Structural Engineering

(CE-401)

Structural Engineering

- Structural engineering is the science and art of *analyzing*, *designing* and making, with economy and elegance, the structures like buildings, bridges, frameworks, and others so that they can safely resist the forces to which they may be subjected during their life span.

Structural Engineering Process

- Determine types and magnitudes of loads
- Determine the structural context like;
  - geometric and geological information
  - cost / schedule / height/ etc. limitations
- Generate alternative structural systems
- Analyze one or more alternatives
- Select and perform detailed design
- Implement (usually done by contractor)

Structural Engineering;

- Identifies the loads to be resisted
- identifies alternatives for providing load paths (arch, truss, frame, ...)
- designs structure to provide safe and economical load paths (material, size, connections)
- to be economical and safe, we must be able to predict what forces are in structure.

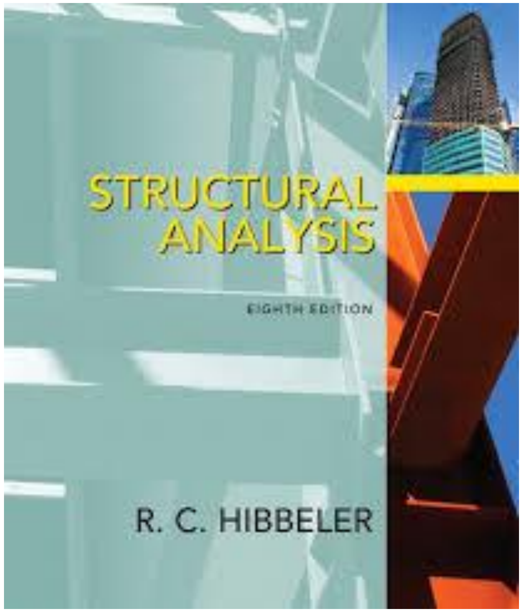
Structural Engineering

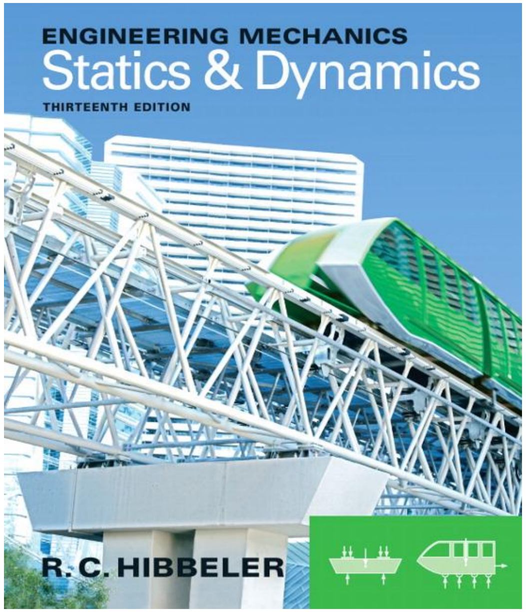
- Total Marks 100
- MID TERM MARKS (30+10)
- END SEMESTER MARKS (40+10)
- ATTENDANCE MARKS (10)

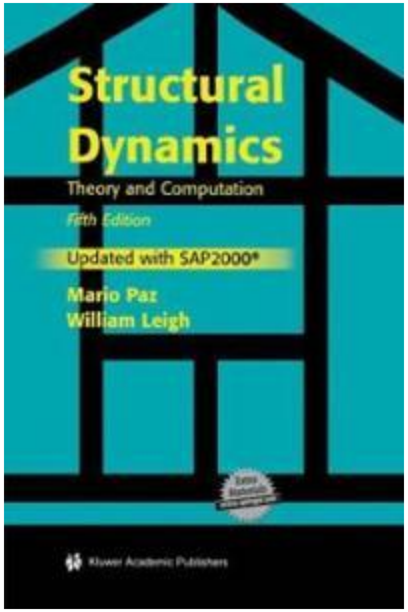
Structural Engineering-COURSE OUTLINE

- Stiffness method (displacement method) or stiffness matrix method for analysis of trusses
- Stiffness method for analysis of beams
- Stiffness method for analysis of frames
- Pre-stressed Concrete
- Structural dynamics
- Application of earthquake loads
- Bridges (slab and R.C. Girder bridge)

Structural Engineering (Books)





