Plain and Reinforced Concrete II

Reference Books:

Concrete Structures by Prof. Dr. Zahid Ahmed Siddiqi

Reinforced concrete mechanics and design by James G. Macgregor

Design of concrete structures by Arthur H. Nilson David Darwin Charles W. Dolan

Reinforced Concrete by Edward G. Nawy

Code:

Building Code Requirements for Structural Concrete (ACI318-11)

American Society for Testing and Materials (ASTM)

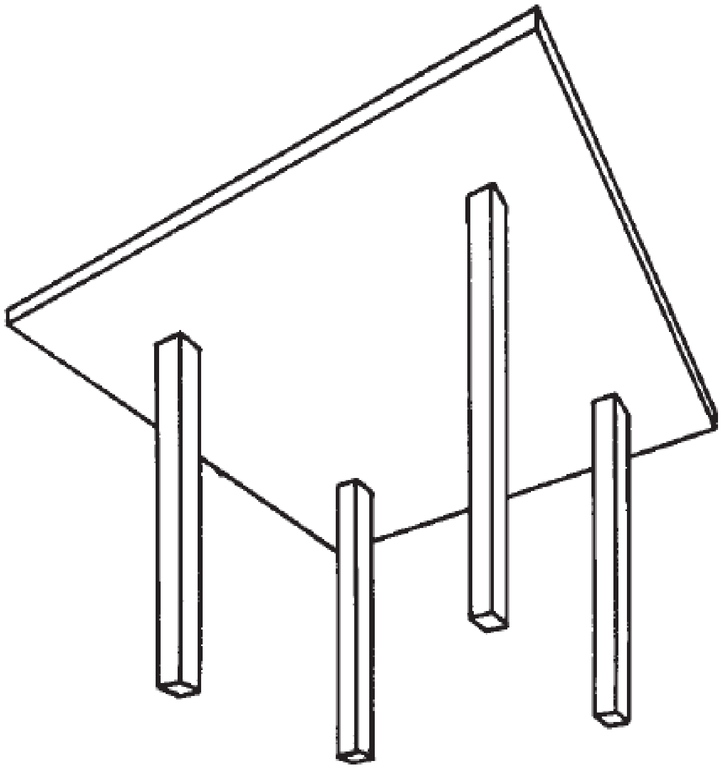
Two Way Slabs, Behavior Analysis and Design

Reference:

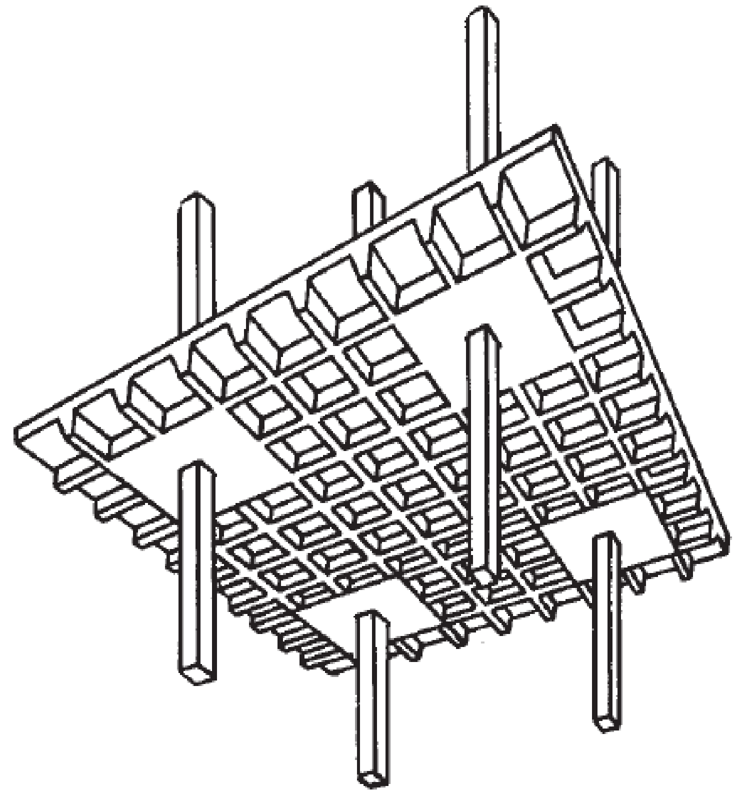
Reinforced concrete mechanics and design

by James G. Macgregor

Type of Two Way Slabs

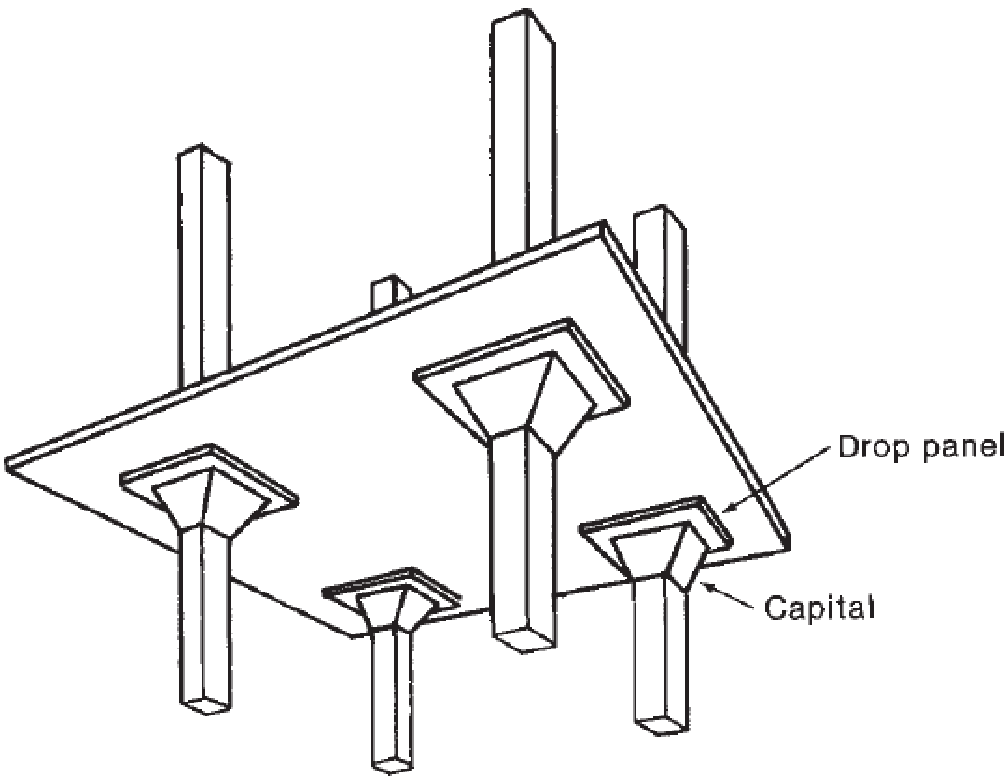


(a) Flat plate.

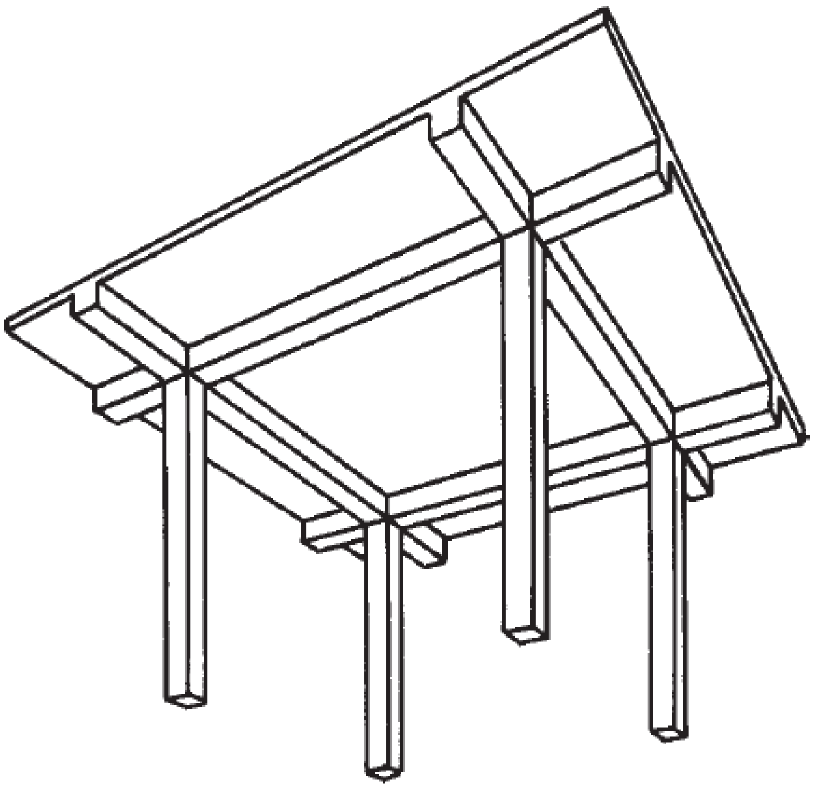


(b) Waffle slab.

Type of Two Way Slabs (Contd.)

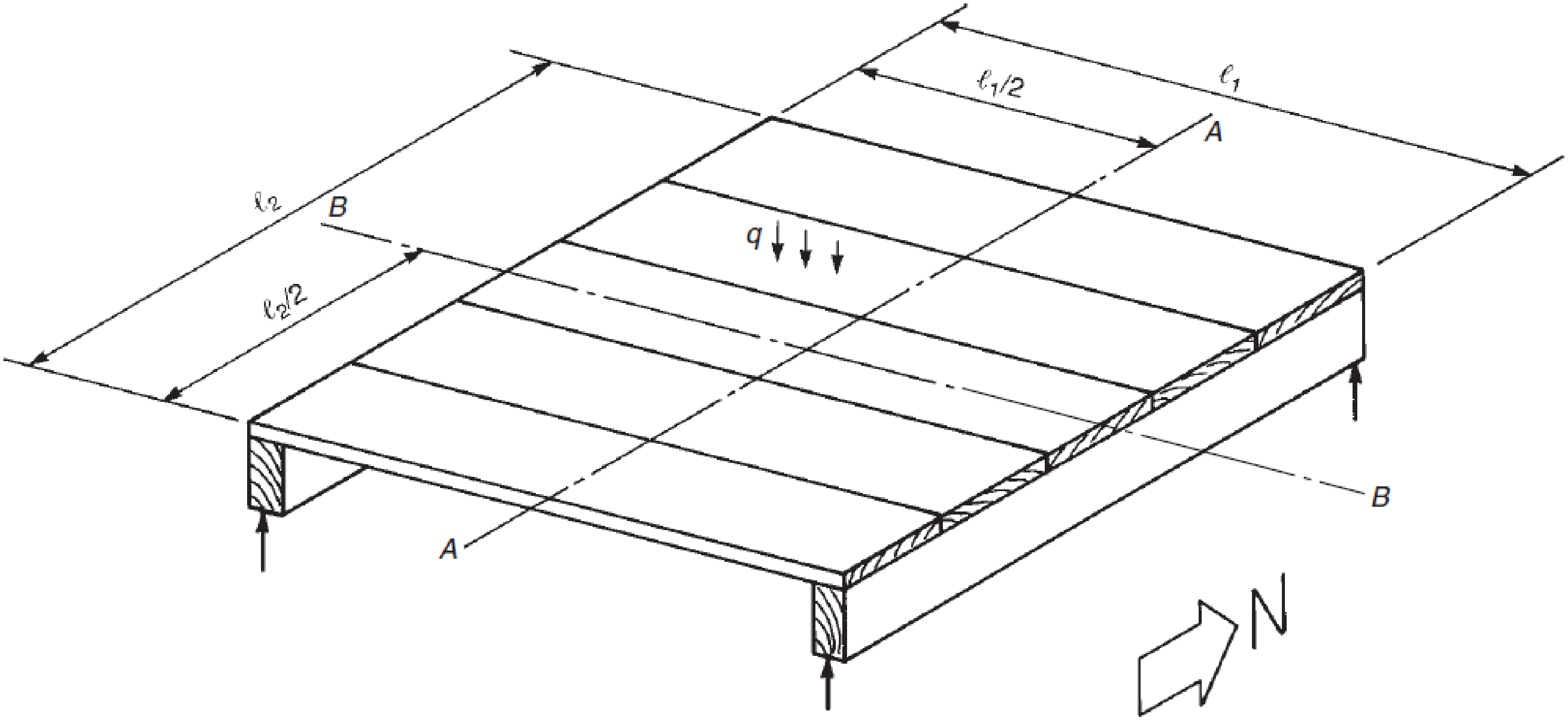


(c) Flat slab.



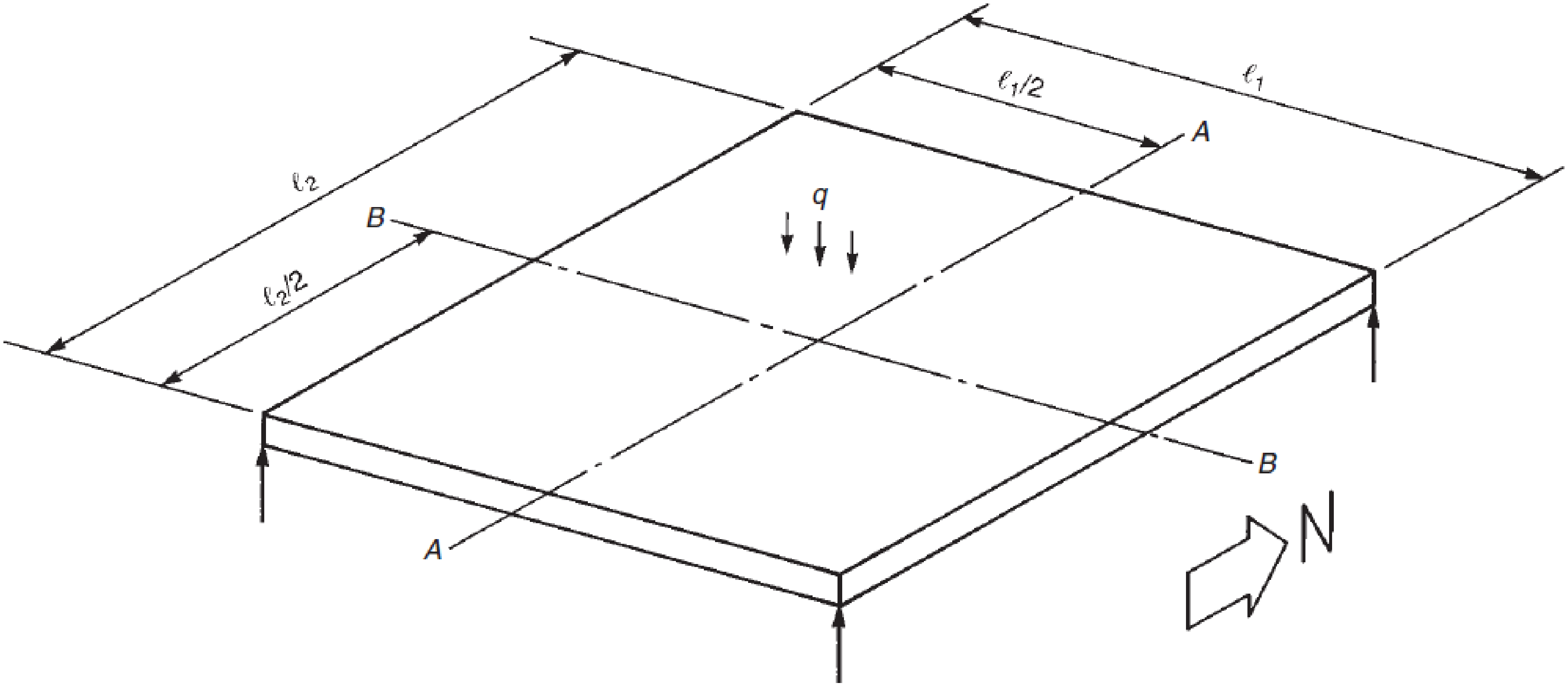
(d) Two-way slab with beams.

