Plain & Reinforced Concrete-1

CE-314

Sixth Term
Civil Engineering

Lecture # 15

Flexural Analysis and Design of Beams

(Ultimate Strength Design of Beams)

Plain & Reinforced Concrete-1

Design of T & L Beams

Known: h_f, L, f_c', f_y, Loads or M_u

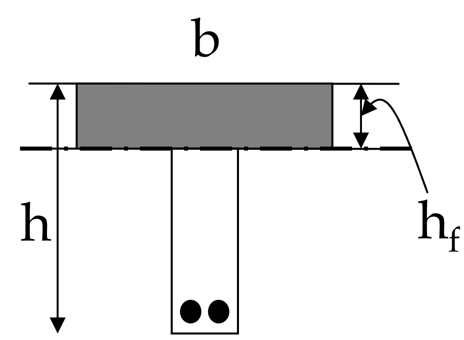
Required: beam size, A_s, detailing, bar bending schedule

Step # 1: Select trial dimension of the beam, b_w & h, nearly same as for the rectangular section.

Step # 2: Calculate the effective width "b".

Step # 3: Assume the N.A. to be at the junction of flange and web, means

$$a_1 = \beta_1 h_f \Rightarrow A_s = \frac{M_u}{0.9 f_y (d - a/2)} \Rightarrow a_1 = \frac{A_s f_y}{0.85 f_c' b}$$



If **$a_1 \le \beta_1 h_f$** Our assumption is correct and beam will be designed as rectangular beam. (b x h)

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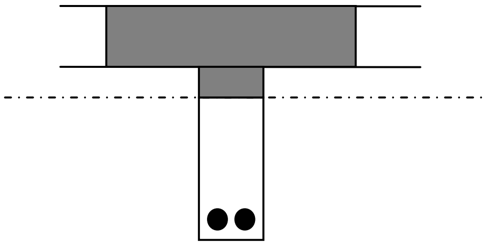
Design of T & L Beam (contd…)

If $a_1 > \beta_1 h_f$ and beam will be designed as T-beam.



Step # 4

$$A_{sf} = 0.85\beta_1 h_f \frac{f_c'}{f_y}(b - b_w)$$

Step # 5: Calculate

$$M_f = \phi_b M_{nf} = 0.9 A_{sf} f_y \left(d - \frac{\beta_1 h_f}{2} \right)$$

Step # 6 Calculate

$$M_w = M_u - M_f$$

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Design of T & L Beam (contd…)

Step # 7

$$A_{sw} = \frac{M_w}{0.9 f_y (d - a/2)}$$

$$a = \frac{A_{sw} f_y}{0.85 f_c' b_w}$$

2 to 3 trials

Step # 8

$$A_s = A_{sf} + A_{sw}$$

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Design of T & L Beam (contd…)

Step # 9: Calculate

$$\rho_w = \frac{A_s}{b_w d}$$

Step # 10: Check for

$$(\rho_w)_{min} \ \& \ (\rho_w)_{max}$$

Step # 11: Perform detailing, make sketches and prepare bar bending schedule if required.

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Example:

Design a **T-Beam** to be used as interior simply supported beam of span 5m. The slab panels on both sides of the beam are 3.5m x 5m. Factored slab load is 15 kN/m^2. f_c' 17.25 MPa and f_y = 420 MPa.

b_w = 300 mm h_f = 125 mm h = 575 mm

Use SI bars

- Design for given loading
- Design for M_u= 800 kN-m

Solution:

- $\ell_y = 5$ m
- $\ell_x = 3.5$ m
- Factored slab load = 15 kN/m^2
- $h = 575$mm
- $d = h - 75 = 500$ mm
- $b_w = 300$ mm
- $h_f = 125$ mm
- $f_c' = 17.25$ MPa
- $f_y = 420$ MPa

Case (i)

Factored self weight of the beam

$$= 1.2 \times 0.300 \times \frac{575 - 125}{1000} \times 2400 \times \frac{9.81}{1000} = 3.82\ kN/m$$

$R = \ell_x / \ell_y = 3.5 / 5.0 = 0.7$

Width of slab supported by the beam

$$= \left(1 - \frac{R^2}{3}\right)\ell_x = \left(1 - \frac{0.7^2}{3}\right) \times 3.5 = 2.93m$$

Factored slab load = 2.93 × 15.00 = 43.95 kN/m

Total factored load = w_u = 43.95 + 3.82 = 47.77 kN/m

$$M_u = \frac{w_u \ell^2}{8} = \frac{47.77 \times 5^2}{8} = 149.3 \text{ kN-m}$$

Effective slab width, b, is the smallest of the following:

- $L / 4 = 5000 / 4 = 1250$ mm
- $16\, h_f + b_w = 16 \times 125 + 300 = 2300$ mm
- $b_w + 0.5\, S_{CL} + 0.5\, S_{CR}$

  $= 300 + 0.5 \times (3500 - 300) \times 2$

  $= 3500$ mm

$\therefore \quad b = 1250$ mm

Assuming $a = \beta_1 h_f$

$= 0.85 \times 125 = 106.3$ mm

$$\text{Trial } A_s = \frac{M_u}{\phi_b\, f_y\,(d - \frac{a}{2})} = \frac{149.3 \times 10^6}{0.9 \times 420 \times (500 - \frac{106.3}{2})}$$

$= 884 \text{ mm}^2$

$$a = \frac{A_s f_y}{0.85\, f_c'\, b} = \frac{884 \times 420}{0.85 \times 17.25 \times 1250} = 20.3$$

$c = a / \beta_1 = 23.9$ mm $< h_f$, N.A. lies within the flange.

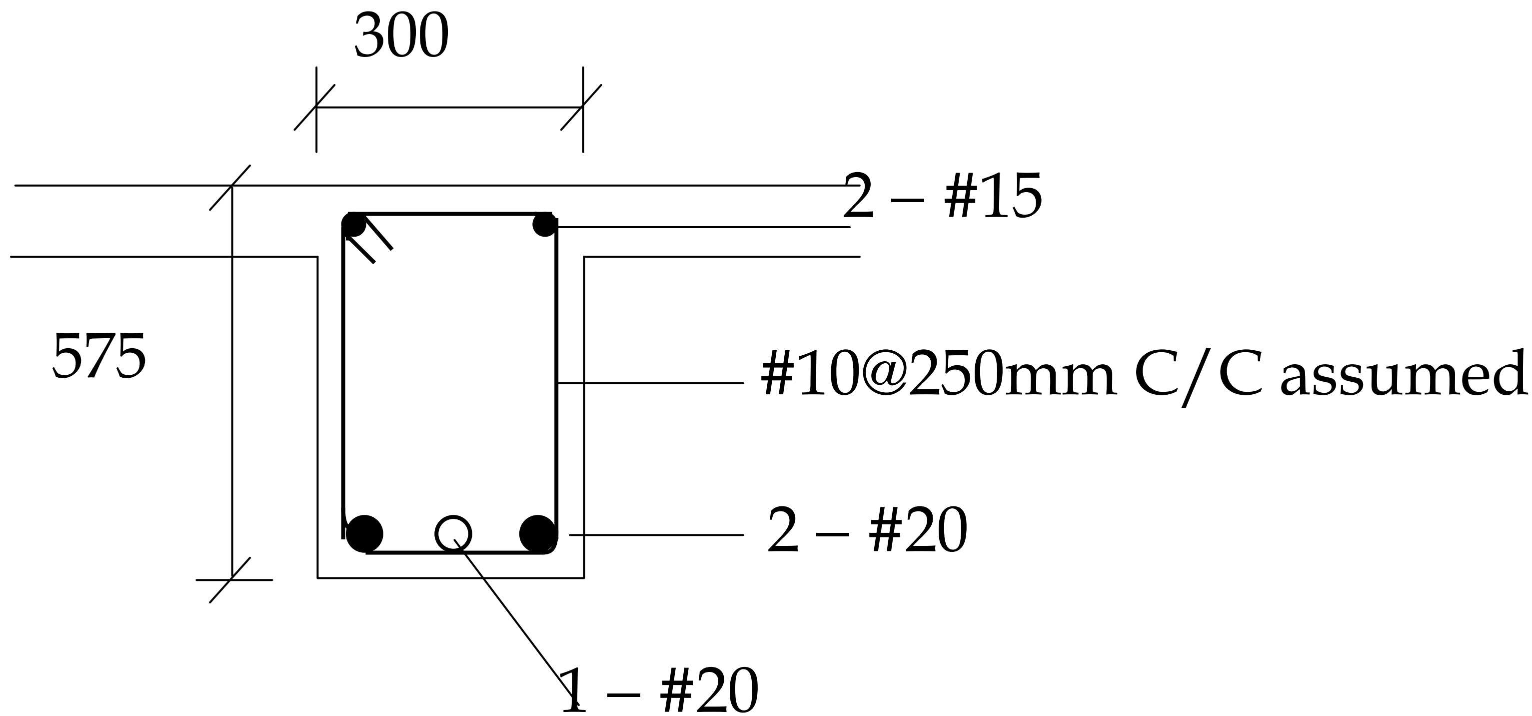
- Revised A_s = 806 mm^2
- a = 18.5 mm
- A_s = 805 mm^2
- a_ℓ = β_1 ×(3/8)× d = 0.85 × 0.375 × 500

  = 159.4 mm
- $a \leq a_\ell$, the limiting tensile strain is produced in the steel and ϕ_b = 0.9

