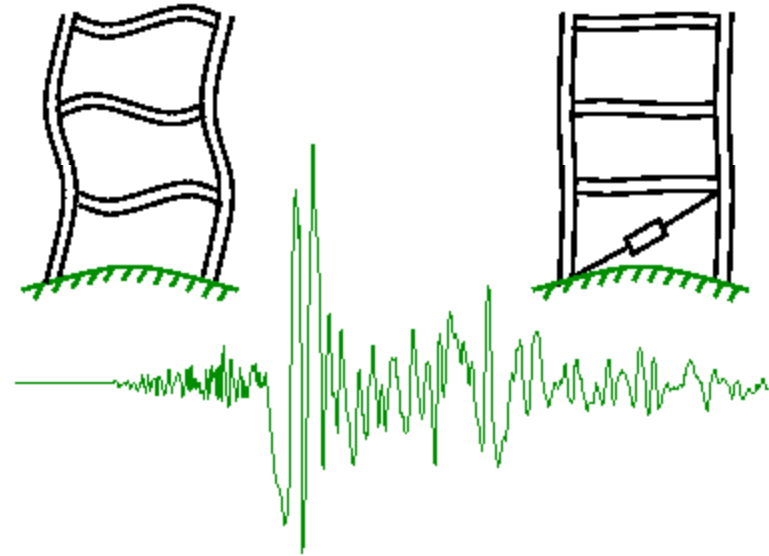


CE-412: STRUCTURAL ENGINEERING

$$\begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} = 0 \\ f_{y2} \\ f_{x3} = f_x \\ f_{y3} = f_y \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \begin{bmatrix} u_{x1} = 0 \\ u_{y1} = 0 \\ u_{x2} \\ u_{y2} = 0 \\ u_{x3} \\ u_{y3} \end{bmatrix}$$



CE-412: STRUCTURAL ENGINEERING

CHAPTER OUTLINE

Matrix methods of analysis:

Virtual force principle and flexibility method, flexibility of bar, beam and general flexural elements, analysis of 2D framed structures with temperature, support settlement and lack of fit Virtual displacement principle and displacement method, element stiffness matrix for bar, beam and plane frame element, coordinate transformation Compatibility and equilibrium Assembly of structure stiffness matrix **Analysis** by stiffness method of 2D trusses, beams and frames including temperature effects, lack of fit and settlement of supports Reliability of computer results Computer applications of above using interactive computer programs Analysis by stiffness method of 2D- Reliability of computer results Computer applications of above using interactive computer programs

CE-412: STRUCTURAL ENGINEERING

CHAPTER OUTLINE

Introduction to Structural Dynamics and Earthquake Engineering:

Vibration of SDOF lumped mass systems, free and forced vibration with and without viscous damping ,Natural vibration of SDOF systems , Response of SDOF systems: to harmonic excitation, to specific forms of excitation of ideal step, rectangular, pulse and ramp forces, Unit impulse response Vibration of MDOF systems with lumped mass Hamilton's principle, modal frequency and mode shapes ,Computer applications of above Introduction to basic terminology in EQ engineering ,Form of structures for EQ resistance Ductility demand, damping etc ,Seismic zoning of Pakistan ,Equivalent lateral force analysis ,Detailing of RC structures for EQ resistance.

Prestressed Concrete: Principles, techniques and types, tendon profiles etc Losses of prestress, Analysis of Prestressed concrete for service load, cracking load and ultimate strength Design and detailing of simply support post-and pre-tensioned beams.

Bridge Engineering: Site selection for a bridge, types and structural forms of bridges, Construction methods Vehicle load transfer to slab and stringers Design and detailing of simple RC deck and girder bridges.

Structural Dynamics and Earthquake Engineering

- **Reference Books**

1. Structural Dynamics: Theory and Computation by Mario Paz, 5th Edition
2. Dynamics of Structures: Theory and Applications to Earthquake Engineering, Anil K Chopra 4th Edition.
3. Dynamics of Structures, R. W. Clough and J. Penzien.
4. Dynamics of Structures, J. L. Humar, 2nd Edition
5. Concrete Structures Part I (Chapter 10: Lateral Loads) by Zahid Ahmad Siddiqi

STRUCTURAL DYNAMICS

- Conventional structural analysis considers the external forces or joint displacements to be **static and resisted only by the stiffness of the structure**. Therefore, the resulting displacements and forces resulting from structural analysis do not vary with time.
- Structural Dynamics is an extension of the conventional static structural analysis. **It is the study of structural analysis that considers the external loads or displacements to vary with time and the structure to respond to them by its stiffness as well as inertia and damping.**

FUNDAMENTAL OBJECTIVE OF STRUCTURAL DYNAMICS ANALYSIS

- Concepts discussed in courses related to structural engineering that you have studied till now is based on the basic assumption that the **either the load (mainly gravity) is already present or applied very slowly on the structures.**
- This **assumption work well most of the time as long no vibration/acceleration is produced due to applied forces.** However, in case of structures/ systems subjected to dynamics loads due to rotating machines, winds, suddenly applied gravity load, blasts, earthquakes, using the afore mentioned assumption provide misleading results and may result in structures/ systems with poor performance that can sometime fail.
- This **course provides fundamental knowledge about how the dynamic forces influences the structural/systems response**

FUNDAMENTAL OBJECTIVE OF STRUCTURAL DYNAMICS ANALYSIS

- The primary purpose of **STRUCTURAL DYNAMICS** is to analyze the stresses and deflections developed in any given type of structure when it is subjected to dynamic loading.
- Dynamics play an important role in many fields of structural engineering. Earthquakes, fast moving trains on bridges, traffic generated or machine induced vibrations, etc.
- Modern materials enable the fabrication of lighter, more flexible structures, where the effects of vibrations can be significantly high.
- Additionally, investment companies desire cost effective structures, which also tends the engineers towards more accurate computations, which implies dynamical analysis, too.