Analysis of Moments in a Plank and Beam Floor



m	$= q\, l_1^2/8$	kN-m/m	per Unit Width
$M_{A\text{-}A}$	$= (q\, l_2)\, l_1^2/8$	kN-m	at Section A-A
M_{1b}	$= (q\, l_1/2)\, l_2^2/8$	kN-m	at Section B-B in one Beam
$M_{B\text{-}B}$	$= (q\, l_1)\, l_2^2/8$	kN-m	at Section B-B in both Beams

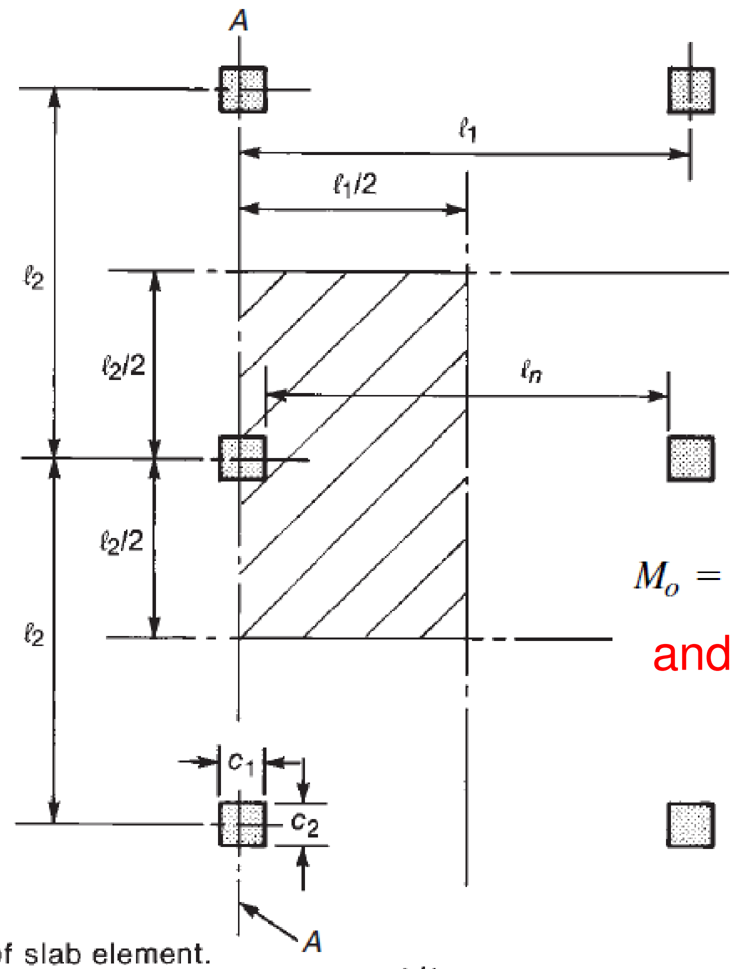
Analysis of Moments in Two Way Slabs



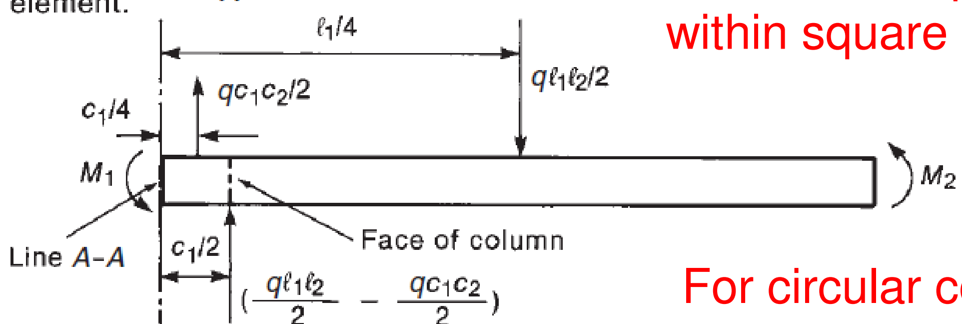
$M_{A\text{-}A} = (q\ l_2)\ l_1^2/8$ kN-m at Section A-A

$M_{B\text{-}B} = (q\ l_1)\ l_2^2/8$ kN-m at Section B-B

Nichol's Analysis of Moments in Slabs



Plan of slab element.



Side view of slab element

Total Statical Moment Mo is sum of negative moment M_1 and positive moment M_2 as computed by summing moments about line A-A

$$M_o = M_1 + M_2 = \left(\frac{q\ell_1\ell_2}{2}\right)\frac{\ell_1}{4} - \left(\frac{qc_1c_2}{2}\right)\frac{c_1}{4} - \left(\frac{q\ell_1\ell_2}{2} - \frac{qc_1c_2}{2}\right)\frac{c_1}{2}$$

and

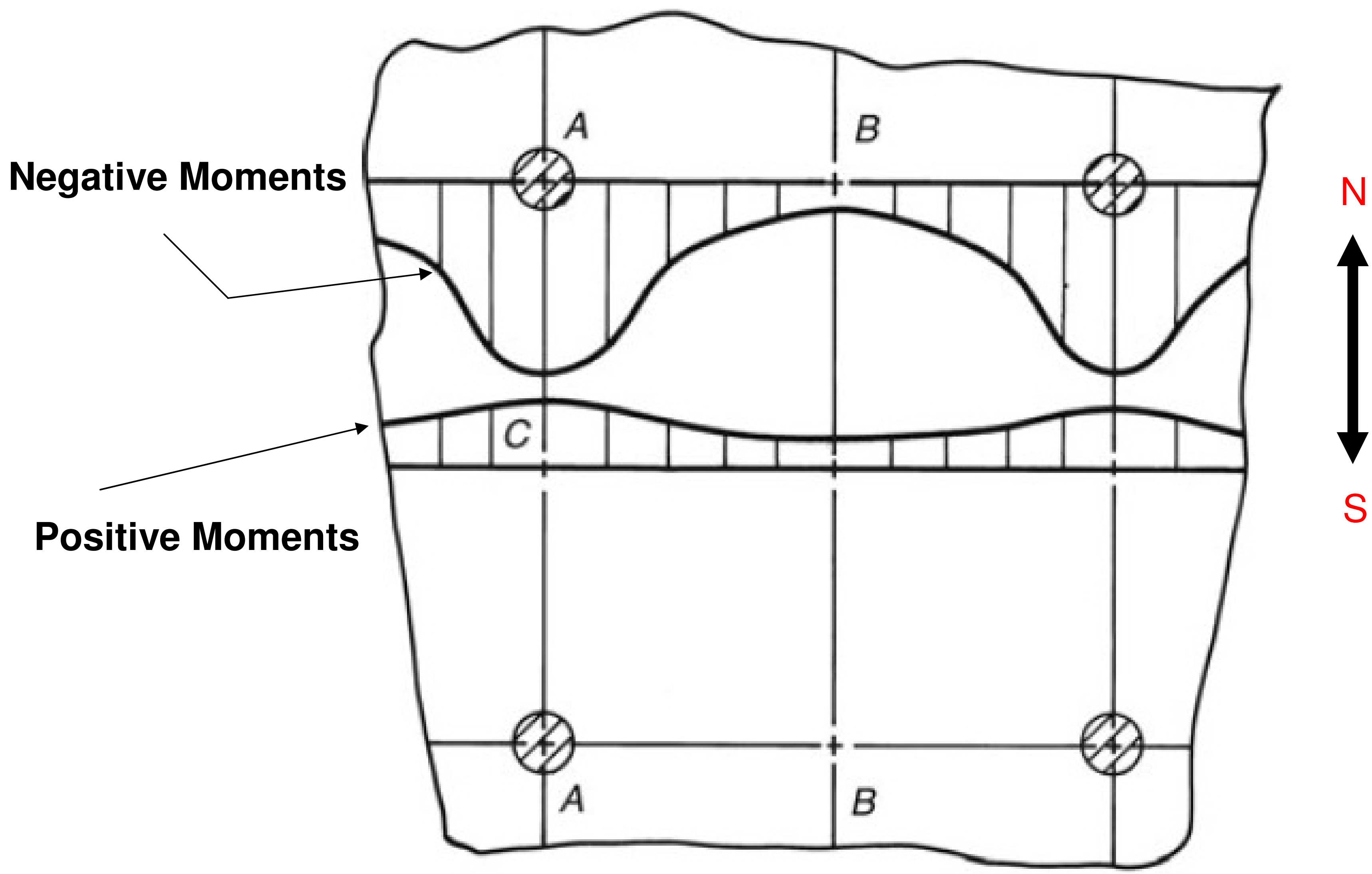
$$M_o = \frac{q\ell_2}{8}\left[\ell_1^2\left(1 - \frac{2c_1}{\ell_1} + \frac{c_2c_1^2}{\ell_2\ell_1^2}\right)\right]$$

ACI code simplified the expression within square brackets with l_n^2, hence

$$M_o = \frac{q\ell_2\ell_n^2}{8}$$

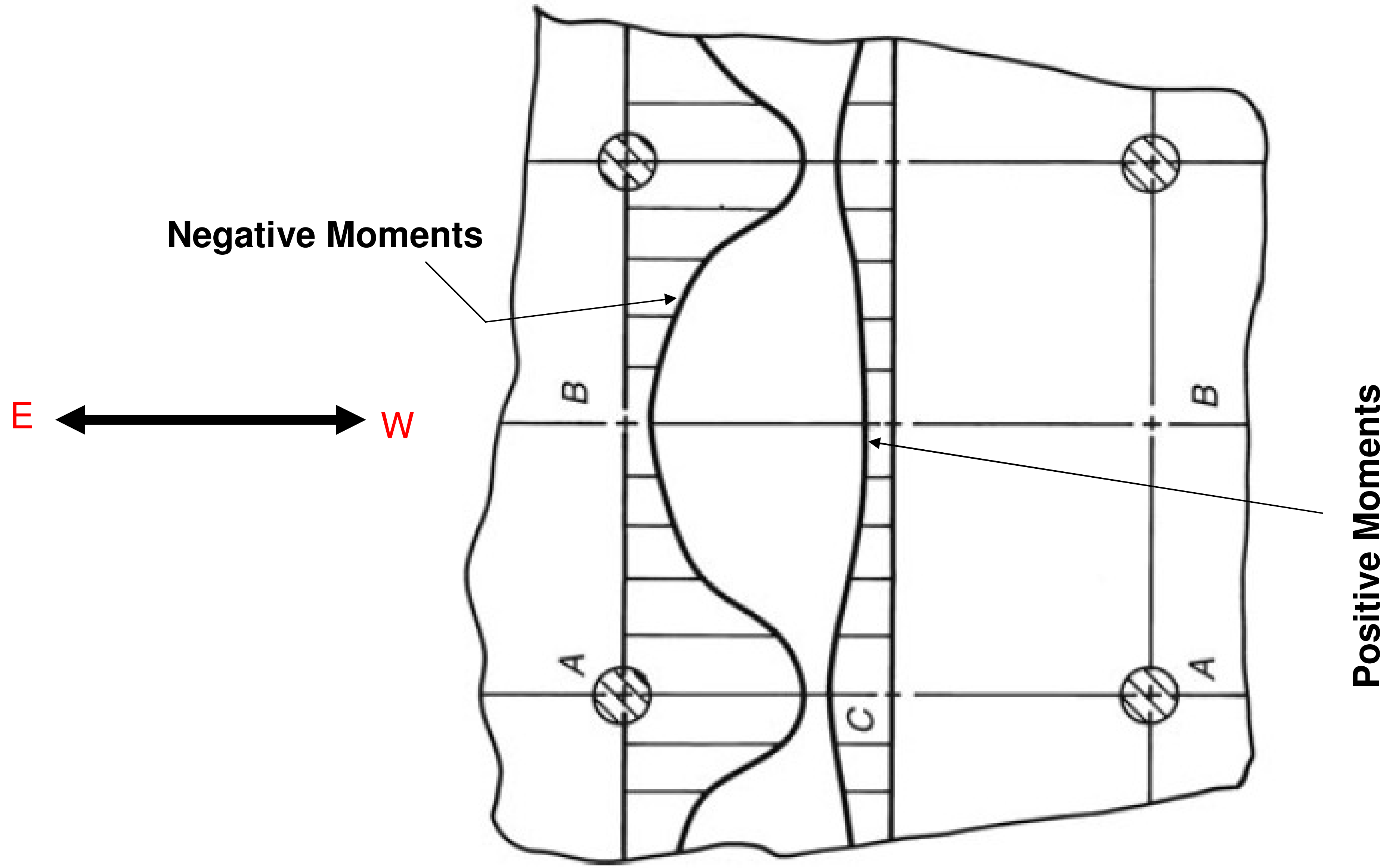
For circular columns ACI expresses it based on equivalent square column

Moments in Slabs Supported by Isolated Columns



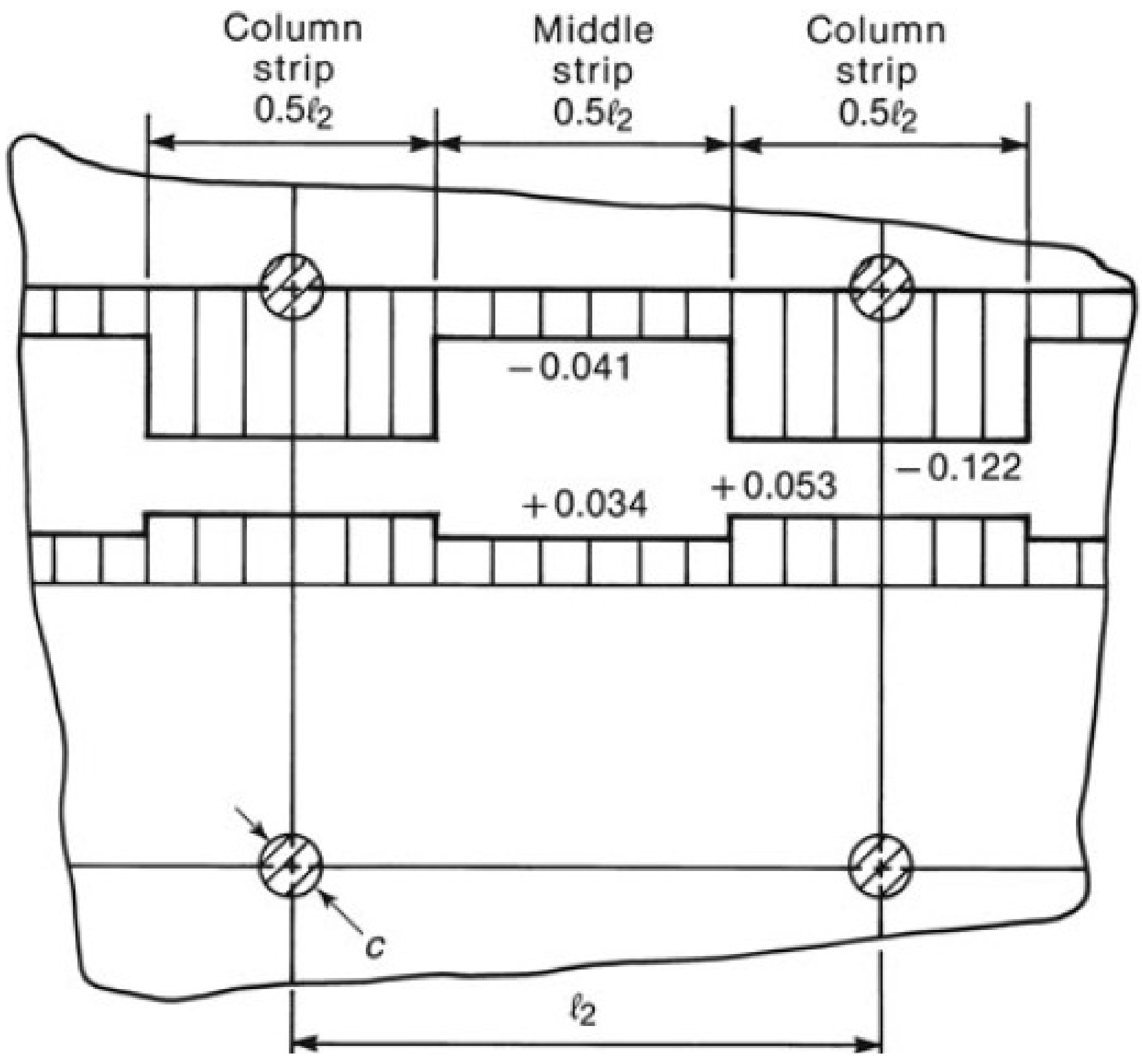
Moments from elastic analysis

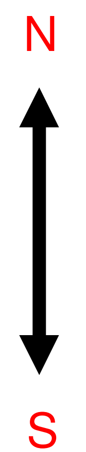
Moments in Slabs Supported by Isolated Columns



Moments from elastic analysis

Moments in Slabs Supported by Isolated Columns

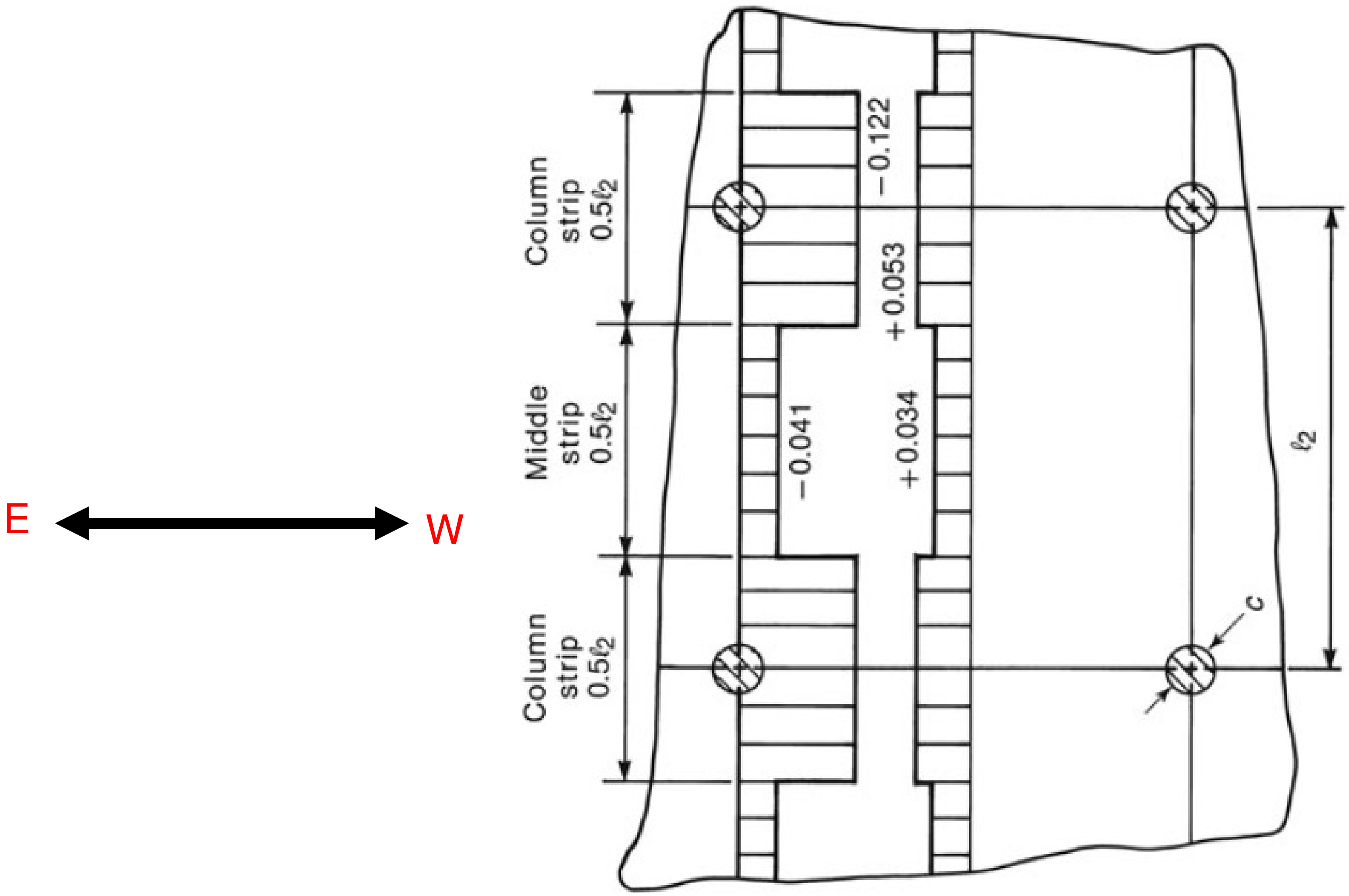




Mo= $q\ell_n^2[(0.122 \times 0.5\ell_2) + (0.041 \times 0.5\ell_2) + (0.053 \times 0.5\ell_2) + (0.034 \times 0.5\ell_2)] = 0.125q\ell_2\ell_n^2$

Elastic moments averaged over strips

Moments in Slabs Supported by Isolated Columns



Mo= $q\ell_n^2[(0.122 \times 0.5\ell_2) + (0.041 \times 0.5\ell_2) + (0.053 \times 0.5\ell_2) + (0.034 \times 0.5\ell_2)] = 0.125 q \ell_2 \ell_n^2$

Elastic moments averaged over strips

Design of Slabs

ACI Code 318-14 section 13.5 allows slabs to be designed by any procedure that satisfies both equilibrium and geometric compatibility, provided that every section has a strength at least equal to the required strength and that serviceability conditions are satisfied. Two procedures for the flexural analysis and design of two way floor systems are presented in detail in the ACI Code 318-14 Chapter 13. These are the

- Direct Design Method (DDM) and
- Equivalent Frame Design Method (EFM)

These two methods differ primarily in the way in which the slab moments are computed.

