Structural Dynamics

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4. Newtons Law of motion
5. Free body Diagram
6. D’ Al. Embert’s principle
7. Solution of Differential Equation of motion
8. Frequency and Period
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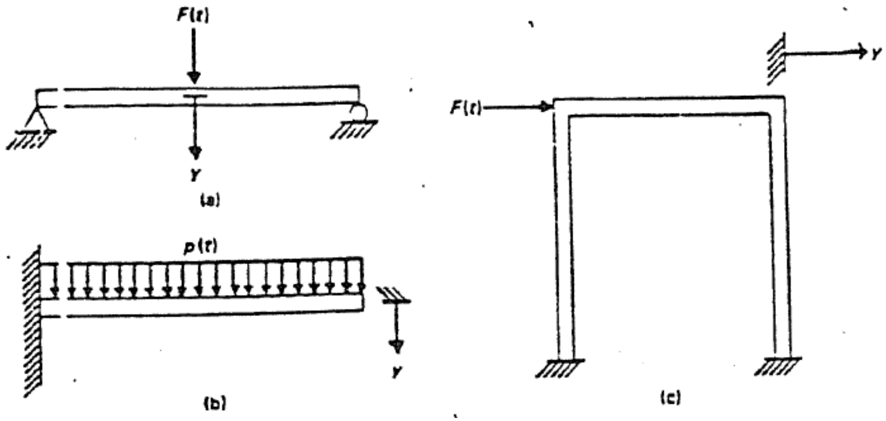
Degree of Freedom

- It is not always possible to obtain a rigorous mathematical solution for engineering problems.
- The analytical solution is obtained for a certain simplified situations.
- The problems involving complex material properties, loading and boundary conditions, the engineer introduces assumptions and idealizations to make the problem mathematically manageable.
- The link b/w the real physical system and the mathematically feasible solution is provided by the mathematical model based on the assumptions.

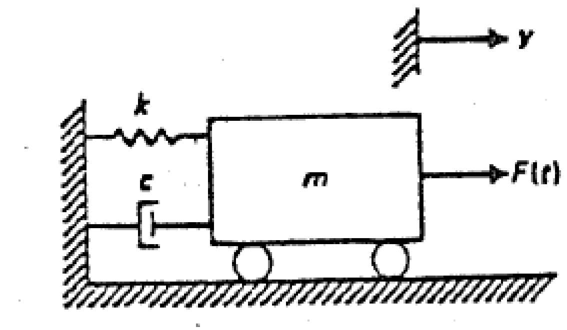
Degree of Freedom

- In structural dynamics the number of independent coordinates necessary to specify the configuration or position of a system at any time is referred to as the number of degrees of freedom.
- A continuous structure has infinite no. of degree of freedoms.
- The process of mathematical modeling decreases the no. of degrees of freedom even to one.



Degree of Freedom

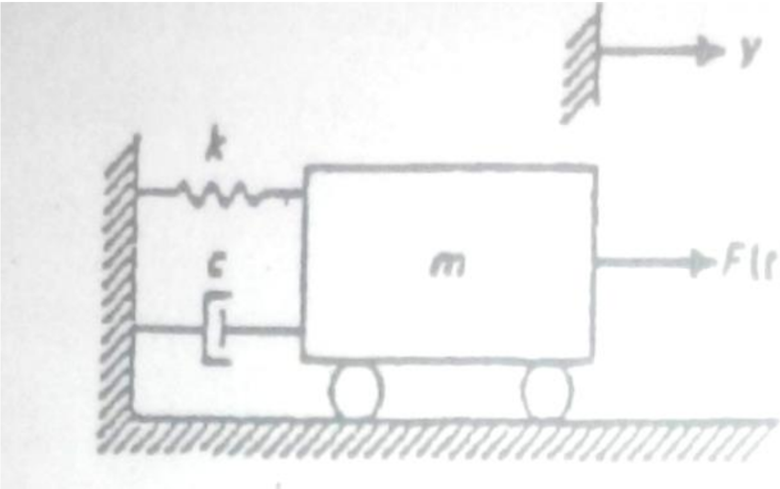
- In the last slide figures SDOF structures have been shown i.e. structures modeled as system with single displacement coordinate.
- SDOF system represented by mathematical model may be demonstrated by the given figure on right.
- This figure has the following elements.