STRUCTURAL DYNAMICS- Loads

- There are two types of forces/loads that may act on structures, namely static and dynamic loads.

Static Loads

- those that are gradually applied and remain in place for longer duration of time.
- These loads are either not dependent on time or have less dependence on time.
- Live load acting on a structure is considered as a static load because it usually varies gradually in magnitude and position.
- Similarly moving loads may also be considered as statically applied forces.

STRUCTURAL DYNAMICS- Loads

Dynamic Loads

- are those that are very much time dependent and these either act for small interval of time or quickly change in magnitude or direction.
- Dynamic force, $F(t)$, is defined as a force that changes in magnitude, direction or sense **in much lesser time interval** or it has continuous variation with time.
- Earthquake forces, machinery vibrations and blast loadings are examples of dynamic forces.

Dynamic Response of Structures

Structural response

is the deformation behavior of a structure associated with a particular loading.

Dynamic response of Structures

- is the deformation pattern related with the application of dynamic forces. In case of dynamic load, response of the structure is also time-dependent and hence varies with time.
- Dynamic response is usually measured in terms of deformations (displacements or rotations), velocity and acceleration.

Dynamic Response of Structure

Dynamic response of a structure may be estimated in two different ways:

Deterministic Estimate of Dynamic Response:

- It is the response in which time variation of loading is fully known whether in case of prescribed oscillatory motion or in case of already recorded earthquake.
- The response to such dynamic force may be determined exactly.

Non-Deterministic Estimate of Dynamic Response:

- It is analysis for random dynamic loading to estimate the structural response
- **Random dynamic loading** is a loading in which the exact variation of force with time is not fully known but can only be approximately defined in a statistical way with some probability of occurrence.

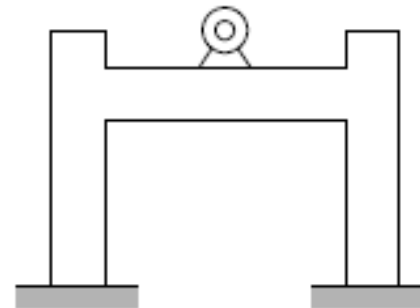
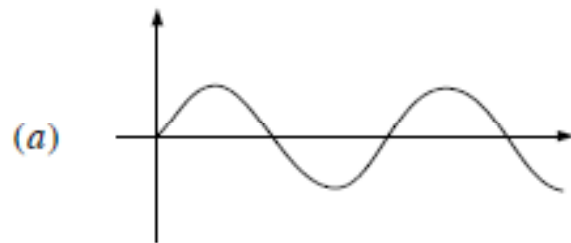
Prescribed Dynamics Loading

The prescribed dynamic loading may be

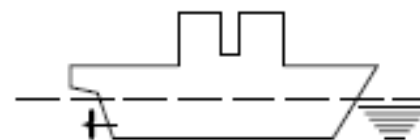
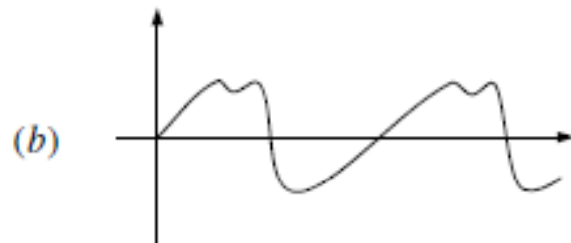
- Periodic Loading
- Non-Periodic Loading

Periodic loading is the loading that repeats itself after equal intervals of time.

Periodic



Unbalanced rotating machine in building

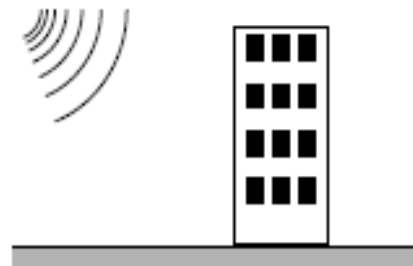
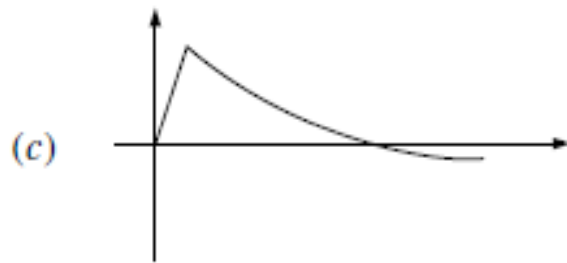


Rotating propeller at stern of ship

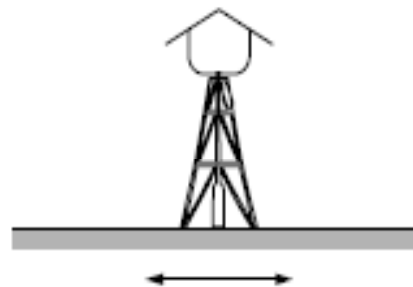
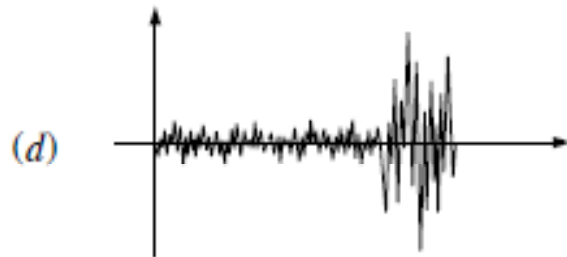
Prescribed Dynamics Loading

Non-periodic loading is not repeated in a fixed pattern and magnitude.

