt{f_c'} \; b_w \, d$$
$$= 0.75 \times 0.17 \times 1 \times \sqrt{20} \times 1000 \times 103 / 1000$$
$$= 58.73 \text{ kN}$$

- The applied shear force is significantly lesser than even $\phi_c V_c / 2$.

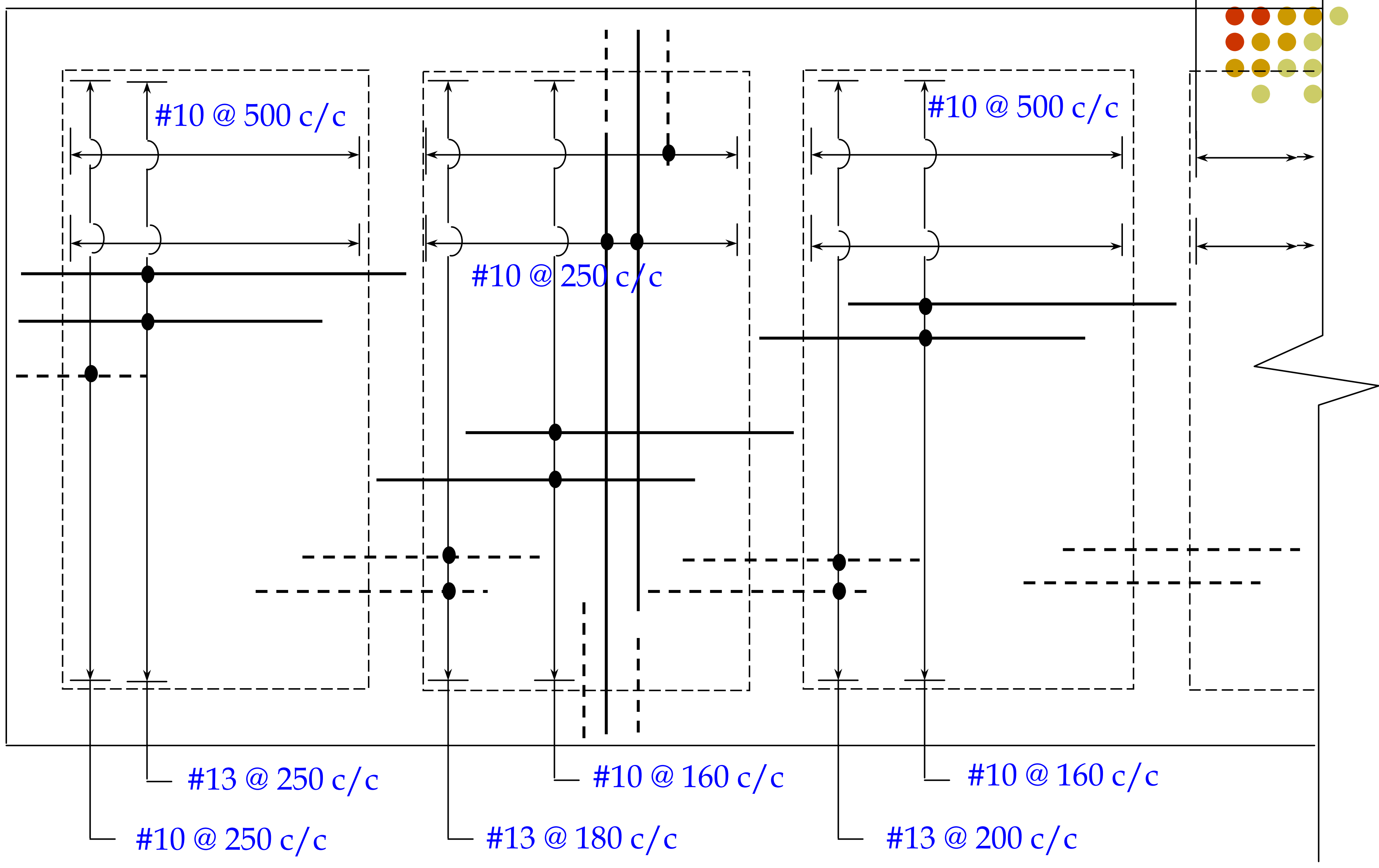

#10 @ 500 c/c
#10 @ 250 c/c
#13 @ 250 c/c
#10 @ 160 c/c
#10 @ 160 c/c
#10 @ 500 c/c
#10 @ 300 c/c
#10 @ 350 c/c



c) Bent-Up Bars

Note: For slabs the distances $\ell_n/4$ and $\ell_n/3$ for top extensions may be reduced to $0.22\ell_n$ and $0.3\ell_n$, respectively. Similarly, the bottom distance $\ell_n/4$ may be reduced to $0.22\ell_n$.