- Mass element (m) representing mass and inertial characteristics of structure
- Spring element (k) representing elastic restoring force and P.E. capacity of str.

Degree of Freedom

- A damping element (c) representing the frictional characteristic and energy loss of the structure.
- Excitation force (F(t)), external force acting on the system.
- In structural dynamics each component represent its own property. i.e. “m” will be used for mass or inertial concept but not for representing elasticity.



Mathematical model for one-degree-of-freedom systems.

Un-damped System

- Starting with fundamental and simple system, SDOF system in which we neglect frictional forces or damping effect i.e. “c”.
- Considering the system is free from external actions or forces.
- The system is under initial conditions; i.e. The given displacement and velocity at time t=0 when the study of the system is initiated.
- This undamped SDOF is called as simple undamped oscillator.
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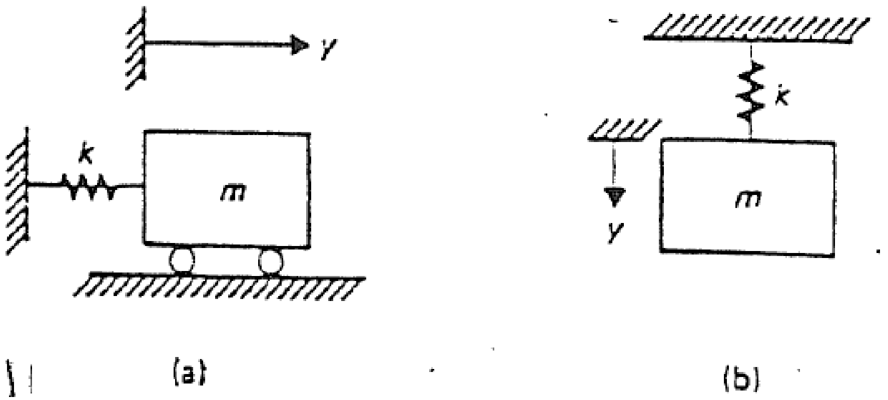
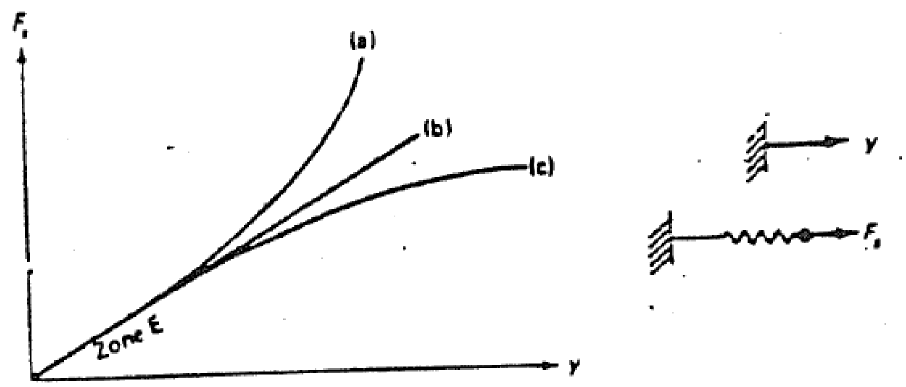
Degree of Freedom

- These two figures represent mathematical model which are dynamically equivalent.
- In these models , ‘m’ represents mass and spring ‘k’ represents linear motion along one co-ordinate axis.
- The property of spring is given by the graph.
  - Hard spring which needs more force for specific displacement.



Degree of Freedom

- The property of spring is given by the graph.
  - Hard spring which needs more force for specific displacement.
  - Line spring in which deformation is directly proportional to force. (Fs= ky)
  - Soft spring in which incremental force required to produce additional deformation decreases as the deformation increases.



Un-damped System

- The linear spring is the simplest to handle in modeling.
- It should be noted that in many cases, the displacement produced in the structure by the action of the external forces or disturbances are small in magnitude, therefore linear approximation is close the actual structural behavior.