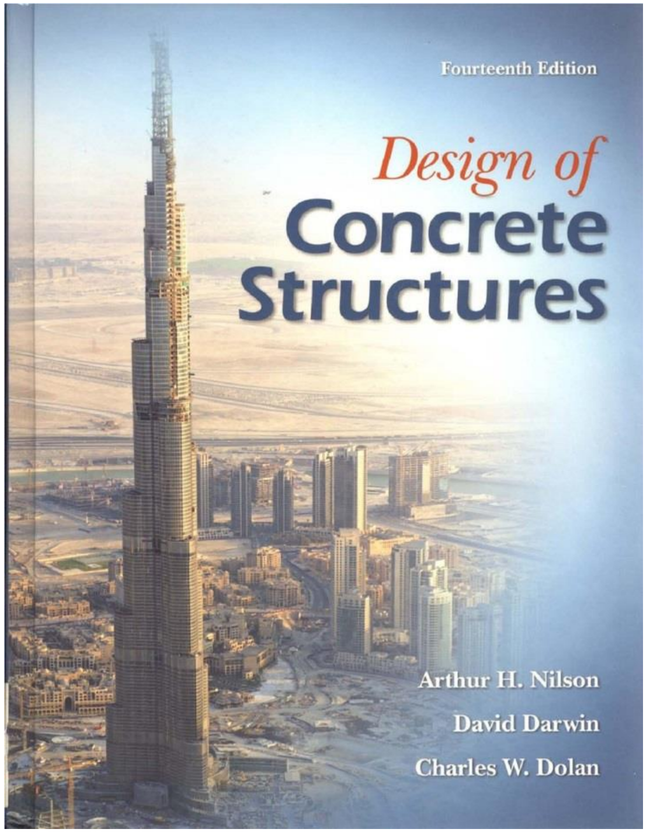


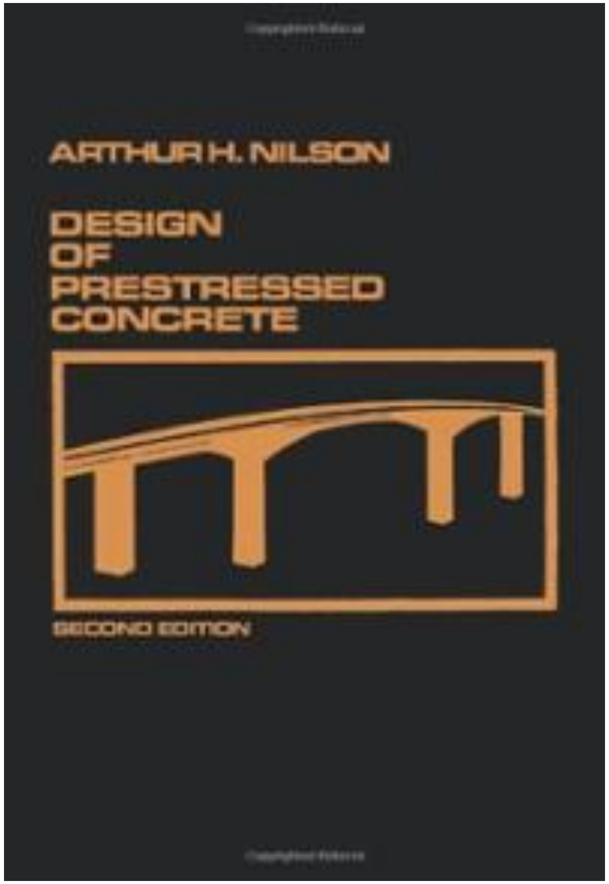
R.C. Hibbler

R.C. Hibbler

Mario Paz

Structural Engineering (Books)





Arthur H. Nilson

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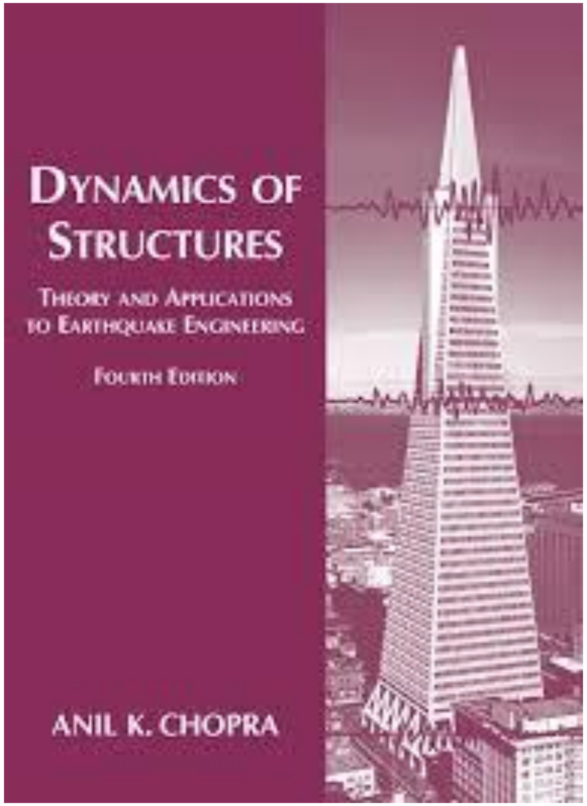
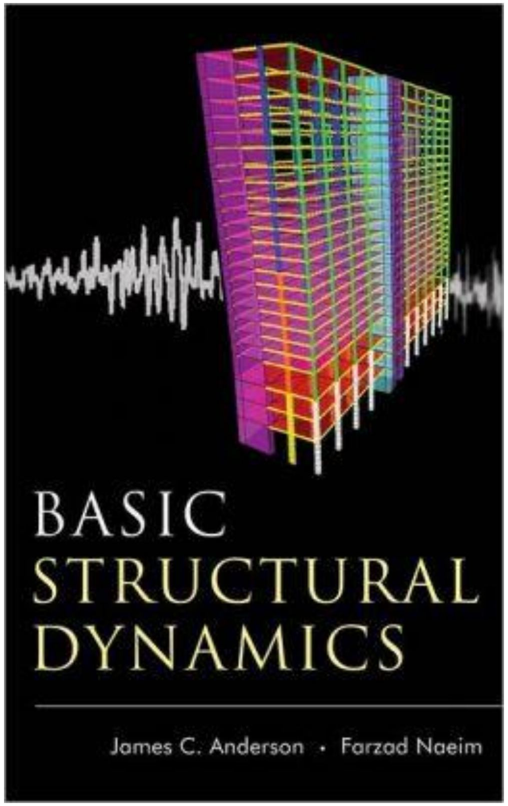
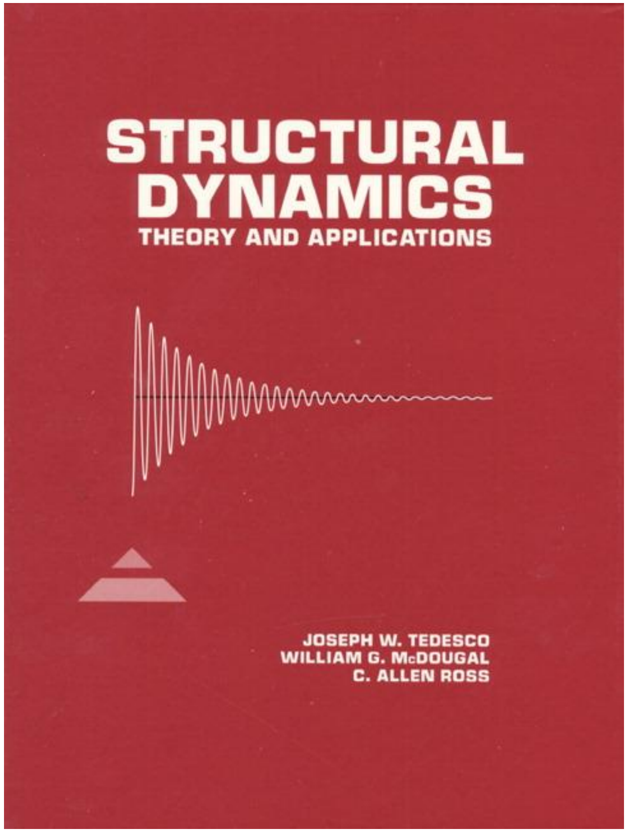
Structural Engineering (Books)