Type of Steel Reinforcement	***Value***	***Length (mm)***
Top steel extension from face of exterior support, for short and long directions.	$\ell_n / 5$	700
Top steel extension of bent-up bars on opposite side from face of supports.	$0.3\ell_n$	1050
Bottom bars bent-up point from face of supports, for short and long directions.	$0.22\ell_n$	770
Top extra steel on interior supports, on both sides, from the face of supports.	$0.22\ell_n$	770



#10 @ 500 c/c
#10 @ 250 c/c
#10 @ 500 c/c
#13 @ 250 c/c
#10 @ 160 c/c
#10 @ 160 c/c
#10 @ 250 c/c
#13 @ 180 c/c
#13 @ 200 c/c

Type of Steel Reinforcement	*Value*	*Length (mm)*
Top steel extension from face of exterior support, for short and long directions.	$\ell_n / 5$	700
Alternate bottom bars curtailed from face of inner supports, in short direction.	$\ell_n / 8$	435
Alternate bottom bars curtailed from face of supports, in long direction.	same as in short direction	435
Extension of top extra steel on interior supports, from the face of supports, for alternate bars.	$0.3\ell_n$	1050
Extension of top extra steel on interior supports, from the face of supports, for remaining alternate bars.	$0.22\ell_n$	770

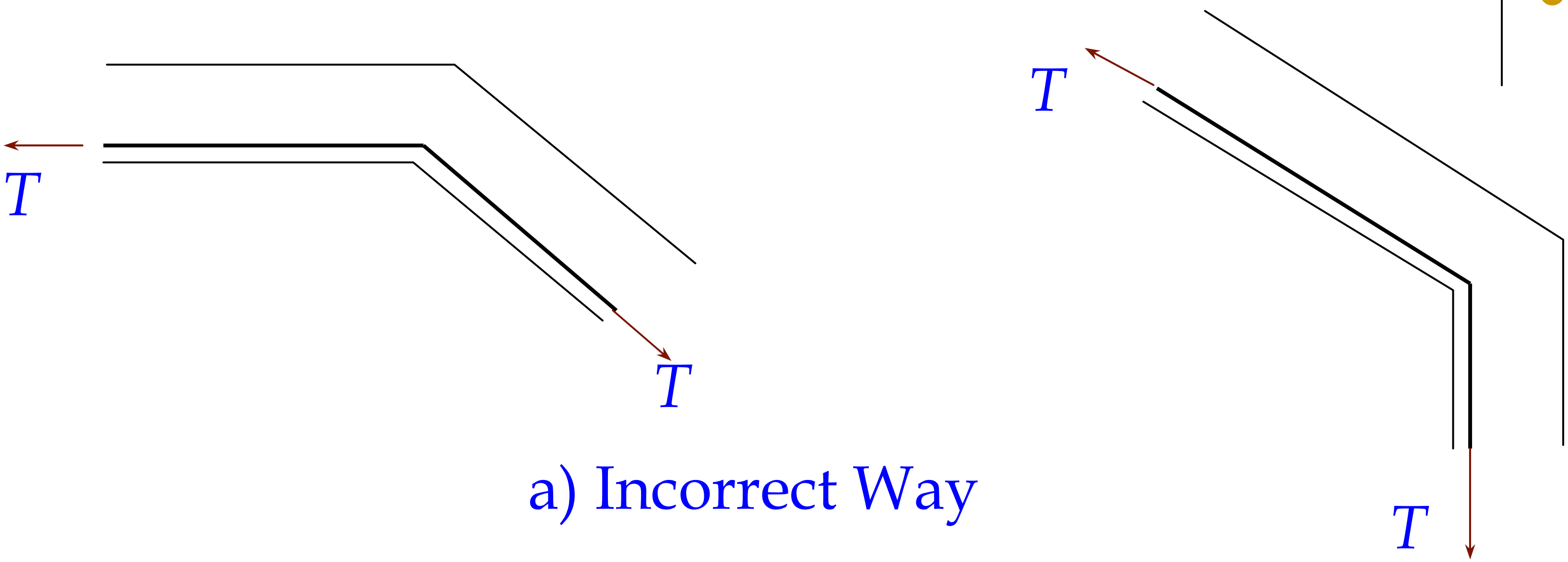
Design Of Stair Slab

- The slab underneath the stairs is designed as one-way slab for the expected live loading, dead load of R. C. slab, dead load of steps and dead load of floor finishes.
