Plain & Reinforced Concrete-1

Sixth Term
Civil Engineering

CE-314

Lecture # 16

Shear Design of Beams

INTRODUCTION

- Shear stress is defined as the magnitude of the internal force acting parallel to the area divided by the area.
- This stress is produced due to the sliding of various layers of the material on one another.
- Shear stress along the depth of cross-section for homogeneous, elastic and uncracked beams may be calculated by using the following equation:

$$v = \frac{VQ}{Ib}$$

Where,

V = applied shear force at the section,

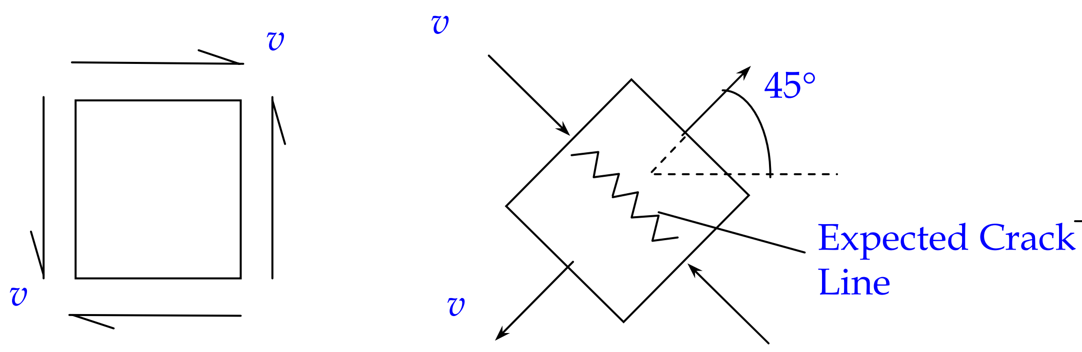
Q = first moment of area of the part of the section lying between the point where the shear stress is required and the nearest outermost fibers about the neutral axis,

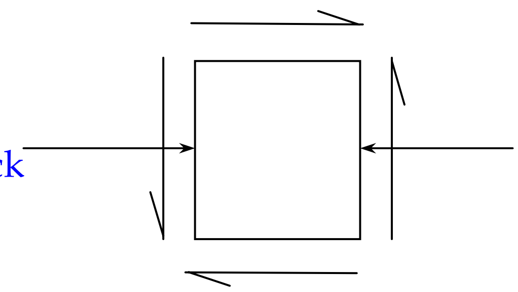
I = moment of inertia of the cross section about the corresponding axis of bending,

And

b = width of the section at the location where the shear stress is required.

- The shear stresses acting on the vertical sections are complementary to the horizontal shear stresses in the longitudinal direction of the beams.
- At the neutral axis, a differential element has only the shear stresses and hence the principal axes are oriented at 45° to the longitudinal axis of the member, as shown in Fig. 5.1.
- Away from the neutral axis, the principal stress angle increases until it becomes horizontal on the tension face, and decreases until it becomes vertical on the compression face.
- The tensile stress trajectories appear as shown in Fig. 5.2(a).

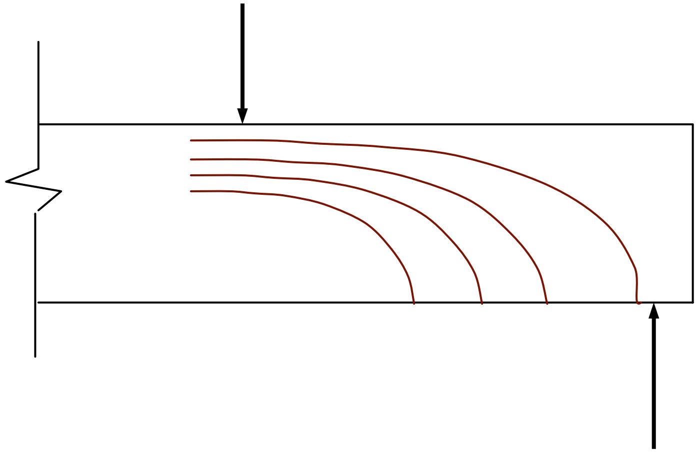


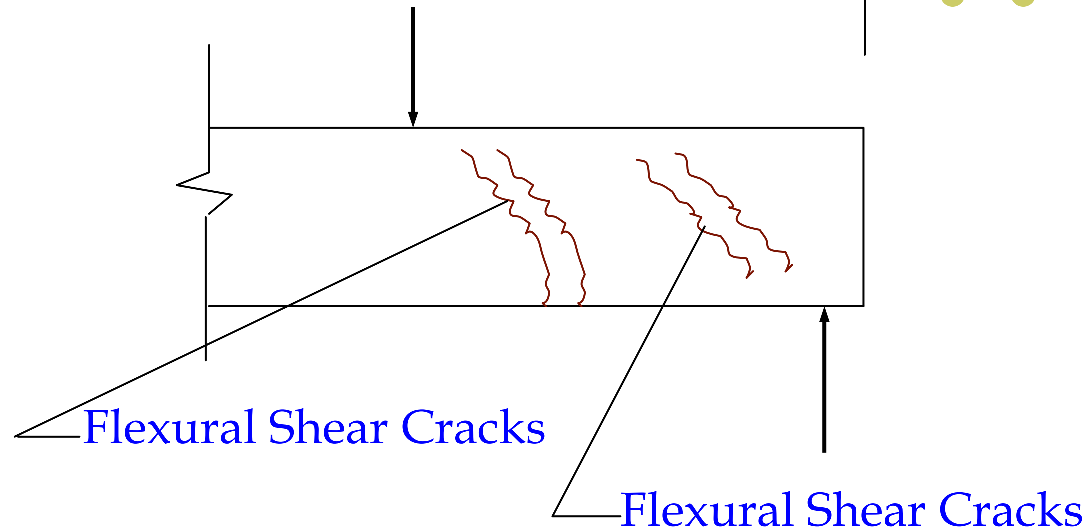


a) Differential Element at Neutral Axis Closer to Right Support.

b) Differential Element Away From Neutral Axis towards Compression Side.

