Structural Mechanics (CE- 312)
Shear Stress in Thin Walled Members

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Review of Shear Formula

There are two types of the stresses that act over the transversal section of a beam subjected bending

1. Bending (flexure) stresses which act parallel to the longitudinal axes and vary directly with bending moment.

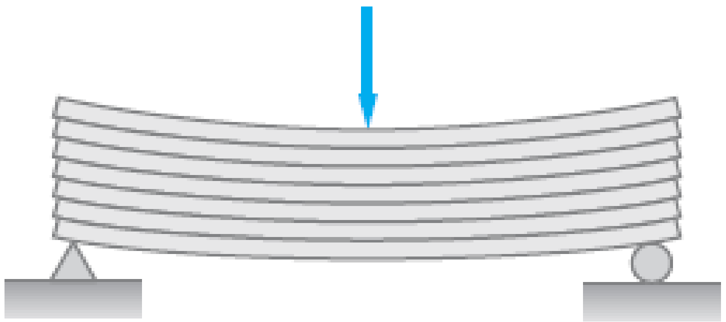
$$\sigma = \frac{M.y}{I}$$

2. Shear stresses (which act perpendicular to the longitudinal axes of the member) that vary directly with shear.

   shear force (V) is the sum of all vertical components of the external load acting on either side of section

$$\tau = \frac{VA'\overline{Y}}{Ib}$$

When a beam is subjected to transversal loading the, shear stress produced in longitudinal axes tends to slide the grains with respect to each other.