Anil K. Chopra



James C. Anderson



Joseph E. Tedesco

Structural Engineering

- THE FINITE ELEMENT METHOD BY O.C. ZEINKIEWICZ
- COMPUTATIONAL METHOD FOR THE SOLUTION OF ENGINEERING PROBLEMS BY BREBBIA
- STRUCTURAL AND STRESS ANALYSIS BY MEGSON

Course Learing Outcome (CLO's)

Sr. No.	Objective	PLO
1	**The student will be able to analyze and solve any type of 2D truss, beam and frame.**	1,2,3,5
2	**The student will be able to familiarize with basic structural dynamics terms and their applications along with computation of dynamic forces for high rise buildings.**	1,2,3
3	**The student will be able to analyze and design pre-stressed / post tensioned beams for bridges.**	1,2,3
4	**The student will be able to familiarize with different bridge terms along with design of slab and cast in situ girder bridge.**	1,2,3

Program Learning out come (PLO)

PLO	Description	PLO	Description
1	**Engineering Knowledge**	7	**Environment and Sustainability**
2	**Problem Analysis**	8	**Ethics**
3	**Design / Development of solution**	9	**Individual and Team work**
4	**Investigation**	10	**Communication**
5	**Modern tool usage**	11	**Project Management**
6	**The Engineers and Society**	12	**Life long Learning**

Statically Indeterminate Structures

Indeterminate structural analysis

Three basic types of equations are available for structural analysis:

Equilibrium equations	This is the sum of forces or moments for equillibrium
Compatibility equations	Solid body should remain continuous while being deformed satisfying force-displacement relationship
Constitutive equations	These are physical laws of mechanics, express the relationship of Force–displacement and stress-strain eqs. etc

$\sigma = F/A$

$k = \frac{F}{\delta}$

Indeterminate structural analysis

- **Constitutive relation** is a relation between two physical quantities that is specific to a material or substance, and approximates the response of that material to external stimuli, like as the connection between applied stress (force, moments etc.) to strains (deformations / settlements / rotations etc.).
- The constitutive equations are complementary equations to the eqb. and compatibility equations. Taken together with the loading and boundary conditions, these are the sufficient, but not always the necessary, equations in order to formulate a complete boundary value problem, from which the unknown of a given body can be calculated. Principle of Superposition is its application.

Review

- Strength and stiffness are not the same thing.
- Here's an analogy: A rubber band is stretched to failure.
- The rubber band failed, say, at five pounds of force, but it stretched more than double its length before failure. The rubber band was not very stiff. In fact, it was elastic.
- Next, we stretch a kite string and find that it also fails at five pounds.

Review

- It only stretched five percent before failure. It is very stiff.
- Both the rubber band and the kite string have the same ultimate strength.
- However, one is very stiff and the other is very flexible.
- This demonstrates that strength and stiffness are not the same thing, and they are dependent upon the chosen material.

Review

- Furthermore, the shape of the material also determines its stiffness without affecting its ultimate strength.
- For instance, if we take a plastic ruler that is 1/8" thick and 1" wide and bend it in the flat long direction it is obvious that it is flexible.
- However, if we try to bend across the 1" thickness we find that it is very stiff.

Review

- This demonstrates that the shape of the material causes the stiffness to change.
- We can take a piece of metal with a given weight and length and change its stiffness by making it narrower and thicker.
- Conversely, we can make it more flexible by making it wider and thinner.

Review

- Structural Analysis has covered the following topics (for Statically Ind. Strs.) so far:
  - Force Method
  - Moment Distribution Method
  - Slope Deflection Method
  - Plastic analysis and

    Three moment equation

Review

- Structural Analysis is an integral part of structural Engineering.
- It is the process of predicting the performance of a given structure under a prescribed loading.
- The performance characteristics which are of interest to the structural Engineer is:
  - Stresses due to (axial forces, shear or moments)
  - Deflections
  - Support reactions

Review

- Most design offices today used the software’s but the main objective of this subject is to understand the solution of framed structures using matrix approach which is also basis of all the available structural analysis and design software's.

Review of methods

- Historical Back Ground
  - The foundation of ***matrix method*** was laid by James C Maxwell in 1864 who also invented method of consistent deformation (force method).
  - Later on many researchers contributed towards the present available matrix method, the prominent among them are S.S. Archer, C.K. Wang, H.C. Martin, E. L. Wilson.

Review

- Classical vs Matrix Method
  - Both methods are based on the same fundamental principles but the fundamental relationships of equilibrium, compatibility and member stiffness are now expressed in the form of matrix equations so that it can be programmed on computer.

Review

- Classical vs Matrix Method
  - Most classical methods were developed to analyze particular types of structure and since those were intended for hand calculations. For e.g. MDM is used only to analyze beams and plane frames under going bending deformations.
  - Matrix method are systematic and can be easily programmed.

Review

- Classical vs Matrix Method
  - In case of analysis of large structures, classical methods are very time consuming, however with the use of matrix method it becomes feasible to analyze such structures.

Review

- Classical vs Matrix Method
  - Classical methods may also be used for preliminary designs for checking the results of computerized analysis.
  - A study of classical methods is considered to be essential for developing and understanding the structural behavior.

Review

Stiffness Method	Flexibility Method
1. Stiffness method uses matrices right from the start.	1. This method may use matrices but after some manual calculations.
2. Stiffness method has a similar procedure both for statically determinate and indeterminate structures.	2. Flexibility method has a different procedure both for statically determinate and indeterminate structures.
3. Stiffness method generates forces and displacements directly.	3. This method does not generate the forces and displacements directly.
4. Stiffness method can be easily programmed for computers.	4. This method cannot be easily programmed for computers.

Review

- Matrix Method vs Finite Element method
  - Matrix method can be used to analyze frame structures only.
  - FEM which is originated as an extension of matrix analysis is used for analysis of diverse structures and plates and shells and now developed to such an extent that it is applicable to solids of practically any shape or form.