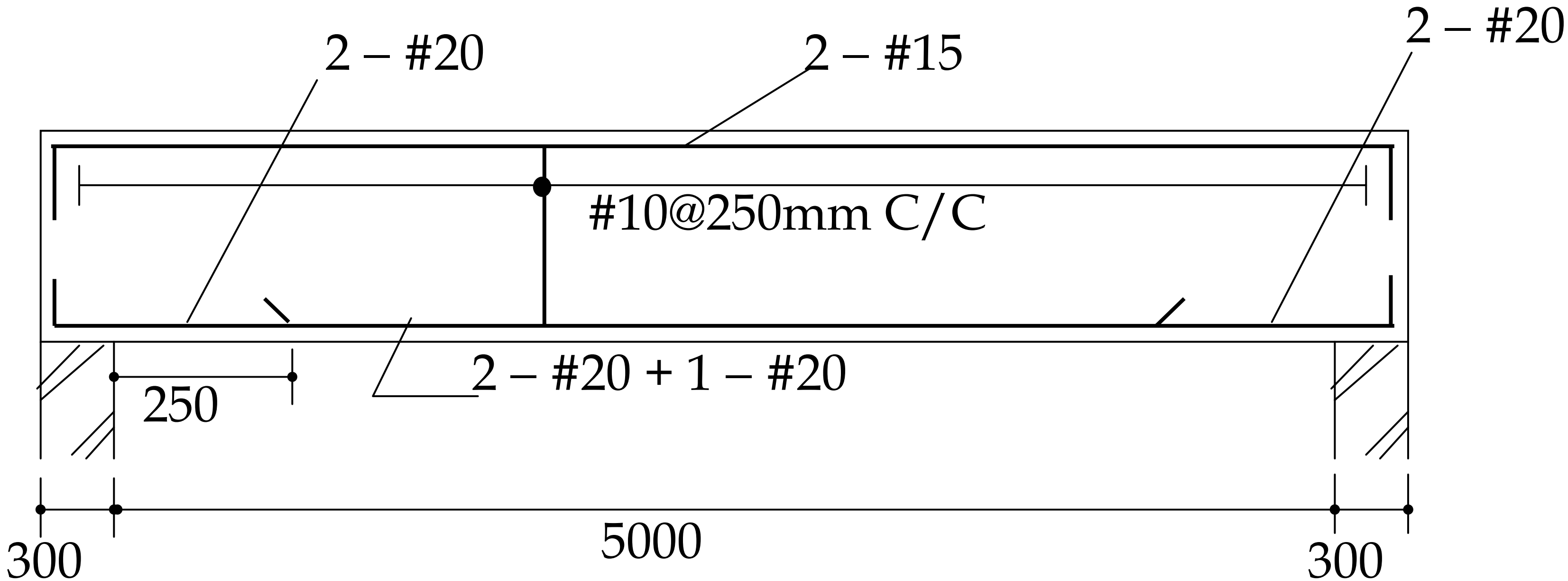
$$\rho_w = \frac{A_s}{b_w d} = 0.00545$$

- $\rho_{w,\,min}$ = 1.4 / f_y = 1.4 / 420 = 0.00333

- $\rho_w > \rho_{w,\,min}$ (OK)
- $A_s = 805\ mm^2$ [3–#20]



a) Cross Section



b) Longitudinal Section

Case (ii)

M_u = 800 kN-m

Assuming $a = \beta_1 \, h_f$

$= 0.85 \times 125 = 106.3$ mm

$$\text{Trial } A_s = \frac{M_u}{\phi_b \, f_y \, (d - \frac{a}{2})} = \frac{800 \times 10^6}{0.9 \times 420 \times (500 - \frac{106.3}{2})} = 4736 mm^2$$

$$a = \frac{A_s f_y}{0.85 \, f_c' b} = \frac{4736 \times 420}{0.85 \times 17.25 \times 1250} = 108.5 mm$$

$c = a / \beta_1 = 128 \text{ mm} > h_f$,

N.A. lies outside the flange.

- $A_{sf} = 0.85\ \beta_1\ h_f\ f_c'/\ f_y\ (b - b_w)$
  $= 0.85 \times 0.85 \times 125 \times (17.25/420) \times (1250 - 300) = 3524\ \text{mm}^2$
- $M_f = \phi_b\ M_{nf} = \phi_b\ A_{sf}\ f_y\ (d - \beta_1\ h_f\ /\ 2)$
  $= 0.9 \times 3524 \times 420 \times (500 - 0.85 \times 125\ /\ 2)\ /\ 10^6$
  $= 595.3\ \text{kN-m}$
- $M_w = M_u - M_f$
  $= 800.0 - 595.3 = 204.7\ \text{kN-m}$
- Assume $a = 106.3\ \text{mm}$

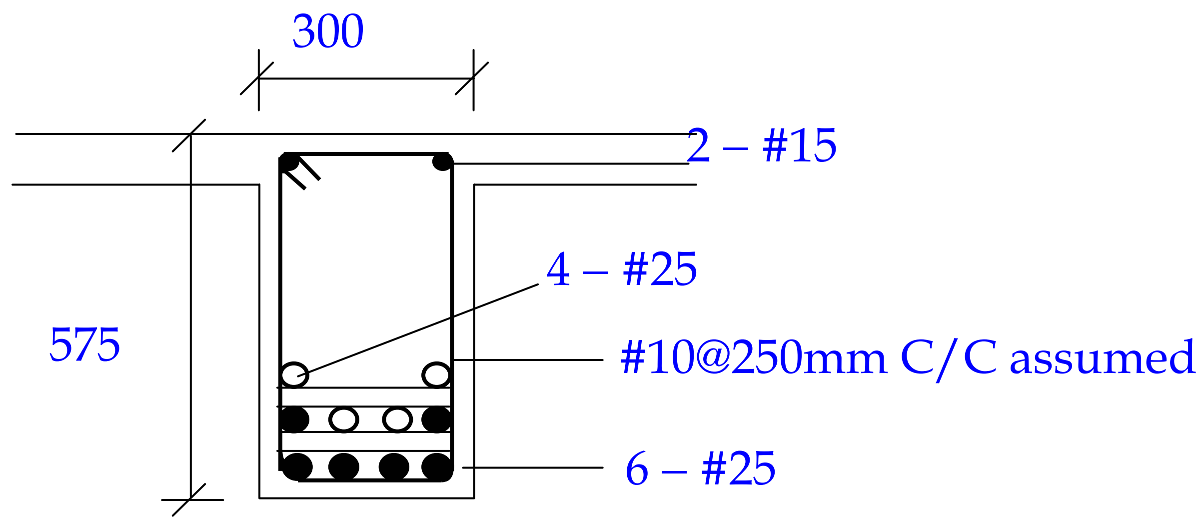
$$A_{sw} = \frac{M_w}{\phi_b f_y (d - \frac{a}{2})} = \frac{204.7 \times 10^6}{0.9 \times 420 \times (500 - \frac{106.3}{2})} = 1212 mm^2$$

$$a = \frac{A_{sw} f_y}{0.85 f_c' b_w} = \frac{1212 \times 420}{0.85 \times 17.25 \times 300} = 115.7 mm$$

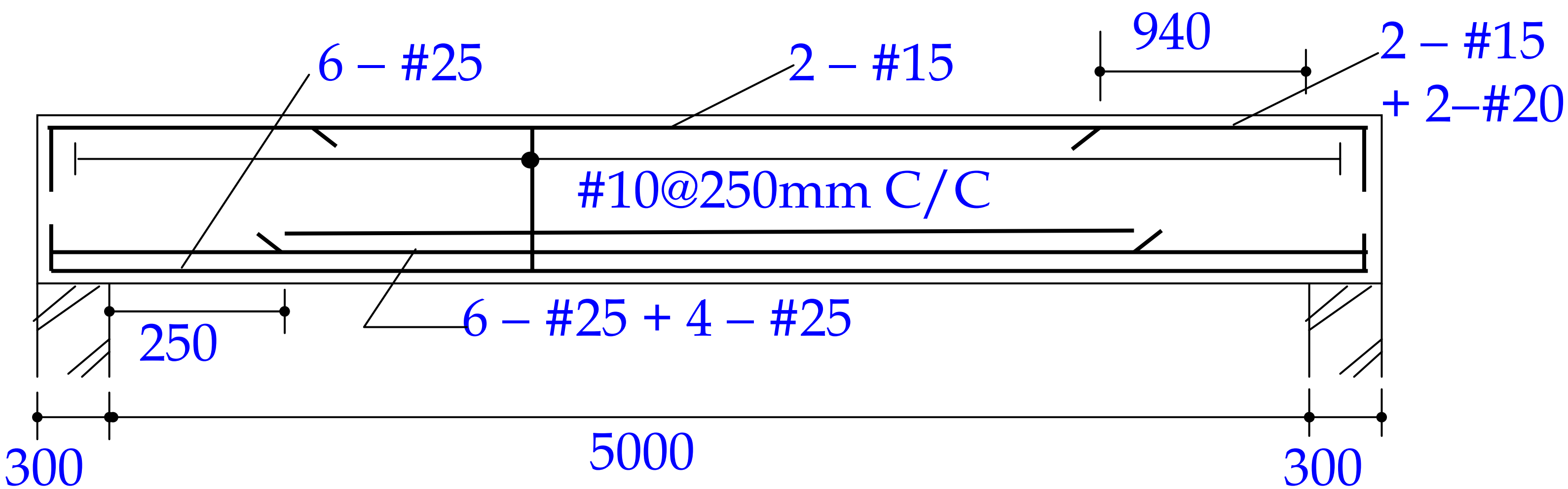
- A_{sw} = 1225 mm^2
- a = 117 mm
- A_{sw} = 1227 mm^2
- A_s = A_{sw} + A_{sf}