Nonperiodic



Bomb blast pressure on building



Earthquake on water tank

Loading histories

Typical examples

Basic Concepts of Vibration

- Any motion that **repeats itself after an interval of time is called *vibration or oscillation***. The swinging of a pendulum and the motion of a plucked string are typical examples of vibration.
- The structures designed to support heavy centrifugal machines, like motors and turbines, or reciprocating machines, like steam and gas engines and reciprocating pumps, are also subjected to vibration.
- The structure or machine component subjected to vibration can fail because of material fatigue resulting from the cyclic variation of the induced stress.
- The vibration causes more rapid wear of machine parts such as bearings and gears and also creates excessive noise.
- In machines, vibration can loosen fasteners such as nuts.

Basic Concepts of Vibration

- Whenever the natural frequency of vibration of a machine or structure coincides with the frequency of the external excitation, there occurs a phenomenon known as *resonance*, which leads to excessive deflections and failure. The literature is full of accounts of system failures brought about by resonance and excessive vibration of components and systems
- Failures of such structures as buildings, bridges, turbines, and airplane wings have been associated with the occurrence of resonance

Tacoma Narrows bridge during wind-induced vibration. The bridge opened on July 1, 1940, and collapsed on November 7, 1940.



Free and Forced Vibration

Free Vibration.

If a system, after an initial disturbance, is left to vibrate on its own, the ensuing vibration is known as free vibration. No external force acts on the system. The oscillation of a simple pendulum is an example of free vibration.

Forced Vibration.

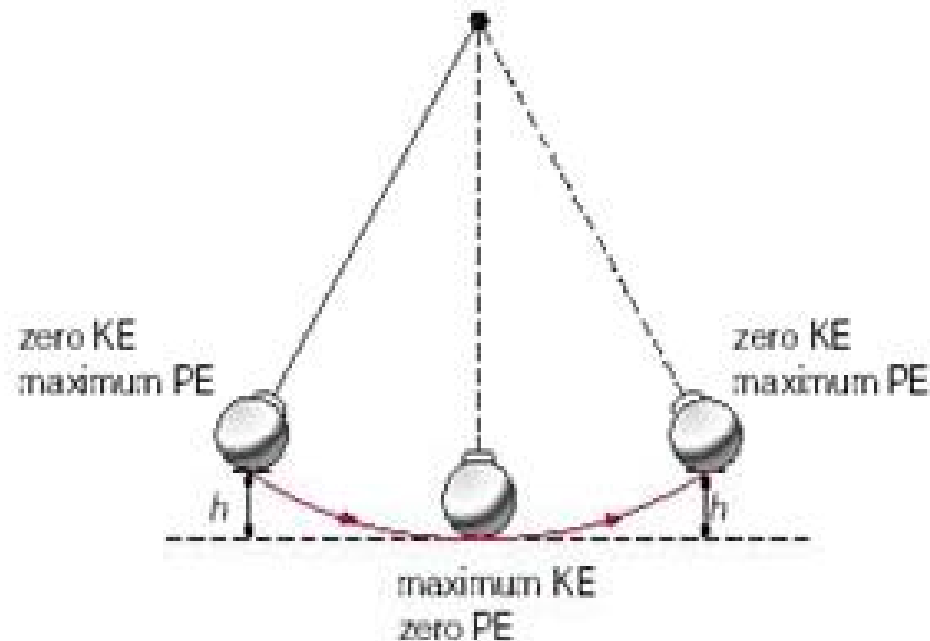
If a system is subjected to an external force (often, a dynamic force), the resulting vibration is known as forced vibration. The oscillation that arises in machines such as diesel engines is an example of forced vibration.

Undamped and Damped Vibration

- If no energy is lost or dissipated in friction or other resistance during oscillation, the vibration is known as *undamped vibration*. If any energy is lost in this way, however, it is called *damped vibration*.
- *In many physical systems, the amount of damping is so small that it can be disregarded for most engineering purposes. However, consideration of damping becomes extremely important in analyzing vibratory systems near resonance.*

Vibrating Systems

- The vibration of a system involves the transfer of its potential energy to kinetic energy and of kinetic energy to potential energy, alternately. If the system is damped, some energy is dissipated in each cycle of vibration and must be replaced by an external source if a state of steady vibration is to be maintained.



Dynamic Analysis Procedure

A dynamic system the excitations (inputs) and responses (outputs) are time dependent. The dynamic response of a system generally depends on the initial conditions as well as on the external excitations.

Most practical vibrating systems are very complex, and it is impossible to consider all the details for a mathematical analysis.

Only the most important features are considered in the analysis to predict the behavior of the system under specified input conditions.

Often the overall behavior of the system can be determined by considering even a simple model of the complex physical system.

The analysis of a dynamic system usually involves mathematical modeling, derivation of the governing equations, solution of the equations, and interpretation of the results.