Review

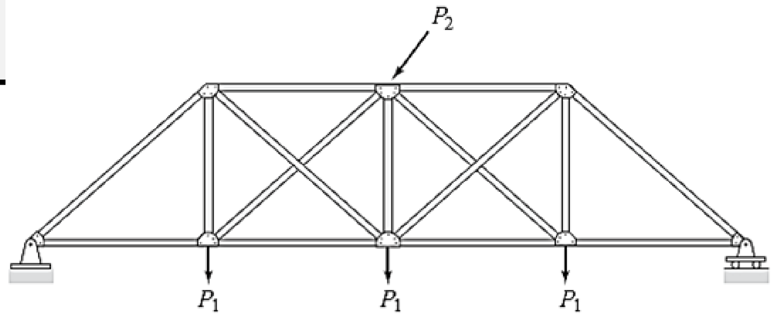
- Matrix Method vs Finite Element method
  - The basic difference b/w two is; In matrix method the member force relationship is based on exact solutions of the underlying differential equations however in FEM they are based on assumed displacement or stress function (Material behavior and their Stress–Strain relationship).
  - Within elastic range both methods gives same solution.
  - In FEM, the solution accuracy depends upon the number of iterations

Review (Framed Structures)

- Plane Truss
  - A truss is defined as assemblage of straight members connected at their end by flexible connections, and subjected to loads and reactions only at the joints.
  - The member of such an ideal truss develop only axial forces when the truss is loaded.
  - If all the members of the truss as well as loads lie in a single plane, it is called plane truss.

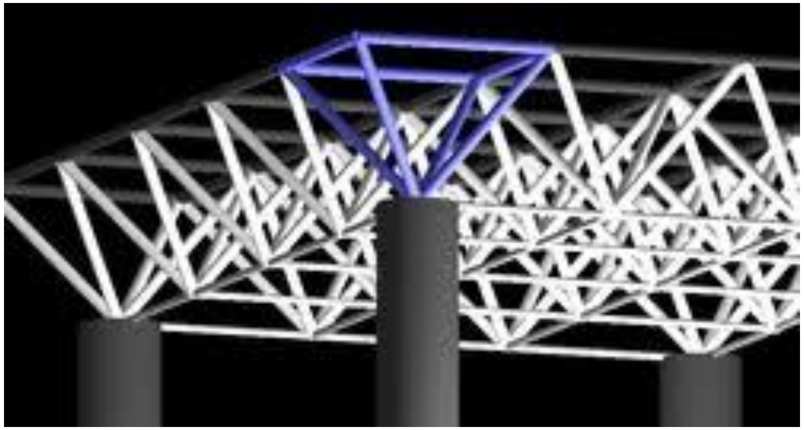
Review (Framed Structures)

- Plane Truss
  - Due to the deviation from the idealized condition, trusses are then also subjected to secondary bending moments and shears comparably very small in comparison to axial forces.
  - If large bending moments are to be anticipated then it shall be designed as a **rigid frame** for analysis and design.



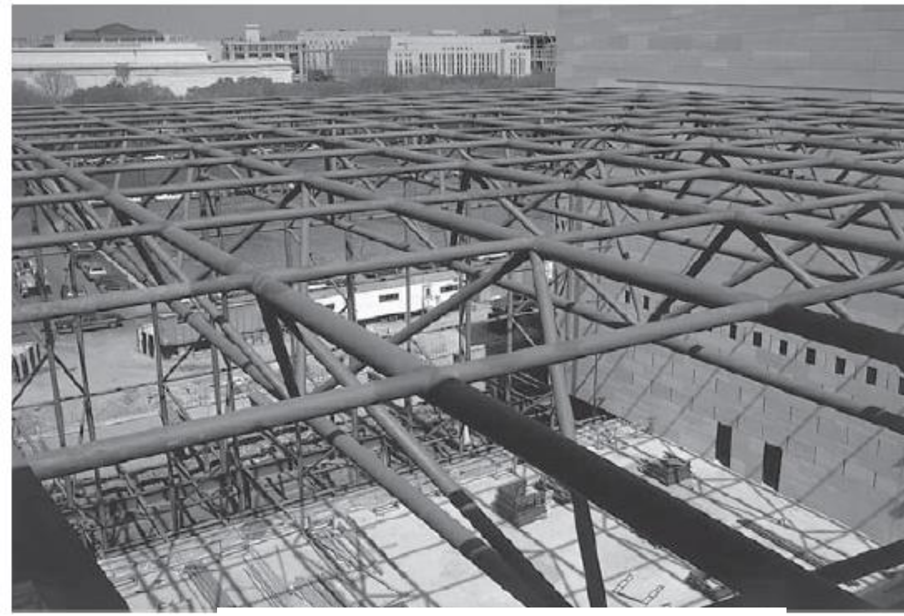
Review (Framed Structures)

- Plane Truss
  - Majority of our trusses are plane trusses used in buildings, bridges or used as roof truss.
  - The analysis of plane truss is simpler than space truss.
  - Generalizing the structure of planar trusses to 3D results in **space trusses**. The most elementary 3D **space truss** structure is the tetrahedron.



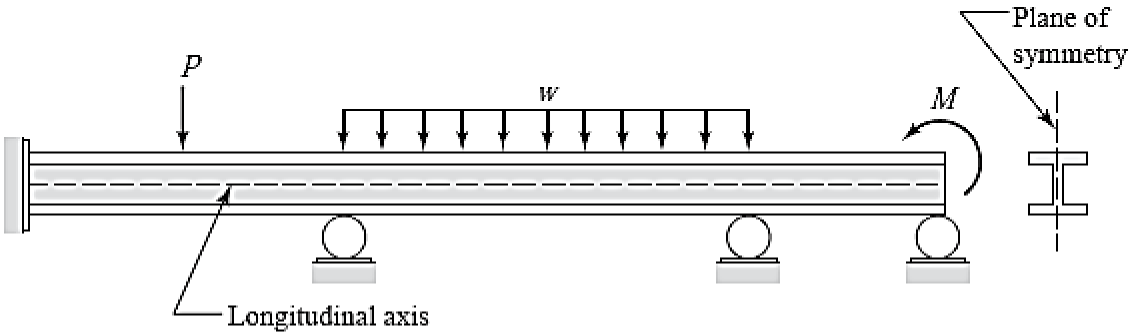
Review (Framed Structures)

- Space Trusses
  - For transmission towers and certain aerospace structures plane trusses cannot be used. Such structures are called space structures.
  - They are 3D structures with loading in three directions.
  - Like plane trusses space trusses also develop axial forces.