  = 1227 + 3524 = 4751 mm^2 [10–#25]

- a_ℓ = β_1 (3/8) d = 0.85 × 0.375 × 500
  = 159.4 mm
- $a \leq a_\ell$, the limiting tensile strain is produced in the steel and ϕ_b = 0.9
- The reinforcement details are shown in Fig. 4.16.



a) Cross Section



b) Longitudinal Section

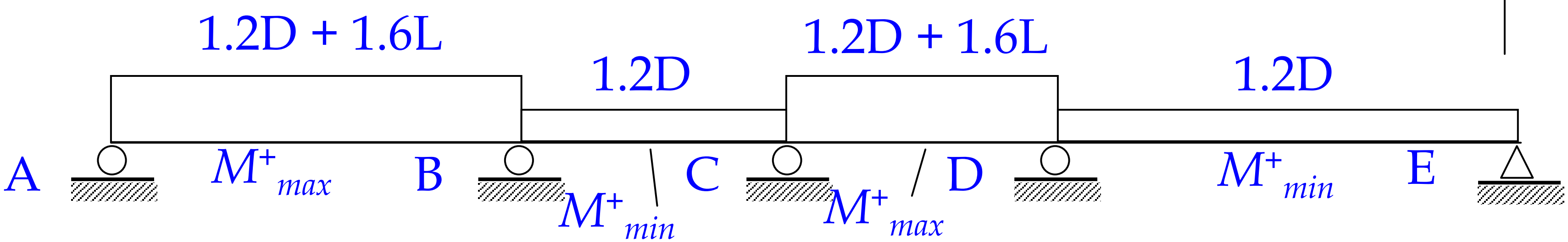
CONTINUOUS BEAMS

- Continuous beams are those beams that have two or more spans built monolithically.
- These beams are indeterminate and require detailed procedures for plotting shear force and bending moment diagrams.
- The beams in frames are to be analyzed for various load combinations of different loads such as dead, live, wind and earthquake loads.
- For each combination of these loads involving live load, pattern loading is to be considered to get maximum force effect at any point.

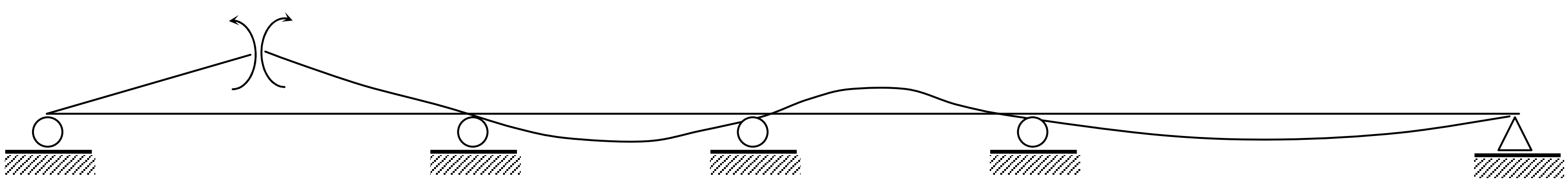
- In pattern loading, live load may be applied in adjacent panels or alternate panels in order to produce critical magnitude of a particular moment or shear.
- This means that the analysis is to be performed many times for each combination involving different pattern loads.
- Maximum values at various points are then picked out of these results.

According to ACI 8.11.2, it is allowed to consider only the following two arrangements of live loads:

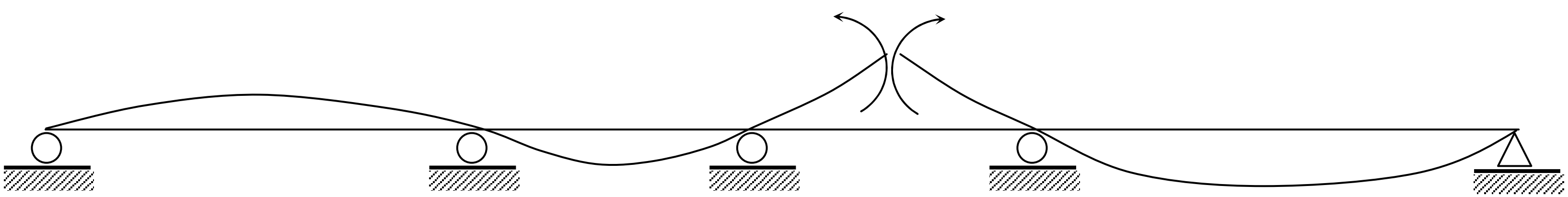
- Factored dead load acts on all spans with full factored live load on two adjacent spans. This arrangement gives the maximum negative moment at the central support.
- Factored dead load acts on all spans with full factored live load on alternate spans. This arrangement gives the maximum positive moment within the fully loaded spans.



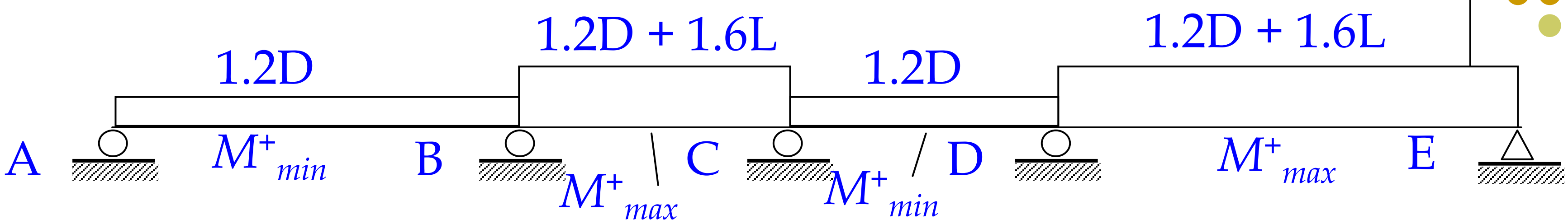
Pattern Loading for M^{+}_{AB} and M^{+}_{CD}



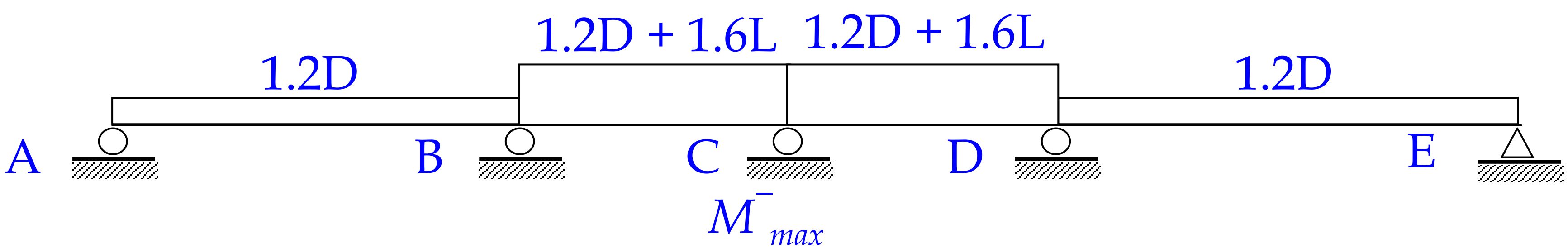
ILD For M^{+}_{AB}



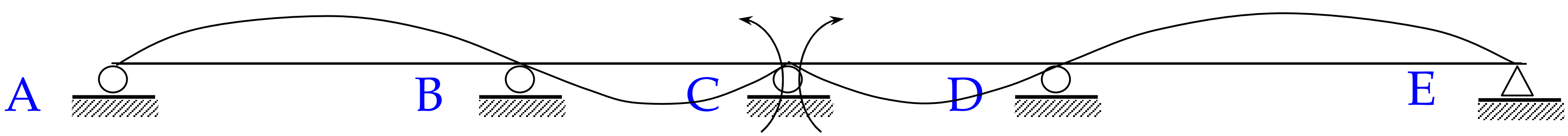
ILD For M^{+}_{CD}



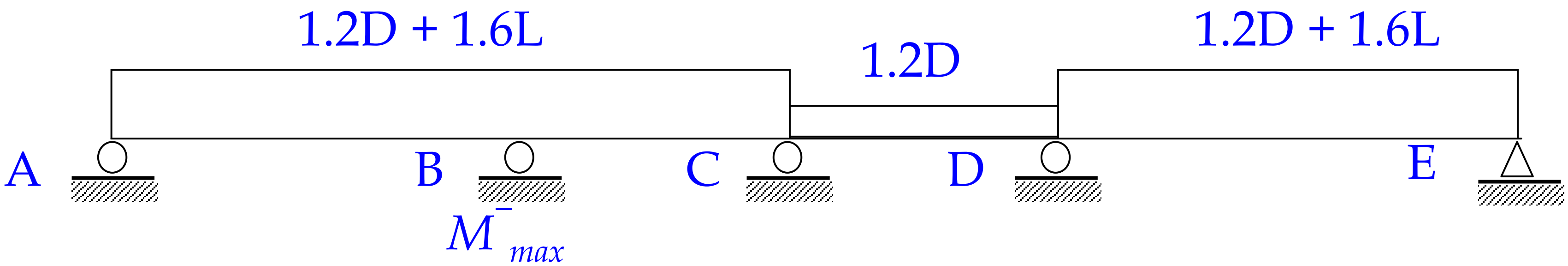
Pattern Loading for M^+_{BC} and M^+_{DE}



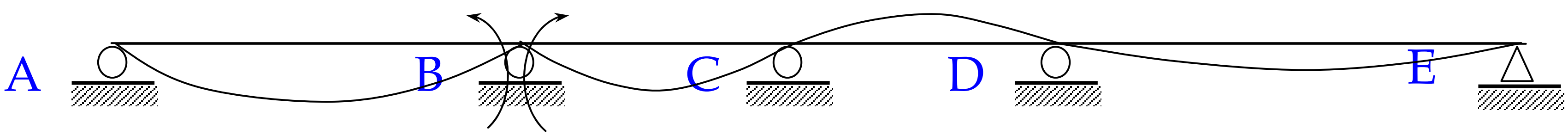
Pattern Loading for M_C^-



ILD For M^-_C



Pattern Loading for M_B^-



ILD For M^{-}_{B}

- The actual number of loading arrangements becomes greater when the adjacent two-span-loaded condition is applied for each support.
- Similarly, the alternate spans loaded condition is reversed once to load those panels that were previously without any live load.
- It is to be noted that to get the influence line diagram with negative moment ordinates on the lower side of the beam, bending moment and rotation at the section are to be applied in the positive direction.