The calculation of moments in the DDM is based on the total statical moment (Mo). In DDM, the slab is considered panel by panel.

The Mo is divided between positive and negative moments, and these are further divided between middle strips and column strips.

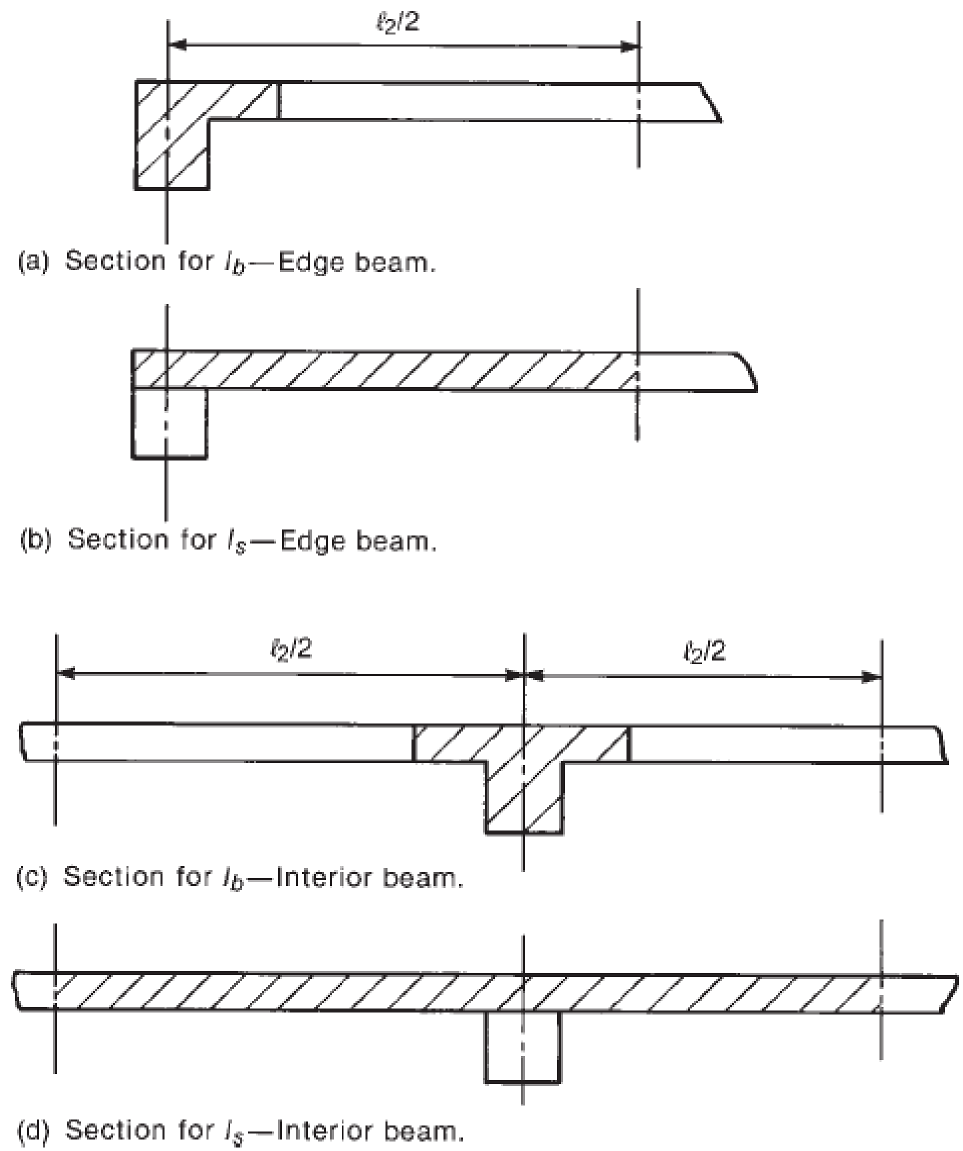
Design of Slabs (Contd.)

In the ACI Code, the effects of beam stiffness on deflections and the distribution of moments are expressed as a function of *αf* defined as the flexural stiffness, *4EI/l*, of the beam divided by the flexural stiffness of a width of slab bounded laterally by the centerlines of the adjacent panels on each side of the beam

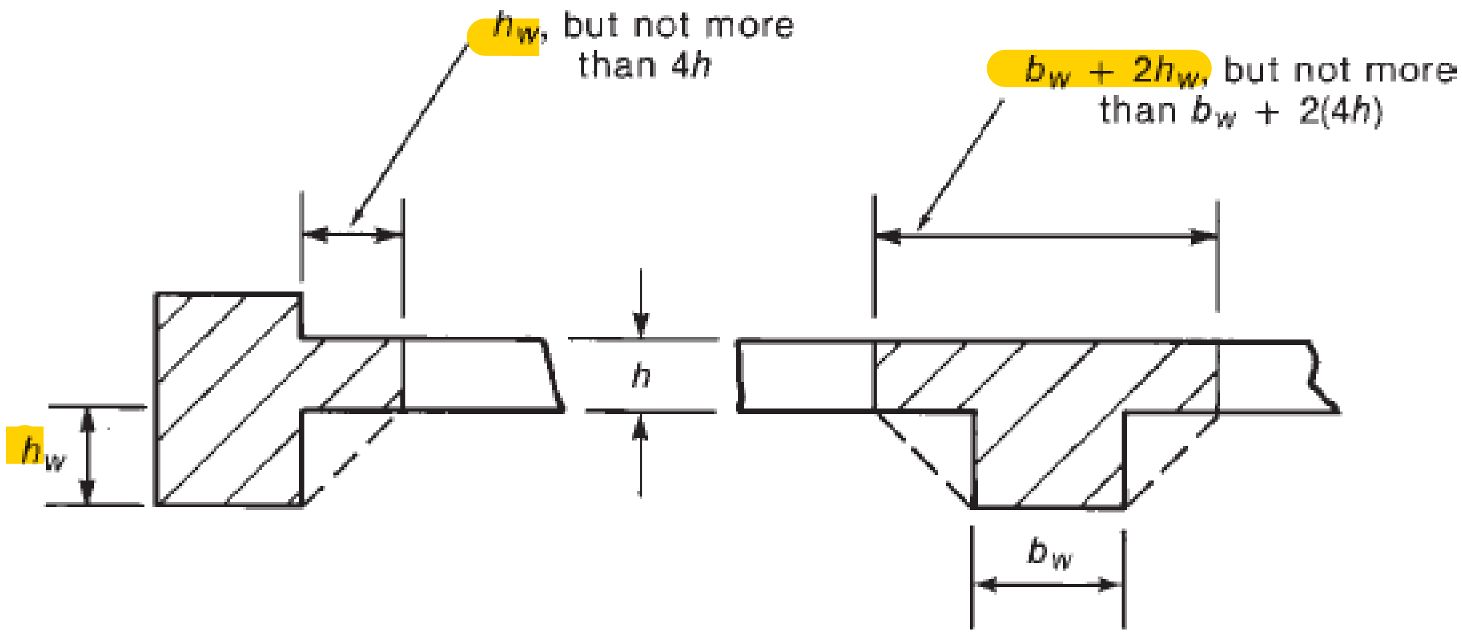
Beam to slab stiffness ratio, α_f

$$\alpha_f = \frac{4E_{cb}I_b/\ell}{4E_{cs}I_s/\ell}$$

$$\alpha_f = \frac{E_{cb}I_b}{E_{cs}I_s}$$



Design of Slabs (Contd.)



Direct Design Method (DDM)

Limitations on the Use of the DDM

1. **There must be a minimum of three continuous spans in each direction. Thus, a nine panel structure (3 by 3) is the smallest that can be considered. If there are fewer than three panels, the interior negative moments from the DDM tend to be too small.**
2. **Rectangular panels must have a long-span/short-span ratio that is not greater than 2. One-way action predominates as the span ratio reaches and exceeds 2.**
3. **Successive span lengths in each direction shall not differ by more than one third of the longer span. This limit is imposed so that certain standard reinforcement cutoff details can be used.**

Direct Design Method (DDM)

Limitations on the Use of the DDM

4. **Columns may be offset from the basic rectangular grid of the building by up to 0.1 times the span parallel to the offset. In a building laid out in this way, the actual column locations are used in determining the spans of the slab to be used in calculating the design moments.**
5. **All loads must be due to gravity only and uniformly distributed over an entire panel. The DDM cannot be used for un-braced, laterally loaded frames, foundation mats, or pre-stressed slabs.**
6. **The service (un-factored) live load shall not exceed two times the service dead load. Strip or checkerboard loadings with large ratios of live load to dead load may lead to moments larger than those assumed in this method of analysis.**

Direct Design Method (DDM)

Limitations on the Use of the DDM

7. **For a panel with beams between supports on all sides, the relative stiffness of the beams in the two perpendicular directions given by $(\alpha_{f1} l_2^2/\alpha_{f2} l_1^2)$ shall not be less than 0.2 or greater than 5.**

Limitations 2 and 7 do not allow use of the DDM for slab panels that transmit load as one-way slabs.

Minimum Thickness of Two Way Slabs

TABLE 9.5(c)—MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS*

f_y, MPa†	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

*For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.

†For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.

‡Drop panels as defined in 13.2.5.

§Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

Minimum Thickness of Two Way Slabs (Contd.)

For slabs without interior beams spanning between the supports and having a ratio of long to short span not greater than 2, the minimum thickness shall be in accordance with the provisions of ACI 318-14 Table 9.5(c) and shall not be less than the following values:

(a) Slabs without drop panels ....125 mm;

(b) Slabs with drop panels ....100 mm

Minimum Thickness of Two Way Slabs (Contd.)

For slabs with beams spanning between the supports on all sides, the minimum thickness, *h*, shall be as follows:

(a) For α_{fm} equal to or less than 0.2, the provisions of 9.5.3.2 (ACI318-14) shall apply;

(b) For α_{fm} greater than 0.2 but not greater than 2.0, *h* shall not be less than

$$h = \frac{ln\left(0.8 + \frac{fy}{1400}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \geq 120mm \cdots\cdots (9-12) \quad \text{ACI}(318\text{-}14)$$

and not less than 125 mm;

α_{fm} = average value of α_f for all beams on edges of a panel

***ln* = length of clear span in long direction measured face to face of beams**

β = ratio of clear spans in long to short direction of slab

Minimum Thickness of Two Way Slabs (Contd.)

(c) For α_{fm} greater than 2.0, *h* shall not be less than

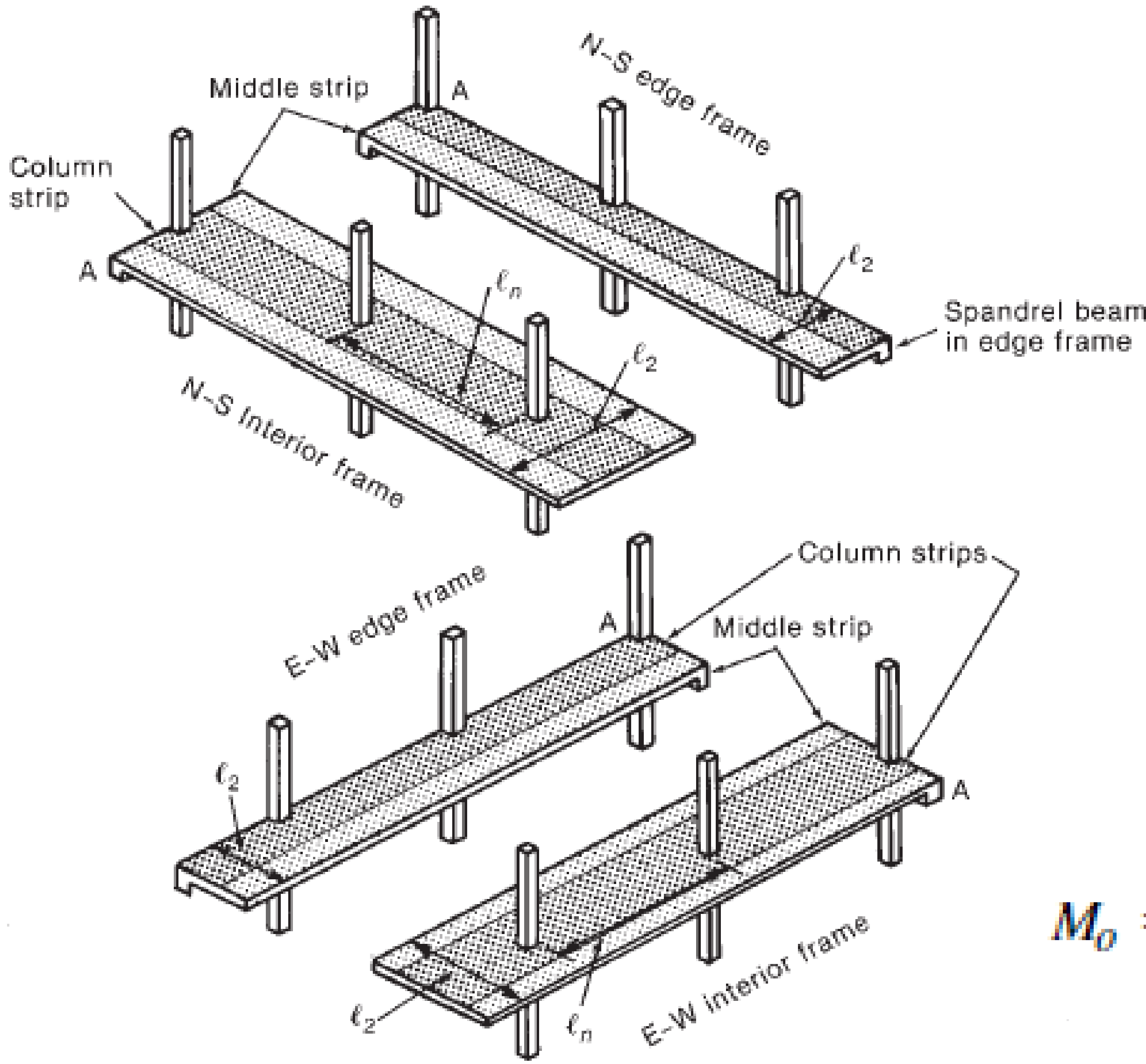
$$h = \frac{ln\left(0.8 + \frac{fy}{1400}\right)}{36 + 9\beta} \geq 90mm \cdots\cdots\cdots (9-13) \;\; \text{ACI (318-14)}$$

and not less than 90 mm.

(d) At discontinuous edges, an edge beam shall be provided with a stiffness ratio α_f not less than 0.80 or the minimum thickness required by Eq. (9-12) or (9-13) shall be increased by at least 10 percent in the panel with a discontinuous edge.

Note: Thickness of a slab may be governed by shear.