- The thickness of the slab for stair is called its waist dimension.
- Following points are to be considered for such a design:

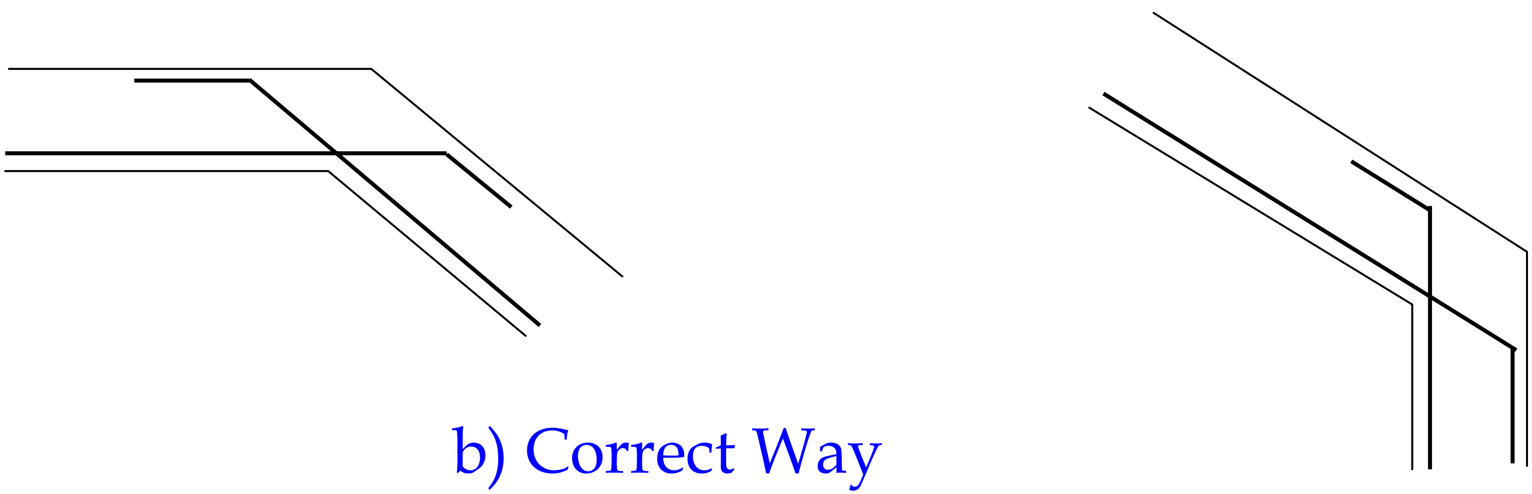
1. The span length and loading with respect to the horizontal plan are considered for the calculation of bending moments.
2. The self-weight of the stair slab is first calculated in the inclined plane and is then multiplied with $\sqrt{R^2 + T^2} / T$ or approximately 1.22 to calculate the load on the horizontal plan, where *R* is the riser and *T* is the tread.

3. In case only one or no edge of steps is supported on walls, the stair is considered to span longitudinally. However, the slab may be assumed to span along the width of the steps if there is newel wall towards the inner side and both edges of the slab are supported.
4. Due to the inclined nature and availability of more stiffness, the waist dimension may be selected equal to both ends continuous one-way slab (*L*/35 for Grades 280 and 300 and *L*/28 for Grade 420). In case the landing is also supported along the other edges, the span of stair may be considered up to the center of the landing. However, in this case, the landing must be designed to carry all the corresponding loads along a direction perpendicular to the stair.