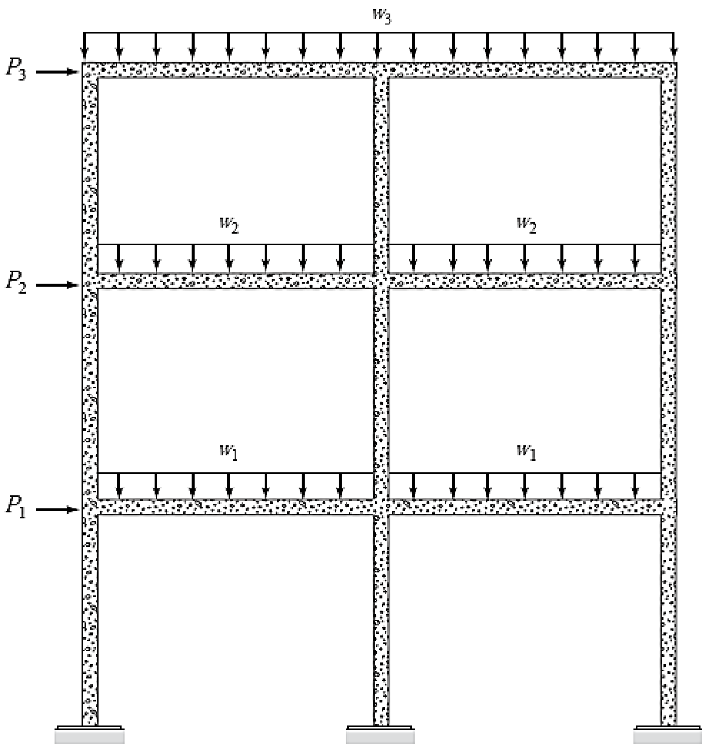
Review (Framed Structures)

- Beam
  - A beam is defined as a long straight structure that is loaded perpendicular to its longitudinal axis. Loads are usually applied in a plane of symmetry of the Beam X-Section, causing its members to be subjected only to bending moments and shear forces.



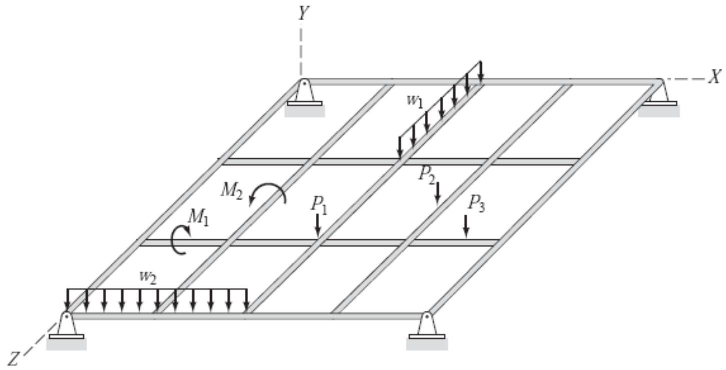
Review (Framed Structures)

- Plane Frames
  - Frames also called rigid frames are composed of straight members connected by rigid (moment resisting) or flexible connections.
  - In frames loads may be applied on joints or on members
  - If all the members of the frame lie in single plane, it is called plane frame and they may be subjected to bending moments shears and axial forces.



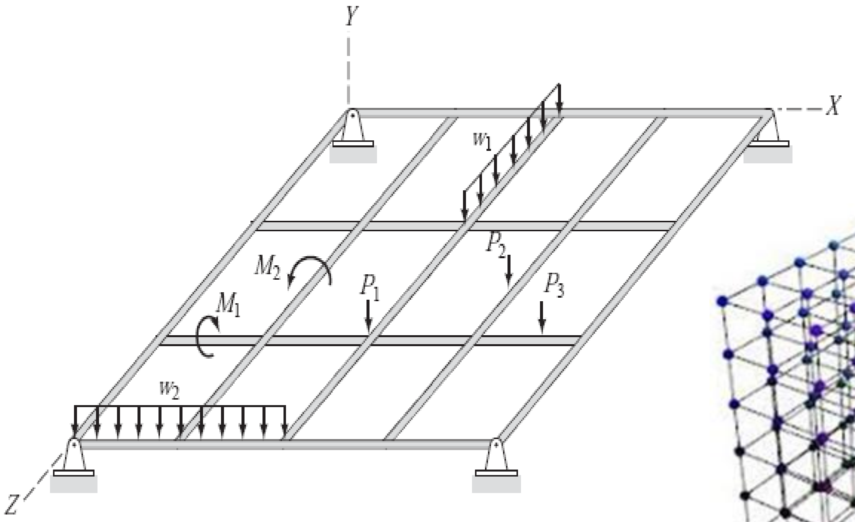
Review (Framed Structures)

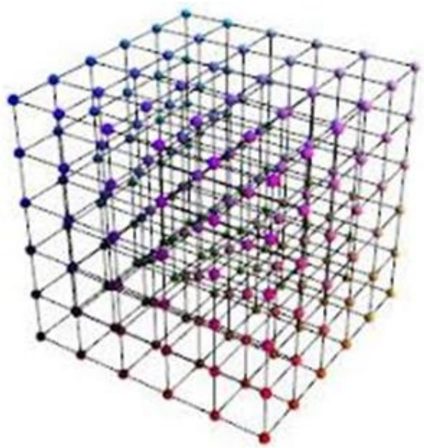
- Grids
- **A grid** is a **structure** (usually two-dimensional) made up of a series of intersecting straight (vertical,horizontal, and angular) or curved guide lines used to structure content.
- A grid, like in a plane frame, is composed of straight members connected together by rigid or flexible connections to form a plane frame work.
- The load on the grid is applied perpendicular to structure plane therefore being subjected to torsional moments along with bending moments and shears.

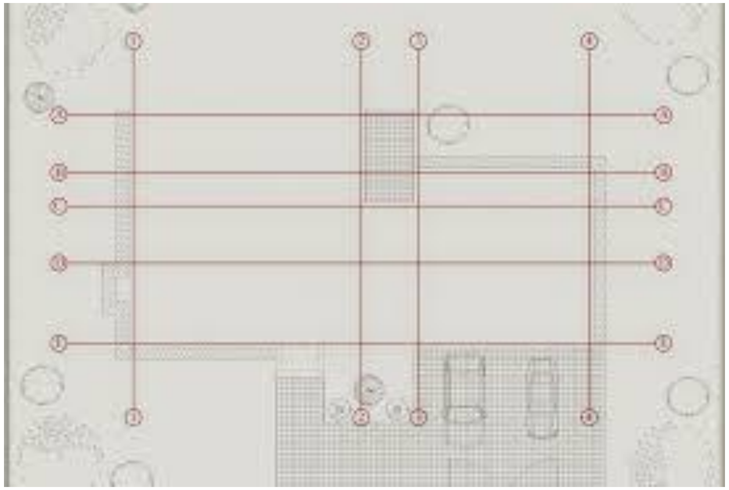


Review (Framed Structures)

- Grids
  - Grids are commonly used to for supporting roofs covering large column free areas in such structures as sports areas, auditoriums and aircraft hangers.







