Example Problem # 1

A thin cylindrical vessel of 2.0 m diameter and 4.0 m length contains a particular gas at a pressure of 1.65 N/mm^2. If the permissible tensile stress of the material of the shell is 150 N/mm^2, find the maximum thickness required.

Data

Permissible tensile stress, σ_{all} = 150 N/mm^2 (Mpa)

L = 4.0 m

d = 2.0 m

p = 1.65 N/mm^2

t = ?

Example Problem # 2

A cylindrical compressed air drum is 2.0 m in diameter with plates 12.5 mm thick. The efficiencies of the longitudinal (η_L) and circumferential (η_c) joints are 85% and 45% respectively. If the tensile stress in the plating is to be limited to 100 MPa, find the maximum safe air pressure.

Data

Permissible tensile stress, σ_{all} = 100 MPa

L = 4.0 m $\qquad \eta_L$ = 85 %

t = 12.5 mm $\qquad \eta_c$ = 45 %

p = ?

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formulas

$$\sigma_H = \frac{pd}{2t\eta_L} \quad \text{longitudinal}$$

$$\sigma_L = \frac{pd}{4t\eta_c} \quad \text{circumferential}$$

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Data:

$D = 2m$ $L = 4m$ $P = 1.65\ N/mm^2$

$\sigma_{all} = 150\ N/mm^2$ $t = ?$

Sol:

$$\sigma_{all} = \sigma_H = \frac{pd}{2t}$$

$t = 11mm$

$$\sigma_{all} = \sigma_L = \frac{pd}{4t}$$

$= 5.5mm$

Res: Safe thickness = 11mm (Bigger one)

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Data:

$D = 2m$ $t = 12.5mm$ $\eta_L = 85\%$ $\eta_c = 45\%$

$\sigma_{all} = 100 MPa$ Safe air pressure $= P = ?$

Sol:

$$100 \times 10^6 = \frac{P \times 2}{2 \times \frac{12.5}{1000} \times 0.85}$$

$P = 1.0625\ MPa$

$$100 \times 10^6 = \frac{P \times 2}{4 \times \frac{12.5}{1000} \times 0.45}$$

$P = 1.125\ MPa$

Example Problem # 3

A cylindrical shell, 0.8 m in a diameter and 3 m long is having 10 mm wall thickness. If the shell is subjected to an internal pressure of 2.5 N/mm^2, determine

(a) change in diameter,

(b) change in length, and

(c) change in volume.

Take E = 200 GPa and Poisson's ratio = 0.25.

Data

Diameter of the shell, $d = 0.8\text{ m} = 800\text{ mm}$.

Thickness of the shell, $t = 10\text{ mm}$.

Internal pressure, $p = 2.5\text{ N/mm}^2$.

δd, δL and δV = ?

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Data.

$D = 0.8m$ $L = 3m$ $t = 10mm$

$P = 2.5 N/mm^2$ $E = 200 GPa$ $v = 0.25$

$\delta_D = ?$ $\delta_L = ?$ $\delta_V = ?$

Sol:

$$\delta_L = \frac{2.5 \times 800 \times (1 - 2 \times 0.25)(3000)}{4 \times 10 \times 200000} \quad 0.375 \text{ mm}$$

$$\delta_D = \frac{pd(2-v)d}{4tE}$$

$$= \frac{2.5 \times 800 \times (2 - 0.25)(800)}{4 \times 10 \times 200000} = 0.35 \text{ mm}$$

$$\varepsilon_v = \frac{\delta V}{V}$$

$$\Rightarrow V = \frac{\pi d^2}{4} \times L$$

$$= \frac{\pi (800)^2}{4} \times 3000 = 1.507 \times 10^9 \text{ mm}^3$$

$$\delta V = \varepsilon_v \times V$$

$$= 1507000 \text{ mm}^3$$

$$= 1.507 \times 10^6 \text{ mm}^3$$

$$\varepsilon_v = \frac{pd}{4tE}(5 - 4\nu)$$

$$= 1 \times 10^{-3}$$

Method 2

$$\delta V = \frac{\pi}{4} L \times 2d\,\delta d + \frac{\pi d^2}{4} \delta L$$

$$= 1.507 \times 10^6 \text{ mm}^3$$

Example Problem # 4

A copper tube of 50 mm diameter and 1200 mm length has a thickness of 1.2 mm with closed ends. It is filled with water at atmospheric pressure. Find the increase in pressure when an additional volume of 32 cc of water is pumped into the tube. Take *E* for copper = 100 GPa, Poisson's ratio = 0.3 and *K* for water = 2000 N/mm^2.