- Cracks also develop on the same pattern along the tensile principal stress paths, Fig. 5.2(b).
- Near the supports, cracks appear at almost 45° originating from the neutral axis, which quickly spread towards the two faces.
- These shear cracks are called pure *shear*, *web* or *inclined shear cracks*.
- When these cracks extend towards the supports, they weaken the anchorage of the main steel.
- Away from the supports but within length of the beam having shear force, flexural cracks appear first on the tension side penetrating vertically upwards.





a) Stress Trajectories in a Beam.

b) Shear Cracks in a Beam.

- When the effective cross-section reduces due to these cracks, shear becomes dominant and the crack become inclined at angles closer to 45°.
- These cracks are termed as *flexural shear cracks*. The average shear stress along the depth of the beam is given by the formula:

$$v = V / b_w d$$

- Where, V is the applied shear force, b_w is the width of the beam and d is its effective depth.

CONCRETE SHEAR STRENGTH

It depends on the following factors:

1. Concrete compressive strength

2. Applied moment to shear ratio and effective depth

3. Longitudinal reinforcement ratio

CONCRETE SHEAR STRENGTH

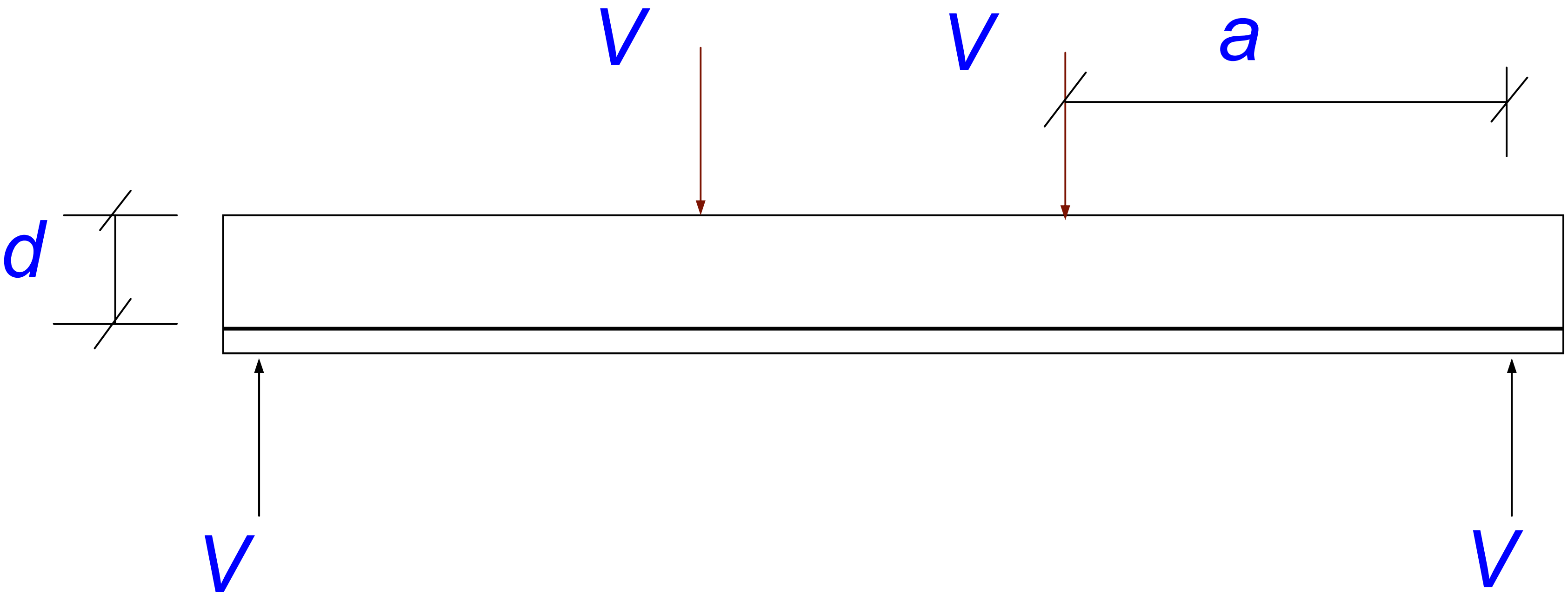
Concrete Compressive Strength

- Greater concrete strength usually corresponds to more tensile and shear strengths.
- This remains true even after initial cracking of the concrete at certain locations.
- The maximum value of allowed by the ACI code is 8.3 MPa, except in prestressed concrete beams and joists.
- This means that the expressions are valid for f_c' up to 69 MPa.

$$V_c \propto \sqrt{f_c'}$$

Applied Moment To Shear Ratio And Effective Depth

- The ratio of the applied moment (*M*) and shear (*V*) at a particular section may simply be calculated as *M*/*V*.
- Greater is this ratio at a particular section of the member, flexural cracks appear quite earlier reducing the net area to resist shear.
- Consequently, the effective shear strength is reduced.
- The opening of the initial flexural cracks may be controlled by providing greater effective depth (*d*) and in-turn the shear strength may vary as directly proportional to the effective depth.


V
V
a
d
V
V

- This means that the shear strength of concrete may be considered as inversely proportional to the factor (*M*/*Vd*).
- In case of field beams usually subjected to uniformly distributed loads, the ratio *M*/*V* is to be calculated at the cross-section of concern.
- However, for experiments, usually concentrated loads are applied symmetrically, as shown in Fig. In such cases, the *M*/*Vd* ratio simplifies to the shear span over effective depth ratio (*a*/*d*).