5. If the steps are made up of reinforced concrete, some minimum steel is to be provided within these steps.
6. A small and usually concealed beam, in-between the landing and the flight of stair, is beneficial to keep the depth of stair slab and the required reinforcement in the economical range.
7. Tension steel making an angle less than 180° and present on the inner side of this angle may cause falling of the concrete cover and loss of tensile force (Fig. 6.8). The detailing must be carried out to eliminate this situation.



a) Incorrect Way

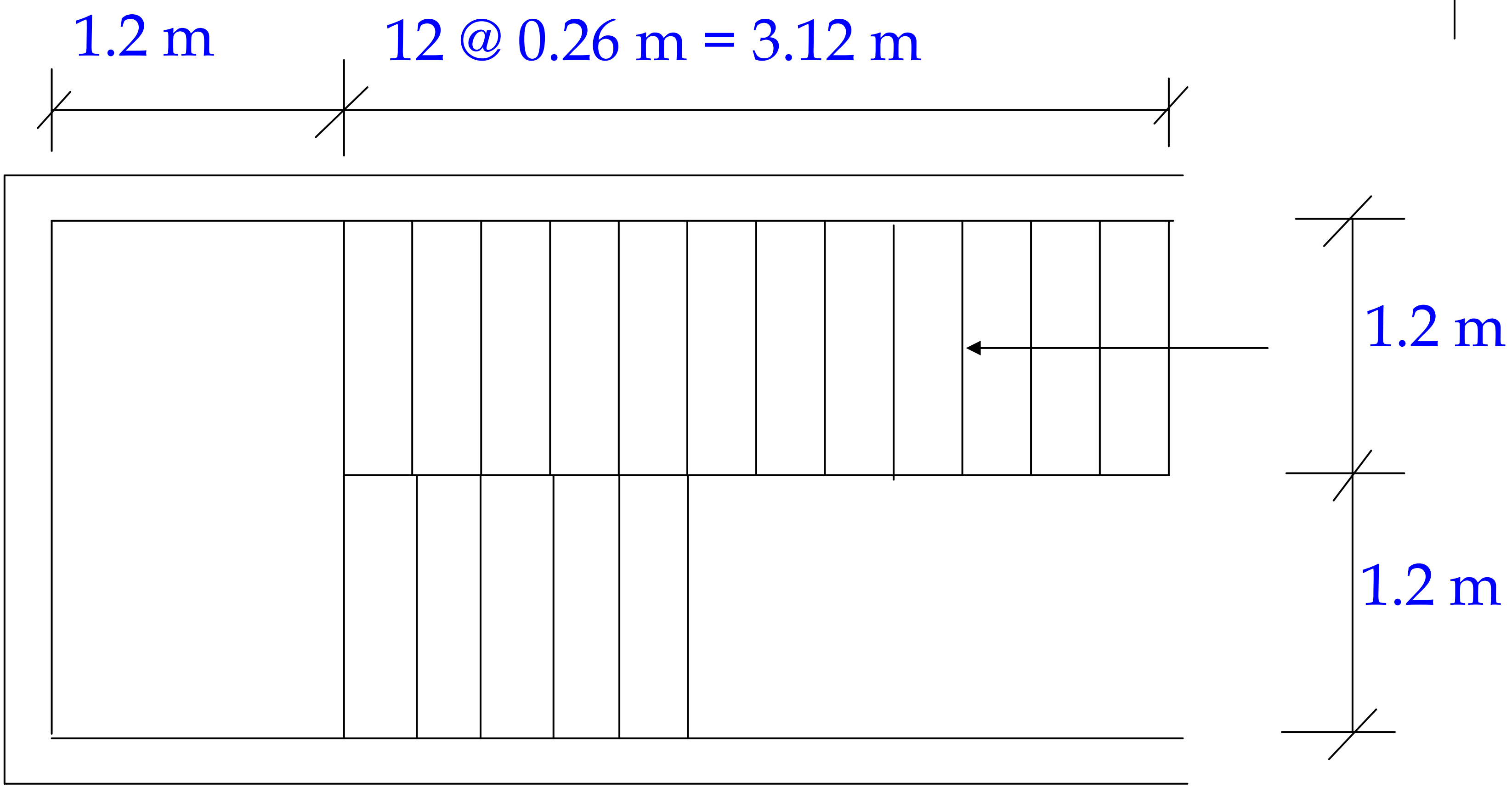


b) Correct Way

Example 6.4: Design the first flight of the stair shown in Fig. 6.9, having a reinforced concrete footing at the bottom. Use C – 18 concrete and Grade 280 steel. R = 180 mm and T = 260 mm. Select US Customary reinforcement.