Prob = 4 Answer = ($P = 15.2$ MPa)

Data: $D = 50$ mm, $L = 1200$ mm, $t = 1.2$ mm, Rate

Increase in P = ?

Extra vol. of fluid = $32\ cm^3$

$E = 100$ GPa $\quad \nu = 0.3 \quad K = 2000\ N/mm^2$

Sol: $$32 \times 1000 = \frac{P \times 50}{4 \times 1.2 \times 100 \times 1000} \times (5 - 4 \times 0.3) \times 2.356 \times 10^6 + \frac{P \times 2.356 \times 10^6}{2000}$$

Example Problem # 6

The internal and external diameters of a ***thick hollow cylinder*** are 80 mm and 120 mm respectively. It is subjected to an external pressure of 40 N/mm^2 and an internal pressure of 120 N/mm^2. Calculate the circumferential stress at the external and internal surfaces and determine the radial and circumferential stresses at the mean radius.

Data

$d_i = 80$ mm , $d_o = 120$ mm

$p_i = 120\ N/mm^2$, $p_o = 40\ N/mm^2$

$(\sigma_H)_o$, $(\sigma_H)_i$ and $(\sigma_H)_{mean}$ = ?

$(\sigma_r)_{mean}$ = ?

Problem 6

Data:

$D_i = 80mm$ $(\sigma_c)_o = ?$

$D_o = 120mm$ $(\sigma_c)_i = ?$

$P_o = 40 N/mm^2$ $(\sigma_c)_{mean\ r} = ?$

$P_i = 120 N/mm^2$ $(\sigma_r)_{mean\ r} = ?$

Sol:

As $\sigma_r = A - \frac{B}{r^2}$

$-P_i = A - \frac{B}{r_i^2}$ and $-P_o = A - \frac{B}{r_o^2}$

$-120 = A - \frac{B}{40^2}$, $-40 = A - \frac{B}{60^2}$

$A = 24$

$B = 230400$

$(\sigma_c)_o = (\sigma_H)_o = A + \frac{B}{r_o^2} = 88 MPa$

$(\sigma_c)_i = (\sigma_H)_i = A + \frac{B}{r_i^2} = 168 MPa$

$(\sigma_H)_{mean\ radius} = A + \frac{B}{(r_{mean})^2} = 116.16 MPa$

$(\sigma_r)_{mean\ rad.} = A - \frac{B}{r^2_{mean}} = -68.16 MPa$

Example Problem # 7

The cylinder of a hydraulic press has an internal diameter of 0.3 m and is to be designed to withstand a pressure of 10 MPa without the material being stressed over 20 MN/m^2. Determine the thickness of the metal and the hoop stress on the outer side of the cylinder.

Data

$d_i = 0.3 \text{ m} = 300 \text{ mm}$

$\sigma_{all} = 20 \text{ MPa}$

$p_i = 10 \text{ MPa}$,

Thickness , $t = ?$

$(\sigma_H)_o = ?$

Prob. 7

Data:

$D_i = 0.3m = 300mm$

$P_i = 10\ MPa$

$\sigma_{all} = 20\ MPa$

$t = ?$

$(\sigma_H)_o = ?$

Sol:

As. $\sigma_r = A - \frac{B}{r^2}$

So $-P_i = A - \frac{B}{r_i^2}$

$-10 = A - \frac{B}{150^2}$

$(\sigma_H)_i = A + \frac{B}{r_i^2}$

$20 = A + \frac{B}{150^2}$

$A = 5$

$B = 337500$

As for outer radius

$0 = A - \frac{B}{r_o^2}$

$0 = 5 - \frac{337500}{r_o^2}$

$r_o = 260mm$

$t = r_o - r_1$

$= 260 - 150$

$= 110\ mm$

$(\sigma_H)_o = A + \frac{B}{r_o^2}$

$= 5 + \frac{33500}{260^2}$

$= 5.5\ MPa$

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Example Problem # 8

(a) In an experiment on a thick cylinder of 100 mm external diameter and 50 mm internal diameter the hoop and longitudinal strains as measured by strain gauges applied to the outer surface of the cylinder were 240×10^{-6} and 60×10^{-6} respectively, for an internal pressure of 90 MPa, the external pressure being zero.

a Determine the actual hoop and longitudinal stresses present in the cylinder if E = 208 GPa and **v** = 0.29. Compare the hoop stress value so obtained with the theoretical value given by the ***Lame*** equations.