- The negative moments at the supports obtained by the analysis are the centerline moments that are very high in magnitude but quickly diminishes away from the supports.
- The design based on these moments will be unrealistically conservative because the support regions are almost infinitely strong.
- The ACI Code allows the design of beams at the supports to be based on the moments at the edges of supports, which are to be calculated for each load combination and pattern.

- The negative moments calculated in this way are still much higher in magnitude than the positive moments.
- This makes the design of prismatic beams somewhat more difficult.
- Selecting the beam size for negative moments make the beam uneconomical for the larger part, which is subjected to positive moments.
- Similarly, the size selected for positive moment is usually insufficient for negative moment, making the design uneconomical and causing congestion of steel reinforcement.
- ACI Code allows a redistribution of moments up to 20% in which the negative moments may be reduced and the positive moments are increased accordingly.

Hence, in order to accurately and practically design a continuous beam, one has to consider all of the following aspects:

- Load combinations.
- Pattern loading.
- Both directions of wind and earthquake loads.
- Correction of negative moments at faces of supports.
- Redistribution of moments.

- In place of all the above detailed procedure, ACI moment coefficients may be used if the associated limitations are satisfied.
- It is important to note that if all the above procedure is not followed for analysis of beams and frames, the ACI coefficients will give more accurate and realistic design as compared insufficient analysis.
- The general concept of solving the frame by computer only once for all the expected loads applied together using a single combination to get accurate results is incorrect and may lead to unsafe and uneconomical design.

The continuous beam, with monolithic construction of beam with slab on top of it, may be designed as a T-beam for positive moment at mid-spans and as rectangular section (singly or doubly reinforced) at the supports.

ACI MOMENT COEFFICIENTS FOR CONTINUOUS BEAMS

Conditions For ACI Approximate Analysis To Be Applicable

- There must be at least two spans.
- The adjacent spans must not vary by more than 20% of the shorter span.
- Only uniformly distributed loads are applied. For other loads, separate analysis and coefficients must be used.
- The live to dead load ratio must not exceed 3.

- All the beams must be prismatic.
- The beams must have an extreme tensile strain of 0.0075 at the negative moment sections to allow redistribution of moments.
- The beam must be present in a braced frame without significant moments due to lateral loads.
- These coefficients only give moments due to gravity loads.

Moment Coefficients

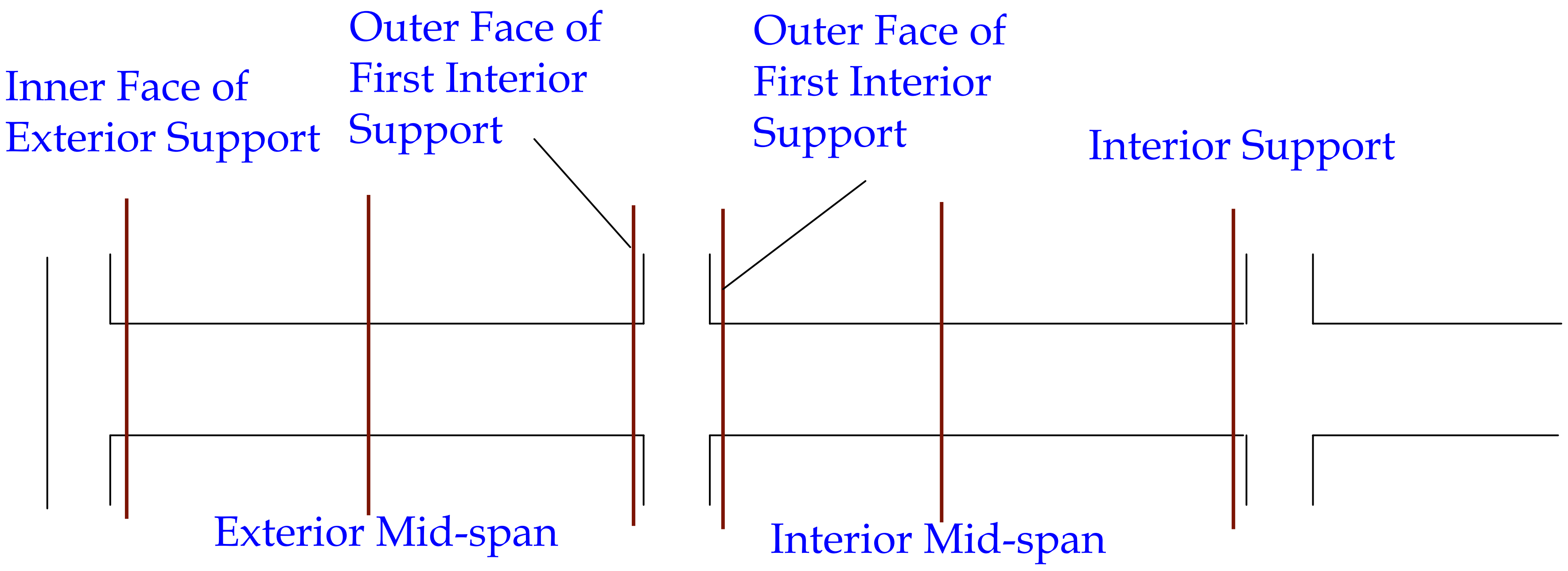


Table 4.1. Moment Coefficients for Slabs Having Spans Lesser Than 3.0 m ***OR*** Beams Having Ratio of Sum of Column Stiffness to Beam Stiffness More Than 8 at Each End of the Span.

1. Negative moments at all supports, integrally built with beams.	$\frac{1}{12} w_u \ell_n^2$
2. Positive moment in end panel.	$\frac{1}{14} w_u \ell_n^2$
3. Positive moment in central panels.	$\frac{1}{16} w_u \ell_n^2$

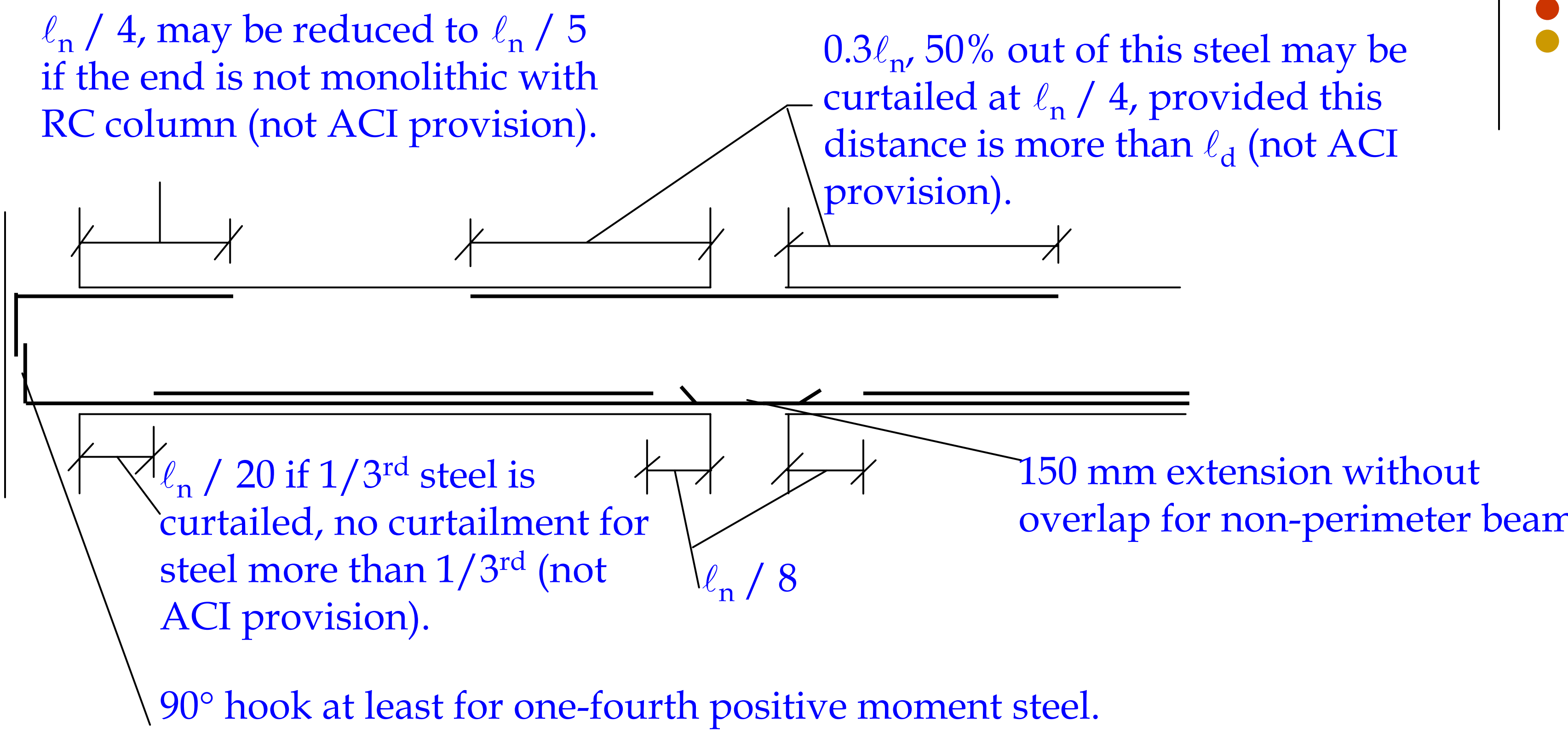
Table 4.2. Moment and Shear Values for Beams and Slabs Having Spans Greater Than 3.0 m.