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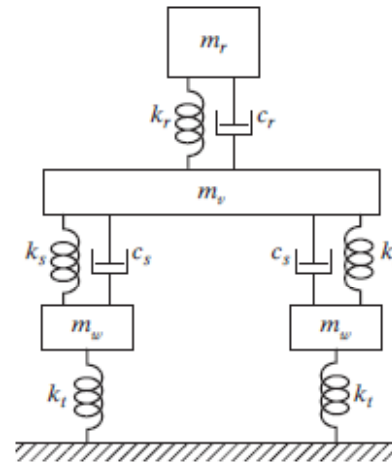
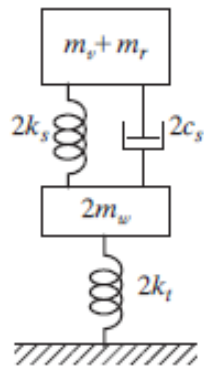
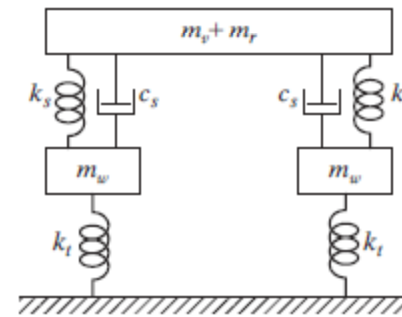
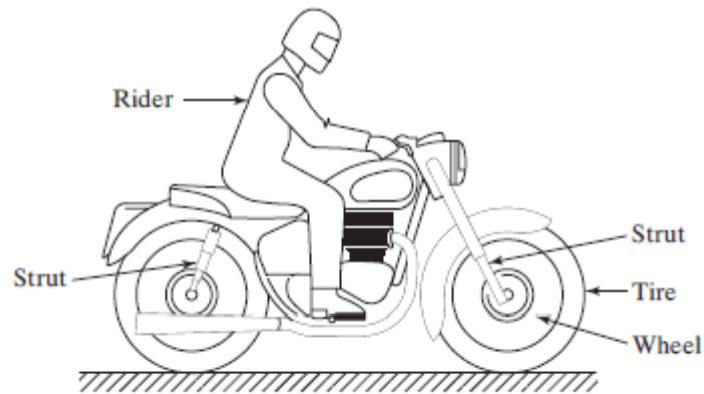
Dynamic Analysis Procedure

Step 1: Mathematical Modeling.

- The purpose of mathematical modeling is to represent all the important features of the system for the purpose of deriving the mathematical (or analytical) equations governing the system's behavior.
- The mathematical model should include enough details to allow describing the system in terms of equations without making it too complex.
- The mathematical model may be linear or nonlinear, depending on the behavior of the system's components. Linear models permit quick solutions and are simple to handle; however, nonlinear models sometimes reveal certain characteristics of the system that cannot be predicted using linear models.
- Great deal of engineering judgment is needed to come up with a suitable mathematical model of a vibrating system.

Dynamic Analysis Procedure

Step 1: Mathematical Modeling.

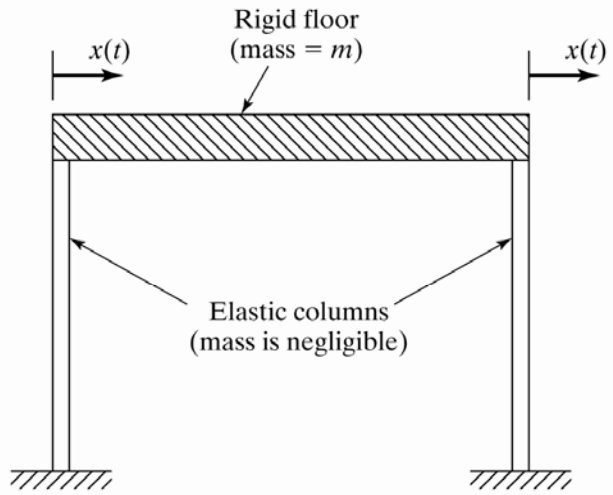


Subscripts
 t : tire v : vehicle
 w : wheel r : rider
 s : strut eq: equivalent

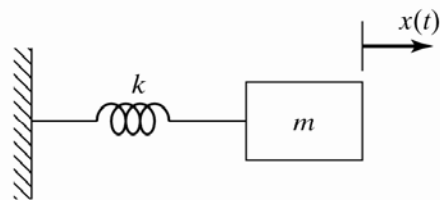
Motorcycle with a rider a physical system and mathematical models.

Dynamic Analysis Procedure

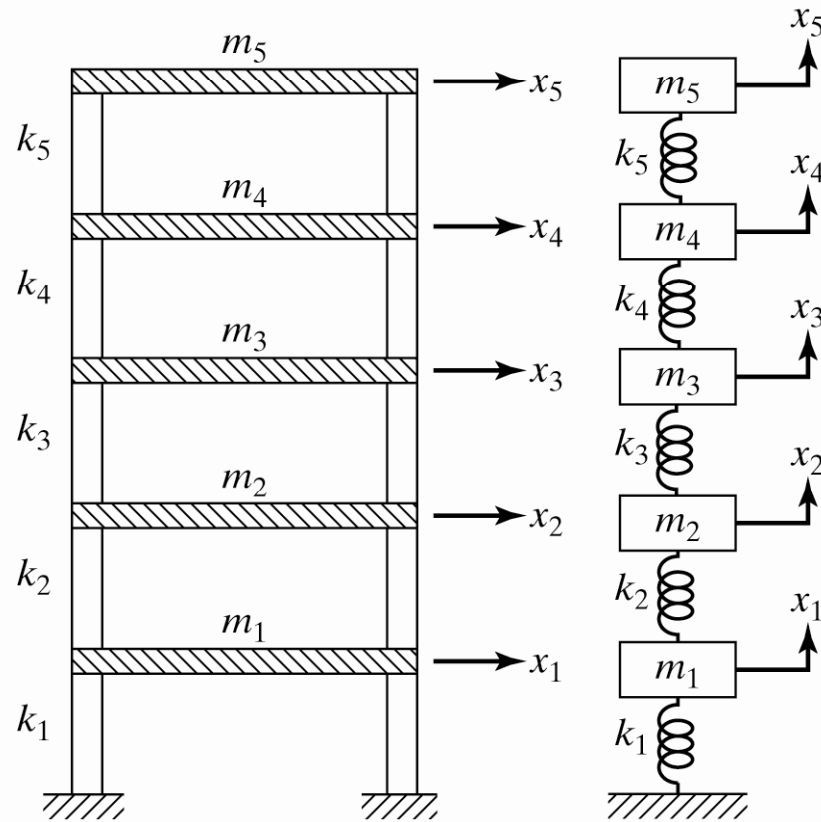
Step 1: Mathematical Modeling.