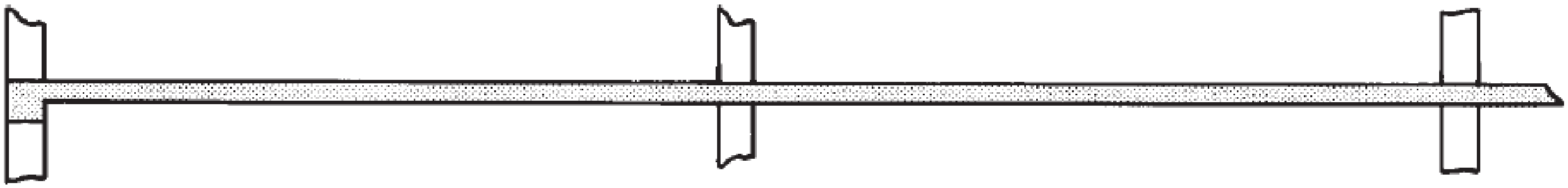
Statical Moment (Mo)

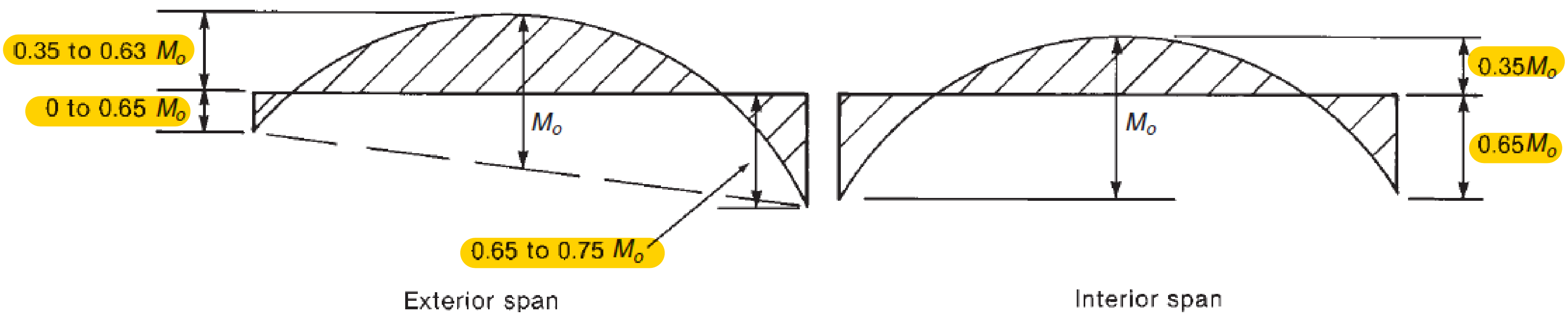


$$M_o = \frac{q_u \ell_2 \ell_n^2}{8}$$

Division of slabs into frames for design

Positive and Negative Moments in Panels





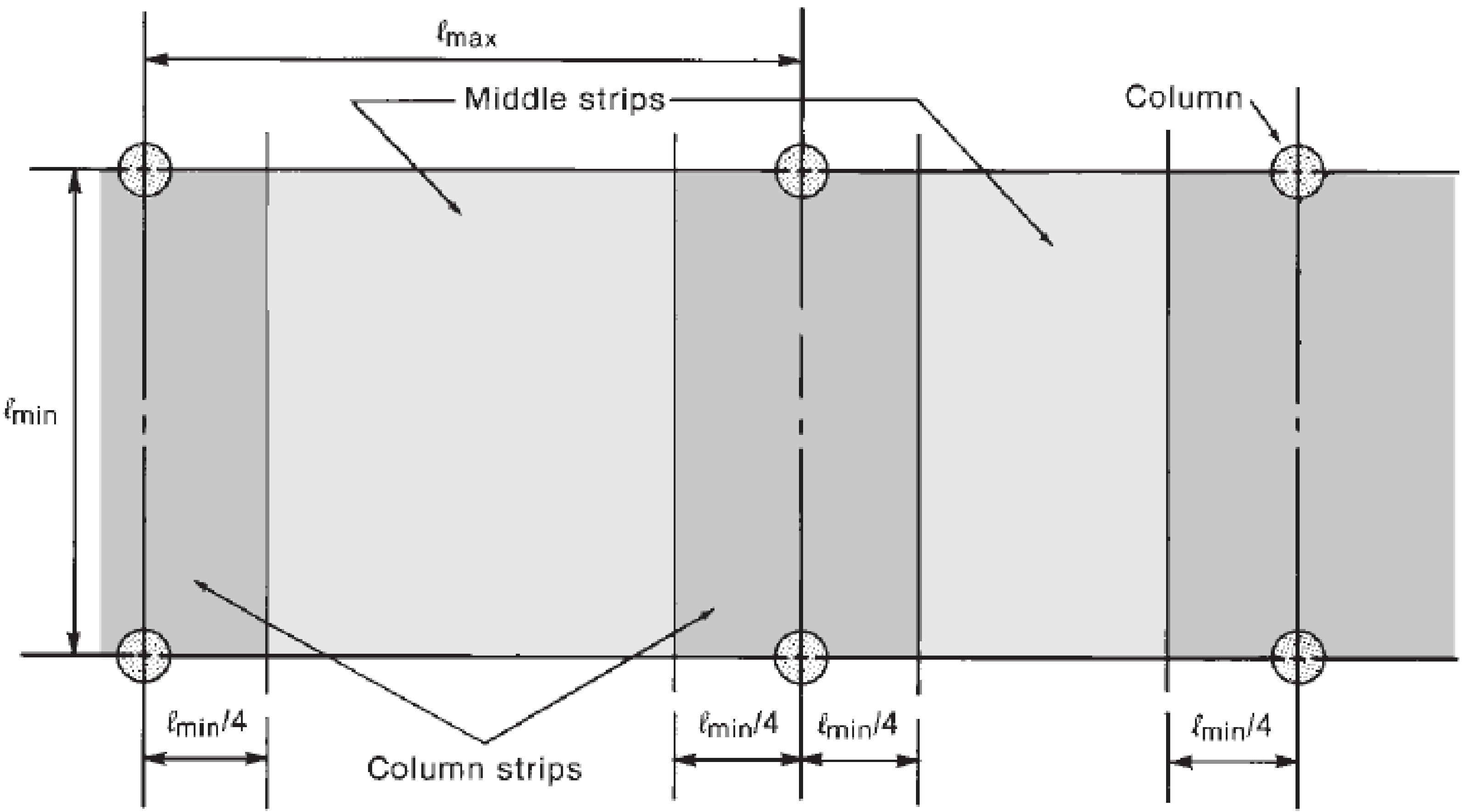
Longitudinal Distribution of Moments

Distribution of Total Factored Static Moment, (Mo), in an Exterior Span

	(1)	(2)	(3)	(4)	(5)
			Slab without beams between interior supports		
	Exterior edge unrestrained	Slab with beams between all supports	Without edge beam	With edge beam	Exterior edge fully restrained
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65

Source: ACI Code Section 13.6.3.3

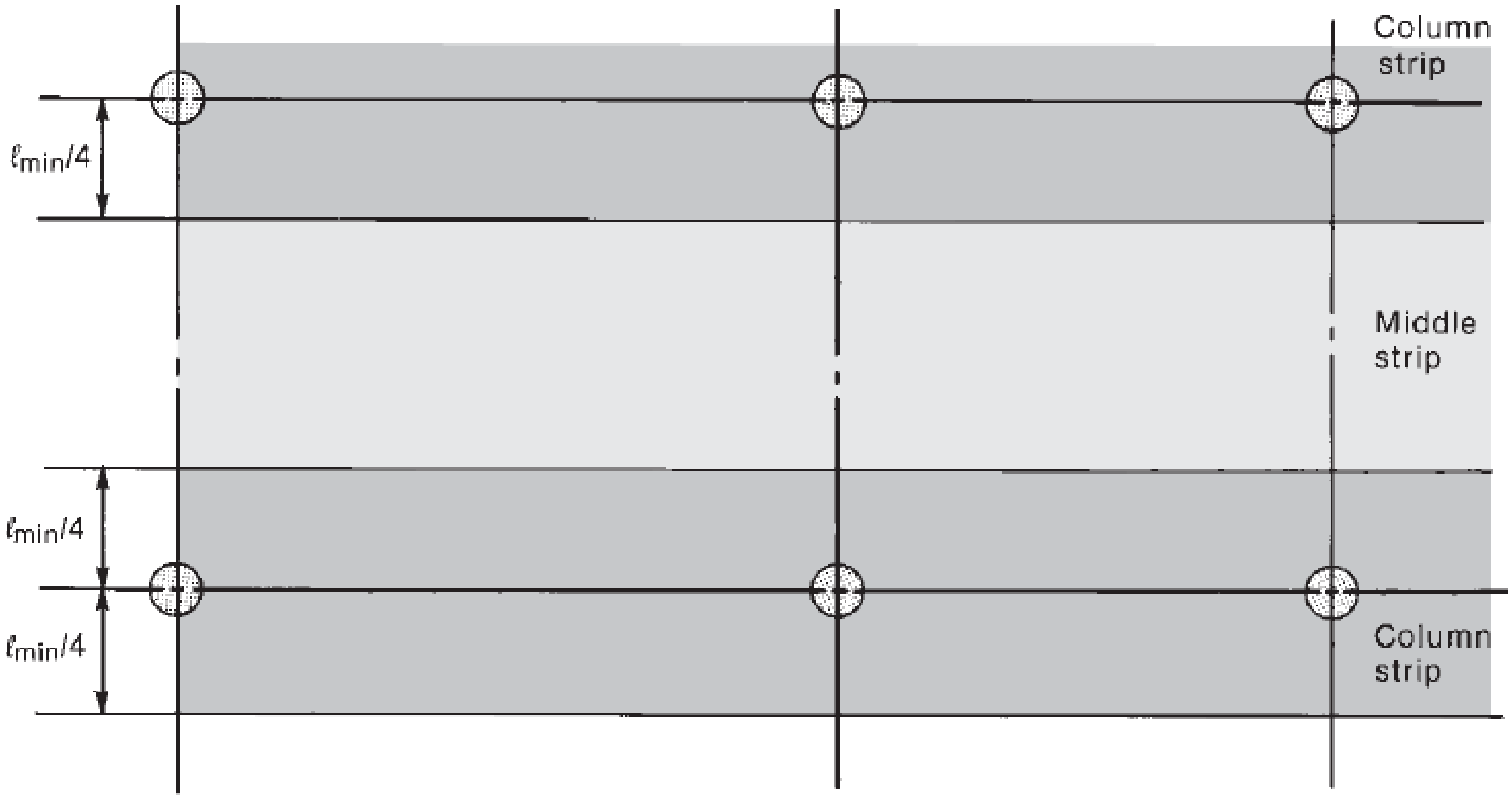
Definitions of Column Strip and Middle Strip



(a) Short direction of panel.

The column strips in both directions extend one fourth, l_{min}, of the smaller span, each way from the column line.

Definitions of Column Strip and Middle Strip (Contd.)



(b) Long direction of panel.

The column strips in both directions extend one fourth, *l_{min}*, of the smaller span, each way from the column line.

Distribution of Moments between Column Strips and Middle Strips

ACI Code Section 13.6.4 defines the fraction of the negative and positive moments assigned to the column strips.

The remaining amount of negative and positive moment is assigned to the adjacent half-middle strips.

The division is a function of $(\alpha_{f1} l_2/l_1)$ which depends on the

- Aspect ratio of the panel (l_2/l_1), and
- The relative stiffness (α_{f1}), of the beams (if any) spanning parallel to and within the column strip.

Factored negative moments in interior column strips(%)

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Source: ACI Code Section 13.6.4.1

Distribution of Moments between Column Strips and Middle Strips

Factored positive moments in exterior / interior column strips (%)

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Source: ACI Code Section 13.6.4.4

Division of the Exterior End Factored Negative Moment

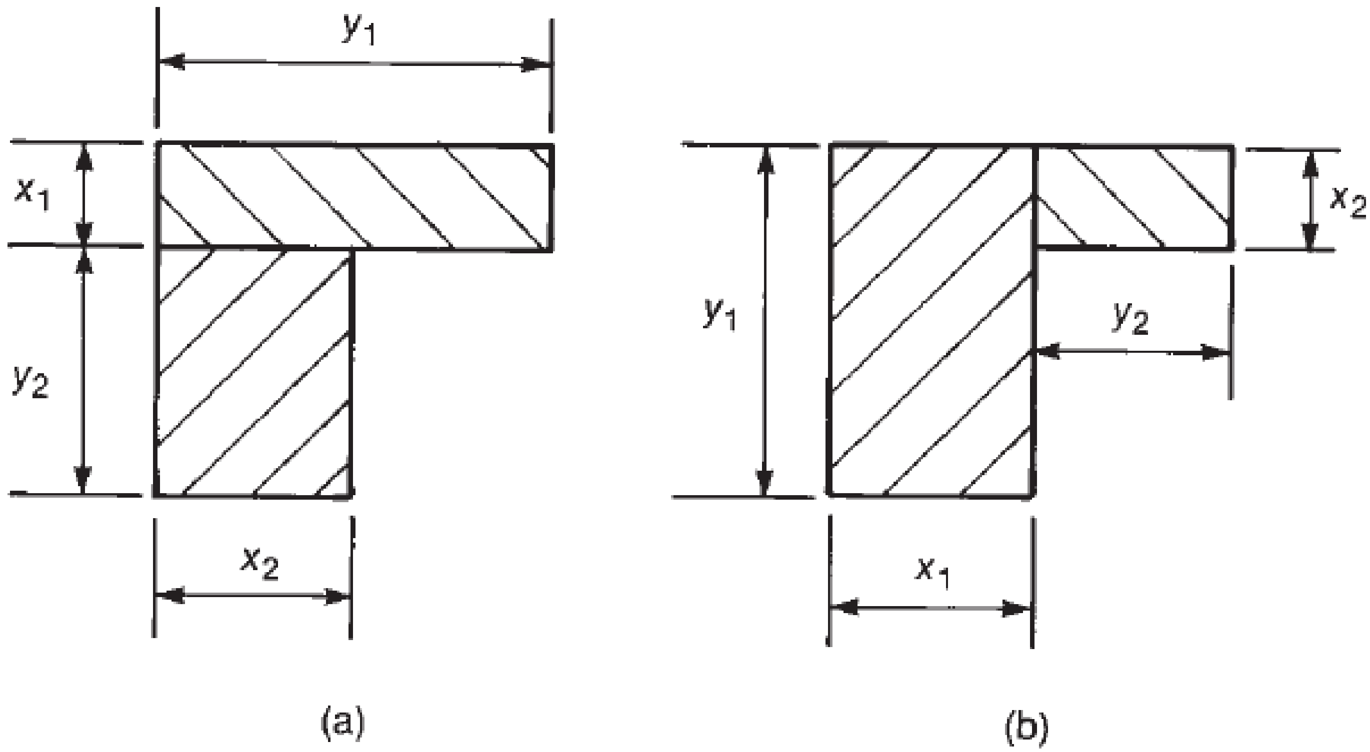
At an exterior edge, the division of the exterior end factored negative moment distributed to the column and middle strips spanning perpendicular to the edge also depends on the torsional stiffness of the edge beam, calculated as the shear modulus, G, times the torsional constant of the edge beam, C, divided by the flexural stiffness of the slab spanning perpendicular to the edge beam (i.e., EI for a slab having a width equal to the length of the edge beam from the center of one span to the center of the other span).

$$\beta t = GC/E_{cs}I_s$$

Assuming that Poisson's ratio is zero gives $G = E_{cb}/2$

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s}$$

Division of Edge Members for Calculation of Torsional Constant, *C*.



$$C = \sum \left[\left(1 - 0.63 \frac{x}{y} \right) \frac{x^3 y}{3} \right]$$

Distribution of Moments between Column Strips and Middle Strips

Factored negative moments in exterior column strips (%)

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

Source: ACI Code Section 13.6.4.2

Transverse Distribution of Moments

Let $l2/l1 = A$ $\qquad 0.5 \le A \le 2.0$

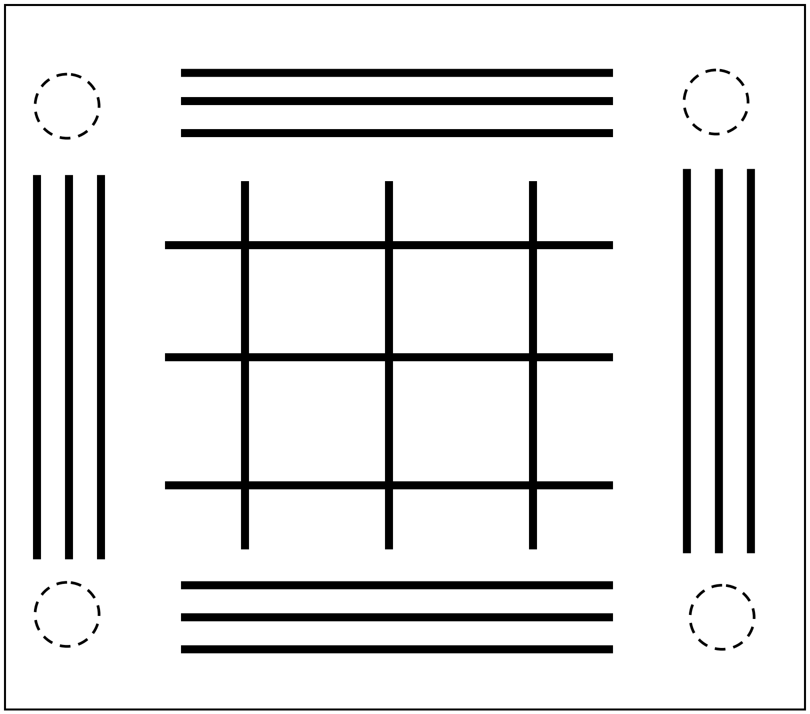
$\beta_t = B$ $\qquad$ If $\beta_t > 2.5,\ B = 2.5$

$\alpha_{f1}\dfrac{l2}{l1} = D$ $\qquad$ If $\alpha_{f1}\dfrac{l2}{l1} > 1.0,\ D = 1.0$

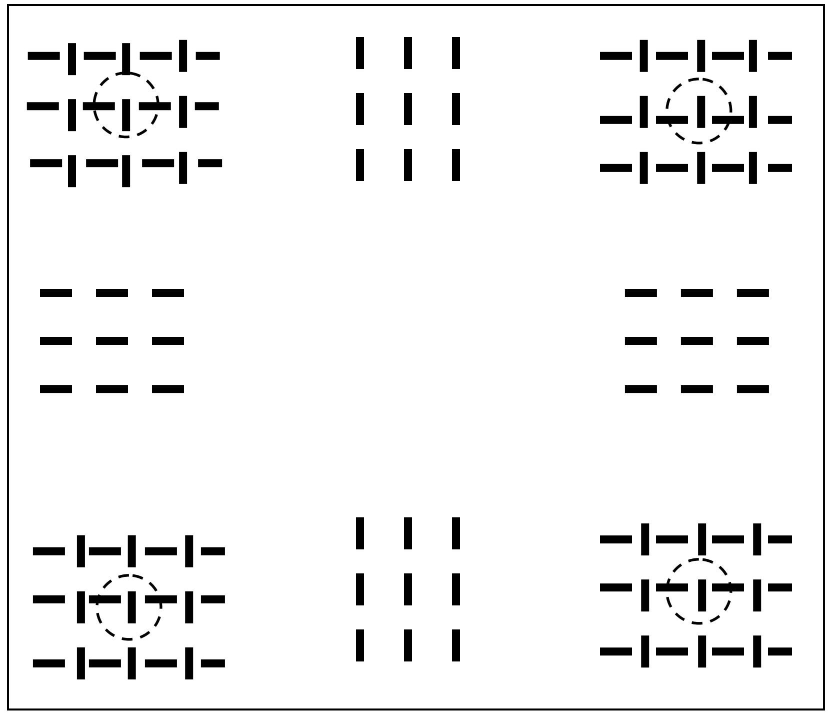
Interior Negative Moment (%) $: 75 + 30(1 - A)D$