Review (Framed Structures)

- Space Frames
  - 3D frame structure is called space frames having forces applied in all directions. (All high rise buildings, Domes etc)

Review (Framed Structures)

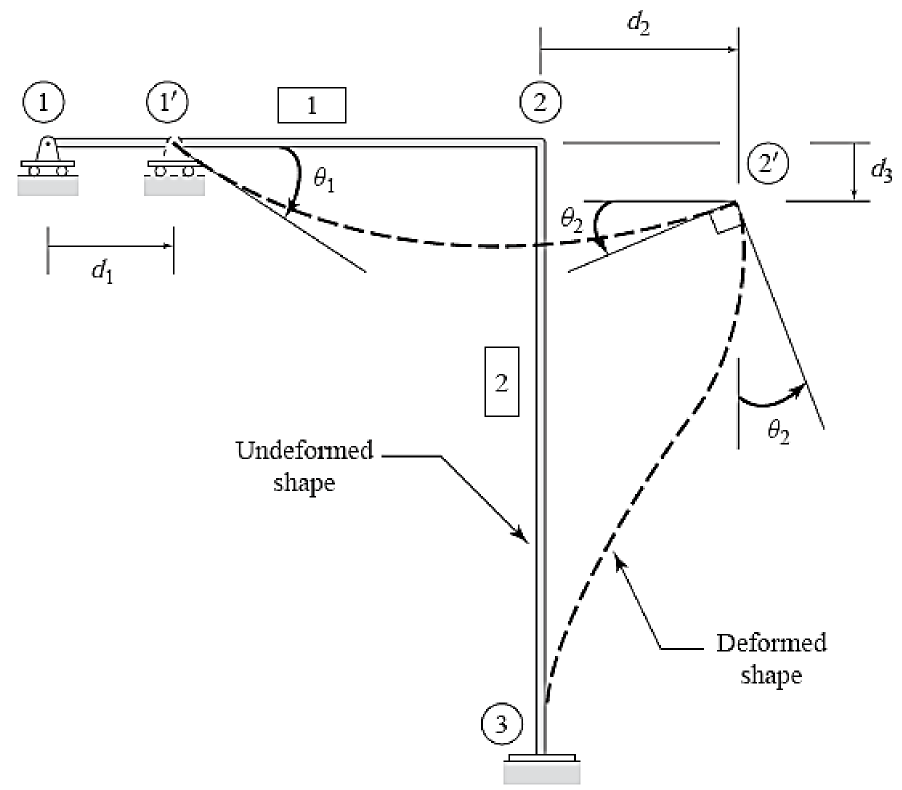
- Equilibrium Conditions
  - Equilibrium conditions for both 2D and 3D structures are shown as follows.

$$\sum F_X = 0 \qquad \sum F_Y = 0 \qquad \sum M = 0$$

$$\sum F_X = 0 \qquad \sum F_Y = 0 \qquad \sum F_Z = 0$$
$$\sum M_X = 0 \qquad \sum M_Y = 0 \qquad \sum M_Z = 0$$

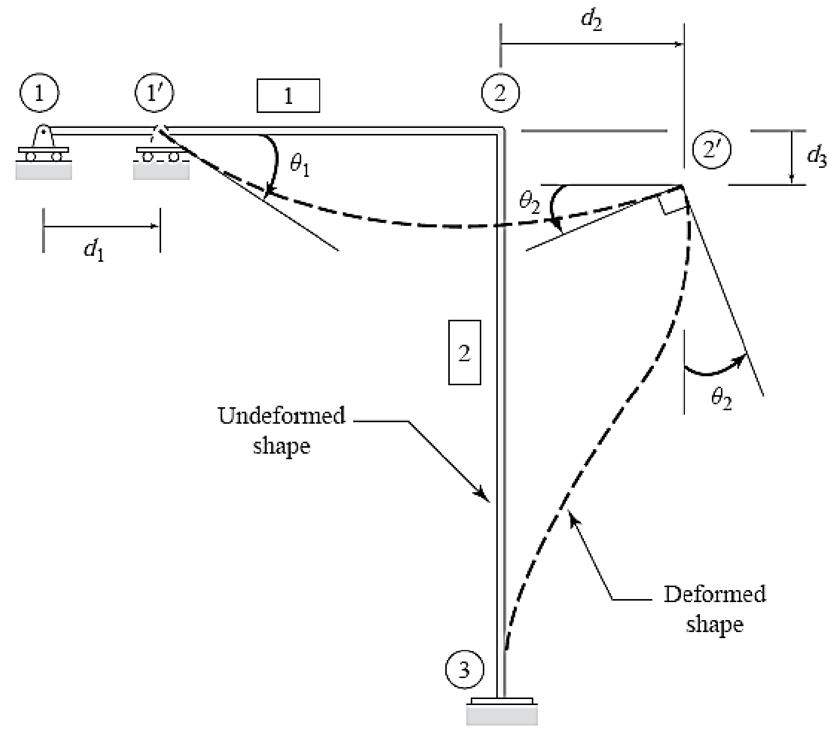
Review (Framed Structures)

- Compatibility Conditions
  - It relates the deformation of a structure so that its various parts (Member, joints , and supports) fit together.
  - These are also called continuity conditions.
  - Consider this frame.
  - Vertical displacement at joint 1 is zero.
  - Vertical and hz disp. at joint 3 is zero



Review (Framed Structures)

- Compatibility Conditions
  - d1 must be equal to d2
  - Theta 2 must be same for hz member-1 and vertical member-2
  - Member 2 and joint 3 will not rotate as it is fixed.



Finally compatibility requires that the deflected shapes of the members of the structures must be continuous and consistent with the displacement at the ends of the member.