1. Positive Moment	
End spans:	
If discontinuous end is unrestrained	$\frac{1}{11} w_u \ell_n^2$
If discontinuous end is integral with the support	$\frac{1}{14} w_u \ell_n^2$
Interior spans:	$\frac{1}{16} w_u \ell_n^2$
2. Negative moment at exterior face of first interior support	
Two spans:	$\frac{1}{9} w_u \ell_n^2$
More than two spans:	$\frac{1}{10} w_u \ell_n^2$

3. Negative moment at other faces of interior supports	$\frac{1}{11} w_u \ell_n^2$
(ℓ_n in no. 3 is the average of clear spans of the two adjacent panels.)	
4. Negative moment at interior faces of exterior supports for members built Integrally with their supports:	
The support is a spandrel beam or girder:	$\frac{1}{24} w_u \ell_n^2$
The support is a column:	$\frac{1}{16} w_u \ell_n^2$
The support is not monolithic:	Zero
5. Shear in end members at first interior support	$1.15 \frac{w_u \ell_n}{2}$
6. Shear at all other supports	$\frac{w_u \ell_n}{2}$

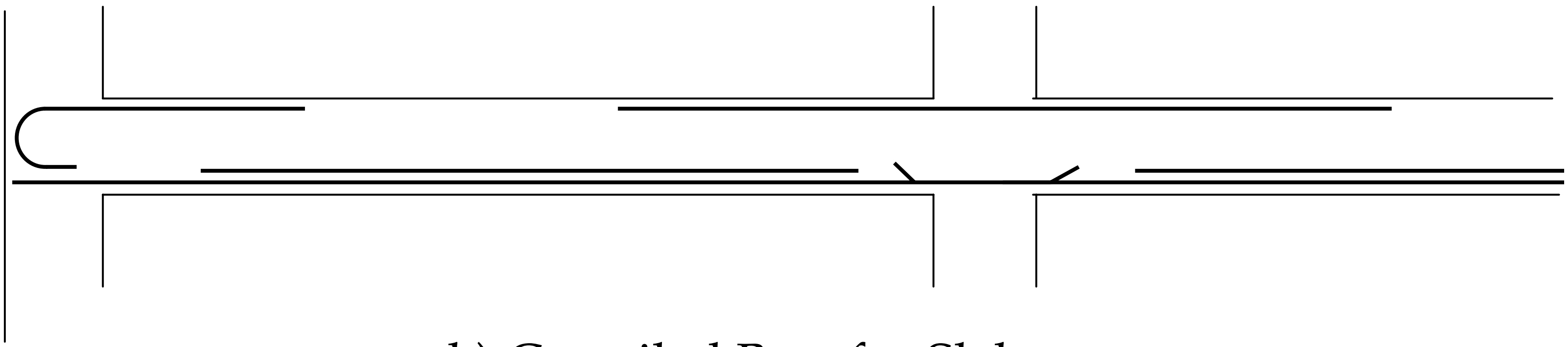
APPROXIMATE CURTAILMENT OF BARS

- According to ACI 12.11.1, at least one-third of the positive reinforcement in simple members and one-fourth the positive reinforcement in continuous members must extend along the same face of the member in to the support.
- However, to use the approximate curtailment methods, design engineers prefer to extend one-half or two-third of the positive steel in to the support.
- According to the code, such reinforcement carried in to the supports must extend at least 150 mm in to the support.
- However, it is better to fully anchor this steel to utilize it in case of hogging due to lateral loads or as compression reinforcement.



ℓ_n = clear distance of the respective span for curtailment of bottom bars and larger of the two adjacent spans for the top steel.

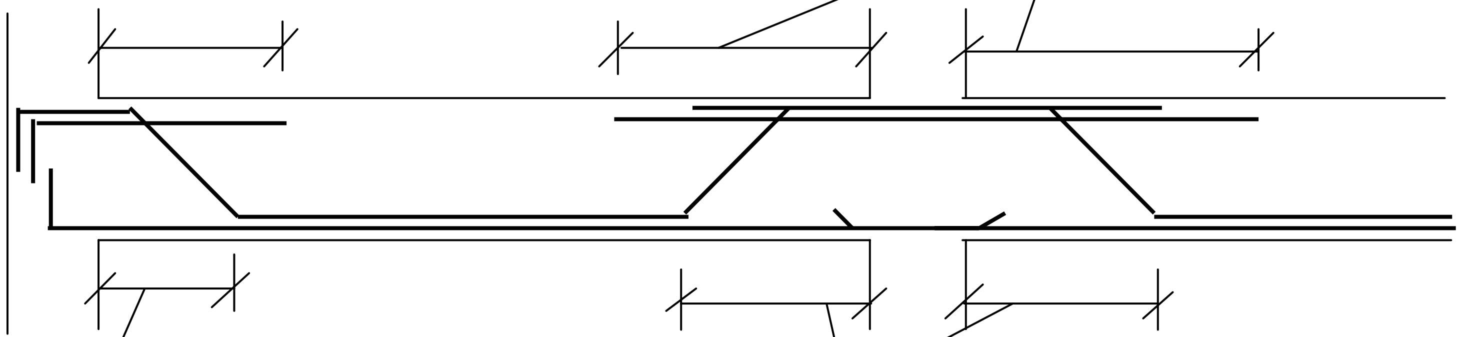
a) Curtailed Bars for Beams



b) Curtailed Bars for Slabs,
Distances Are Same as for Beams.

ℓ_n / 4, may be reduced to ℓ_n / 5 if the end is not monolithic with RC column.

ℓ_n / 3, top additional steel may be curtailed at ℓ_n / 4, not less than ℓ_d (ACI value is ℓ_n / 3).

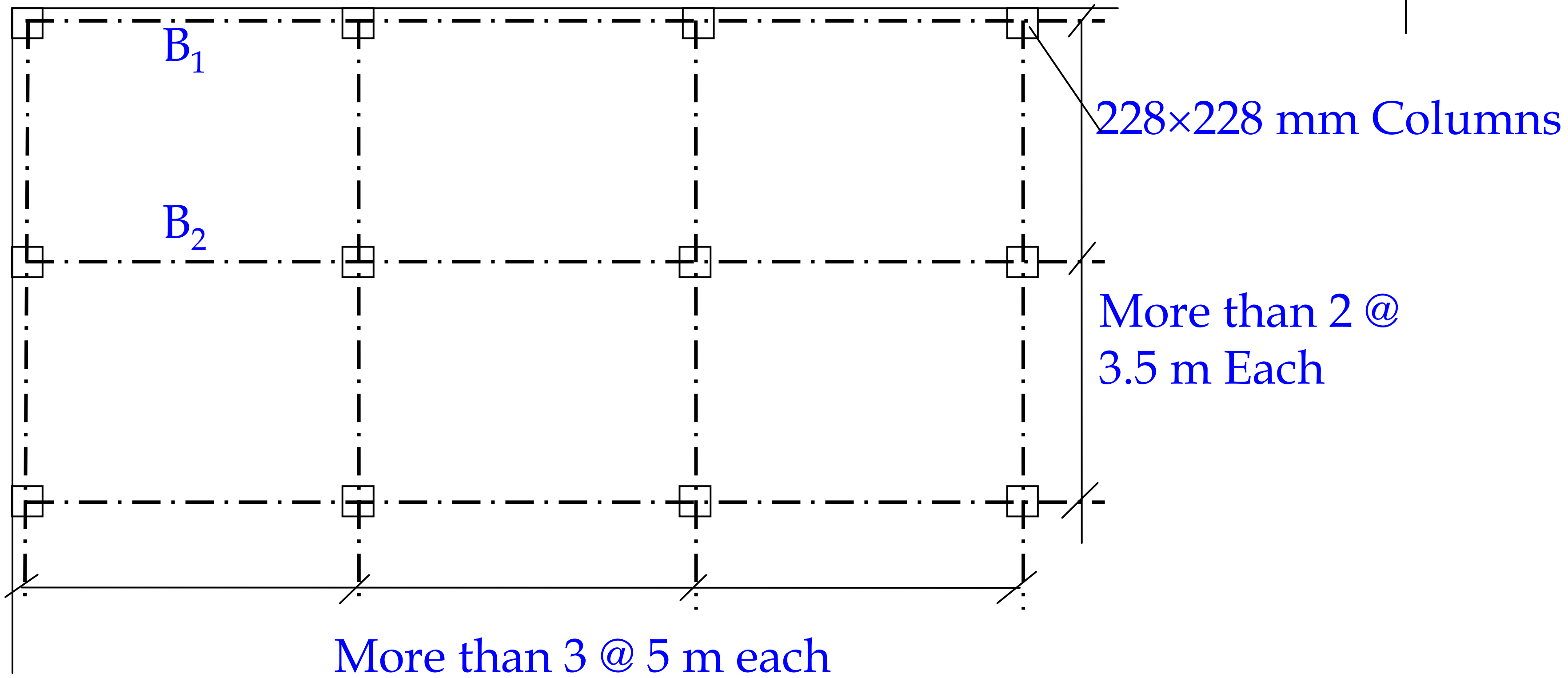


ℓ_n / 7 if less than 50% of the steel is bent up.