Exterior Negative Moment (%) $: 100\text{-}10B + 2BD(1\text{-}A)$

Positive Moment (%) $: 60 + 15(3\text{-}2A)D$



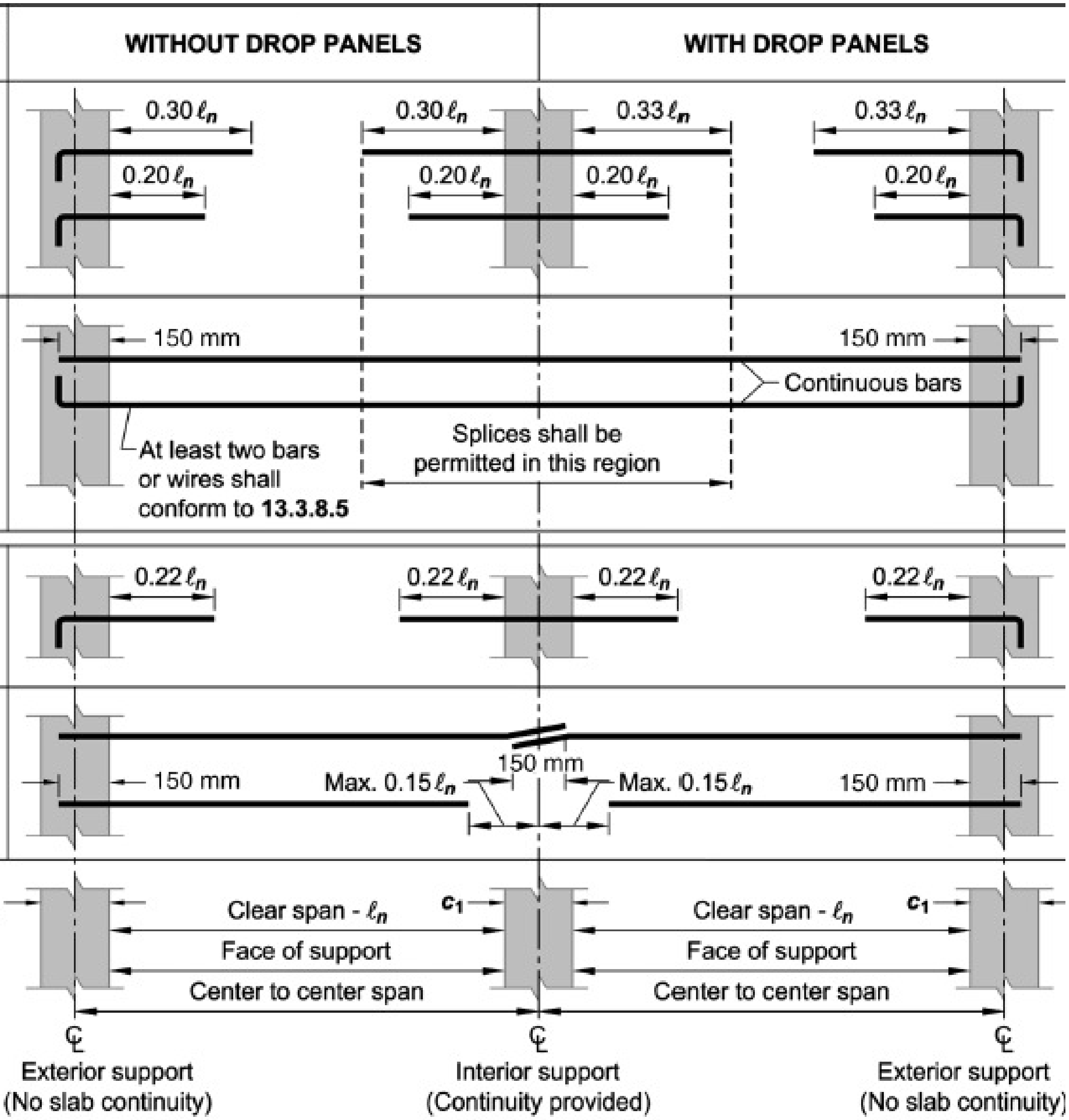
Bottom Steel



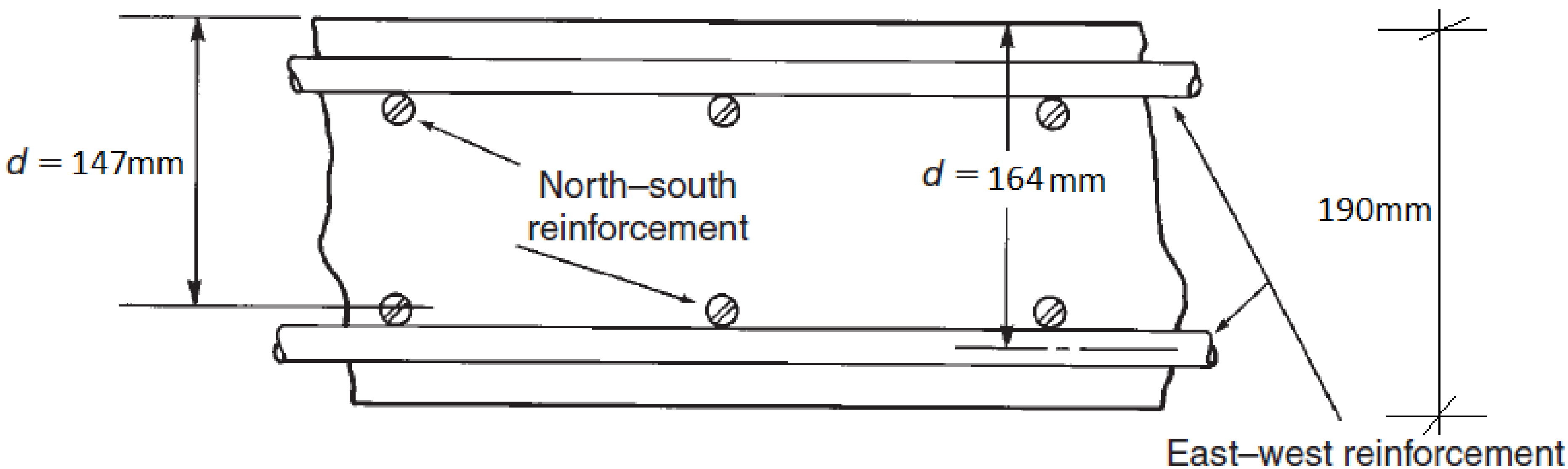
Top Steel

Minimum extensions for reinforcement in slabs without beams

STRIP	LOCATION	MINIMUM - A_S AT SECTION
COLUMN STRIP	TOP	50%
		REMAINDER
	BOTTOM	100%
MIDDLE STRIP	TOP	100%
	BOTTOM	50%
		REMAINDER



Arrangement of bars in a slab



Shear Strength of Slabs

1. Two-way slabs supported on beams

The critical location is found at d distance from the column, where

$$V_{ud} \leq \phi V_C = \phi(\sqrt{f_c'}bd/6) \text{ where } \phi = 0.75$$

The supporting beams are stiff $(\alpha_{f1} l_2/l_1) \geq 1.0$

and are capable of transmitting floor loads to the columns.

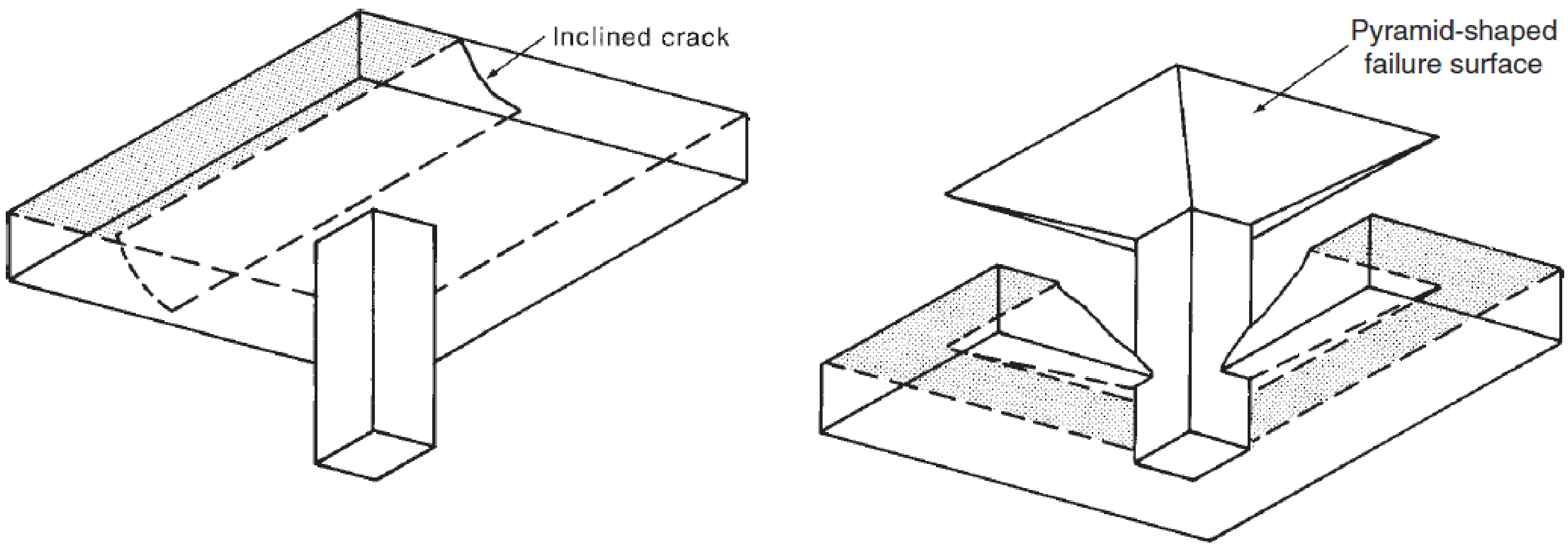
Shear Strength of Slabs

2. Two-Way Slabs without beams

There are two types of shear that need to be addressed

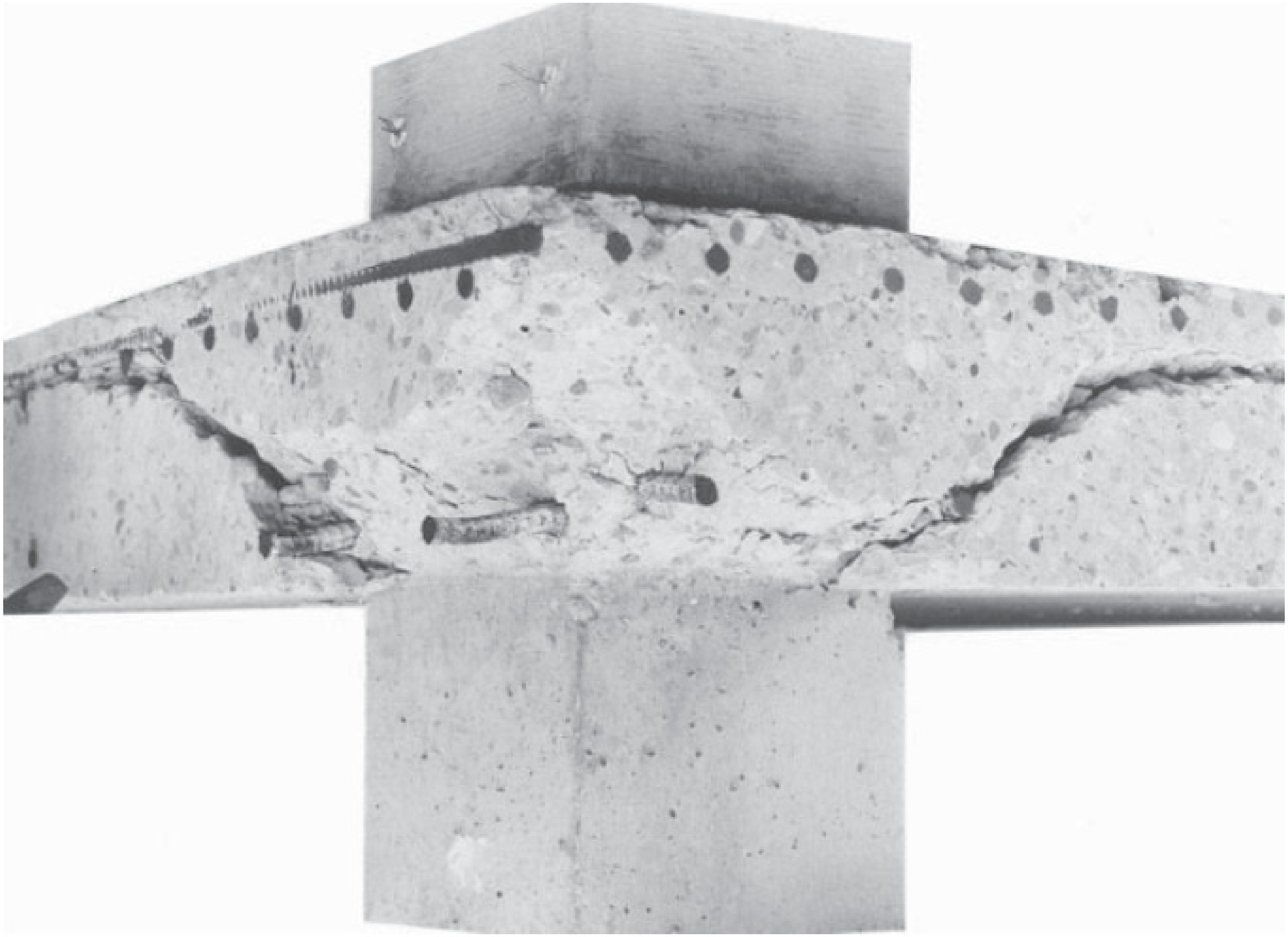
1. One-way shear or beam shear at distance d from the column

2. Two-way or punching shear which occurs along a truncated cone.



(a) One-way shear. (b) Two-way shear.

Two Way Shear



Inclined cracks in a slab after a shear failure. (Photograph courtesy of J. G. MacGregor.)

Design for One Way Shear

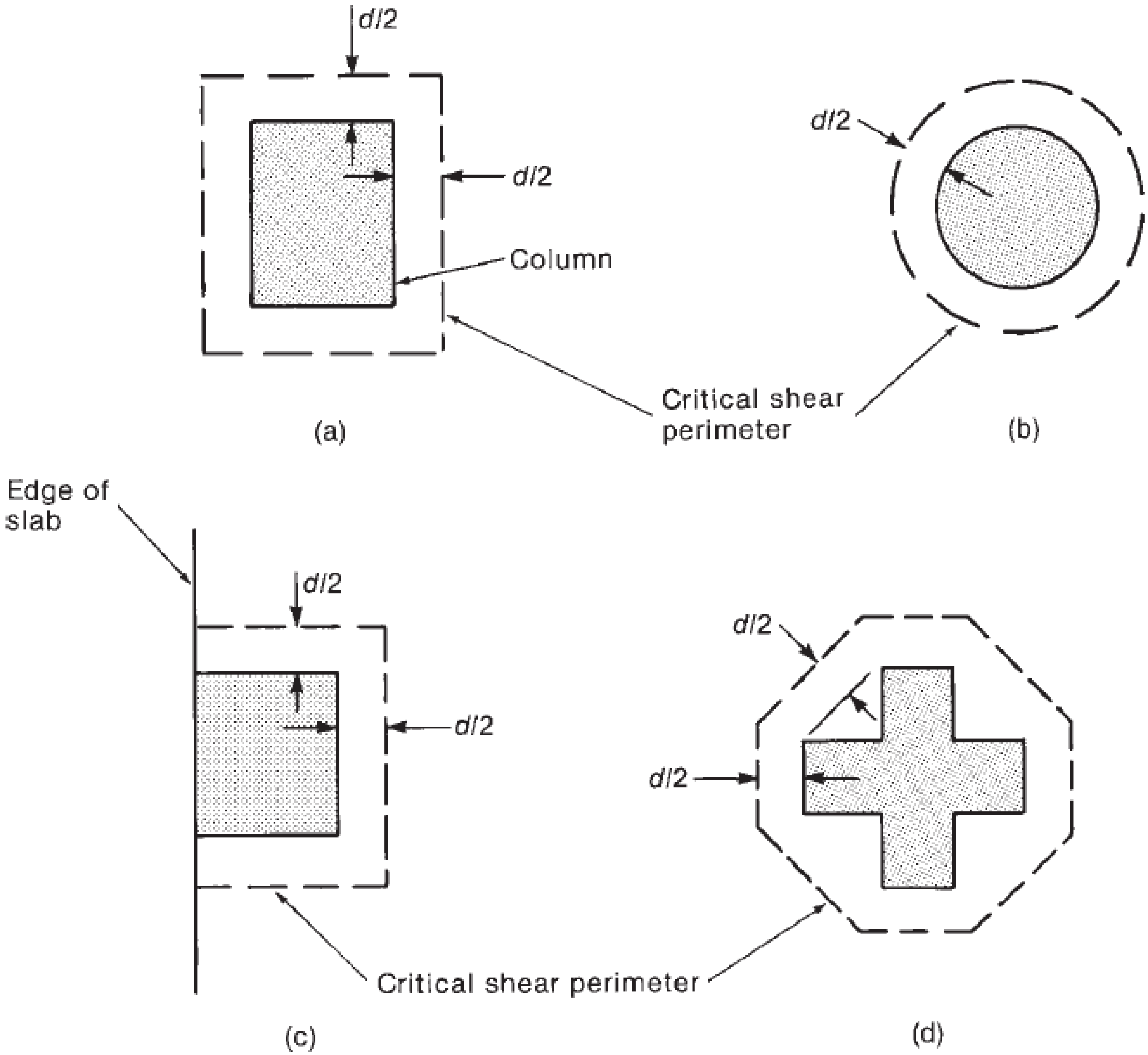
One-way shear considers critical section a distance d from the column and the slab is considered as a wide beam spanning between supports.