Springs in Parallel or Series

- Springs in series or parallel are shown in the figure below.
- The total force is by defination equivalent spring constant and is given by $K_e = K_1 + K_2$
- When springs are parallel as in (a)
- $k_e = \sum_{i=1}^{n} k_i$

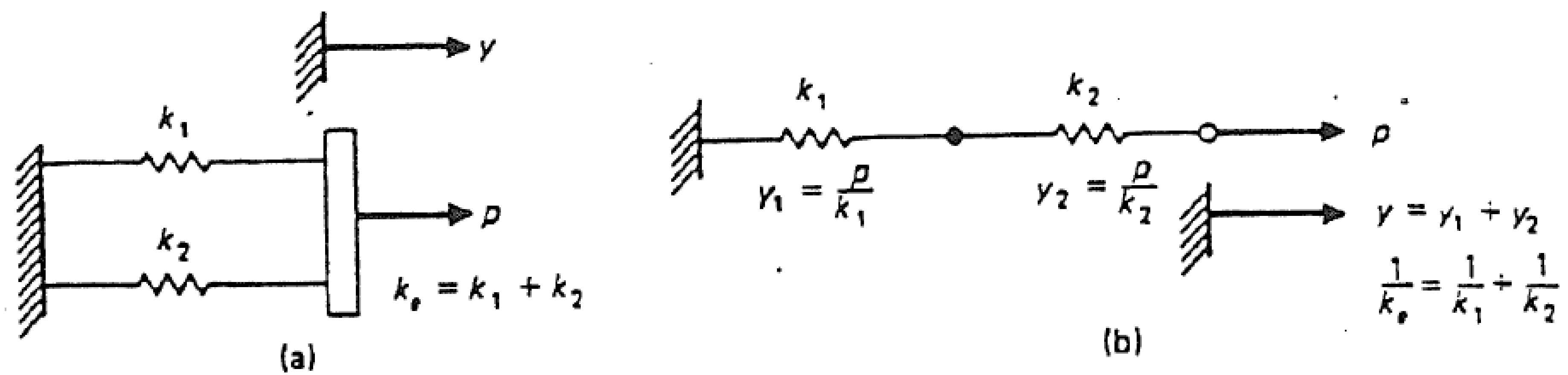


Fig 1.5 Combination of springs. (a) Springs in parallel. (b) Springs in series.

Springs in Parallel or Series

- When springs are in series: $y_1 = \frac{P}{k_1}$; $y_2 = \frac{P}{k_2}$
- The total displacement is : $y = \frac{P}{k_1} + \frac{P}{k_2} = \frac{P}{(k1+k2)}$: $k_e = \frac{P}{y}$
- Substituting "y" from last equation we get

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2} \text{ or } \frac{1}{k_e} = \sum_{i=1}^{n} \frac{1}{k_i}$$