Solution:

- L $\cong$ 1.2 + 3.12 = 4.32 m
- h_{min} considering both ends continuous/fixed = L / 35
  = 4320 / 35 = 124 mm
  (say 125 mm)
- $d \cong h - 27$ = 98 mm



Plan View of Stair for Example 6.4.

Dead Load

- R. C. slab: $0.125 \times 2400 \times (180^2+260^2)^{0.5} / 260$
  = 365 kgs / m^2
- Weight of steps: $(R/2000) \times 2400$ = 216 kgs / m^2
- 15 mm floor finish: 0.015×2300 = 35 kgs / m^2

  q_D = 616 kgs / m^2

Live Load

- For stairs: q_L = 300 kgs / m^2

Factored Slab Load

- $q_u = 1.2\, q_D + 1.6\, q_L$
  $= (1.2 \times 616 + 1.6 \times 300) \times 9.81 / 1000$
  = 11.96 kN / m^2
  = 11.96 kN / m per meter width

Factored Bending Moment

- $M_u \cong$ 1 / 10 q_u L^2 (one end continuous)

  = $1/10 \times 11.96 \times 4.32^2$

  = 22.4 kN-m per meter width
- d_{min} for singly reinforced section

$$= \sqrt{\frac{M_u}{0.205 f_c' b}} = \sqrt{\frac{22.4 \times 10^6}{0.205 \times 18 \times 1000}} = 78mm$$

Main Reinforcement

- M_u / bd^2 = $22.4 \times 10^6 / (1000 \times 98^2)$ = 2.3324 MPa
- f_c' = 18 MPa : f_y = 280MPa
- ρ = 0.0103
- A_s = $0.0103 \times 1000 \times 98$ = 1010 mm^2 per meter width

Diameter And Spacing

- Selected Steel = #13 @ 120 mm c/c
- 2 *h* = 300 mm (OK)

Temperature Reinforcement

- Temperature steel: $0.002 \times b \times h$

  $= 0.002 \times 1000 \times 125 = 250\ mm^2$
- Selected temperature reinforcement: #10 @ 275 mm c/c