$$V_{ud} \leq \phi V_C = \phi(\sqrt{f_c'}bd/6) \text{ where } \phi = 0.75$$

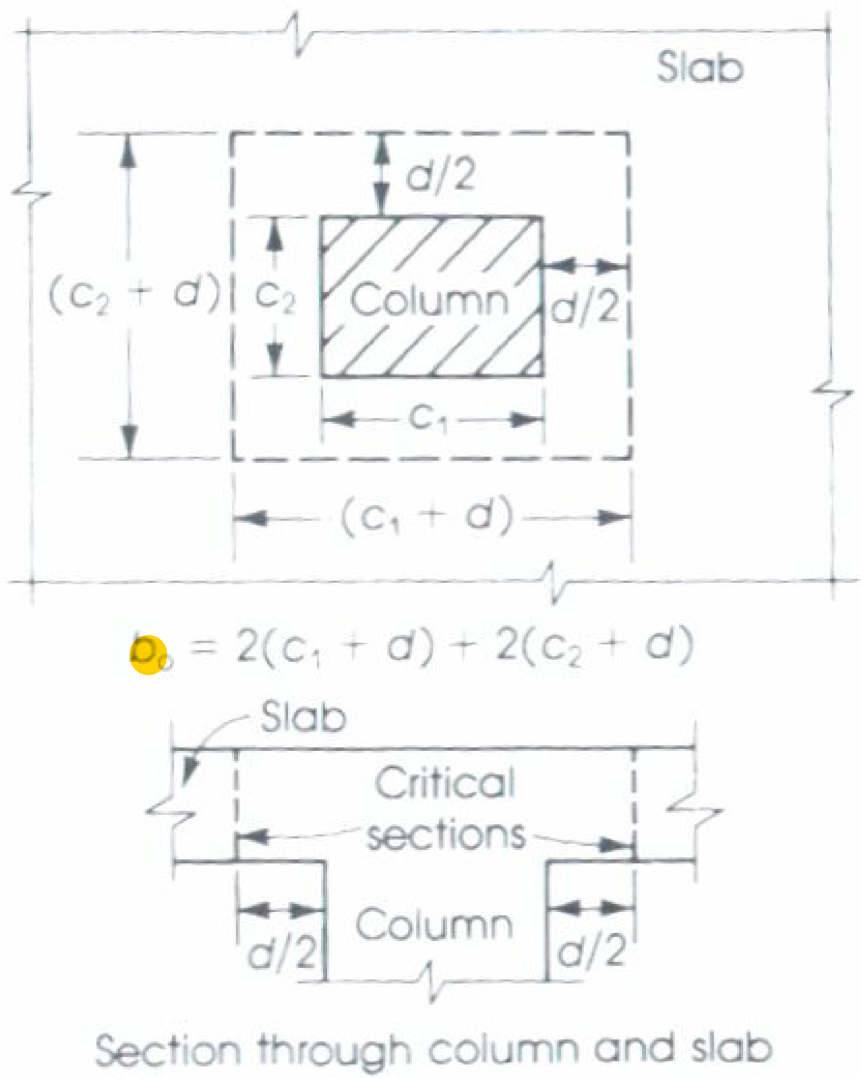
Design for Two-Way Shear

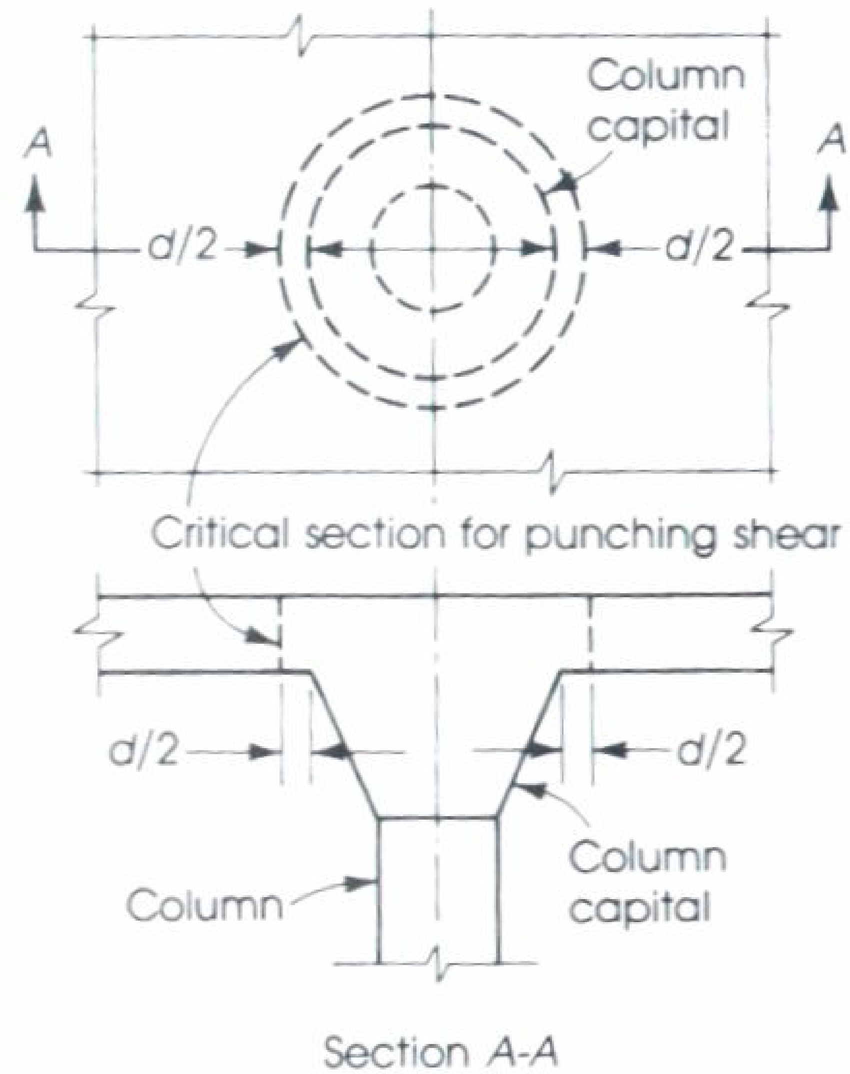
- **From extensive tests, Moe (1961) concluded that the critical section for shear was located at the face of the column.**
- **ACI-ASCE Committee 326 (now 445) (1962) accepted Moe's conclusions, but showed that a much simpler design equation could be derived by considering a critical section at *d*/2 from the face of the column, column capital, or drop panel.**
- **This was referred to as the *pseudo-critical section for shear*. This simplification has been incorporated in the ACI Code.**

Location of critical shear perimeters.

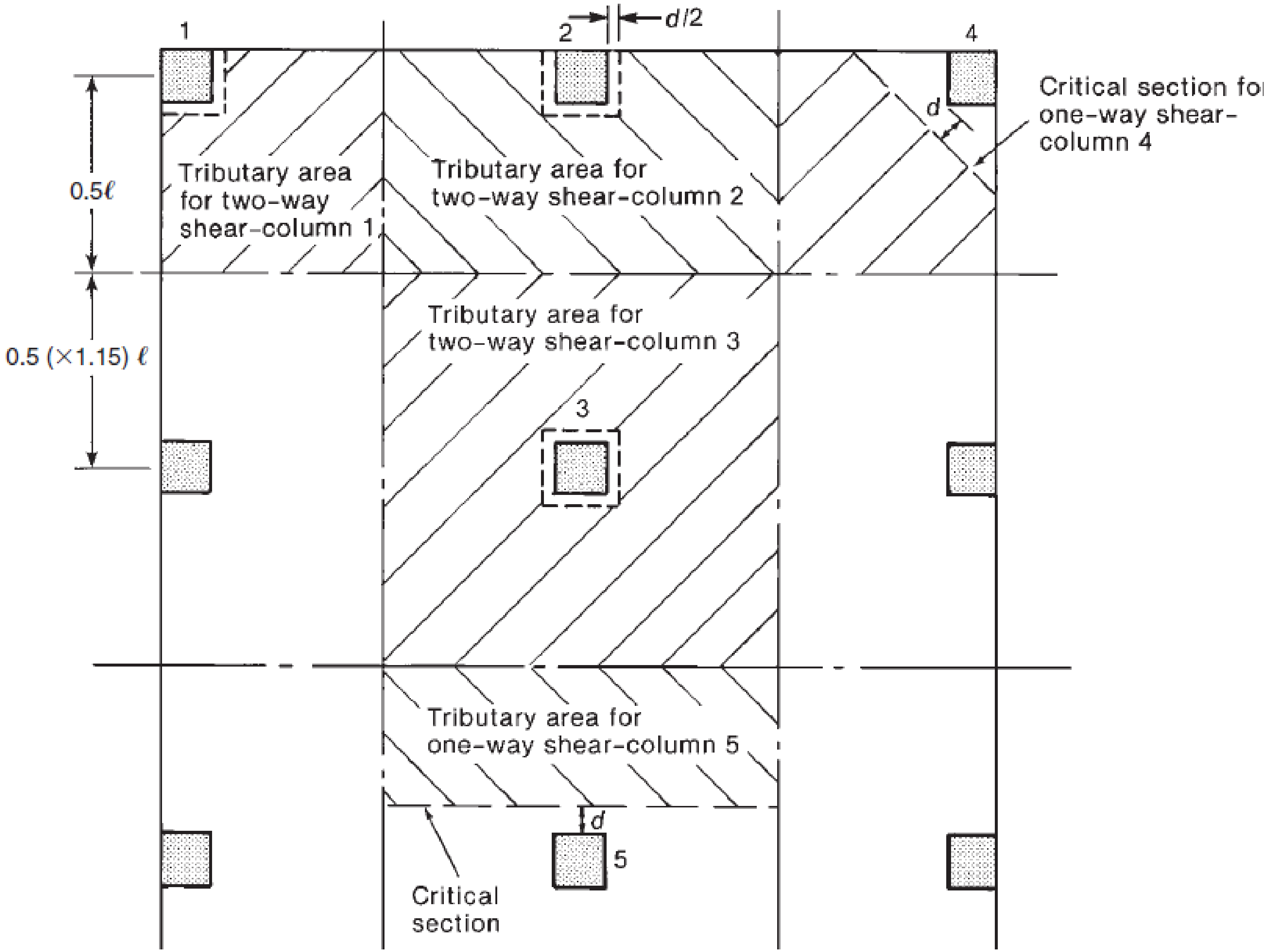


Shear Strength of Slabs





Critical sections and tributary areas for shear in a flat plate



Shear Strength of Slabs

If shear reinforcement is not provided, the shear strength of concrete is the smaller of:

1. $$\phi V_C = \phi \times 0.17\left(1 + \frac{2}{\beta_C}\right)\sqrt{f'_C}\, b_o d$$

b_o = Perimeter of critical section

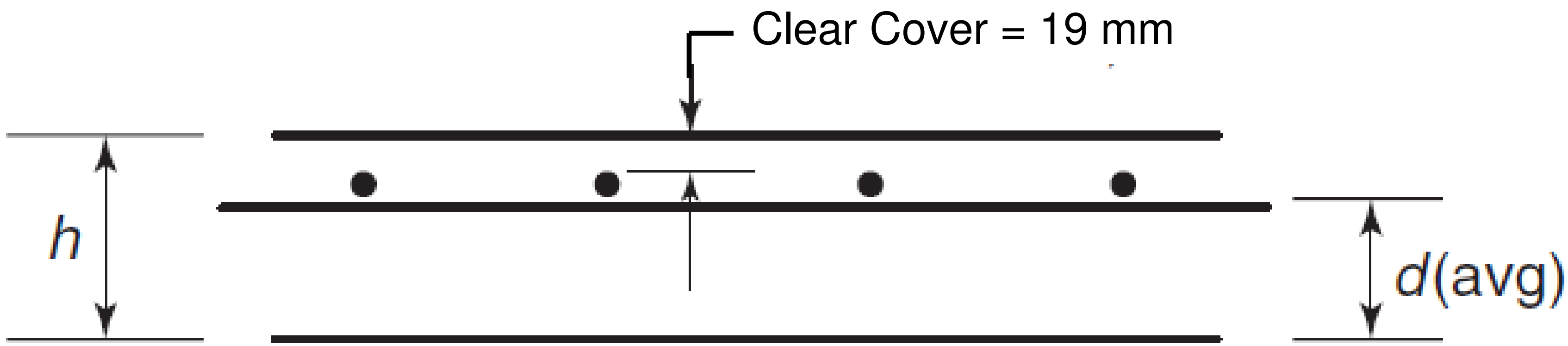
β_C = Ratio of long side of column to short side

2. $$\phi V_C = \phi \times 0.083\left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_C}\, b_o d$$

α_s = 40 for interor columns, 30 for edge columns and 20 for corner columns

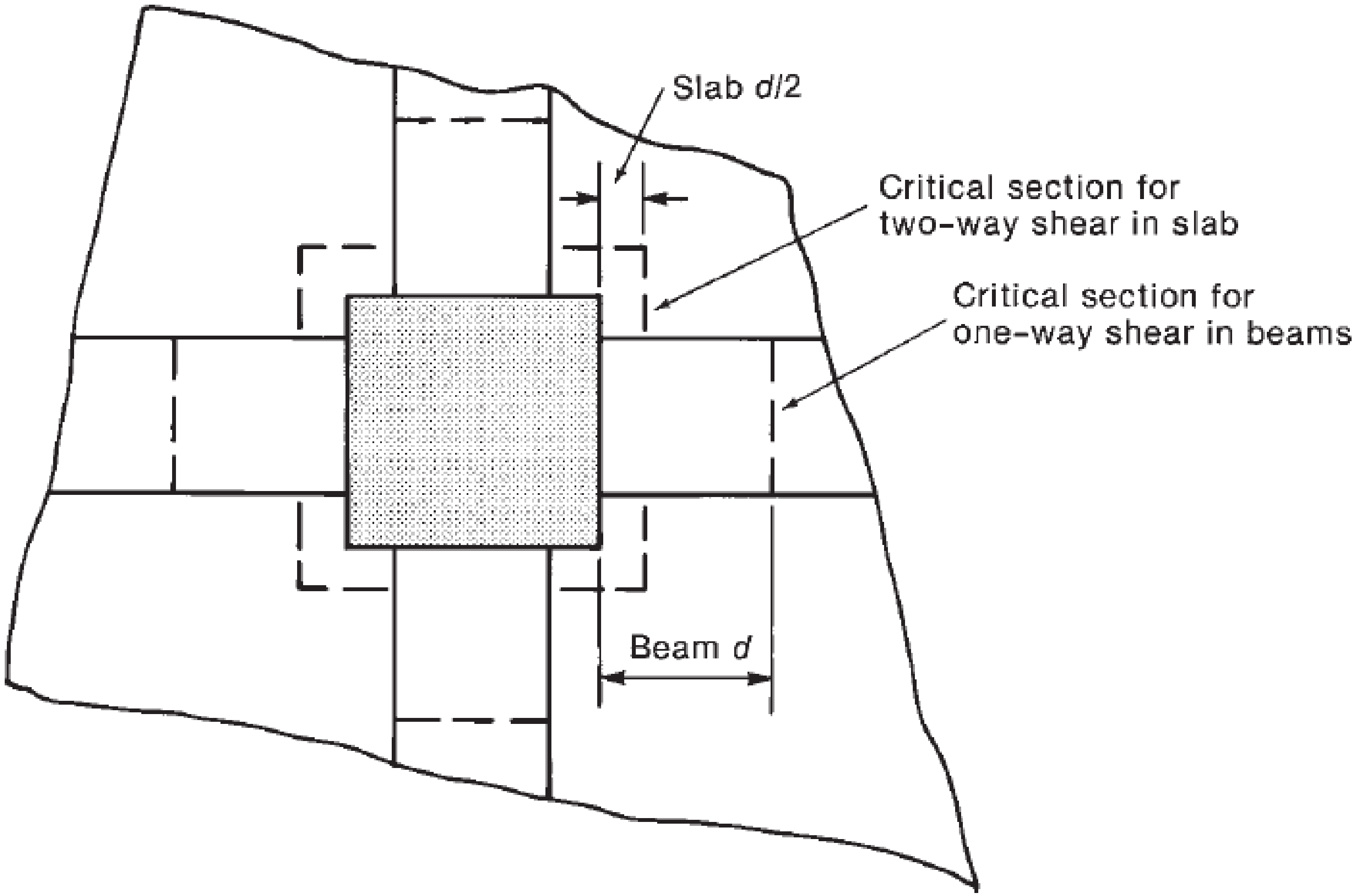
3. $$\phi V_C = \phi \times 0.33\sqrt{f'_C}\, b_o d$$

Determination of *d*(avg) for use in shear strength evaluation of two-way slabs.



$$d\,(avg) = h - 19 - d_b\ (mm)$$

Shear perimeters in slabs with beams



Shear Strength of Slabs

3. Shear Reinforcement in two-way slabs without beams.

For plates and flat slabs, which do not meet the condition for shear, one can either