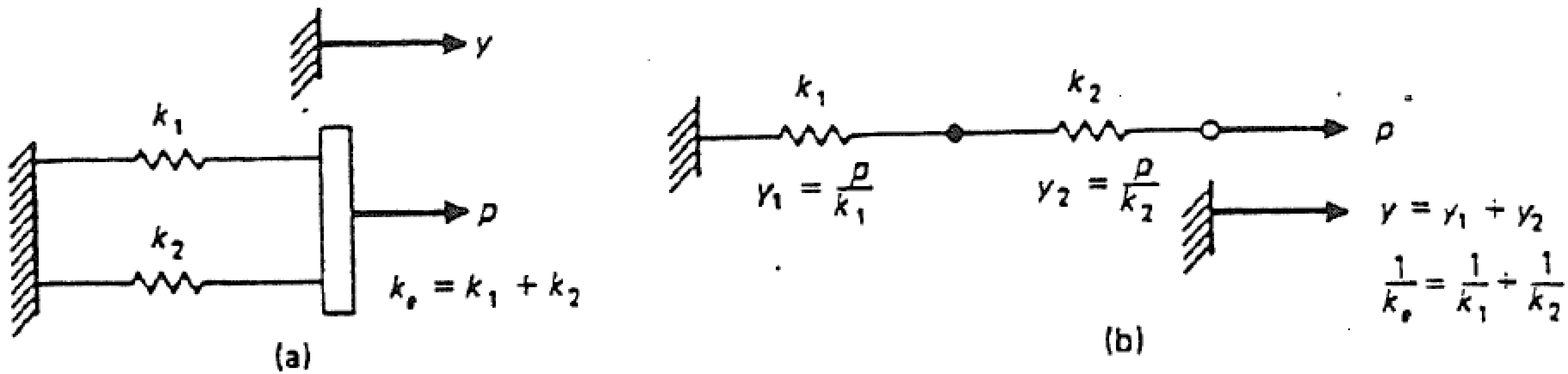


Fig 1.5 Combination of springs. (a) Springs in parallel. (b) Springs in series.

Newton's Second Law of motion

- For simple oscillator, its motion or displacement of mass w.r.t for a given set of initial conditions is represented by Newton's Second law of motion. (F=ma)
- Where "F" is the resultant force acting on a particle of mass 'm' and 'a' is its resultant acceleration. The above relation can be written in terms of its component along x, y and x axis. (i.e. $\sum F_x = ma_x$; $\sum F_y = ma_y$; $\sum F_z = ma_z$)
- The acceletation is the second derivative of displacement w.r.t time. (a= $\frac{d^2y}{dt^2}$) Hence above equations are the differential equations.

Newton's Second Law of motion

- For plane motion of a rigid body which is symmetric with respect to the reference plane of motion (x-y plane) Newton's second law of motion yields the following.
- $\sum F_x = m(a_G)_x$; $\sum F_y = m(a_G)_y$; $\sum M_G = (I_G)_\alpha$

- In above equations $(a_G)_{x/y}$ are the acceleration components along x and y axis of the center of mass G of the body, "α" is the angular acceleration, I_G is the mass moment of inertia of the body w.r.t an axis through "G", the center of mass.
- M_G is the sum of moments of all the forces acting the body w.r.t an axis through G, perpendicular to x-y plane.

Newton’s Second Law of motion

- Alternatively the last equation may be written as
  - $\sum M_0 = (I_0)_\alpha$
- In above equations the mass moment of inertia and moment of forces are determined w.r.t to the fixed axis of rotation.
- The general motion of a rigid body is described by two vector eqs., one expressing the relation b/w the force and the acceleration of the mass center and another relating the moment of the force and the angular motion of the body.

FREE BODY DIAGRAM

- First step in structural dynamics is to draw a free body diagram of the system, prior to writing a mathematical description of the system.
- For oscillating system its equivalent FBD is shown below for mass ‘m’ displaced in ‘y’ direction and acted upon by spring force (F_s=ky) (A linear spring)

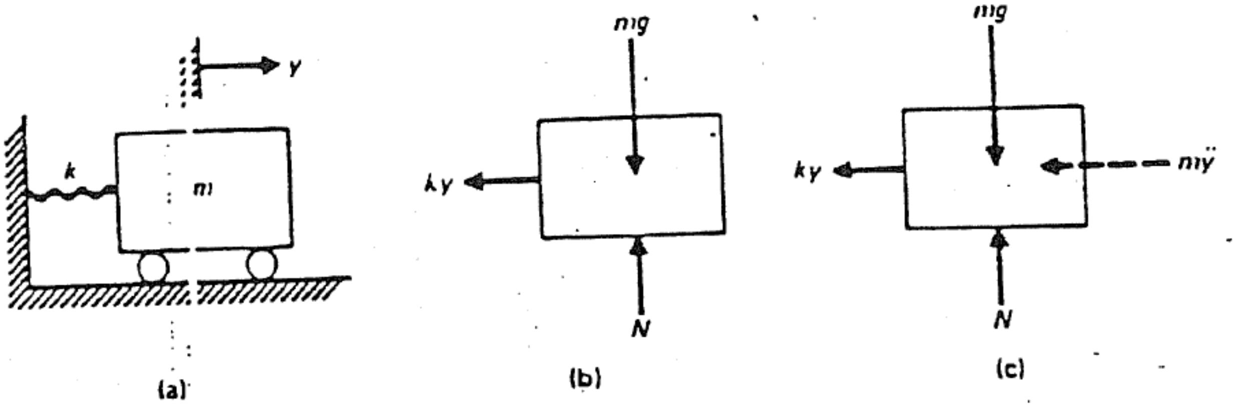


Fig. 1.6 Alternate free body diagrams: (a) Single degree-of-freedom system. (b) Showing only external forces. (c) Showing external and inertial forces.