ℓ_n / 4 if less than ½ of steel is bent up, more than 50% must not be bent up for approximate detailing.

c) Bent-Up Bars

Example 4.5: A roof system consists of 228 × 228 mm columns at a spacing of 5m and 3.5m in two mutually perpendicular directions, as shown in Fig. 4.20. Beams run in both directions over the columns having more than two spans in each direction. Design the first interior long beam (B_2) if the factored slab load is 12.00 kN/m^2, thickness of slab = 125 mm, E–20 concrete, Grade 420 steel, and live load ≤ 3 × dead load. Select US customary bars.


B1
B2
228×228 mm Columns
More than 2 @ 3.5 m Each
More than 3 @ 5 m each

Solution:

- ℓ_y = 5 m
- ℓ_x = 3.5 m
- Factored slab load = 12 kN/m^2
- b_w = 228 mm, equal to the size of the column
- h_f = 125 mm
- f_c' = 20 MPa
- f_y = 420 MPa
- Beam B_2 is to be designed.

Slab Load

- Total factored load: $= 12.00\ \text{kN/m}^2$

Approximate Self Weight

- Factored dead load

  $= 1.2 \times 2400 \times 0.228 \times (5/12 - 0.125) \times (9.81/1000)$

  $= 1.9\ \text{kN/m}$

Equivalent Width Of Slab Supported By Beam B1

- $\ell_y = 5$ m : $\ell_x = 3.5$ m : $R = \ell_x / \ell_y = 3.5/5 = 0.7$
- Equivalent slab width supported $= (1 - R^2/3)\ \ell_x$

  $= (1 - 0.7^2 / 3) \times 3.5 = 2.93$ m

Factored Slab Load Acting On Beam

- Factored slab load on beam

  = 1.1 × width of slab × slab load per unit area

  = 1.1 × 2.93 × 12.00 = 38.6 kN/m

Note: First interior beam will have to support 10% more width than the interior one.

Total Factored Load

- w_u = 38.6 + 1.9

  = 40.5 kN/m

Total Factored Bending Moments

- $\ell_n = 5 - 0.228 = 4.772$ m
- Exterior support M_u^- $= \dfrac{w_u\,\ell_n^2}{16} = \dfrac{40.5 \times 7.772^2}{16} = 57.7 kN - m$
- Exterior span M_u^+ $= \dfrac{w_u\,\ell_n^2}{14} = \dfrac{40.5 \times 7.772^2}{14} = 65.9 kN - m$
- First interior support M_u^- $= \dfrac{w_u\,\ell_n^2}{10} = \dfrac{40.5 \times 7.772^2}{10} = 92.3 kN - m$
- Interior support M_u^- $= \dfrac{w_u\,\ell_n^2}{11} = \dfrac{40.5 \times 7.772^2}{11} = 83.9 kN - m$
- Interior span M_u^+ $= \dfrac{w_u\,\ell_n^2}{16} = \dfrac{40.5 \times 7.772^2}{16} = 57.7 kN - m$

Selection Of Beam Depth

- Many different options are available for sizing the beam.
- One option may be to size the beam as a singly reinforced section for exterior span positive moment and then design as doubly reinforced section for negative moments.
- Second option may be to proportion the section for maximum negative moment, which may significantly increase the beam depth.
- Third option may be to select dimensions in-between the earlier stated two options.

- Minimum depth of beam for deflection control for exterior panel $= \ell / 18.5$

  $= 5000 / 18.5 = 270$ mm
- $\ell / 12 = 5000 / 12 = 417$ mm
- d_{min} for M_u^+ in the exterior panel

$$= \sqrt{\frac{M_u}{0.205 \times f_c' \times b}} = \sqrt{\frac{65.9 \times 10^6}{0.205 \times 20 \times 228}} = 266mm$$

- $h_{min} = d_{min} + 75 = 341$ mm
- d_{min} for M_u^-, $_{max}$ $= \sqrt{\dfrac{M_u}{0.205 \times f_c' \times b}}$

$$= \sqrt{\frac{92.3 \times 10^6}{0.205 \times 20 \times 228}} = 315mm$$

- $h_{min} = d_{min} + 75 = 390$ mm
- Let $h = 4 \times 75 + 125 = 425$ mm

$\Rightarrow d = 425 - 75 = 350$ mm

Maximum Capacity As Singly Reinforced Rectangular Section At Support

- For negative moment at support, ρ_{max} corresponding to an extreme tensile strain of 0.0075 may be used.

$$\rho_{max} = 0.85\beta_1 \frac{2}{7}\frac{f_c'}{f_y} = 0.85 \times 0.85 \times \frac{2}{7} \times \frac{20}{420} = 0.00983$$

- $A_{s1} = \rho_{max}\, b\, d = 0.00983 \times 228 \times 350 = 784\ \text{mm}^2$

$$a = \frac{A_{s1}\, f_y}{0.85\, f_c'\, b} = \frac{784 \times 420}{0.85 \times 20 \times 228} = 85mm$$

- $M_1 = \phi_b\, M_n = \phi_b\, A_{s1}\, f_y\, (d - a / 2)$

  $= 0.9 \times 784 \times 420 \times (275 - 85 / 2) / 10^6$

  $= 91.1$ kN-m

Effective Flange Width For T-Beam Behavior

- The effective flange width, b, is the minimum of the following three dimensions:

a) $\ell / 4 = 5000 / 4 = 1250$ mm

b) $16\, h_f + b_w = 16 \times 125 + 228 = 2228$ mm

c) S = center-to-center spacing of the beams = 3500 mm

$\therefore \quad b = 1250$ mm

Minimum Steel Ratio

- $\rho_{min} = 1.4 / f_y = 1.4 / 420 = 0.00333$
- $A_{s,\ min} = 0.00333 \times 228 \times 350 = 266\ \text{mm}^2$

Design For Positive Moment In Exterior Span

- For positive moment, the flange of the T-beam will be in compression.
- However, it is more likely that for this smaller moment the N.A. will lie within the flange.
- The beam will act like a rectangular section of dimensions 1250 $\times$ 425 mm.
- Area of steel can not be calculated by using tables or curves as the steel ratio for the full section is much less.

Assume *a* = 75 mm

$$A_s = \frac{M_u}{\phi_b\, f_y\,(d - \frac{a}{2})} = \frac{65.9 \times 10^6}{0.9 \times 420 \times (350 - 75/2)} = 558mm^2$$

$$a = \frac{A_s\, f_y}{0.85 f_c'\, b} = \frac{558 \times 420}{0.85 \times 20 \times 1250} = 11mm$$

$\leq \beta_1 h_f$ = 106 mm ∴ Assumption is correct.

$$A_s = \frac{65.9 \times 10^6}{0.9 \times 420 \times (350 - 11/2)} = 506mm^2$$

$$a = \frac{506 \times 420}{0.85 \times 20 \times 1250} = 10mm$$

A_s = 505 mm^2 > $A_{s,\,min}$ [2–#16 + 1–#13]

Design For Positive Moment In Interior Span

M_u^+ = 57.7 kN-m, Assume a = 10 mm

$$A_s = \frac{M_u}{\phi_b f_y (d - \frac{a}{2})} = \frac{57.7 \times 10^6}{0.9 \times 420 \times (350 - 10/2)} = 442 mm^2$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{442 \times 420}{0.85 \times 20 \times 1250} = 8.7 mm$$

$\leq \beta_1 h_f$ = 106 mm ∴ Assumption is correct.

$$A_s = \frac{57.7 \times 10^6}{0.9 \times 420 \times (350 - 8.7/2)} = 442 mm^2$$

> $A_{s,\,min}$ [2–#16 + 1–#13]

Increased to [2–#16 + 1–#13] for compatibility.