Matrices

Matrix. A *matrix* is a rectangular arrangement of numbers having m rows and n columns. The numbers, which are called *elements*, are assembled within brackets. For example, the $\mathbf{A}$ matrix is written as:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \vdots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Such a matrix is said to have an *order* of $m \times n$ (m by n). Notice that the first subscript for an element denotes its row position and the second subscript denotes its column position. In general, then, a_{ij} is the element located in the ith row and jth column.

Matrices

Row Matrix. If the matrix consists only of elements in a single row, it is called a *row matrix*. For example, a $1 \times n$ row matrix is written as

$$\mathbf{A} = [a_1 \quad a_2 \quad \cdots \quad a_n]$$

Here only a single subscript is used to denote an element, since the row subscript is always understood to be equal to 1, that is, $a_1 = a_{11}$, $a_2 = a_{12}$, and so on.

Column Matrix. A matrix with elements stacked in a single column is called a *column matrix*. The $m \times 1$ column matrix is

$$\mathbf{A} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$$

Here the subscript notation symbolizes $a_1 = a_{11}$, $a_2 = a_{21}$, and so on.

Matrices

Square Matrix. When the number of rows in a matrix equals the number of columns, the matrix is referred to as a *square matrix*. An $n \times n$ square matrix would be

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \vdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Symmetric Matrix. A *square matrix* is symmetric provided $a_{ij} = a_{ji}$. For example,

$$\mathbf{A} = \begin{bmatrix} 3 & 5 & 2 \\ 5 & -1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

Matrices

Diagonal Matrix. When all the elements of a square matrix are zero except along the main diagonal, running down from left to right, the matrix is called a *diagonal matrix*. For example,

$$\mathbf{A} = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$

Unit or Identity Matrix. The *unit* or *identity matrix* is a diagonal matrix with all the diagonal elements equal to unity. For example,

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix Operations

Equality of Matrices. Matrices **A** and **B** are said to be equal if they are of the same order and each of their corresponding elements are equal, that is, $a_{ij} = b_{ij}$. For example, if

$$\mathbf{A} = \begin{bmatrix} 2 & 6 \\ 4 & -3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 2 & 6 \\ 4 & -3 \end{bmatrix}$$

then $\mathbf{A} = \mathbf{B}$.

Addition and Subtraction of Matrices. Two matrices can be added together or subtracted from one another if they are of the same order. The result is obtained by adding or subtracting the corresponding elements. For example, if

$$\mathbf{A} = \begin{bmatrix} 6 & 7 \\ 2 & -1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -5 & 8 \\ 1 & 4 \end{bmatrix}$$

Matrix Operations

Addition and Subtraction of Matrices. Two matrices can be added together or subtracted from one another if they are of the same order. The result is obtained by adding or subtracting the corresponding elements. For example, if

$$\mathbf{A} = \begin{bmatrix} 6 & 7 \\ 2 & -1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -5 & 8 \\ 1 & 4 \end{bmatrix}$$

then

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 1 & 15 \\ 3 & 3 \end{bmatrix} \qquad \mathbf{A} - \mathbf{B} = \begin{bmatrix} 11 & -1 \\ 1 & -5 \end{bmatrix}$$

Matrix Operations

Multiplication by a Scalar. When a matrix is multiplied by a scalar, each element of the matrix is multiplied by the scalar. For example, if

$$\mathbf{A} = \begin{bmatrix} 4 & 1 \\ 6 & -2 \end{bmatrix} \qquad k = -6$$

then

$$k\mathbf{A} = \begin{bmatrix} -24 & -6 \\ -36 & 12 \end{bmatrix}$$

Matrix Operations

Matrix Multiplication. Two matrices **A** and **B** can be multiplied together only if they are *conformable*. This condition is satisfied if the number of *columns* in **A** *equals* the number of *rows* in **B**. For example, if

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} \qquad \text{(A–1)}$$

then **AB** can be determined since **A** has two columns and **B** has two rows. Notice, however, that **BA** is not possible. Why?

If matrix **A** having an order of $(m \times n)$ is multiplied by matrix **B** having an order of $(n \times q)$ it will yield a matrix **C** having an order of $(m \times q)$, that is,

$$\underset{(m \times n)(n \times q)}{\mathbf{A}\quad\mathbf{B}} = \underset{(m \times q)}{\mathbf{C}}$$

Matrix Operations

The methodology of this formula can be explained by a few simple examples. Consider

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ -1 & 6 & 1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 2 \\ 6 \\ 7 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 49 \\ 41 \end{bmatrix}$$

As a second example, consider

$$\mathbf{A} = \begin{bmatrix} 5 & 3 \\ 4 & 1 \\ -2 & 8 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 2 & 7 \\ -3 & 4 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 47 \\ 5 & 32 \\ -28 & 18 \end{bmatrix}$$

Matrix Operations

1. In general the product of two matrices is not commutative:

$$\mathbf{AB} \neq \mathbf{BA} \tag{A–3}$$

2. The distributive law is valid:

$$\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC} \tag{A–4}$$

3. The associative law is valid:

$$\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C} \tag{A–5}$$

Matrix Operations

Transposed Matrix. A matrix may be transposed by interchanging its rows and columns. For example, if

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}$$

$$\mathbf{A}^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \qquad \mathbf{B}^T = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$$

$$(k\mathbf{A})^T = k\mathbf{A}^T$$

$$(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$$

Matrix Operations

Matrix Partitioning. A matrix can be subdivided into submatrices by partitioning. For example,

$$\mathbf{A} = \left[\begin{array}{c:ccc} a_{11} & a_{12} & a_{13} & a_{14} \\ \hdashline a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{array}\right] = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}$$

Here the submatrices are

$$\mathbf{A}_{11} = [a_{11}] \qquad \mathbf{A}_{12} = [a_{12} \quad a_{13} \quad a_{14}]$$

$$\mathbf{A}_{21} = \begin{bmatrix} a_{21} \\ a_{31} \end{bmatrix} \qquad \mathbf{A}_{22} = \begin{bmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \end{bmatrix}$$

Matrix Operations

Determinants

A determinant is a square array of numbers enclosed within vertical bars. For example, an nth-order determinant, having n rows and n columns, is

$$|A| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \vdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} \qquad \text{(A–9)}$$