FREE BODY DIAGRAM

- The weight of the body 'mg' and eqv reaction 'N' is also shown. These forces are acting in the vertical direction. The application of NSLM gives

$$-ky = m\ddot{y}$$

Where the spring force acting the –ve direction has minus sign and 'y' double dot indicates second derivative w.r.t time i.e. acceleration.

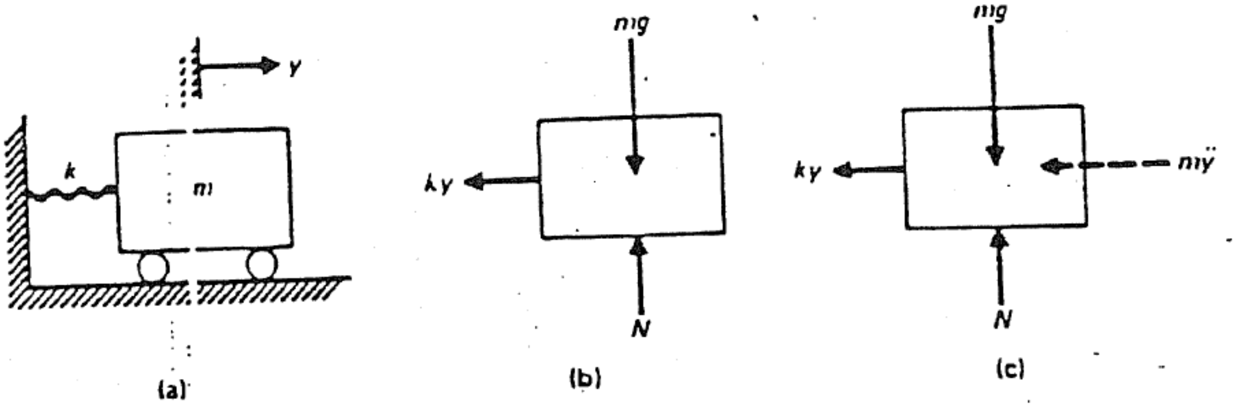


Fig. 1.6 Alternate free body diagrams: (a) Single degree-of-freedom system. (b) Showing only external forces. (c) Showing external and inertial forces.

D' ALEMBERT'S PRINCIPLE

- An alternate approach to obtain $(-ky = m\ddot{y})$ is to make use of D' Alembert's principle which states that a system may be set in state of dynamic equilibrium by adding to the external forces a fictitious force which is commonly known as inertial force.
- Figure 1.6 c shows the FBD with inclusion of inertial force my. This force is equal to the mass into acceleration and should always be directed –vely w.r.t the corresponding coordinate.
- Under equilibrium the above equation may be written as using D' Alembert's principle

$$ky + m\ddot{y} = 0$$

SOLUTION OF DIFFERENTIAL EQ. OF MOTION

- The equation ($-ky = m\ddot{y}$) is in differential form.
- In order to solve this equation we need to assume functions as being devised in case of solution of differential equations.
- Let us take $y = \mathrm{A}\cos(\omega t)\, or\ y = B\sin(\omega t)$ Where A and B are the constants.
- Using cosine function into the above equation after differentiating we get.

$$(m\omega^2 + k) * A\cos(\omega t) = 0$$

Also $w = \sqrt{\frac{k}{m}}$ Where w = natural frequency of the system

SOLUTION OF DIFFERENTIAL EQ. OF MOTION

- Since $y = \text{A}\cos(\omega t)\ or\ y = B\sin(\omega t)$ is a solution of $(-ky = m\ddot{y})$ and since this differential eq. is linear , the super position of these two solutions having constants ‘A’ and ‘B” is the general solution for this second order differential equation.

$$y = \text{A}\cos(\omega t) + B\sin(\omega t)\ldots\ldots\text{(1)}$$

Also velocity $\dot{y} = -\text{A}\omega\sin(\omega t) + B\omega\cos(\omega t)$ …..(2)

- We should determine the constants of integration ‘A’ & ‘B’.
- These constants are determined using the initial conditions i.e. when t=0, y=0. These conditions are referred to as initial conditions.