Design For Negative Moment At Exterior Support

- M_u- = 57.7 kN-m
- The T-beam will act like a rectangular section of dimensions 228 × 425, because the flange comes under tension.

$$\frac{M_u}{bd^2} = \frac{57.7 \times 10^6}{228 \times 350^2} = 2.066 MPa$$

- ρ = 0.0059 < ρ_{max} (***OK***)
- A_s = 0.0059 × 228 × 350 = 471 mm^2 > $A_{s,\,min}$

Design For Negative Moment At First Interior Support

- $M_u^- = 92.3$ kN-m $> \phi_b M_n = 70.6$ kN-m
- Design as a doubly reinforced rectangular section of dimensions 228 × 425.
- Let $d' = 60$ mm
- $M_2 = M_u - M_1$

  $= 92.3 - 91.1 = 1.2$ kN-m
- Assuming compression steel to be yielding,

$$A_s' = \frac{M_2}{\phi_b f_y (d - d')} = \frac{1.2 \times 10^6}{(0.9)(420)(350 - 60)} = 11 mm^2$$

- $A_{s2} = A_s' = 11 \text{ mm}^2$
- $A_s = A_{s1} + A_{s2}$

  $= 784 + 11 = \underline{795 \text{ mm}^2}$
- $a = 85$ mm (as calculated before)

$$f_s' = 600\,\frac{a - \beta_1 d'}{a} = 600\,\frac{85 - 0.85 \times 60}{85} = 240.00\,MPa$$

$< f_y$, compression steel is not yielding

$$A_s'{}_{,\ \text{revised}} = A_s'{}_{,\ \text{trial}}\,(f_y / f_s')$$

$$= 11 \times 420 / 240 = \underline{20 \text{ mm}^2}$$

Design For Negative Moment At Interior Supports

- $M_u^- = 83.9$ kN-m $< \phi_b M_n = 91.1$ kN-m
- Design as a singly reinforced rectangular section of dimensions 228 × 425.

$$\frac{M_u}{bd^2} = \frac{83.9 \times 10^6}{228 \times 350^2} = 3.004$$

- $\rho = 0.0090$
- $A_s = 0.0090 \times 228 \times 350 = \underline{718 \text{ mm}^2}$

Adjustment Of Steel For Bent-Up Option

- Top steel available at exterior support = 3–#13 (387 mm^2)

  Extra steel required = 471 – 387

  = 84 mm^2 [1–#13]

- Top steel available at first interior support

  = 2–#13 + 1–#13 + 1–#13 (516 mm^2)

  Extra steel required = 795 – 516 = 279 mm^2 [2–#16]

- Bottom steel available at first interior support (exterior face) = 2–#16 (398 mm^2)

  Extra steel required = 20 – 398 = Code Minimum

- Bottom steel available at first interior support (Interior face) = 2–#13 (258 mm^2)

  Extra steel required = 0 – 258

  = Code Minimum

- Top steel available at interior support

  = 2–#13 + 2–#16 (656 mm^2)

  Extra steel required = 718 – 656 = 62 mm^2 [1–#13]

- Bottom steel available at interior support

  = 2–#13 (258 mm^2)

- Extra steel required = 0 – 258

  = Code Minimum

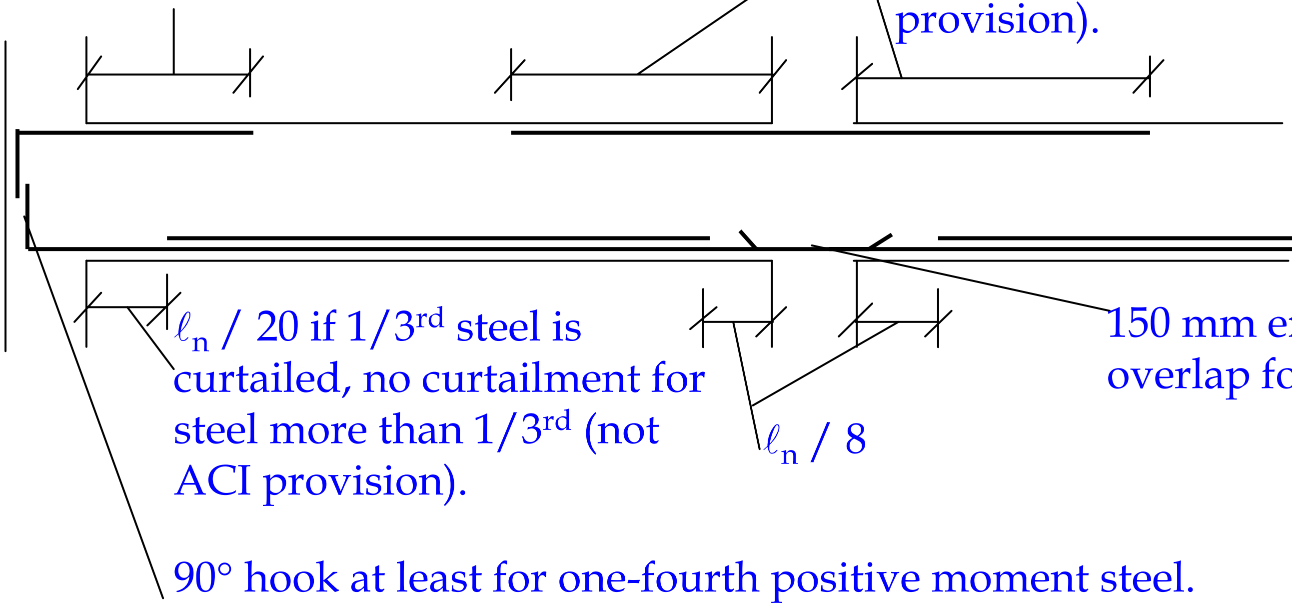
Typical Curtailment Lengths

$\ell_n = 5000 - 338 = 4772$ mm

Value	*Length (mm)*
$\ell_n / 5$	950
$\ell_n / 4$	1190
$\ell_n / 20$	240
$\ell_n / 7$	680
$\ell_n / 8$	600
$\ell_n / 3$	1590
$0.3\ell_n$	1430

ℓ_n / 4, may be reduced to ℓ_n / 5 if the end is not monolithic with RC column (not ACI provision).

0.3ℓ_n, 50% out of this steel may be curtailed at ℓ_n / 4, provided this distance is more than ℓ_d (not ACI provision).



ℓ_n / 20 if 1/3rd steel is curtailed, no curtailment for steel more than 1/3rd (not ACI provision).

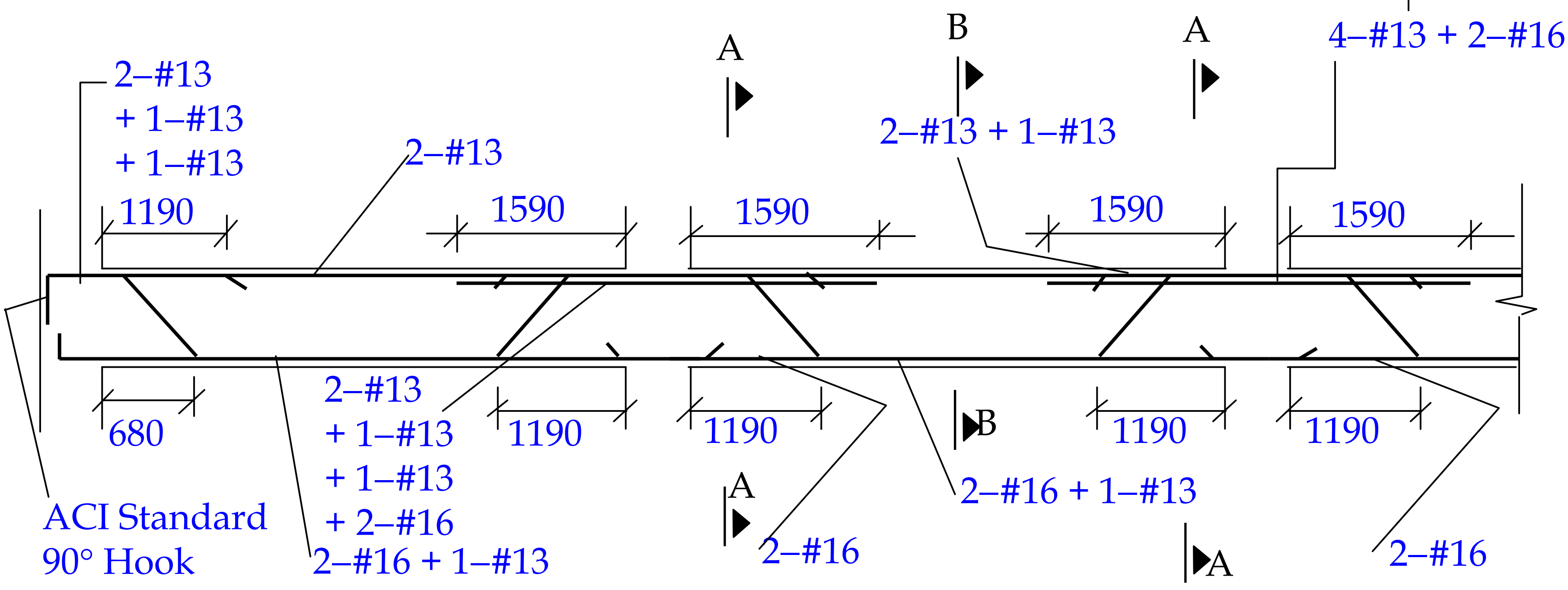
ℓ_n / 8

150 mm extension without overlap for non-perimeter beam

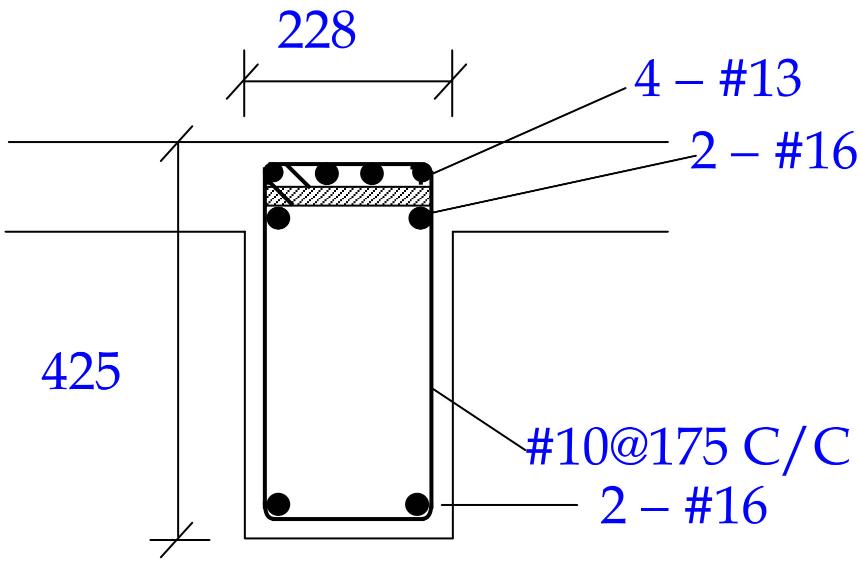
90° hook at least for one-fourth positive moment steel.

