Prob 2 (slide)

Sol:

a) $\sigma_{cr} = \frac{P_{cr}}{A}$ $\quad I = \frac{bh^3}{12} = 72000\ mm^4$

$= \frac{\pi^2 EI}{A\, Le^2}$

$L_e = 507.4\ mm$

$Le = KL$

$Le = (1)L$ $\quad$ Hence $L = Le = 507.4\ mm$

b) $P_{cr} = \frac{\pi^2 EI}{Le^2}$ $\quad L = 2m = 2000\ mm$

$= 35.53\ kN$

Slide 22

a) $P_{cr} = 2.5 \times 100 = 250\ kN$ $\quad P = 100{,}000\ N$

$E = 130{,}000\ MPa$

$L = 2000$

$P_{cr} = 250000\ N$

$P_{cr} = \frac{\pi^2 EI}{L^2}$

$I = 7.79 \times 10^6\ mm^4$

$I = \frac{a^4}{12}$ $\quad a = 93.33\ mm \approx 100\ mm$

As $\sigma = \frac{P}{A} = \frac{100{,}000}{100 \times 100} = 10\ MPa$

$\sigma < \sigma_{all}$

$10 < 12$ $\quad$ Hence $a = 100\ mm$ (OK)

b) $P_{cr} = 2.5 \times 200 = 500\ kN$

$I = 15.59 \times 10^6\ mm^4$

$I = \frac{a^4}{12}$ $\quad a = 116.95\ mm$

as $\sigma = \frac{P}{A} = \frac{200{,}000}{(116.95)^2} = 14.62\ MPa$

$\sigma > \sigma_{all}$ $\quad$ Hence repeat dimensions

$\sigma_{all} = \frac{P}{A}$

$A = \frac{200{,}000}{12} = 16.67 \times 10^{+3}\ mm^2$

$a = 129\ mm$

Rankine formula

$P_{cr} = \frac{\sigma_y A}{1 + a\left(\frac{L}{r}\right)^2}$

Data:

$P_e = ?$

$D_o = 120$ mm , $t = 20$ mm

$L = 4.2$ m Hinged $K = 1$

$E = 80$ KN/mm² $\sigma_y = 550$ N/mm² $a = \frac{1}{1600}$

Compute euler load with crushing load using Rankine

For what length Euler formula cease to apply = ? (L_{limit}?)

$= 1336$ mm

Sol:

$D_i = 80$ mm

$I = 8.17 \times 10^6$ mm⁴

$\Rightarrow P_E = 365690$ N

$A = 6283$ mm²

$r = \sqrt{\frac{I}{A}} = 36$ mm.

$\Rightarrow P_R = 363487$ N

E = 80,000 MPa

Euler's formula.

$P_{cr} = \frac{\pi^2 EI}{(L_e)^2} = \frac{\pi^2 EI}{(KL)^2}$

$= \frac{\pi^2 E (Ar^2)}{(KL)^2}$

$= \frac{\pi^2 EA}{\left(\frac{KL}{r}\right)^2}$

$L_e = KL$

$\frac{P_E}{P_R} = 1.006$

$P_{cr} = \frac{\pi^2 EI}{L_e^2}$

$\sigma_y \times A = \frac{\pi^2 EI}{L_e^2}$

$550 \times 6283 = \frac{\pi^2 \times 80000 \times 8.17 \times 10^6}{L_e^2}$

$L_e = 1336$ mm

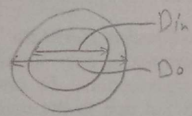
$\frac{3455650}{1 + 4.82 \times 10^{-7} L^2} = \frac{6.45 \times 10^{12}}{L^2}$