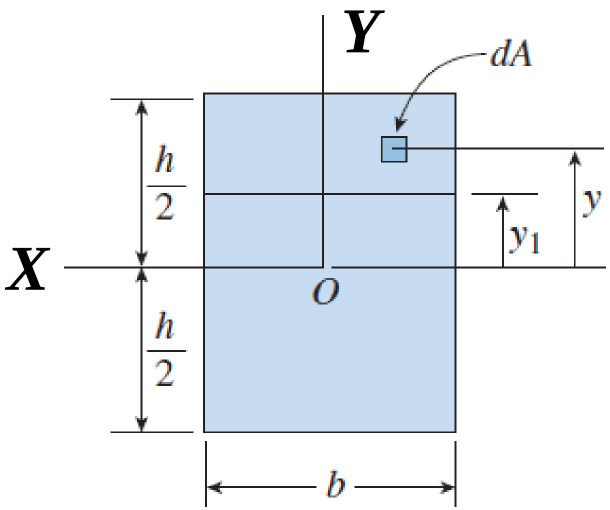


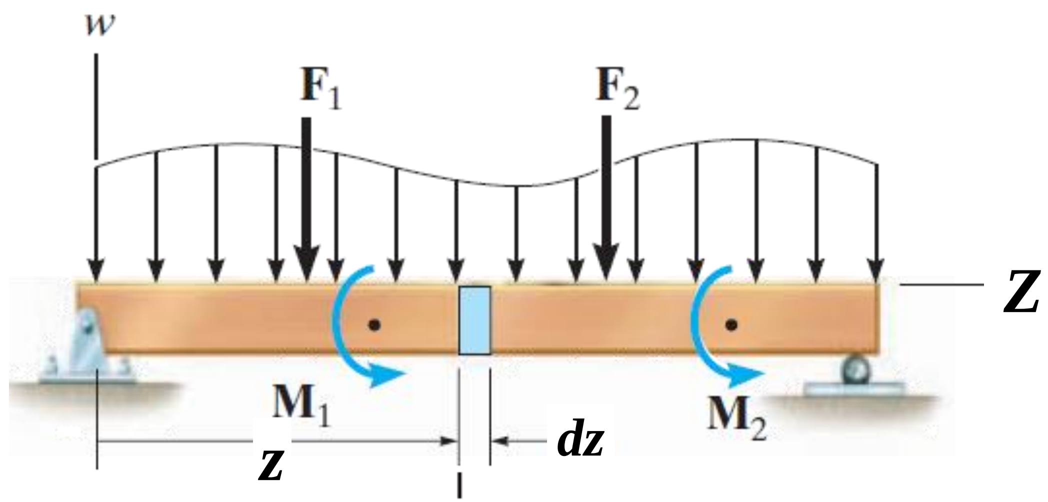
DERIVATION OF SHEAR FORMULA

Assumptions

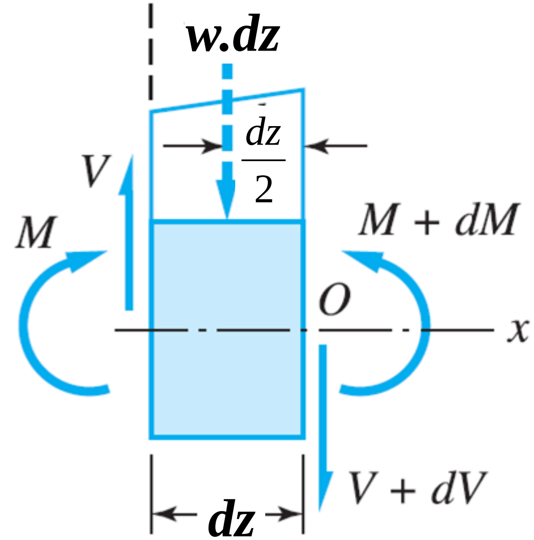
1. Material is homogeneous and all the stresses are within the elastic range (Hooks law implies).
2. Shear stresses are uniformly distributed over the entire cross-section.
3. The formula is being derived for the rectangular cross section but it may also be applied to any other cross section with a plane of symmetry to calculate the approximate value of shearing stress.

Let consider a differential segment of thickness ***dz*** along the length of a beam of rectangular cross-section (***bxh***). The beam is subjected to transversal loading as shown in figure.