SOLUTION OF DIFFERENTIAL EQ. OF MOTION

- At t=0, $y = y_0$ and $\dot{y} = v_o$ into eqs. (1) and (2) we get

$Y_0 = A$ and $v_o = B\omega$

- Substituting back into the equation we get

$$y = y_0 cos\omega t + \frac{v_0}{\omega} \sin\omega t \text{........(3)}$$

- The above expression denotes displacement 'y' of the simple oscillator as a function the time variable.

FREQUENCY AND PERIOD

- Equation (3) is harmonic and peroidic i.e. it can be expressed by sine or cosing function of the same frequency ‘ω’.
- This perod may easily be found since the functions sine and cosine both have a period of 2π.
- The period ‘T” of the motion is determined from

$$\omega T=2\pi \text{ or } T= 2\pi /\omega$$

- The period is usually expressed in ther seconds per cycle or simply in seconds.
- Inverse of ‘T’ is called natural frequency represented by small ‘f” $f= 1/T$

FREQUENCY AND PERIOD

- Frequency is measured in hertz or cycles / second. To avoid confusion ‘ω’ is called angular frequency measure in radians / second.

AMPLITUDE

- Amplitude is the maximum displacement.
- It is measured by calculating the height of the functional wave, considering one half to be triangle as shown in the figure.

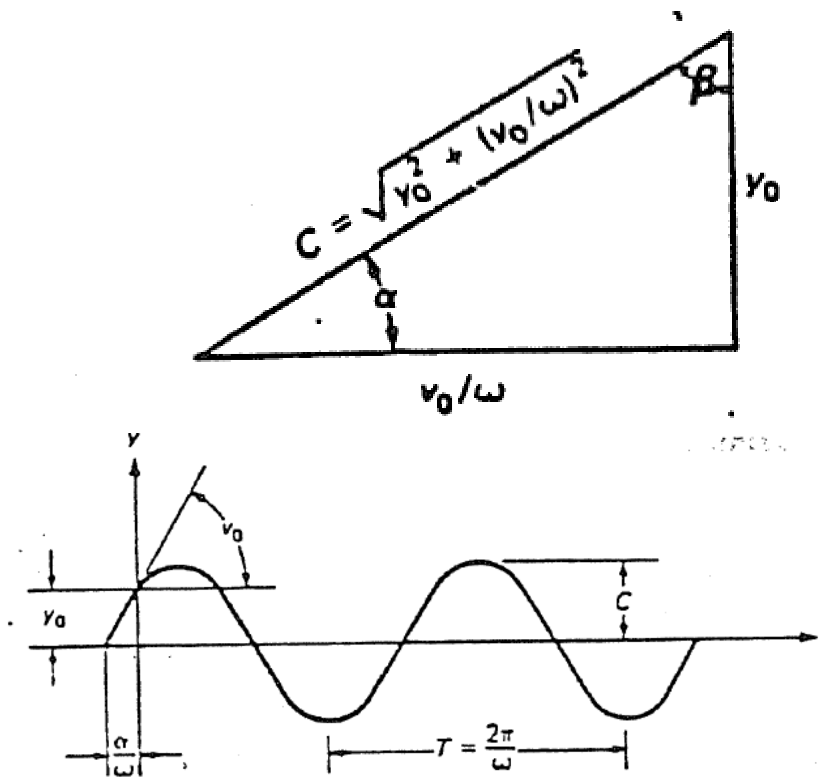
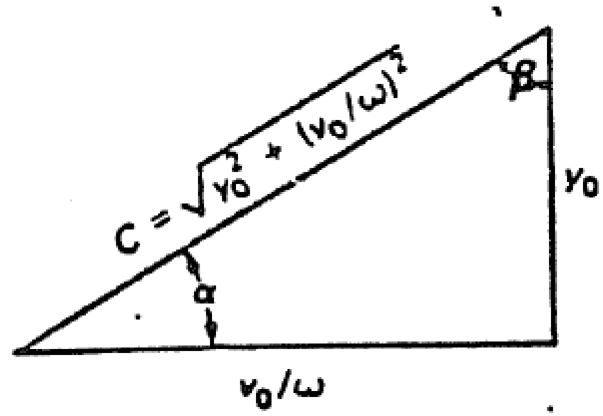


Fig. 1.10 Undamped free-vibration response.