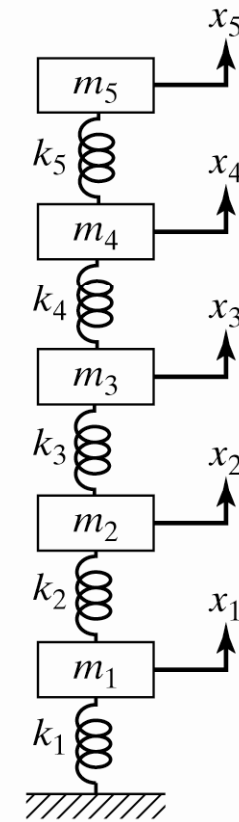
(a) Building frame



(b) Equivalent spring-mass system



(a)



(b)

Idealization of the Building Frame Multistory Building and Equivalent Spring Mass Models

Dynamic Analysis Procedure

Step 2: Derivation of Governing Equations.

- Once the mathematical model is available, we use the principles of dynamics and derive the equations that describe the dynamic response of the system.
- The equations of motion can be derived conveniently by drawing the free-body diagrams of all the masses involved. The free-body diagram of a mass can be obtained by isolating the mass and indicating all externally applied forces, the reactive forces, and the inertia forces.
- The equations of motion of a vibrating system are usually in the form of a set of ordinary differential equations for a discrete system and partial differential equations for a continuous system.
- The equations may be linear or nonlinear, depending on the behavior of the components of the system.
- Several approaches are commonly used to derive the governing equations. Among them are Newton's second law of motion, D'Alembert's principle, and the principle of conservation of energy

Dynamic Analysis Procedure

Step 3: Solution of the Governing Equations.

- The equations of motion must be solved to find the dynamic response of the system.
- Depending on the nature of the problem, we can use standard methods of solving differential equations, Laplace transform methods and numerical methods.
- If the governing equations are nonlinear, they can seldom be solved in closed form. Furthermore, the solution of partial differential equations is far more involved than that of ordinary differential equations.
- Numerical methods involving computers can be used to solve the equations.

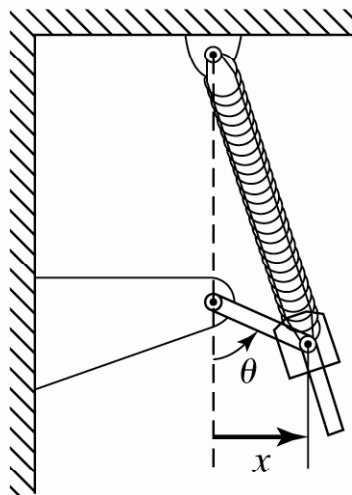
Dynamic Analysis Procedure

Step 4: Interpretation of the Results.

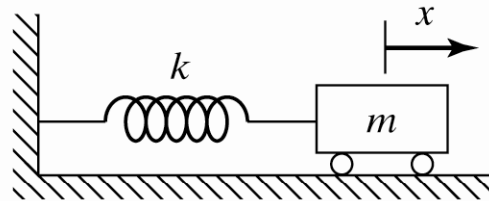
- The solution of the governing equations gives the displacements, velocities, and accelerations of the various masses of the system.
- The results must be interpreted with a clear view of the purpose of the analysis and the possible design implications of the results.

Dynamic Degrees of Freedom

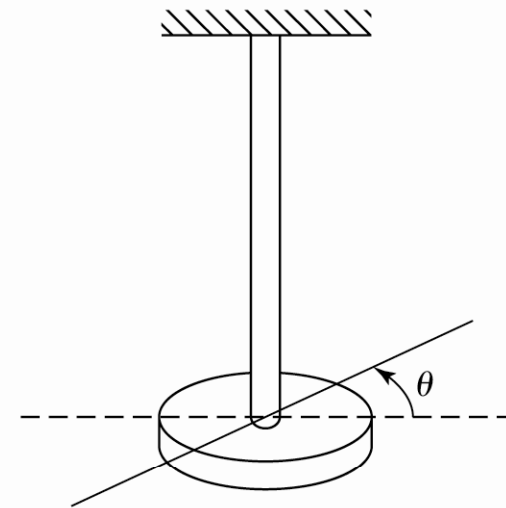
- The minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time defines the number of dynamic degrees of freedom of the system.