- Increase slab thickness

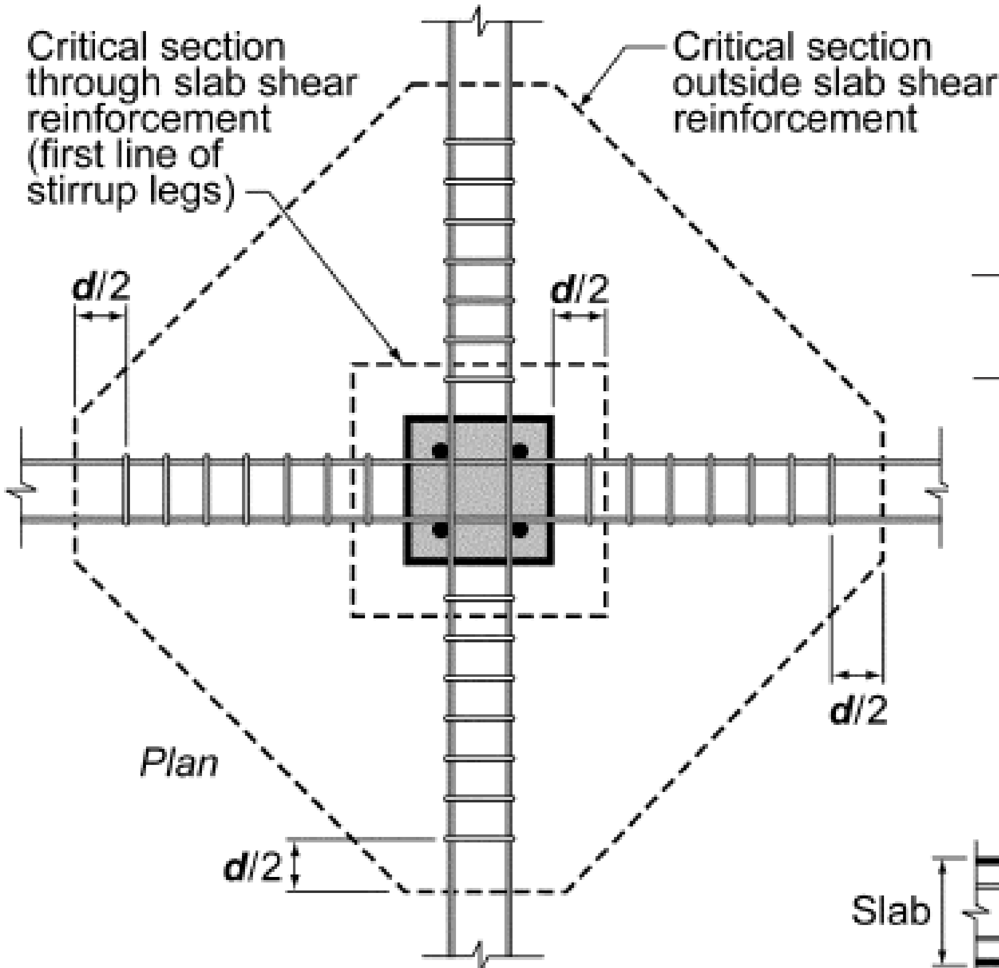
- Use a drop panel to thicken the slab adjacent to the column.

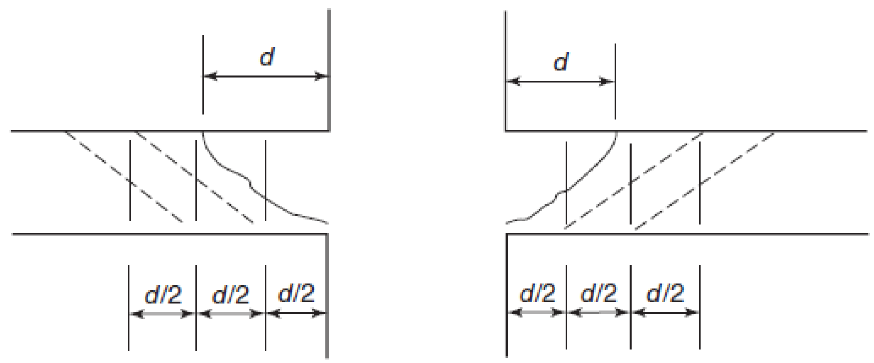
- Increase by increasing the column size or by adding a shear capital around the column.

- Add reinforcement

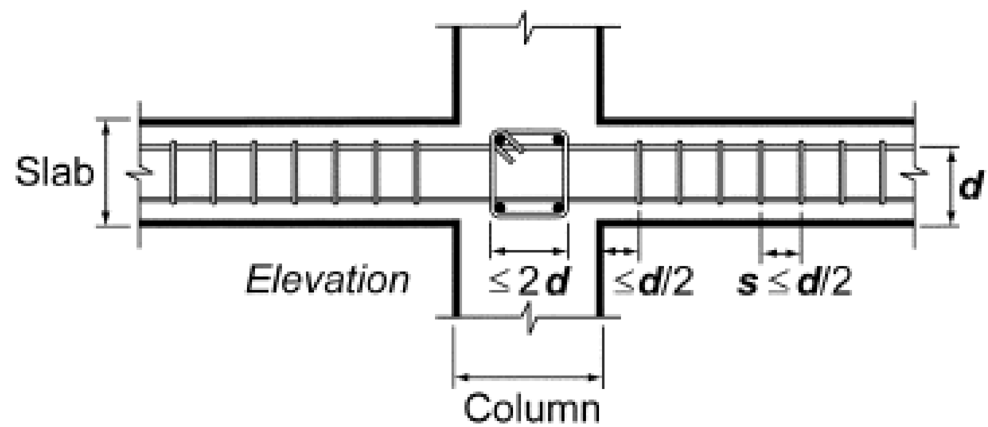
Reinforcement can be done by shearheads, anchor bars, conventional stirrup cages and studded steel strips (see ACI 11.11.4.

Arrangement of stirrup shear reinforcement, interior column

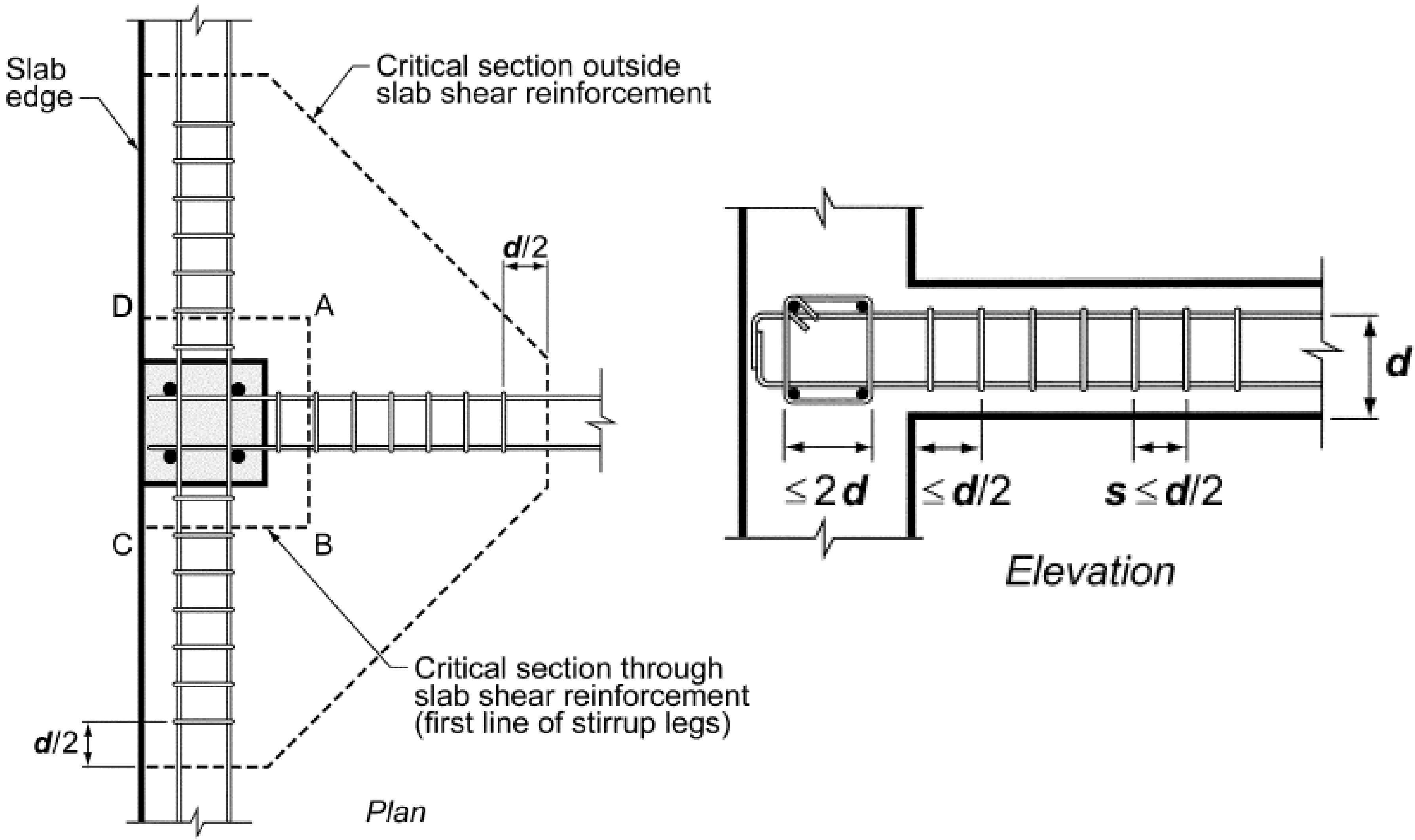




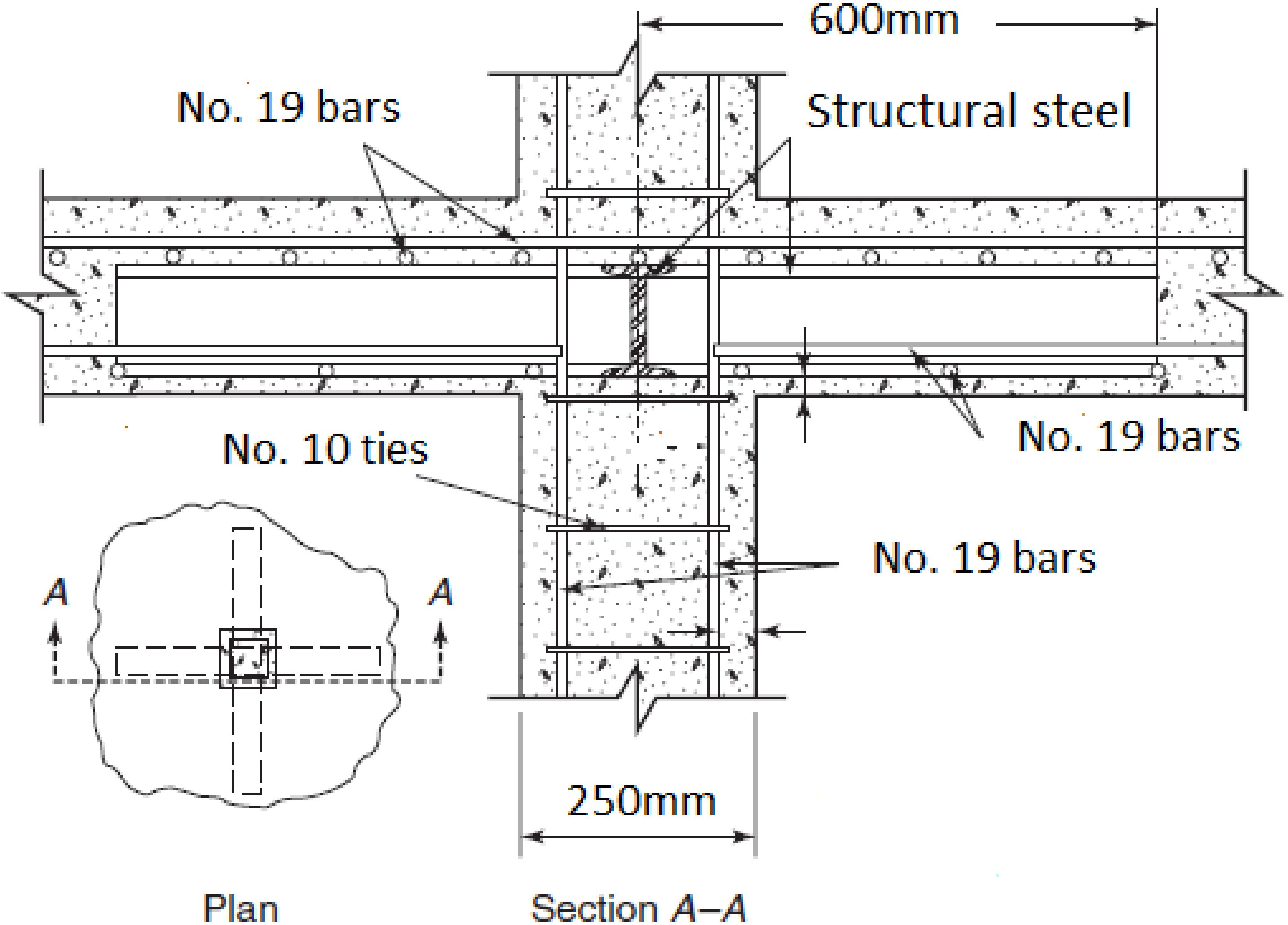
Potential shear cracks and stirrup spacing at slab–column connection



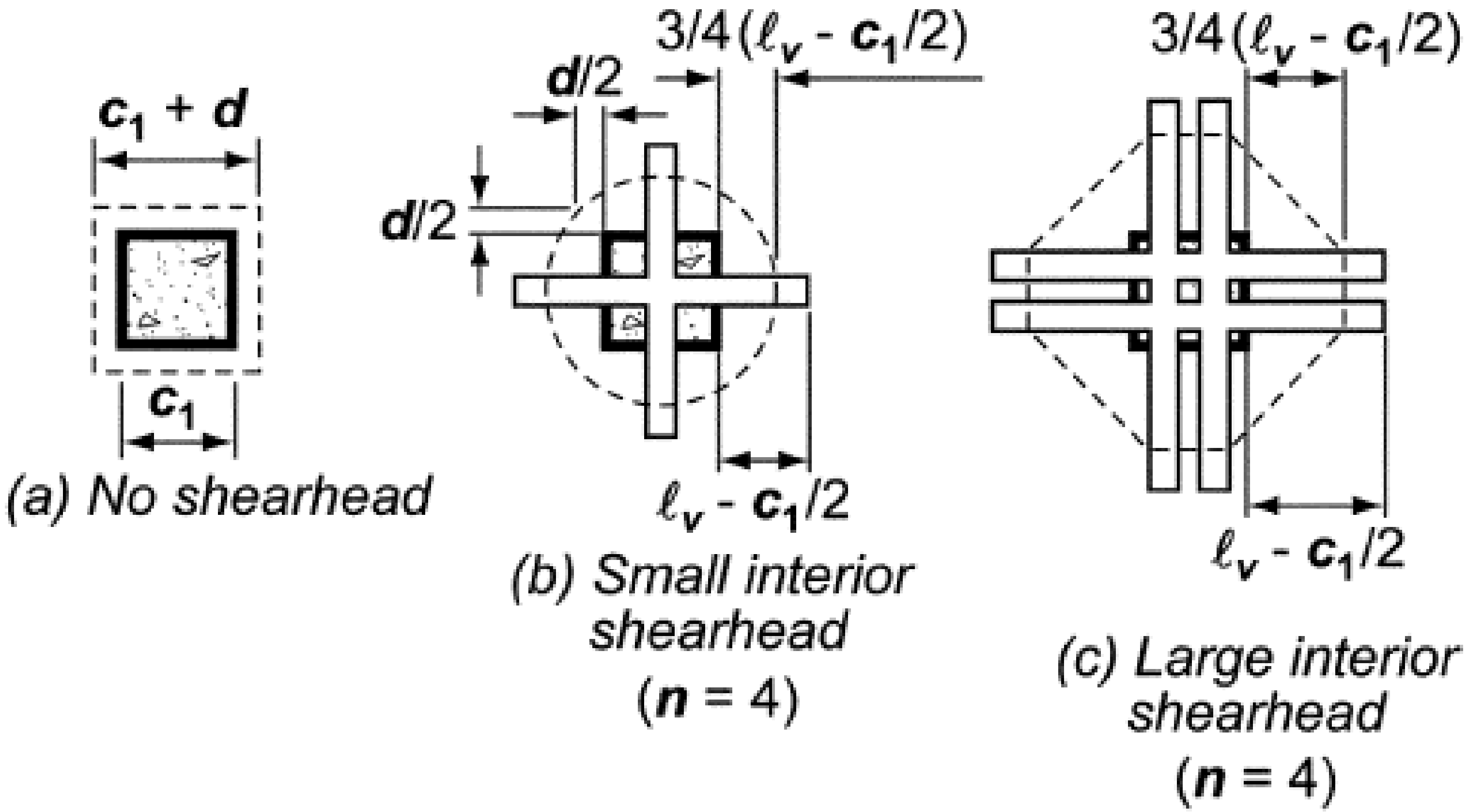
Arrangement of stirrup shear reinforcement, edge column