(b) Assuming that the above strain readings were obtained for a thick cylinder of 100 mm external diameter but unknown internal diameter calculate this internal diameter.

a)

$D_o = 100mm$

$D_i = 50mm$

$\varepsilon_H = 240\times10^{-6}$

$\varepsilon_L = 60\times10^{-6}$

$P_i = 90MPa$

$P_o = 0$

$\sigma_H = ?$

$\sigma_L = ?$

$E = 208GPa$

$\nu = 0.29$

Sol:

$\varepsilon_H = \frac{1}{E}(\sigma_H - \nu\sigma_L - \nu\sigma_r)$, $\varepsilon_L = (\sigma_L - \nu\sigma_H - \nu\sigma_r)$

$\because \sigma_r = 0$

Solving for σ_H and σ_L

$\sigma_H = 58.4 MPa$

$\sigma_L = 29.4MPa$

By Lames equation

$$\sigma_r = A - \frac{B}{r^2}$$

i) $-P_i = A - \frac{B}{r_i^2}$

$-90 = A - \frac{B}{25^2}$

$A = 30$

$B = 75000$

ii) $-P_o = A - \frac{B}{r_o^2}$

$0 = A - \frac{B}{50^2}$

As

$\sigma_r + \sigma_H = 2A$

$\sigma_H = 2(30)$

$\boxed{\sigma_H = 60MPa}$

$\sigma_L = \frac{\sigma_H}{2}$

$\boxed{\sigma_L = 30MPa}$

b) As $\sigma_H = A + \frac{B}{r^2}$

$(\sigma_H) = 58.4$

$\sigma_r = 0$

At $r_o = 50mm$

$\sigma_H = A + \frac{B}{r_o^2}$, $\sigma_r = A - \frac{B}{r_o^2}$

$58.4 = A + \frac{B}{50^2}$, $0 = A - \frac{B}{50^2}$

$A = 29.35$

$B = 73375$

Now As $\sigma_r = A - \frac{B}{r_i^2}$

$-P_i = A - \frac{B}{r_i^2}$

$-90 = 29.35 - \frac{73375}{r_i^2}$

$r_i = 24.79mm$

$D_i = 49.6mm$

Past papers

Data: Thin cylinder

$D_i = 230mm$

$t = 5mm$

$L = 1m$

change in internal vol. $\delta V = 12 \times 10^{-6} m^3$

$E = 200 GPa$

$\nu = 0.25$

Rigid end plates

i) σ_H, $\sigma_L = ?$

ii) σ_H, σ_L when $\eta_H = 45\%$

$\eta_L = 85\%$

Sol:

$$\sigma_H = \frac{Pd}{2t} = \frac{(1.26)(230)}{2 \times 5} = 28.98 \text{ MPa}$$

$$\sigma_L = \frac{\sigma_H}{2} = 14.49 \text{ MPa}$$

As

$$\delta V = \frac{Pd}{4tE}[5 - 4\nu]V$$

Volume of cylinder $= \pi r^2 h$

$= \pi (115)^2 (1000)$

$= 41.55 \times 10^6 mm^3$

$= 41.55 \times 10^{-6} m^3$

$$12 \times 10^{-6} = \frac{P \times 0.23}{4 \times 0.005 \times 200 \times 10^9}[5 - 4 \times 0.25] 41.55 \times 10^{-6}$$

$P = 1255763 \text{ N/m}^2$

$P = 1.26 \text{ MPa}$

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iii) Change in $P = ?$

further increase in internal vol. by 15%

$\delta V = 1.38 \times 10^{-5} m^3$

$P = 1.45 \text{ MPa}$