AMPLITUDE

$$y = C \sin(\omega t + \alpha)$$



$$y = C \cos(\omega t - \beta)$$

$$C = \sqrt{y_0^2 + (v_0/\omega)^2},$$

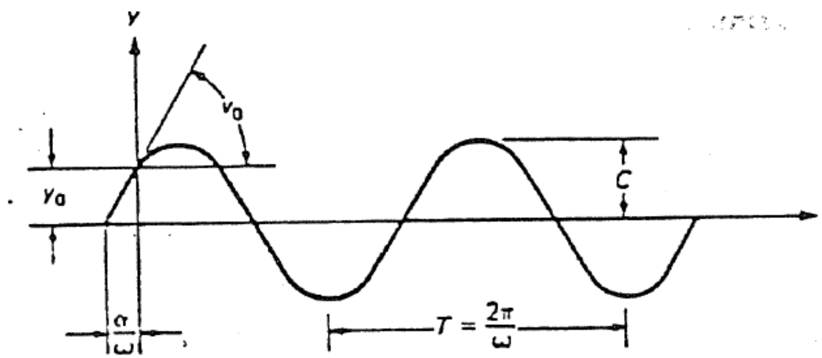


Fig. 1.10 Undamped free-vibration response.

$$\tan \alpha = \frac{y_0}{v_0/\omega},$$

$$\tan \beta = \frac{v_0/\omega}{y}.$$

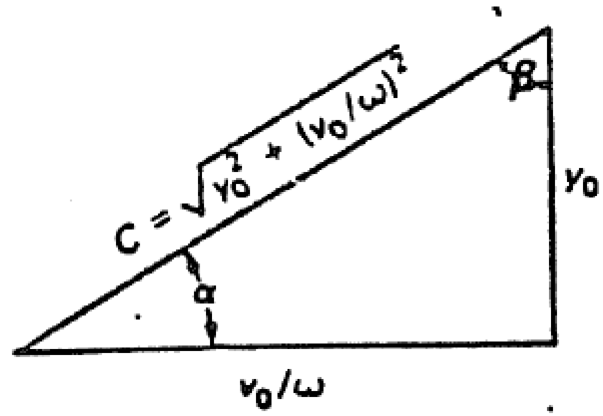
AMPLITUDE

$$y = C\left(\frac{y_0}{C}\cos\omega t + \frac{v_0/\omega}{C}\sin\omega t\right).$$

$$\sin\alpha = \frac{y_0}{C} \qquad \cos\alpha = \frac{v_0/\omega}{C}.$$

$$y = C(\sin\alpha\cos\omega t + \cos\alpha\sin\omega t).$$

- The value ‘C’ in above eq. is amplitude motion and angles (Alpha and Beta) are called phase angles.



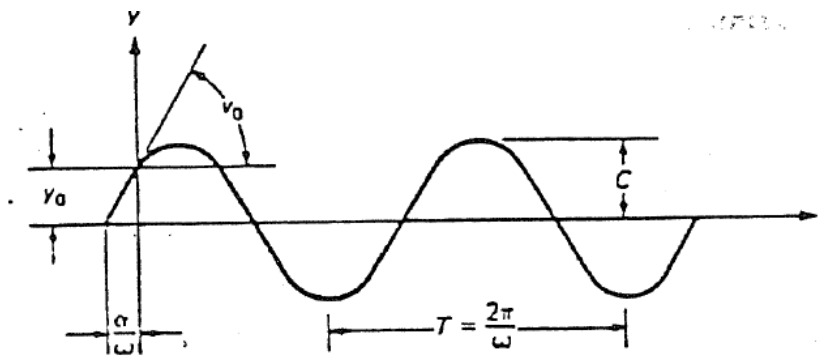


Fig. 1.10 Undamped free-vibration response.