M = maximum moment = $V\,a$

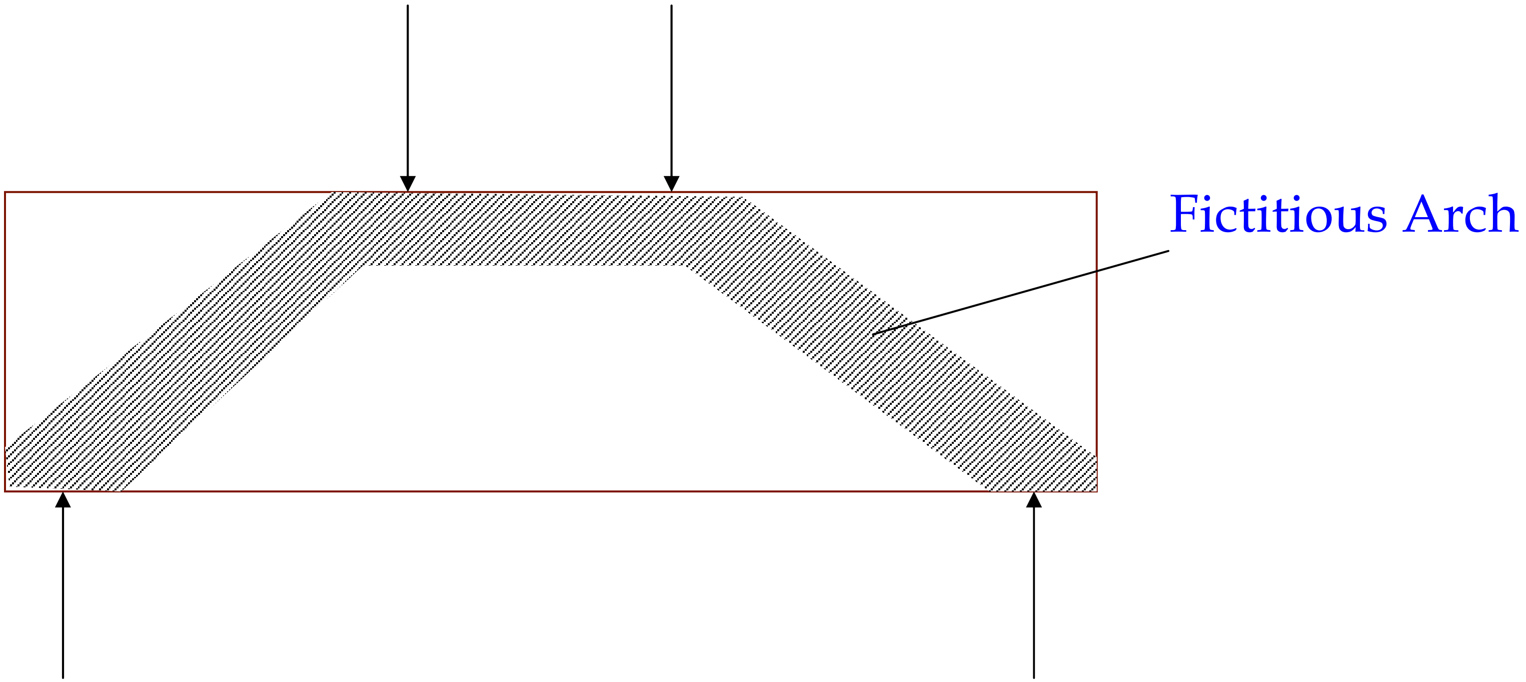
$M / V = a$

$$\frac{M}{Vd} = \frac{a}{d}$$

Where a = the shear span

The shear span to effective depth ratio (a/d ratio) in experimental beams have the same effect as M/Vd ratio in beams subjected to any other types of loads.

- Very short shear spans ($a/d \leq 1.0$) show *tied arch action* at failure in place of beam action as in Fig.
- Inclined cracks joining the load and the support are produced damaging the horizontal shear flow from the longitudinal steel to the compression zone.
- The arch is tied at the bottom by the flexural reinforcement and the final failure mode is commonly the anchorage failure at the ends.


Fictitious Arch

- Short shear spans (a/d from 1.0 to 2.5) also develop arch action after the formation of inclined cracks but the total load is partly carried by this arch action and partly by the *dowel action* of the main reinforcement combined with the mechanical interlocking between the cracked inclined surfaces.
- The failure takes place by *bond or dowel failure* along the flexural reinforcement or by the *shear compression failure*.
- In shear compression failure, the inclined crack rises higher into the beam than a flexural crack, reduces the compression area, and causes crushing of concrete over the crack.

- Slender shear span (*a*/*d* from 2.5 to 6.0) do not develop arch action and failure occurs purely by the flexural shear cracks.
- The resistance offered by concrete in shear after the initial crack is only due to mechanical interlocking of concrete surfaces at cracks and dowel action of horizontal steel.
- Very slender shear span beams (*a*/*d* > 6.0) usually fail in flexure without the formation of inclined cracks.

Longitudinal Reinforcement Ratio

- Smaller steel ratio (ρ_w) causes flexural cracks to extend higher into the beam and to open more.
- This reduces the shear capacity of the remaining smaller section. Longitudinal steel provides dowel action and prevents relative vertical movement of two parts of the beam formed by the inclined cracks.
- It also helps to provide more interlocking between the cracked surfaces in case of hairline cracks.
- The dowel action ends after splitting of concrete horizontally at the level of the main reinforcement.
- Longitudinal steel also acts as a tie if arch action has to develop for smaller *a*/*d* ratios.

$$V_c \propto \rho_w$$

ACI CODE PROVISIONS FOR CONCRETE SHEAR STRENGTH

For members without axial force, the shear strength (V_c) provided by the concrete alone is given by:

$$V_c = \left[0.16\lambda\sqrt{f_c'} + 17\rho_w \frac{V_u d}{M_u} \right] b_w d \quad \leq \quad 0.29\lambda\sqrt{f_c'}\, b_w d$$

Where,

- V_c = Concrete shear strength, N
- V_u = Factored shear at the section, N

- M_u = Factored moment at the section, N-mm
- d = Effective depth, mm
- $V_u d / M_u \leq 1.0$, the maximum value to be used in the formula must be 1.0, even if it is actually more
- λ = modification factor for light-weight concrete, 1.0 for normal weight concrete
- ϕ_v = 0.75

- The above equation involves calculation of V_u and M_u at each section and hence the manual calculations become lengthy.
- The ACI Code gives an alternate and easy equation to estimate the shear strength of concrete, which is as follows:

$$V_c = 0.17\lambda\sqrt{f_c'}\,b_w d$$

Average Safe Ultimate Shear Stress for No Shear Reinforcement.