ℓ_n = clear distance of the respective span for curtailment of bottom bars and larger of the two adjacent spans for the top steel.

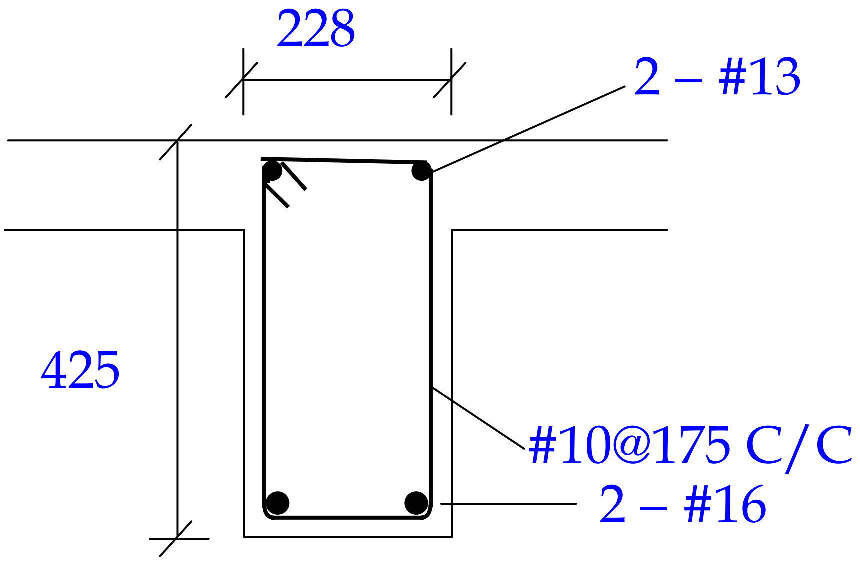
a) Curtailed Bars for Beams



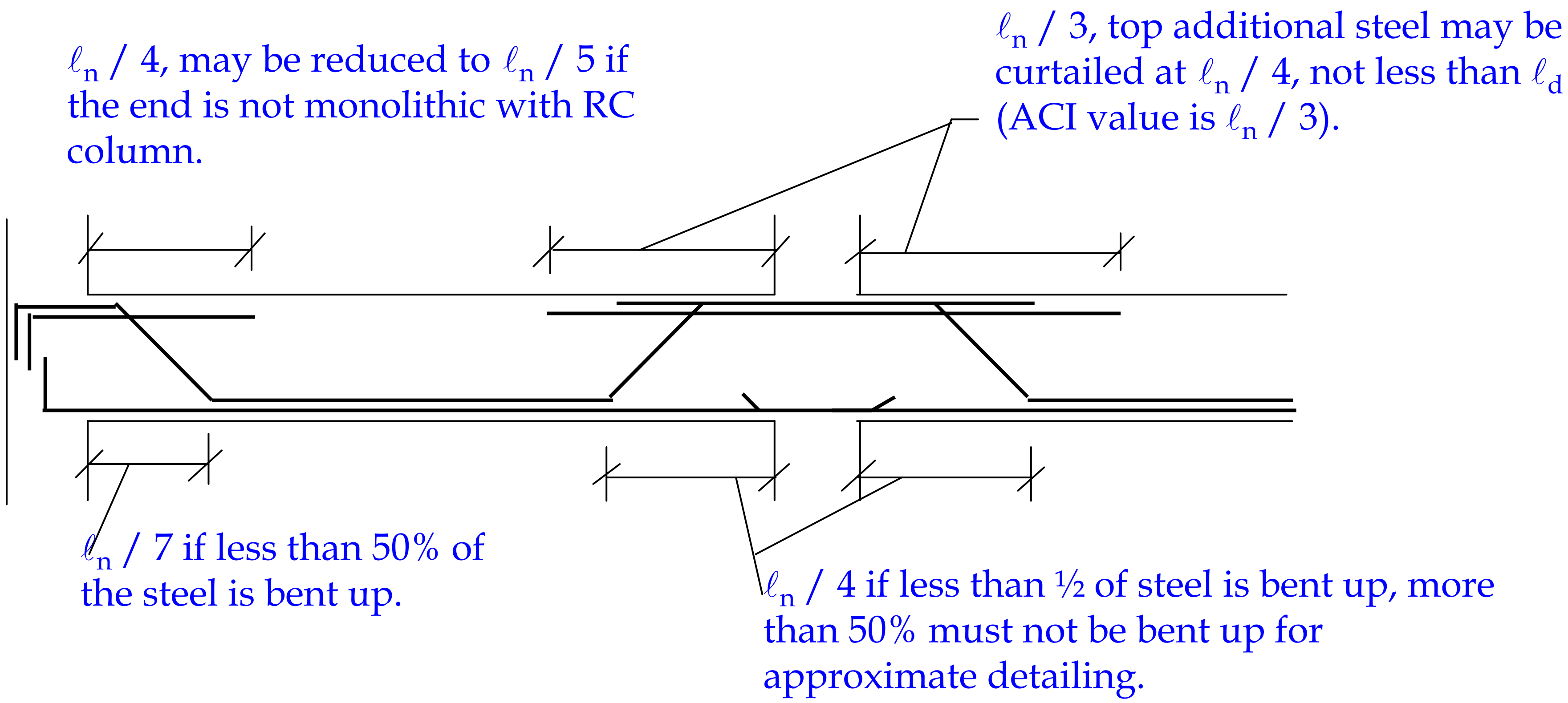
Longitudinal Section



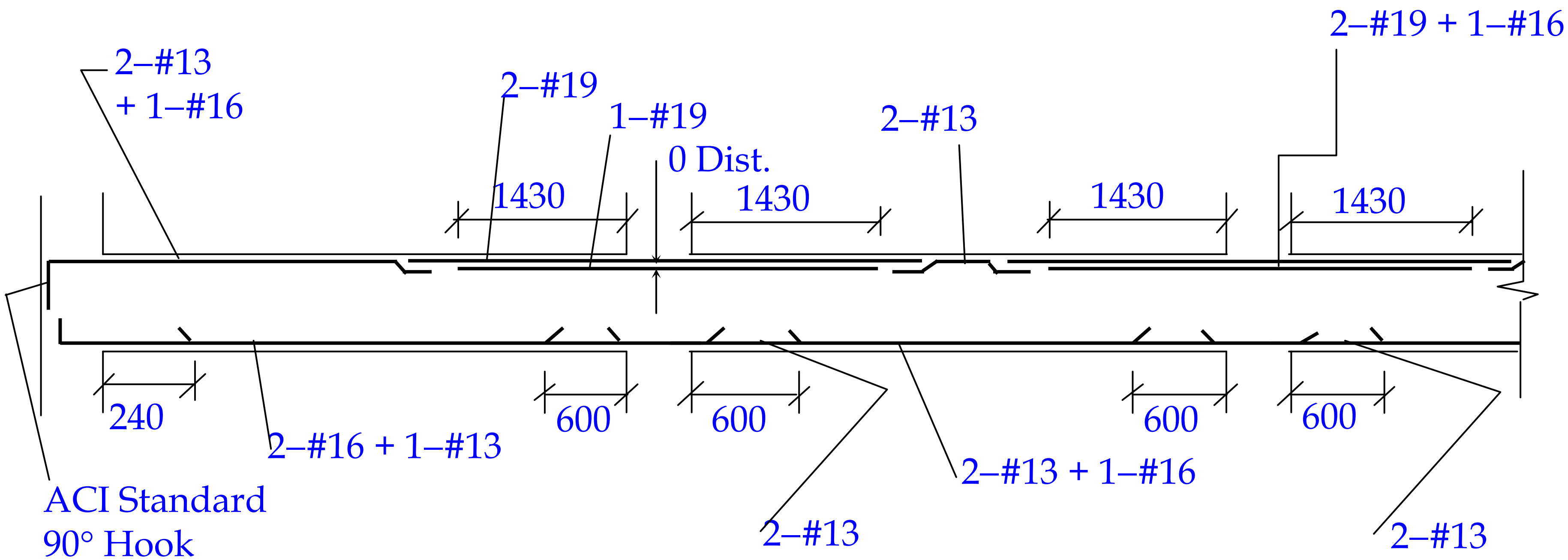
<u>Cross Section AA</u>



<u>Cross Section BB</u>



c) Bent-Up Bars



Better Reinforcement Details Using Curtailment

Assignment Chapter 4, 5 Problems