Matrix Operations

Evaluation of this determinant leads to a single numerical value which can be determined using *Laplace's expansion*. This method makes use of the determinant's minors and cofactors. Specifically, each element a_{ij} of a determinant of nth order has a *minor* M_{ij} which is a determinant of order $n-1$. This determinant (minor) remains when the ith row and jth column in which the a_{ij} element is contained is canceled out. If the minor is multiplied by $(-1)^{i+j}$ it is called the cofactor of a_{ij} and is denoted as

$$C_{ij} = (-1)^{i+j} M_{ij} \qquad \text{(A–10)}$$

For example, consider the third-order determinant

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Matrix Operations

The cofactors for the elements in the first row are

$$C_{11} = (-1)^{1+1}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$C_{12} = (-1)^{1+2}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$C_{13} = (-1)^{1+3}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Laplace's expansion for a determinant of order n, Eq. A–9, states that the numerical value represented by the determinant is equal to the sum of the products of the elements of any row or column and their respective cofactors, i.e.,

$$D = a_{i1}C_{i1} + a_{i2}C_{i2} + \cdots + a_{in}C_{in} \qquad (i = 1, 2, \ldots, \text{or } n)$$

or (A–11)

$$D = a_{1j}C_{1j} + a_{2j}C_{2j} + \cdots + a_{nj}C_{nj} \qquad (j = 1, 2, \ldots, \text{or } n)$$

Matrix Operations

Inverse of a Matrix

Consider the following set of three linear equations:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = c_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = c_3$$

which can be written in matrix form as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \qquad \text{(A–12)}$$

$$\mathbf{Ax} = \mathbf{C} \qquad \text{(A–13)}$$

Matrix Operations

One would think that a solution for x could be determined by dividing **C** by **A**; however, division is not possible in matrix algebra. Instead, one multiplies by the inverse of the matrix. The *inverse* of the matrix **A** is another matrix of the same order and symbolically written as $\mathbf{A}^{-1}$. It has the following property,

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

where **I** is an identity matrix. Multiplying both sides of Eq. A–13 by $\mathbf{A}^{-1}$, we obtain

$$\mathbf{A}^{-1}\mathbf{A}\mathbf{x} = \mathbf{A}^{-1}\mathbf{C}$$

Since $\mathbf{A}^{-1}\mathbf{A}\mathbf{x} = \mathbf{I}\mathbf{x} = \mathbf{x}$, we have

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{C} \qquad \text{(A–14)}$$

Matrix Operations

To illustrate how to obtain $\mathbf{A}^{-1}$ numerically, we will consider the solution of the following set of linear equations:

$$\begin{aligned} x_1 - x_2 + x_3 &= -1 \\ -x_1 + x_2 + x_3 &= -1 \\ x_1 + 2x_2 - 2x_3 &= 5 \end{aligned} \qquad \text{(A–16)}$$

Here

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix}$$

Matrix Operations

The cofactor matrix for **A** is

$$\mathbf{C} = \begin{bmatrix} \begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} & -\begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} & \begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} -1 & 1 \\ 2 & -2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \end{bmatrix}$$

Evaluating the determinants and taking the transpose, the adjoint matrix is

$$\mathbf{C}^T = \begin{bmatrix} -4 & 0 & -2 \\ -1 & -3 & -2 \\ -3 & -3 & 0 \end{bmatrix}$$

Matrix Operations

Since

$$A = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 2 & -2 \end{vmatrix} = -6$$

The inverse of **A** is, therefore,

$$\mathbf{A}^{-1} = -\frac{1}{6}\begin{bmatrix} -4 & 0 & -2 \\ -1 & -3 & -2 \\ -3 & -3 & 0 \end{bmatrix}$$

Solution of Eqs. A–16 yields

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = -\frac{1}{6}\begin{bmatrix} -4 & 0 & -2 \\ -1 & -3 & -2 \\ -3 & -3 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}$$

Matrix Operations

Solution of Eqs. A–16 yields

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = -\frac{1}{6}\begin{bmatrix} -4 & 0 & -2 \\ -1 & -3 & -2 \\ -3 & -3 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}$$

$$x_1 = -\tfrac{1}{6}[(-4)(-1) + 0(-1) + (-2)(5)] = 1$$

$$x_2 = -\tfrac{1}{6}[(-1)(-1) + (-3)(-1) + (-2)(5)] = 1$$

$$x_3 = -\tfrac{1}{6}[(-3)(-1) + (-3)(-1) + (0)(5)] = -1$$

Review

Stiffness:

- The stiffness or spring coefficient, *k*, of a body is a measure of the resistance offered by an elastic body to deformation. For an elastic body, the stiffness is the force required to produce unit displacement i-e $k = \frac{F}{\delta}$ Where, *F* is the force applied on the body, δ is the displacement .
- A body may also have a *rotational stiffness*, *k*, given by $k = \frac{M}{\theta}$ Where *M* is the applied moment and *θ* is the rotation
- **Relationship to elasticity**: $k = \frac{AE}{L}$ *A* is the cross-sectional area, *E* is the (tensile) elastic modulus (or Young's modulus),*L* is the length of the element.

Flexibility:

It is inverse of stiffness; The displacement per unit force; i-e $f = 1/k = \frac{\delta}{F}$

Review

Stiffness and Flexibility

If the material of the spring is linearly elastic, the load P and elongation δ are proportional, or $P = k\,\delta$.

$k = P/\delta$ is the stiffness (or "spring constant") with units N/m
$f = \delta/P$ is the flexibility (or "compliance") with units m/N

For a prismatic bar,

$$\sigma = E\varepsilon \qquad \left(\frac{P}{A_o}\right) = E\left(\frac{\delta}{L_o}\right) \qquad \delta = \frac{PL_o}{EA_o}$$

$$k = \frac{P}{\delta} = \frac{EA_o}{L_o} \qquad f = \frac{\delta}{P} = \frac{L_o}{EA_o}$$

k and *f* play an important role in computational analysis of large structures, where they are assembled into stiffness and flexibility matrices for the entire structure.

Review

- Discuss the trusses their degree of indeterminacy, stability, global and local system etc.

Review

Stiffness Method (Displacement /slope deflection method)	Flexibility Method (force method)
1. Stiffness method uses matrices right from the start. 2. Stiffness method has a similar procedure both for statically determinate and indeterminate structures. 3. Stiffness method generates forces and displacements directly. 4. Stiffness method can be easily programmed for computers.	1. This method may use matrices but after some manual calculations. 2. Flexibility method has a different procedure both for statically determinate and indeterminate structures. 3. This method does not generate the forces and displacements directly. 4. This method cannot be easily programmed for computers.