Table 5.1. Safe Ultimate Shear Stress for No Shear Reinforcement.	
f_c' *MPa*	**Average Shear Stress** MPa
17.25	0.265
20	0.285
22	0.299
25	0.319
28	0.337
30	0.349

SHEAR STRENGTH PROVIDED BY VERTICAL REINFORCEMENT

The shear reinforcement commonly consists of No. 10 to 16 mm bars anchored against the flexural reinforcement and hanger bars. These stirrups may be single, double or four legged having the shapes as shown in Fig.

Let

A_v = combined area of all transverse legs of the stirrup at a particular cross-section,

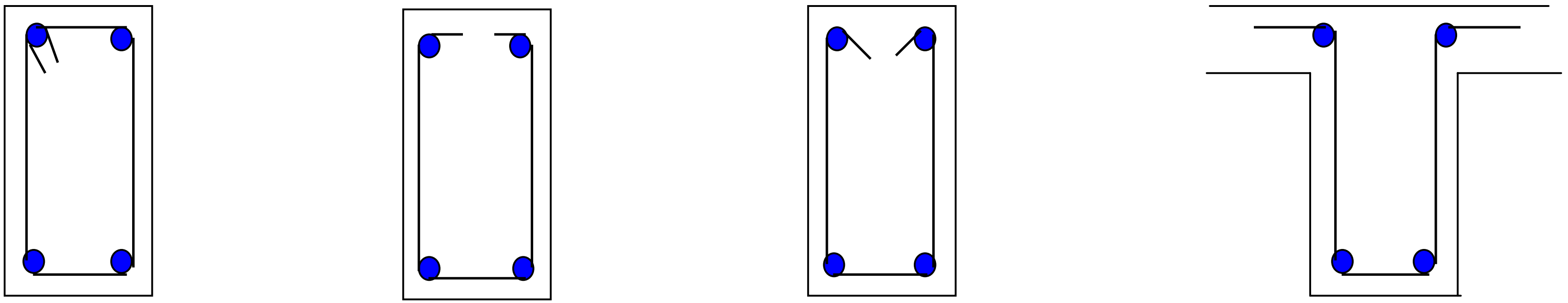
s = longitudinal spacing of the stirrups,

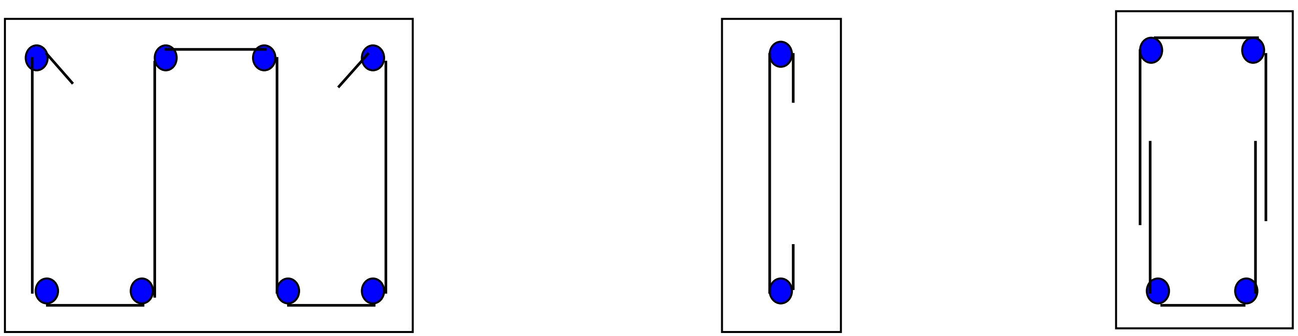
and

V_s = shear strength provided by the stirrups at a particular cross-section.

- Knowing that both the applied shear force and the stirrups are in the transverse direction, it is expected that, after the formation of cracks, one shear stirrup can resist $A_v f_y$ force.
- The average resistive force provided per unit length of beam then becomes $A_v f_y / s$.
- If the dimension of a crack is assumed to be equal to d in the longitudinal direction, the shear strength provided by the stirrup (V_s) becomes:

$$V_s = \frac{A_v f_y d}{s} \qquad \text{(N)}$$





Used to Cover-up The Deficiency

STRENGTH REDUCTION OR RESISTANCE FACTOR IN SHEAR

- The strength reduction factor for shear (ϕ_v) is taken equal to 0.75.
- It is relatively lesser than the corresponding factor in bending, giving more factor of safety, due to firstly the sudden nature of failure and secondly the larger scatter of the test results.

MINIMUM WEB REINFORCEMENT

- If V_u is lesser than $\phi_v V_c$, theoretically no web reinforcement is required.
- However, if $V_u \geq \phi_v V_c/2$, the ACI Code requires the provision of a minimum area of web reinforcement for extra safety as given below:

$$A_{v,\min} = 0.062\sqrt{f_c{}'}\,\frac{b_w s}{f_{yt}} \quad \geq \quad 0.35\frac{b_w s}{f_{yt}}$$