Structural Steel Shearheads

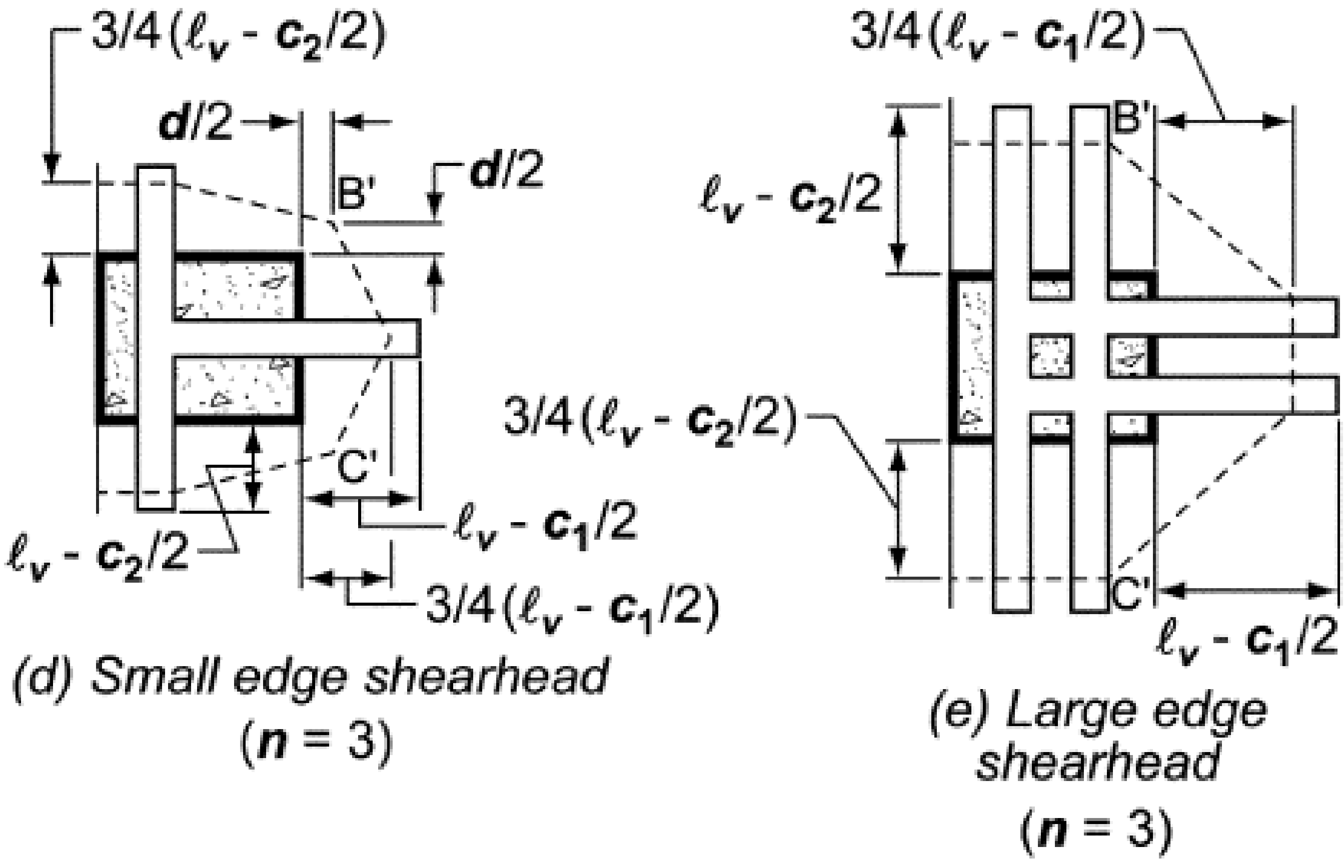


Structural Steel Shearheads



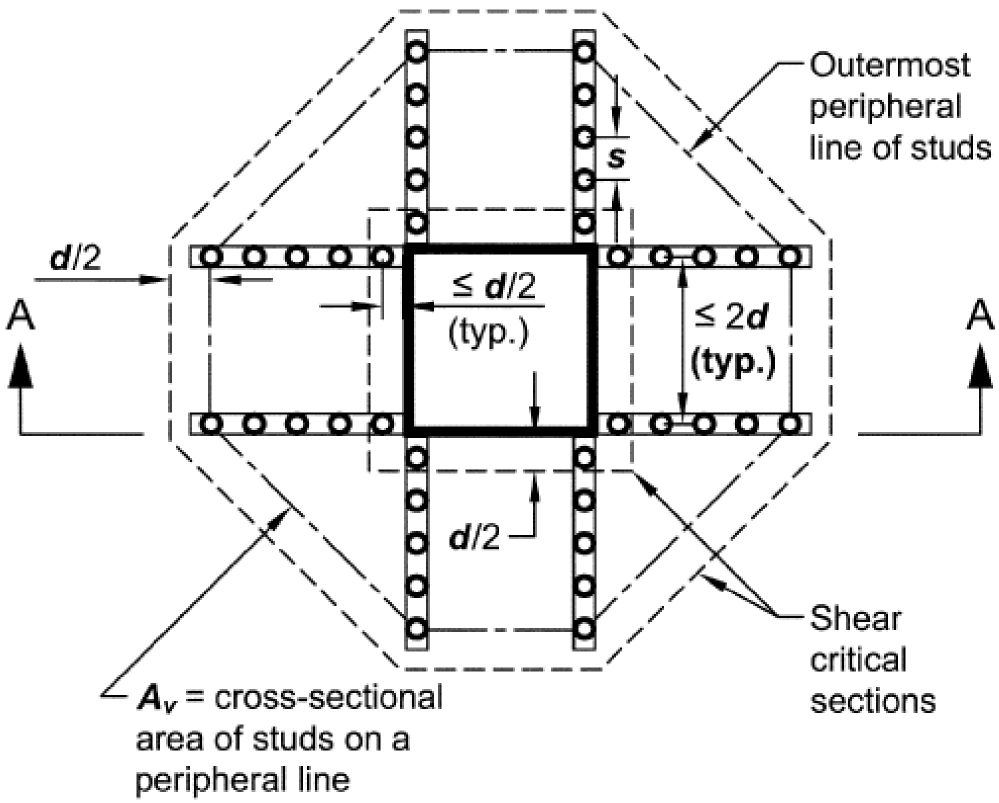
ℓ_v **is minimum length of each shearhead arm**

Structural Steel Shearheads

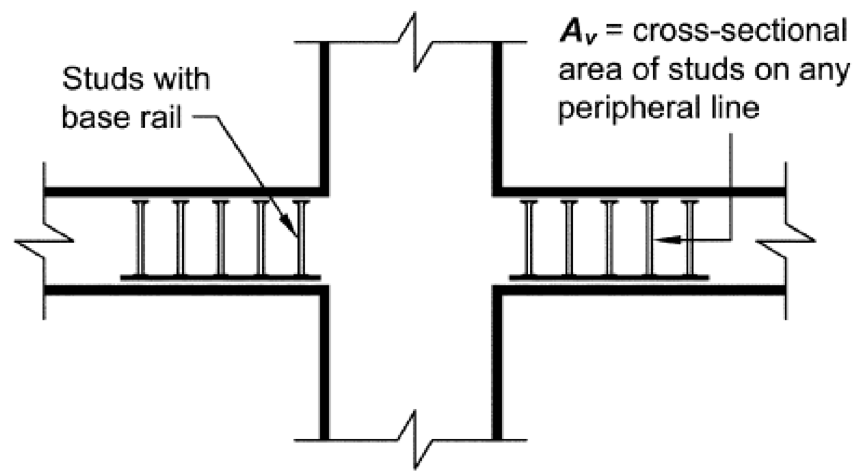


ℓ_v **is minimum length of each shearhead arm**

Headed Shear Studs



Interior column



Section A-A

DESIGN OF SLABS WITH BEAMS IN TWO DIRECTIONS

- Because of its additional depth, a beam is stiffer than the adjacent slab, and thus, it attracts additional loads and moments.
- The average moments in the column strip are almost the same in a flat plate and in a slab with beams between all columns.
- In the latter case, the column-strip moment is divided between the slab and the beam.
- This reduces the reinforcement required for the slab in the column strip because the beam must be reinforced to carry most of the load.

DESIGN OF SLABS WITH BEAMS IN TWO DIRECTIONS

- The greater stiffness of the beams reduces the overall deflections, allowing a thinner slab to be used than in the case of a flat plate.
- Thus, an advantage of slabs with beams in two directions lies in their reduced weight.
- Also, two-way shear does not govern for most two-way slabs with beams, again allowing thinner slabs.
- This is offset by the increased overall depth of the floor system and increased forming and reinforcement-placing costs.

DESIGN OF SLABS WITH BEAMS IN TWO DIRECTIONS

The DDM for computing moments in the slab and beams is the same as the procedure used in slabs without beams, with one additional step. Thus, the designer will, as usual:

1. Compute “Mo”
2. Divide “Mo” between the positive and negative moment regions.
3. Divide the positive and negative moments between the column and middle strips.

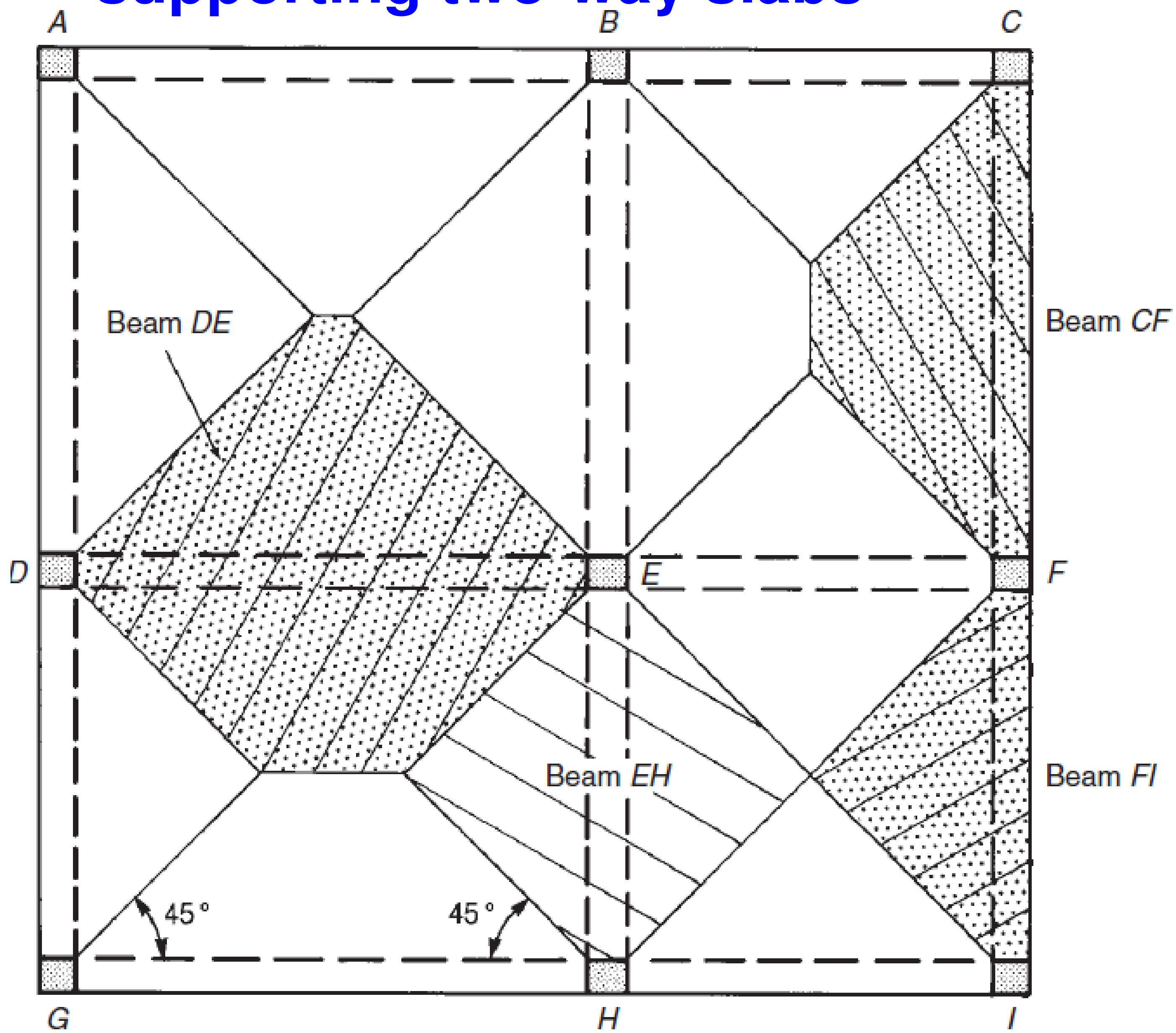
The additional step needed is

4. Divide the column-strip moments between the beam and the slab.

DESIGN OF SLABS WITH BEAMS IN TWO DIRECTIONS

- The amount of moment assigned to the column and middle strips in step 3 and the division of moments between the beam and slab in step 4 are a function of $\alpha_{f1} l_2/l_1$, where, α_{f1} is the beam–slab stiffness ratio in the direction in which the reinforcement is being designed.
- When slabs are supported on beams having $\alpha_{f1} l_2/l_1 \geq 1.0$, the beams must be designed for shear forces computed by assuming tributary areas bounded by 45º lines at the corners of the panels and the centerlines of the panels.

Tributary areas for computing shear in beams supporting two-way slabs



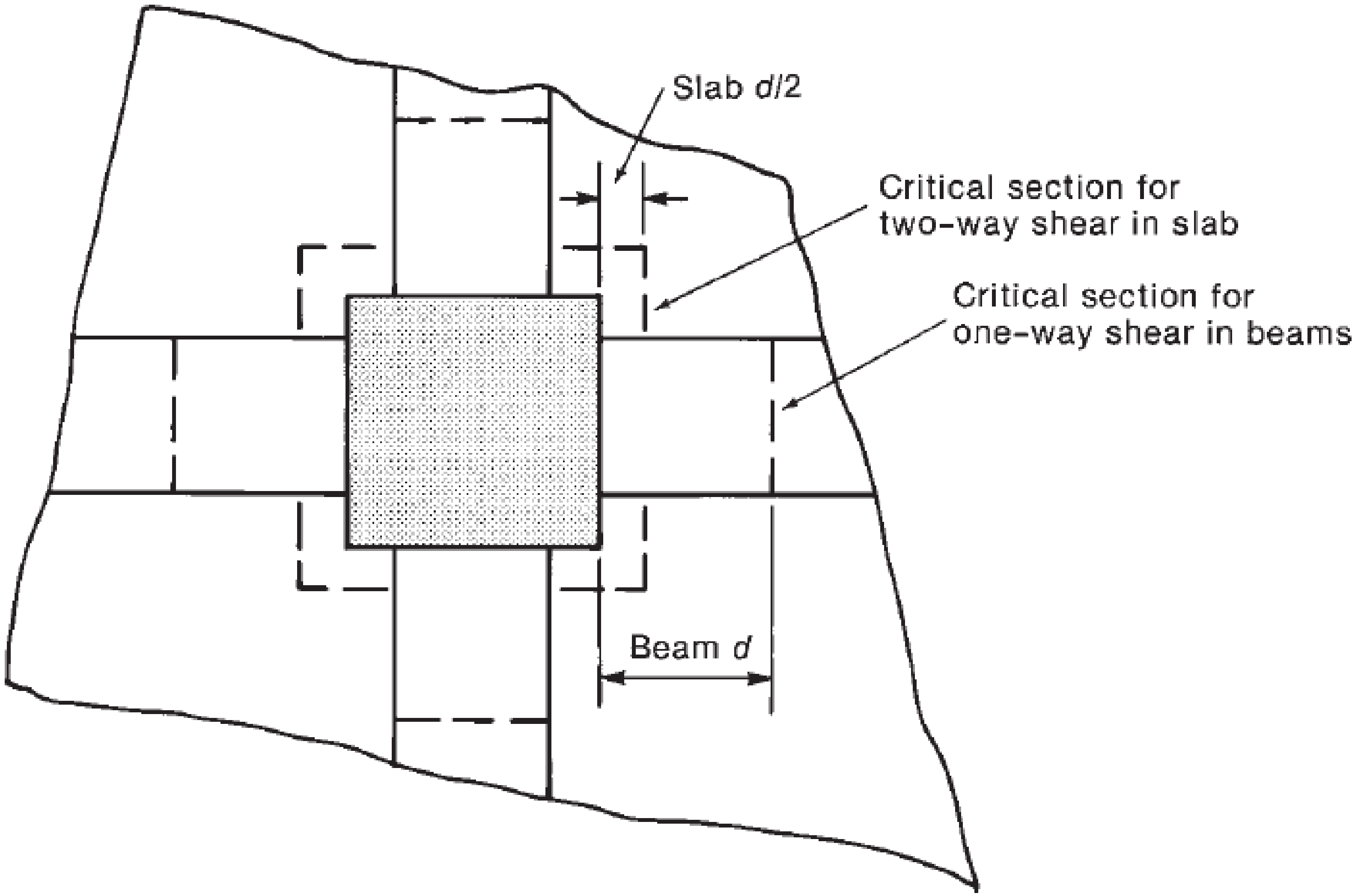
DESIGN OF SLABS WITH BEAMS IN TWO DIRECTIONS

- If the beams have $\alpha_{f1}l_2/l_1$ between 0 and 1.0, the shear forces computed from these tributary areas are multiplied by $\alpha_{f1}l_2/l_1$.
- In such a case, the remainder of the shear must be transmitted to the column by shear in the slab.
- The ACI Code is silent on how this is to be done.

DESIGN OF SLABS WITH BEAMS IN TWO DIRECTIONS

- The most common interpretation involves using two-way shear in the slab between the beams and one-way shear in the beams.
- Frequently, problems are encountered when $\alpha_{f1} l_2/l_1$ is less than 1.0, because the two-way shear perimeter is inadequate to transfer the portion of the shear not transferred by the beams.
- Thus it is recommended to select beam sizes such that $\alpha_{f1} l_2/l_1$ exceeds 1.0 for a two-way slab.

Shear perimeters in slabs with beams



DESIGN OF SLABS WITH BEAMS IN TWO DIRECTIONS

- The size of the beams also is governed by their shear and flexural strengths.
- The cross section should be large enough so that $V_u \leq \phi (V_c + V_s)$.
- where an upper practical limit on $(V_c + V_s)$ would be about $1/2(\sqrt{f_C'} b_w d)$
- The critical location for flexure is the point of maximum negative moment, where the reinforcement ratio, ρ, should not exceed approximately $0.5 \rho_b$