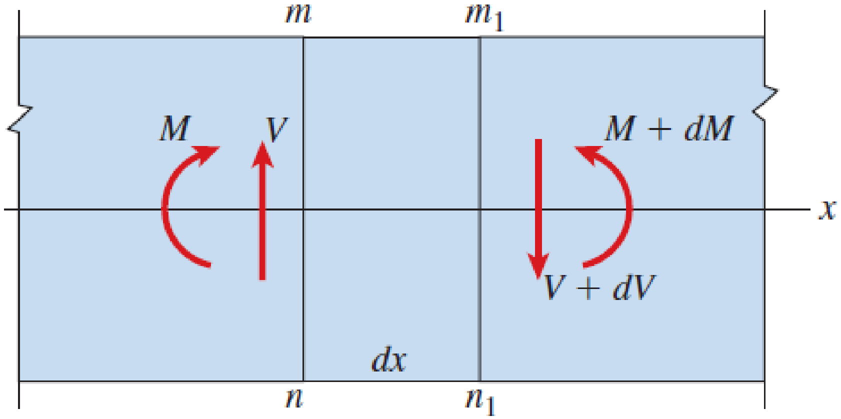


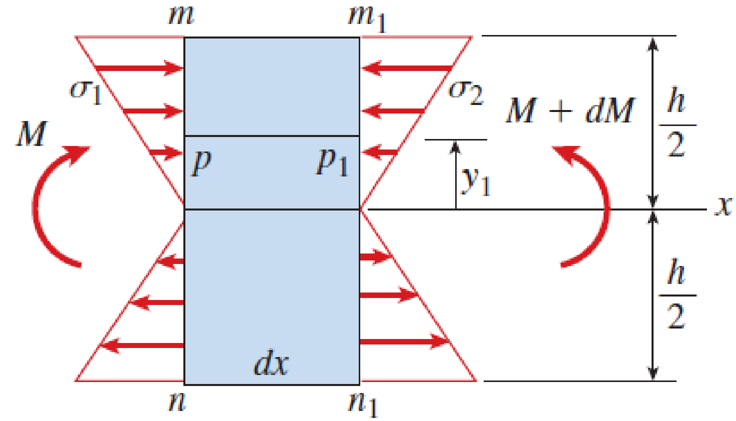


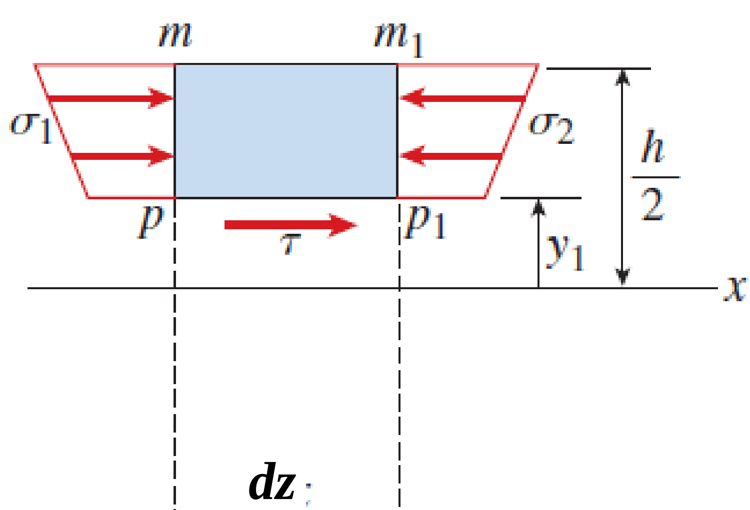
Assuming that due to applied loading. $V_1=V$, $M_1=M$ and $V_2=V+\Delta V$, $M_2=M+\Delta M$ are the resisting shear force and bending moment acting on the either side of the differential segment ***dz***.



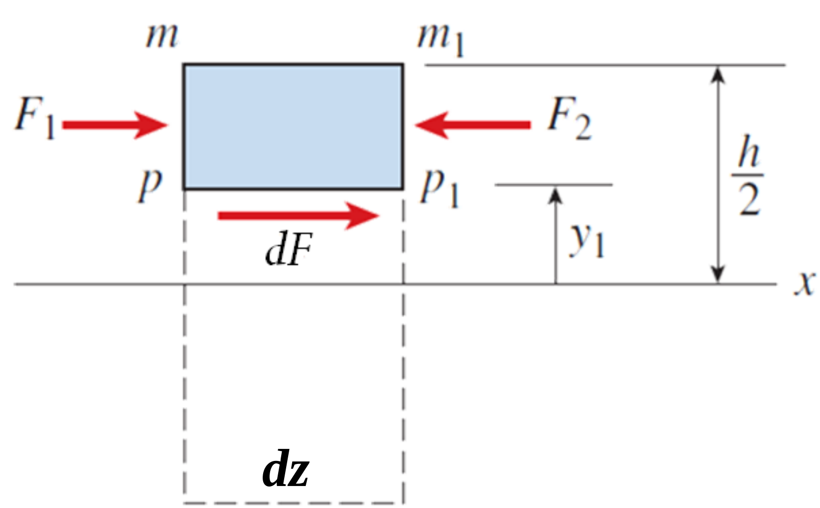
Considering $M_2 > M_1$ and thus compression forces (produced by these bending moment) $F_2 > F_1$. since no force is acting on the top or sides, dF is balancing force acting at the bottom of the any arbitrary layer (p - p_1).







Bending stress above the layer P-P_1



Force diagram

Applying $\Sigma F_z = 0$:

$$dF = F_2 - F_1$$

$$dF = \int_{y_1}^{h/2} \sigma_2 dA - \int_{y_1}^{h/2} \sigma_1 dA$$

$$dF = \frac{M_2}{I_x} \int_{y_1}^{h/2} y.dA - \frac{M_1}{I_x} \int_{y_1}^{h/2} y.dA$$

$$\tau.bdz = \frac{M_2 - M_1}{I_x} \int_{y_1}^{h/2} y.dA$$

$$dF = \frac{dM}{I_x} \int_{y_1}^{h/2} y.dA$$

$$\tau = \frac{dM}{dz.bI_x} \int_{y_1}^{h/2} y.dA$$

$$\therefore \quad \tau = \frac{dF}{bdz}$$

$$\Rightarrow \quad dF = \tau.bdz$$

$$\therefore \quad Q = \int_{y_1}^{h/2} ydA = A'\bar{Y}$$

$$\therefore \quad V = \frac{dM}{dz}$$

$$\tau = \frac{VQ}{Ib} = \frac{VA'\bar{Y}}{Ib}$$

T = the shear stress in the member at the point located a distance ***y*** from the neutral axis. This stress is assumed to be constant and therefore, *averaged across the width* ***b*** *of the member*