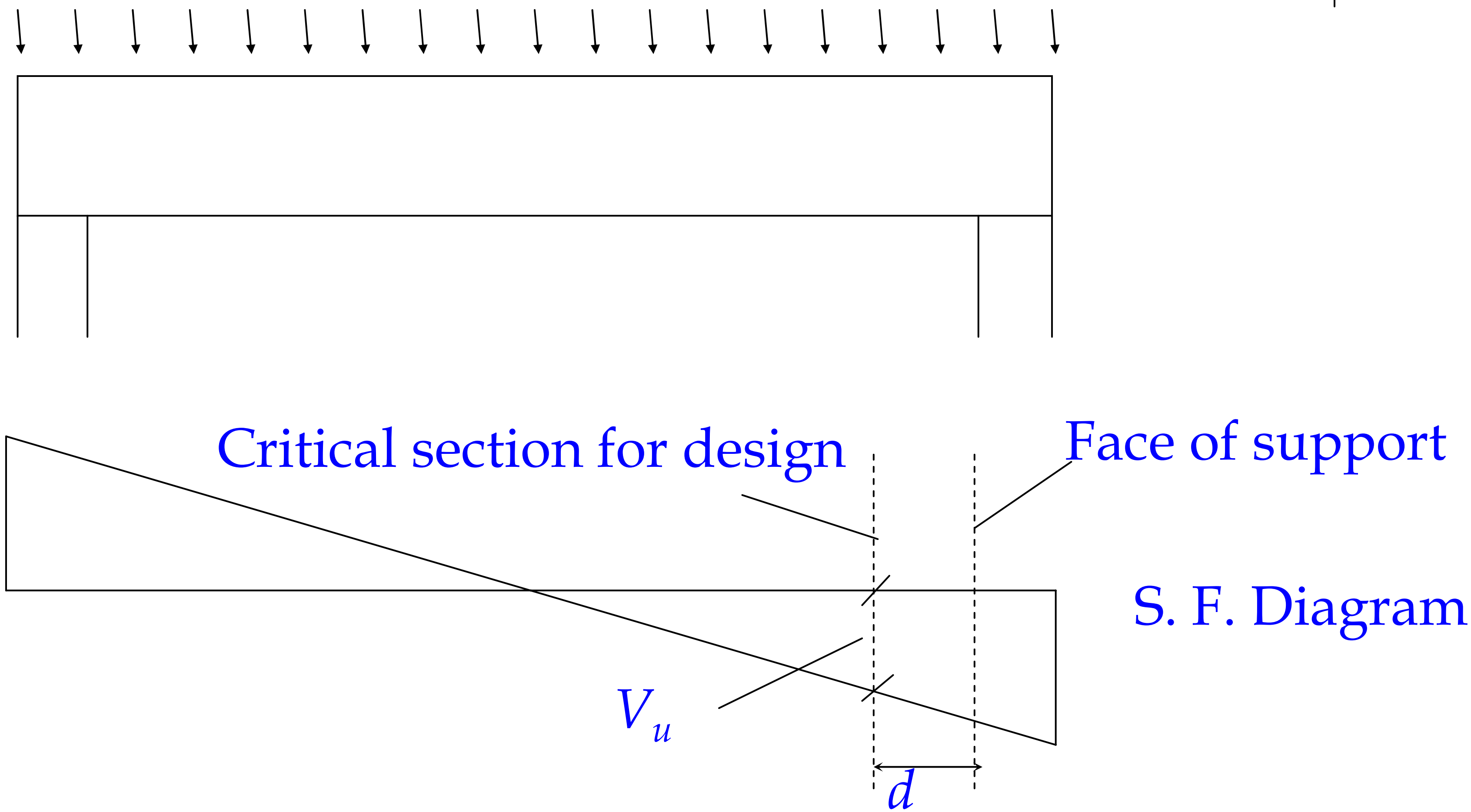
Where $A_{v,min}$ = total cross-sectional area of web steel within distance, s

and f_{yt} = the specified yield strength of transverse reinforcement.

- The second expression governs for $f_c' \leq 31.9$ MPa. This minimum reinforcement provides an average shear stress capacity of 0.35 MPa.
- It is important to note that the maximum spacing limit of $d/2$, given later, is also required to be satisfied for this minimum web reinforcement.
- The exceptions where this minimum reinforcement is not required are the members that have capacity to redistribute stresses across the width of the member or in some cases to adjacent members, having following examples:
  - Slabs and footings.
  - Concrete joist construction.
  - Shallow beams with a total depth not greater than 250mm and beams integral with slabs, having depths not greater than 600 mm and larger of 2.5 times the thickness of the flange and one-half the width of the web.

LOCATION OF MAXIMUM SHEAR FOR DESIGN OF BEAMS

- The non-prestressed sections located less than a distance *d* from the face of the support may be designed for the same shear, V_u, as that computed at a distance *d.*
- This provision should not be used in case of inverted beam with load applied on the slab acting as flange of the beam.
- This is due to the presence of local compressive bearing stresses due to the support reaction, which reduces the chances of development of shear cracks.
- The loads applied to the beam within a distance *d* from the support will be transferred directly to the support by an inverted wedge of concrete in compression formed within 45° cracks.


Critical section for design
Face of support
S. F. Diagram
V_u
d

MINIMUM SECTION FOR DESIGN SHEAR CAPACITY

- If required shear to be resisted by the transverse steel (V_s) exceeds

$$0.66\sqrt{f_c'}\,b_w d \quad \text{(N)}$$

the cross-sectional dimensions must be increased.

MAXIMUM SPACING REQUIREMENTS

When required V_s does not exceed $0.33\sqrt{f_c'}\,b_w d$ (N) the maximum spacing is given by

s_{max} = smaller of the following three:

i) $\dfrac{A_v f_{yt}}{0.35 b_w}$

ii) $d / 2$

iii) 600 mm

When required V_s exceeds

$$0.33\sqrt{f_c'}\, b_w d \quad \text{(N)}$$

the maximum spacing is given by

s_{max} = smaller of the following three:

i) $\dfrac{A_v f_{yt}}{0.35 b_w}$

ii) d / 4

iii) 300 mm

DESIGN OF WEB REINFORCEMENT

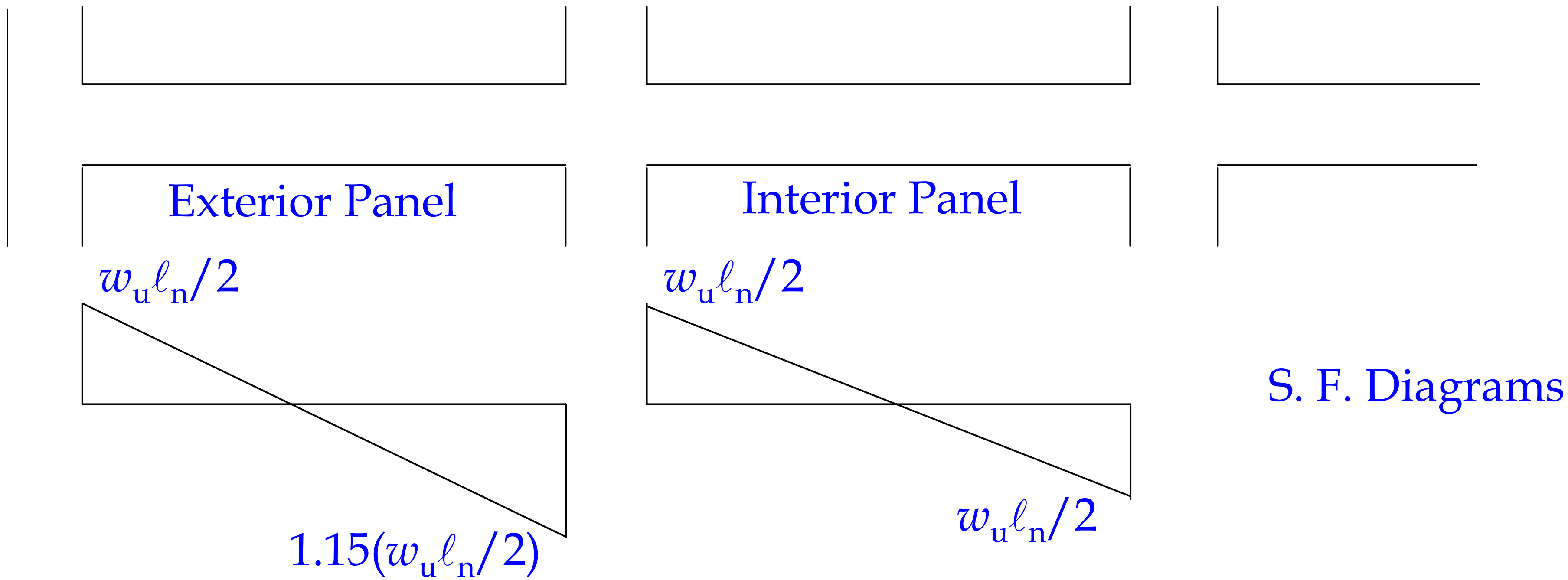
- $V_n = V_c + V_s$
- For design, $V_u \leq \phi_v V_n$ where $\phi_v = 0.75$

$$V_u \leq \phi_v V_c + \frac{\phi_v A_v f_y d}{s}$$

$$s_{\max} = \frac{\phi_v A_v f_y d}{(V_u - \phi_v V_c)}$$

- The size and number of legs of stirrups should be selected to avoid a spacing making the pouring of concrete difficult, which should commonly be not less than 80 to 90 mm.

TYPICAL SHEAR FORCE DIAGRAMS



GENERAL PROCEDURE FOR SHEAR DESIGN

- Plot exact or ACI Code shear force diagram for the beam. Also draw bending moment diagram if the exact equation for V_c is to be used.
- Find maximum design shear, V_u, and factored moment, M_u, at d distance from inner face of the support.
- Calculate concrete shear strength preferably using the exact equation. However, if the data for exact equation is not available, the approximate equation may be used.
- If $V_u \leq \phi_v V_c/2$, then shear reinforcement is not required according to the code. A minimum amount of stirrups may be provided to keep the longitudinal steel in position.

- Decide whether cross-sectional dimensions are all right or the size has to be increased.
- Generally, if $V_s > 0.33\sqrt{f_c'}\, b_w d \quad \text{(N)}$

  the size is to be increased. However, we may go to double of this value in exceptional cases.
- Decide a trial diameter and shape of the stirrup and calculate A_v.

- Calculate the maximum required spacing as follows:

  s_{max} = the smallest out of the following four values:

  i) $\dfrac{A_v f_{yt}}{0.35 b_w}$

  ii) $d / 2$

  iii) 600 mm

  iv) $\dfrac{\phi_v A_v f_{yt} d}{(V_u - \phi_v V_c)}$

If $V_s > 0.33\sqrt{f_c'}\,b_w d$ (N), reduce the values given by the conditions (ii) and (iii) to one-half of the above values.