(a) Slider-crank-spring mechanism



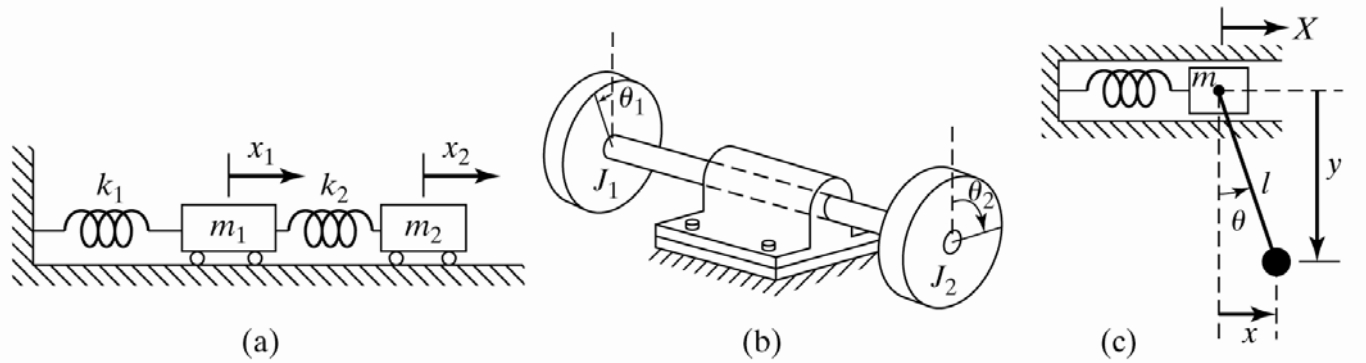
(b) Spring-mass system



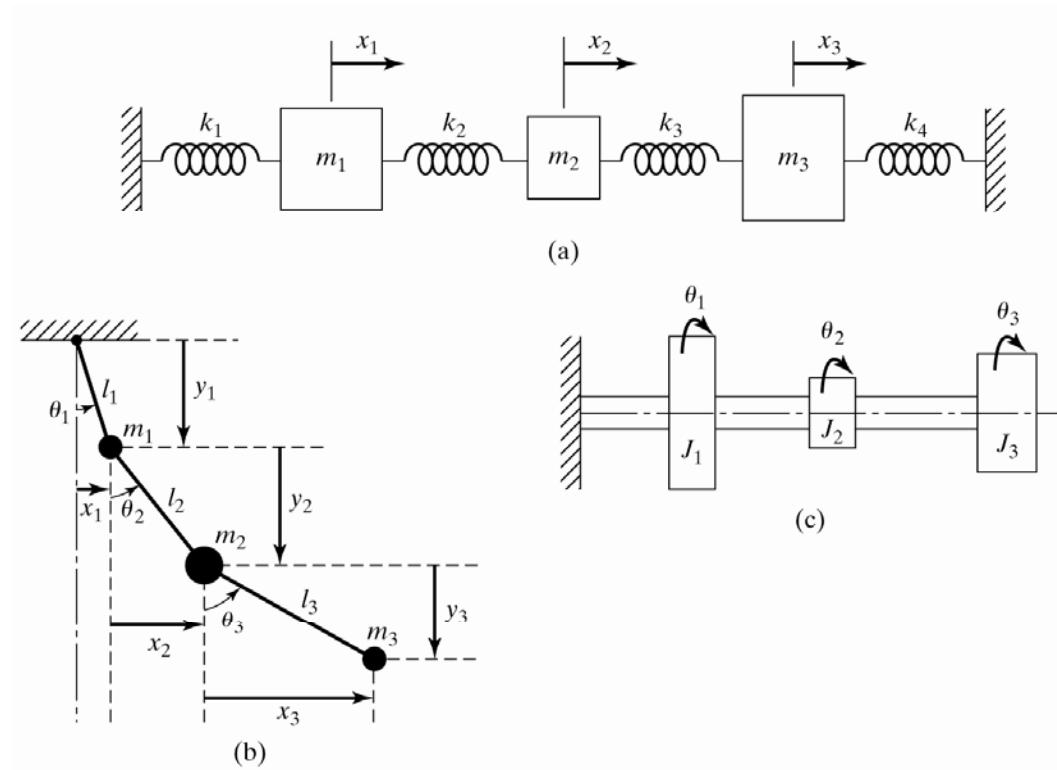
(c) Torsional system

Single Degree of Freedom Systems

Dynamic Degrees of Freedom



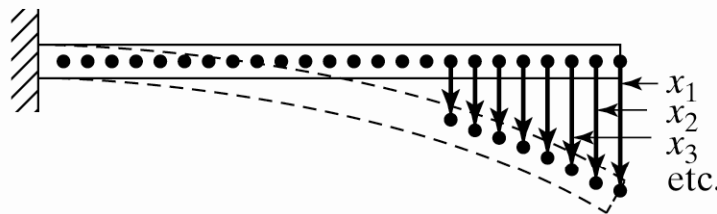
Two Degree of Freedom Systems



Three Degree of Freedom Systems

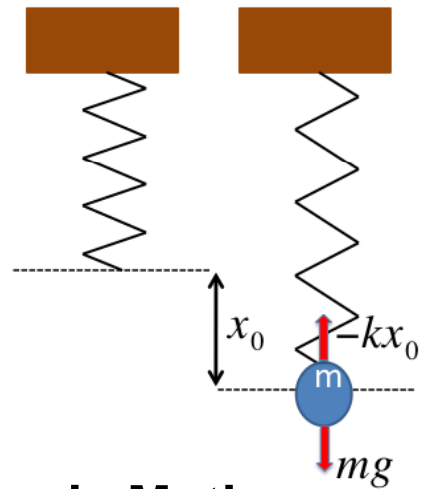
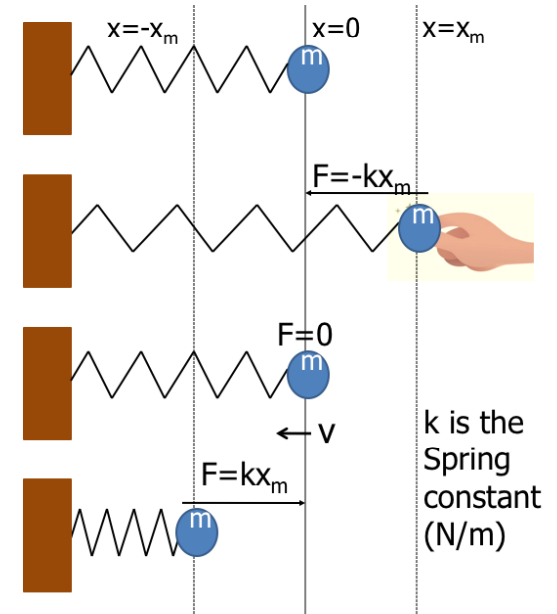
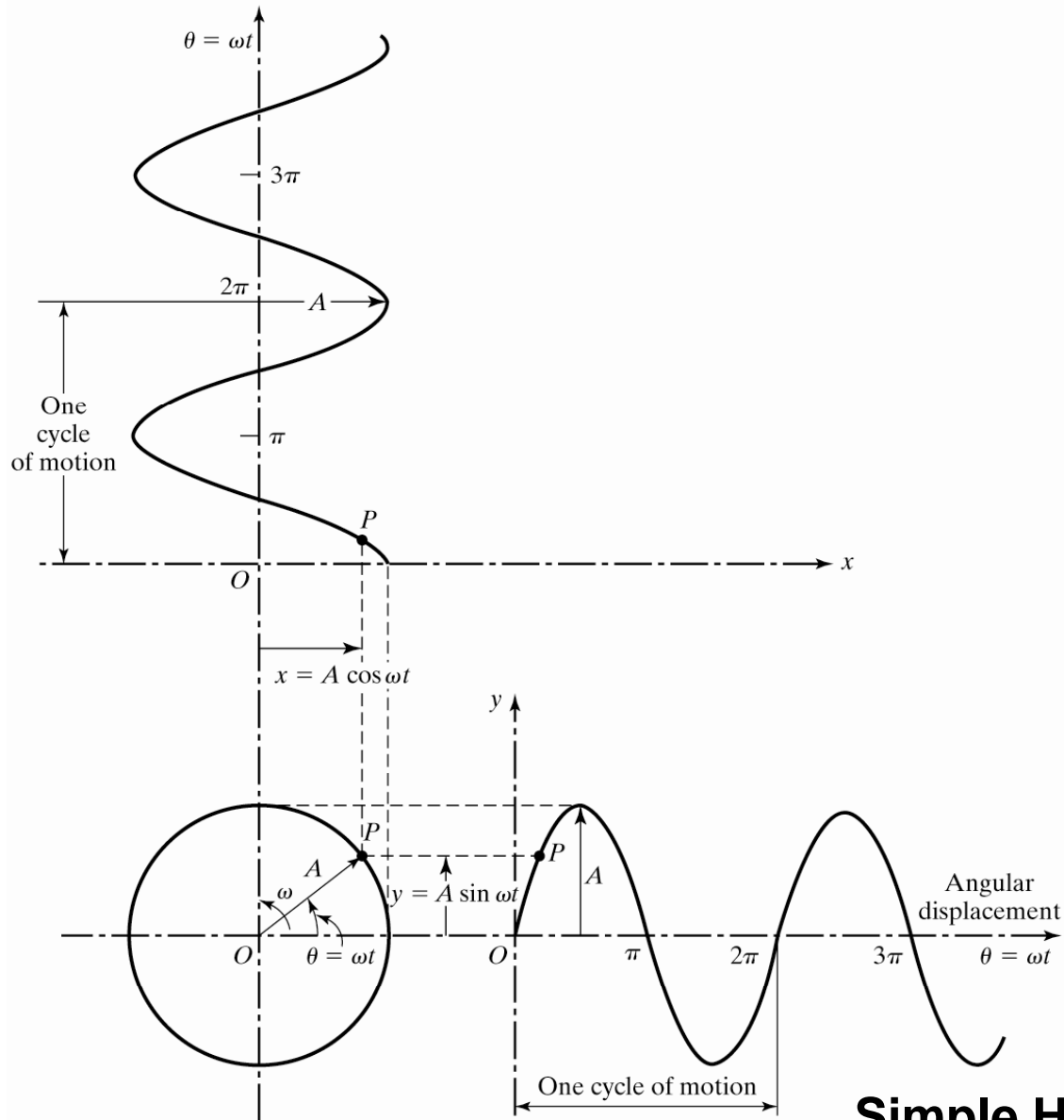
Discrete and Continuous Systems

- Systems with a finite number of degrees of freedom are called *discrete or lumped parameter systems*, and those with an infinite number of degrees of freedom are called *continuous or distributed systems*.
- Most of the time, continuous systems are approximated as discrete systems, and solutions are obtained in a simpler manner. Although treatment of a system as continuous gives exact results.
- Most of the practical systems are studied by treating them as finite lumped masses, springs, and dampers. In general, more accurate results are obtained by increasing the number of masses, springs, and dampers that is, by increasing the number of degrees of freedom.



A cantilever beam (an infinite-number-of-degrees-of-freedom system).

Definitions and Terminology



Simple Harmonic Motion

Definitions and Terminology

Cycle: The movement of a vibrating body from its undisturbed or equilibrium position to its extreme position in one direction, then to the equilibrium position, then to its extreme position in the other direction, and back to equilibrium position is called a *cycle of vibration*. One revolution (i.e., angular displacement of 2π radians) of the point P or one revolution of the vector OP constitutes a cycle.

Amplitude: The maximum displacement of a vibrating body from its equilibrium position is called the *amplitude of vibration*. In Figure the amplitude of vibration is equal to A .

Time period : The time taken to complete one cycle of motion is known as the *time period or period of oscillation* and is denoted by T . It is equal to the time required for the vector OP to rotate through an angle of 2π radians and hence

$$T = 2\pi / \omega$$

Where ω is called the circular frequency

Definitions and Terminology

Frequency of oscillation: The number of cycles per unit time is called the *frequency of oscillation* or simply the *frequency* and is denoted by f . Thus

$$f = 1/T = \omega/2\pi$$

The variable ω denotes the angular velocity of the cyclic motion; f is measured in cycles per second (hertz) while ω is measured in radians per second.