V = the internal resultant shear force, determined from the method of sections and the equations of equilibrium

I = the moment of inertia of the *entire cross-sectional area* calculated about the neutral axis

b = the width of the member's cross-sectional area, measured at the point where shear stress is to be determined

A' = *partial area of the top (or bottom) portion of the* **layer of member's cross-section.**

Y = moment arm of the partial area about the neutral axes.

Q = *A'.Y* = first (static) moment of partial area

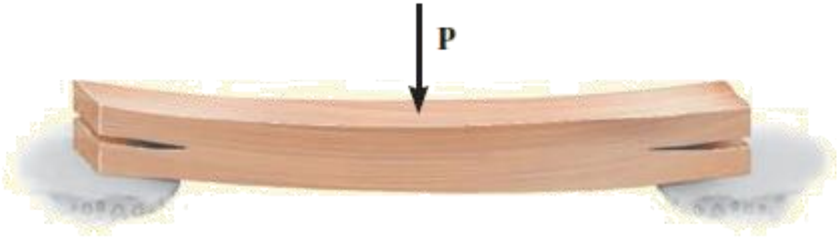
SHEAR FLOW (*q*):

If shear stress (*τ*) is multiplied by the width (***b***) a quantity q known as shear flow is obtained.

Shear Flow represents the shear (longitudinal) force per unit length transmitted across the section at any level (layer) y_1.

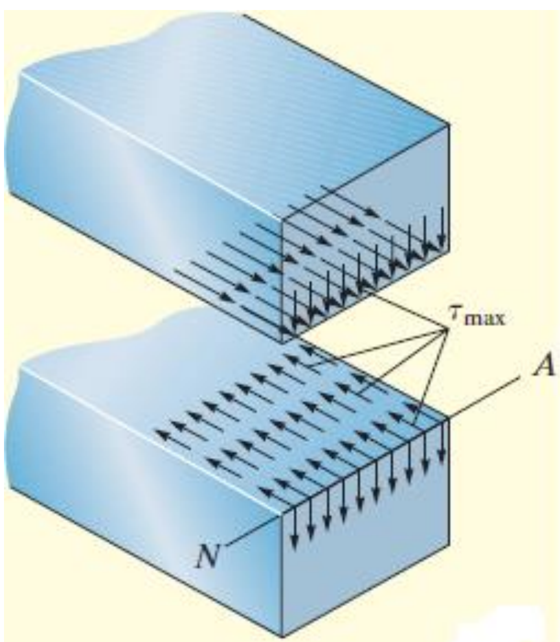
$$q = \tau . b = \frac{VQ}{Ib} . b \quad \Rightarrow \quad q = \frac{VQ}{I} = \frac{VA'\bar{Y}}{I}$$

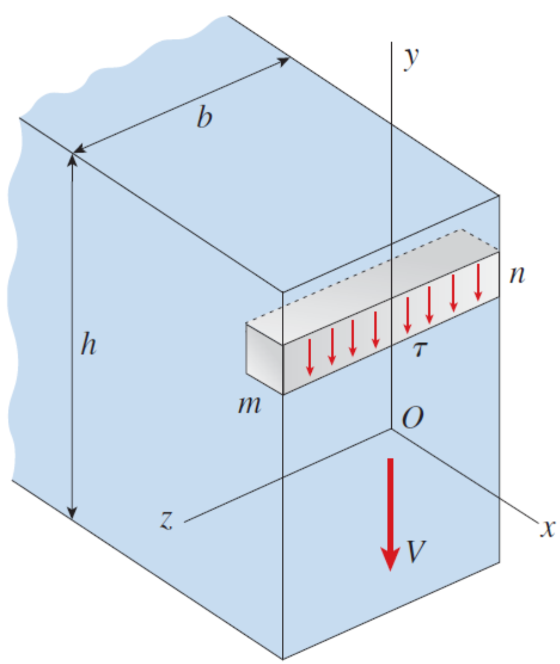
- Shear stress does not occurs in a beam subjected to pure bending
- Sliding or shearing of the fibers does not occur in homogeneous material. However, for wood which is weak in shear, rupture cracks appear at the ends along the neutral plane