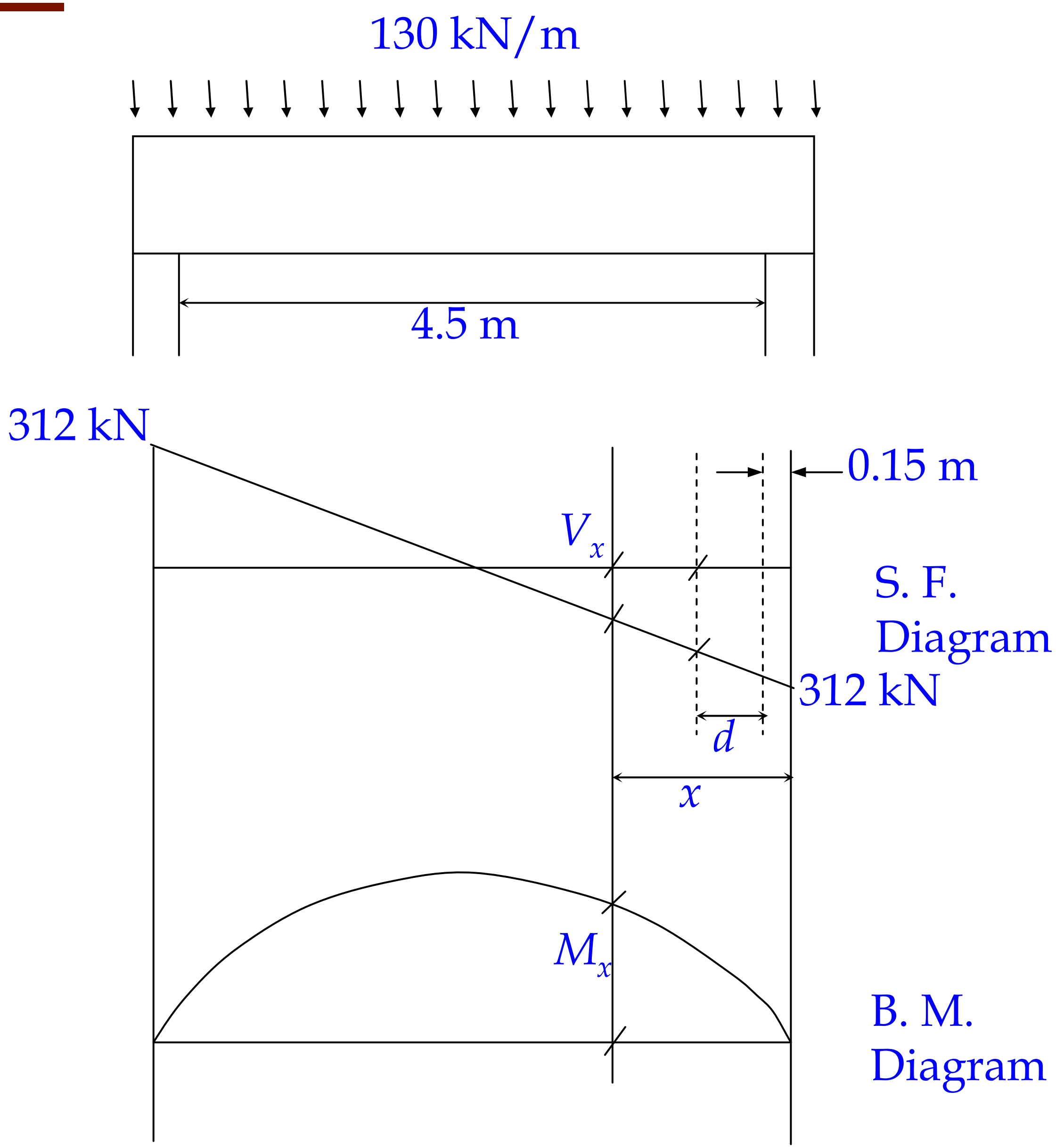
- Round the calculated spacing to the nearest 10mm multiples. If this value is lesser than about 80mm, increase the number of legs of stirrups (if possible), yield strength of stirrup steel (if feasible) or the diameter of the stirrup.
- Place first stirrup at *s* / 2 distance from the inner face of the support and decide how many of such spaces are to be continued towards the center of the beam.

- Usually the spacing for the most critical section is continued up to a distance in-between $\ell_n/5$ to $\ell_n/4$.
- Calculate V_u and M_u at this new location.
- It is to be noted that the spacing may be recalculated after placement of each stirrup if the construction is carried out in a factory under controlled conditions and a computer program is used for the calculations.

- Repeat steps 3 onwards leaving the irrelevant steps to calculate spacing up to the point where no shear reinforcement is required or make calculations for one more point in-between.
- Location beyond which shear stirrups are not required by the ACI Code can be approximately found by equating the expression for the external shear in terms of distance from the support, *x*, to $\phi_v V_c/2$, using the approximate relation for the concrete shear strength.

Example 5.1 (SI Units): A simply supported rectangular beam of size 300×525 mm, having an effective depth of 450mm, carries a total factored load of 130 kN/m. The clear span of the beam is 4.5m (effective span = 4.8m). The flexural reinforcement consists of 8–#25 US customary bars (A_s = 4080 mm^2), four of which are curtailed at 450mm short of face of supports. Use C-20 concrete and Grade 280 steel. Using double legged stirrups, design the web reinforcement.

Solution:



$$V_x = 312 - 130x$$

$$M_x = 312x - 65x^2$$

The shear and moment at the critical section, *d* distance from inner edge of the support, are as follows:

V_u = 234 kN ; M_u = 163.8 kN-m

$$\frac{V_u d}{M_u} = \frac{234 \times 0.45}{163.8} = 0.643 \leq 1.0$$

$$\rho_w = \frac{4080/2}{300 \times 450} = 0.015$$

$$V_c = \left[0.16\lambda\sqrt{f_c'} + 17\rho_w \frac{V_u d}{M_u}\right] b_w d \quad \leq \quad 0.29\lambda\sqrt{f_c'}\, b_w d$$

$$= \left[0.16 \times 1 \times \sqrt{20} + 17 \times 0.015 \times 0.643\right] \frac{300 \times 450}{1000}$$

$$\leq \quad \frac{0.29 \times 1 \times \sqrt{20} \times 300 \times 450}{1000}$$

= 108.7 $\leq$ 175.1 kN

= 108.7 kN

- $\phi_v V_c = 0.75 \times 108.7 = 81.53$ kN ;

  $\phi_v V_c/2 = 40.76$ kN
- $V_u > \phi_v V_c/2$

  $\therefore$ Transverse reinforcement is required.
- $V_u - \phi_v V_c = 234.00 - 81.53 = 152.47$ kN
- $V_s = (V_u - \phi_v V_c) / \phi_v = 152.47 / 0.75 = 203.3$ kN

$$0.33\sqrt{f_c'}\, b_w d = 0.33\sqrt{20} \times \frac{300 \times 450}{1000} = 199.2 kN$$

$$V_s = 0.66\sqrt{f_c'}\, b_w d \qquad \therefore \text{ Cross - sectional dimensions are OK}$$

However, $V_s > 0.33\sqrt{f_c'}\, b_w d$ and the spacing limits are to be used accordingly.