Check For Shear

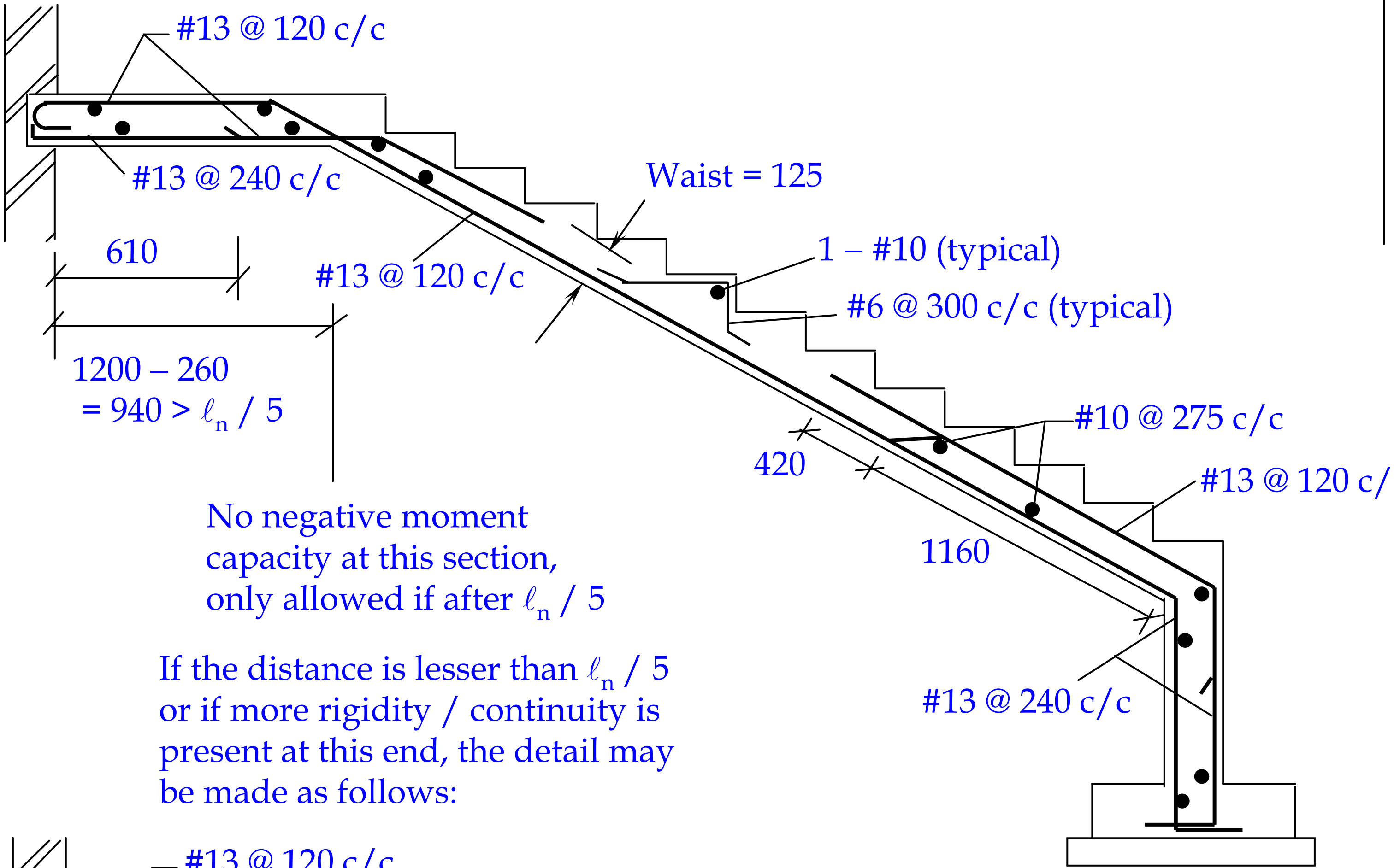
- $V_u = q_u \, (L_n / 2 - d)$

  $= 11.96 \times (4.32 /2 - 0.098) = 24.66$ kN
- $\phi_c V_c = 0.75 \times 0.17\sqrt{f_c'} \, b_w \, d$

  $= 0.75 \times 0.17\sqrt{18} \times 1000 \times 98 / 1000 = 53.0$ kN
- The applied shear force is significantly lesser than even $\phi_c V_c$

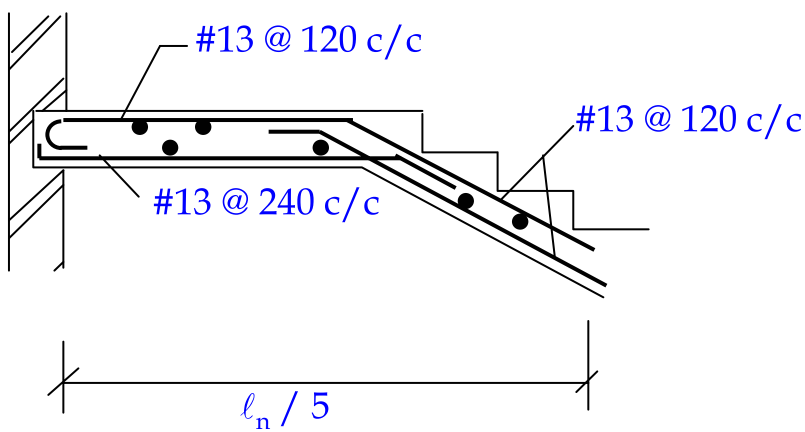
Curtailment Distances

- $L_n / 7 = 4320 / 7 = 617$ mm (say 610 mm)
- $L_n / 5 = 4320 / 7 = 864$ mm (say 870 mm)
- Inclined $0.22 \, L_n = 0.22 \times 4320 \times 1.22 = 1160$ mm
- Inclined $0.30 \, L_n = 0.30 \times 4320 \times 1.22 = 1580$ mm



No negative moment capacity at this section, only allowed if after ℓ_n / 5

If the distance is lesser than ℓ_n / 5 or if more rigidity / continuity is present at this end, the detail may be made as follows:



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