Natural frequency. If a system, after an initial disturbance, is left to vibrate on its own, the frequency with which it oscillates without external forces is known as its *natural frequency*.

A vibratory system having n degrees of freedom will have, in general, n distinct natural frequencies of vibration

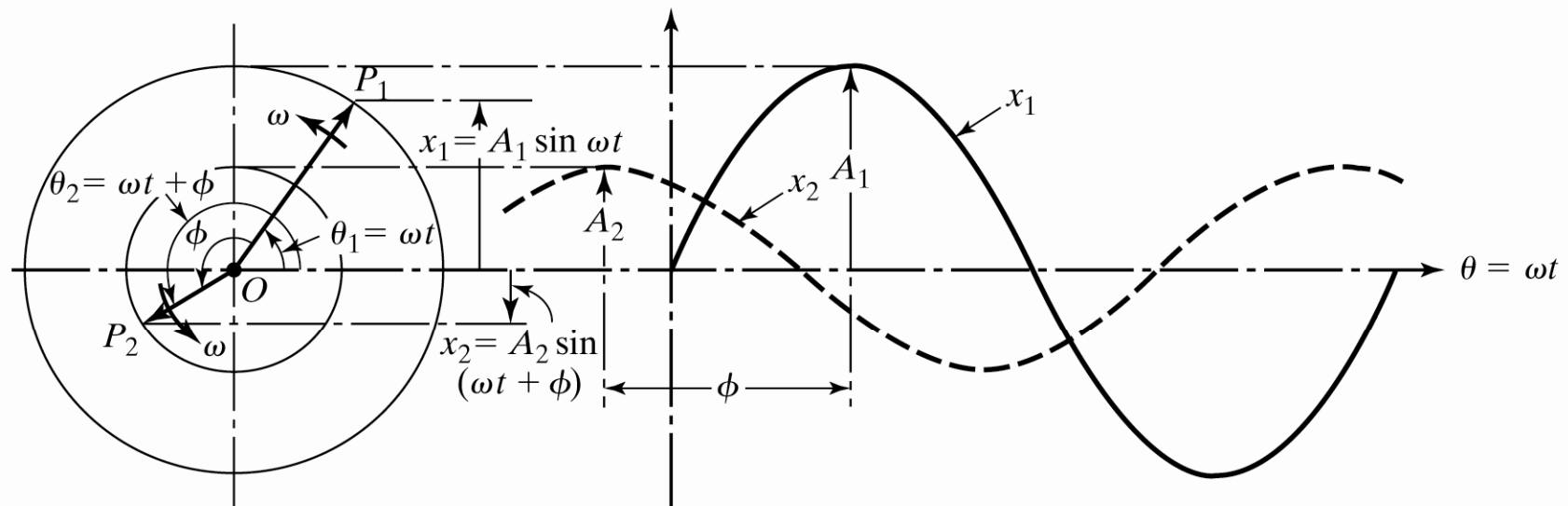
Definitions and Terminology

Phase angle: Consider two vibratory motions denoted by

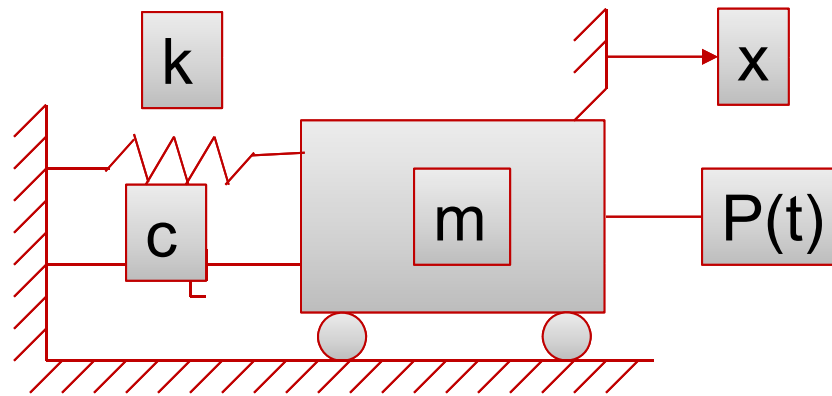
$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin(\omega t + \phi)$$

The two harmonic motions are called synchronous because they have the same frequency or angular velocity. Two synchronous oscillations need not have the same amplitude, and they need not attain their maximum values at the same time. In this figure, the second vector OP_2 leads the first one OP_1 by an angle ϕ known as the phase angle. This means that the maximum of the second vector would occur ϕ radians earlier than that of the first vector.



Mathematical model - SDOF System



- Mass element ,m - representing the mass and inertial characteristic of the structure
- Spring element ,k - representing the elastic restoring force and potential energy capacity of the structure.
- Dashpot, c - representing the frictional characteristics and energy losses of the structure
- Excitation force, P(t) - represents the external force acting on structure.

Mathematical model - SDOF System

Inertial Force: This force tries to retain the original shape or direction of motion of the structure.

$$F_i = \text{mass} \times \text{acceleration}$$

Elastic Restoring Force: is the resisting force that tries to restrict the deformation or tries to regain the original shape.

- For a particular deflected shape, this acts as potential energy.
- It acts as spring constant in the dynamic model.

$$F_e = \text{stiffness} \times \text{displacement}$$

Damping Force : Damping is the process by which free vibration steadily diminishes. This is due to release of energy from the structure, usually in the form of heat. It is produced by opening and closing of micro-cracks, friction between different components and deformations within the inelastic range, etc.

$$F_d = \text{Damping constant} \times \text{velocity}$$

Mathematical model - SDOF System

Mechanical Parameters and Components