- Let diameter of bar = #10 US

  Then, area of double legged stirrup,

  A_v = 142 mm^2
- s_{max} = the smallest out of the following four values:

i) $$\frac{A_v f_{yt}}{0.35 b_w} = \frac{142 \times 280}{0.35 \times 300} = 379 mm$$

ii) d / 4 = 112 mm

iii) General minimum = 300 mm

iv) $$\frac{\phi_v A_v f_{yt} d}{(V_u - \phi_v V_c)} = \frac{0.75 \times 142 \times 280 \times 450}{152.47 \times 1000} = 88 mm$$

= 88 mm

- s = say 80 mm $\geq$ 80 mm (OK)
- First stirrup is placed at $s/2$ = 40mm distance from face of support.
- $\ell_n/5$ = 900mm and $\ell_n/4$ = 1125mm.
- Eleven intervals of stirrups may be provided at a spacing of 80mm.
- The location of the last stirrup is defined as follows:

  $x = 0.150 + 0.040 + 11 \times 0.080 = 1.07$ m
- At x = 1.07 m:

  V_u = 172.9 kN ; M_u = 259.42 kN-m

$$\frac{V_u d}{M_u} = 0.3 \le 1.0 \quad ; \quad \rho_w = 0.30$$

$$V_c = \left[0.16\lambda\sqrt{f_c'} + 17\rho_w \frac{V_u d}{M_u}\right] b_w d \quad \le \quad 0.29\lambda\sqrt{f_c'}\, b_w d$$

$$= \left[0.16 \times 1 \times \sqrt{20} + 17 \times 0.030 \times 0.3\right] \frac{300 \times 450}{1000}$$

$$\le \quad \frac{0.29 \times 1 \times \sqrt{20} \times 300 \times 450}{1000}$$

= 117.25 ≤ 175.08

= 117.25 kN

$\phi_v V_c$ = 0.75×117.25 = 87.94 kN ;

$\phi_v V_c/2$ = 43.97 kN

- V_u > $\phi_v V_c/2$ ∴ Transverse reinforcement is required.
- s_{max} = the smallest out of the following four values:

i) $$\frac{A_v f_{yt}}{0.35 b_w} = \frac{142 \times 280}{0.35 \times 300} = 379mm$$

ii) d / 2 = 125 mm

iii) General minimum = 600 mm

iv) $$\frac{\phi_v A_v f_{yt} d}{(V_u - \phi_v V_c)} = \frac{0.75 \times 142 \times 280 \times 450}{(172.9 - 87.94) \times 1000} = 158mm$$

= say 150 mm

- To find the location after which stirrups are not required theoretically, let $V_x = \phi_v V_c/2$. Approximate relation for V_c may be used here.

- $V_x = 0.17\lambda\sqrt{f_c'}\, b_w d$

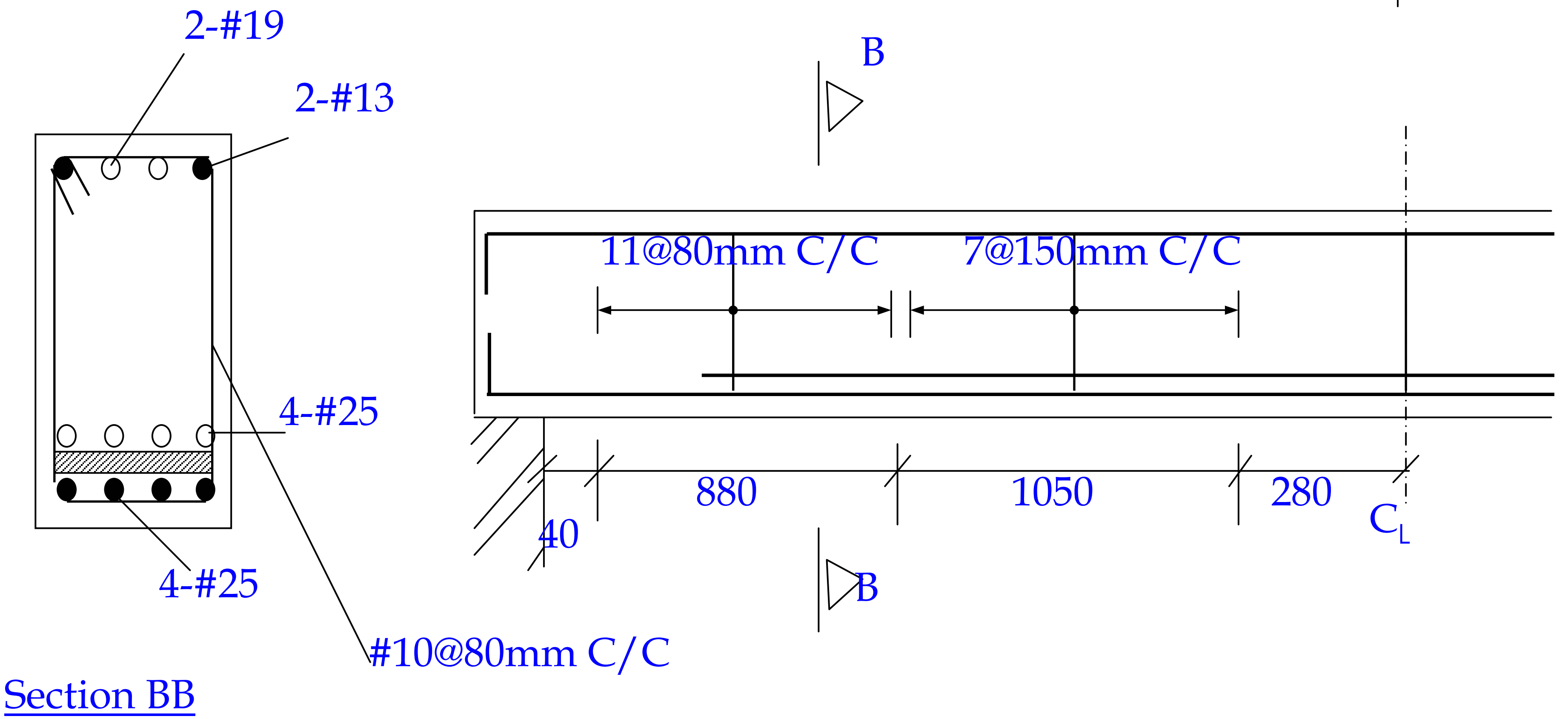
$$312 - 130\,x = 0.75 \times \frac{0.17}{2}\,\frac{\sqrt{20}}{1000} \times 300 \times 450$$

$$0.75 \times 130\,x = 312 - 38.49$$

- $x = 2.10$ m

  $\therefore$ Continue a spacing of 150 mm for 7 intervals.

- An extra stirrup may be provided at the center.


2-#19
2-#13
4-#25
4-#25
#10@80mm C/C
Section BB
B
B
11@80mm C/C
7@150mm C/C
880
1050
280
40
C_L

Assignment Chapter 5