$2297630898 = \sigma_{cr}$

365690.1

Problem slide = 29

$D_{in} = D_o - 2t$
$= 200 - 2(20) = 160 mm$



$A = \frac{\pi}{4}(D_o^2 - D_{in}^2) = 11309.73 mm^2$

$I = \frac{\pi}{64}(D_o^4 - D_{in}^4) = 46.37 \times 10^6 mm^4$

$r = \sqrt{\frac{I}{A}} = 64 mm$

$L_e = KL = 0.5 \times 4.5 = 2.25 m$

$P_R = 3509.4 KN$

$P_{safe} = \frac{P_R}{FOS} = 877 KN$

Slide = 30

$P_R = P_e$

$K = 1$
$L = L_e$

$I = \frac{\pi}{64}(D_o^4 - D_i^4) = 44140.11 mm^4$

$A = \frac{\pi}{4}(D_o^2 - D_{in}^2) = 278.8 mm$

$r = \sqrt{\frac{I}{A}} = 12.58$

$L_{lim} = 980 mm$

(Slide = 35 Problem

a) $L = 6m$ $K = 1$ $L_e = KL = 6m$

limiting slenderness ratio $R_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = 108$

$$\frac{L_e}{r_{min}} = \frac{6000}{78} = 77$$

$$\frac{L_e}{r_{min}} < R_c \text{ (short column)}$$

$\Rightarrow FOS = \frac{5}{3} + \frac{3}{8}\left(\frac{L_e/r}{R_c}\right) - \frac{1}{8}\frac{(L_e/r)^3}{R_c^3}$

$FOS = 1.89$

$\Rightarrow \sigma_{max} = \left(1 - \frac{(L_e/r)^2}{2R_c^2}\right)\sigma_y$

$= 253.6$ MPa

$\Rightarrow \sigma_{all} = \frac{\sigma_{max}}{FOS}$

$= 134.18$ MPa

$\Rightarrow \sigma_{all} = \frac{P_{all}}{A}$

$P_{all} = 2214.135$ KN

b) $L = 9m$ $K = 1$ $L_e = 9m$

$R_c = 108$

$$\frac{L_e}{r_{min}} = \frac{9000}{78} = 115.4$$

$$\frac{L_e}{r_{min}} > R_c \text{ (long column)}$$

$\sigma_{all} = \frac{\left(\frac{\pi^2 E}{(L_e/r)^2}\right)}{1.92}$

$= 77.33$ MPa

$P_{all} = 1276$ KN

Slide = 49

Data:

$P = 350\ kN$ $e = ?$ $K = 2$ (cantilever)

$y = 5\ mm$ $\sigma_{max} = ?$ $L = 3.2\ m$

$E = 200\ GPa$

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2}$$

$$= \frac{\pi^2 (200{,}000)(1870 \times 10^4)}{(2 \times 3200)^2}$$

$$P_{cr} = 901180\ N$$

a) As

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right]$$

$$5 = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{350000}{901180}}\right) - 1\right]$$

↓ Radian

$$5 = e\left[\sec(0.9789) - 1\right]$$

$e = 6.31\ mm$

b) Max. stress

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y}$$

$$M_{max} = P(e + y_{max})$$

$$= 3958500\ Nm$$

$$\sigma_{max} = 69.6\ MPa$$

Data: Steel pipe column

$P_{all} = ?$

$K = 2$

a) $L = 2.8m$ b) $L = 3.2m$

$D_o = 140mm$

$t = 7mm$

$E = 200 GPa$

$\sigma_y = 250 MPa$

Sol:

$D_o - 2t = D_i$

$D_i = 126mm$

$A = 2925 \ mm^2$

$r = \sqrt{\frac{I}{A}}$ $I = 6.49 \times 10^6 mm^4$

$r = 47.1 mm$

a) $L = 2.8m$

$L_e = 5600 \ mm$

$\frac{L_e}{r} = 118.9$

$R_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = 125.67$

$\frac{L_e}{r} < R_c$ Short column

$FOS = 1.92$

$\sigma_{max} = 138.11$

$\sigma_{all} = 71.9 \ MPa$

$P_{all} = 210.4 KN$

b) $L = 3.2m$

$L_e = 6400mm$

$\frac{L_e}{r} = 135.88$

$R_c = 125.67$

$\frac{L_e}{r} > R_c$ long column

$FOS = 1.92$

$\sigma = \frac{P}{A}$

$\sigma_{cr} = \left(\frac{\pi^2 EI}{L_e^2}\right) A$

$= 107$ MPa

$\sigma_{all} = 55.7 \ MPa$

$P_{all} = 163 KN$