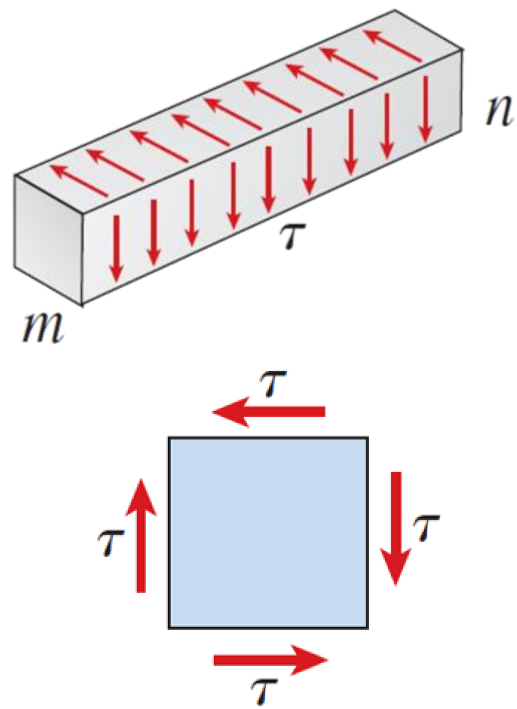


Relationship Between Horizontal and Vertical shear Stresses

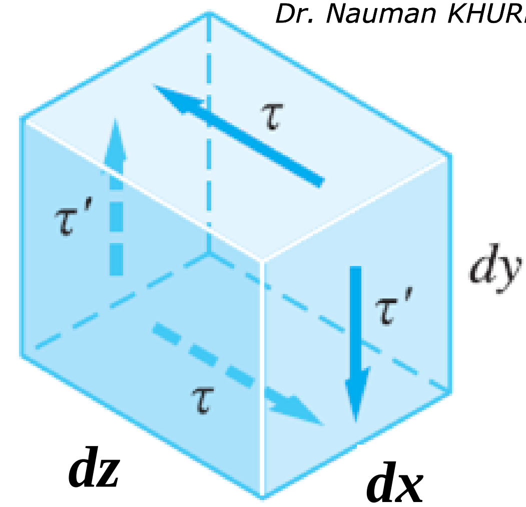
The horizontal shear stress at any point is always accompanied by an equal vertical shear stress and are termed as ***Complementary Stress.***





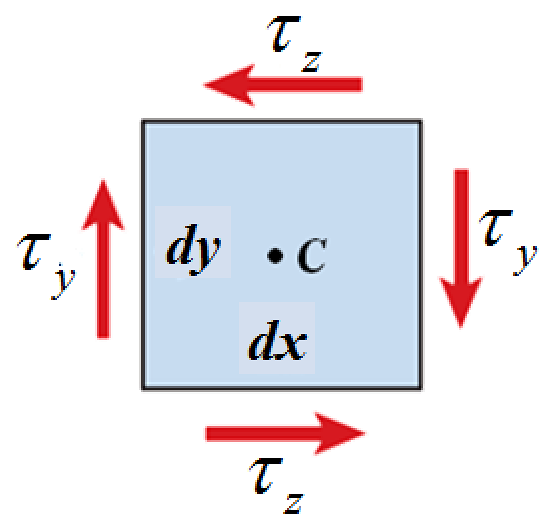


Applying $(\Sigma M)_c = 0$:



$$\tau_z.(dx.dz)dy = \tau_y.(dy.dx)dz$$

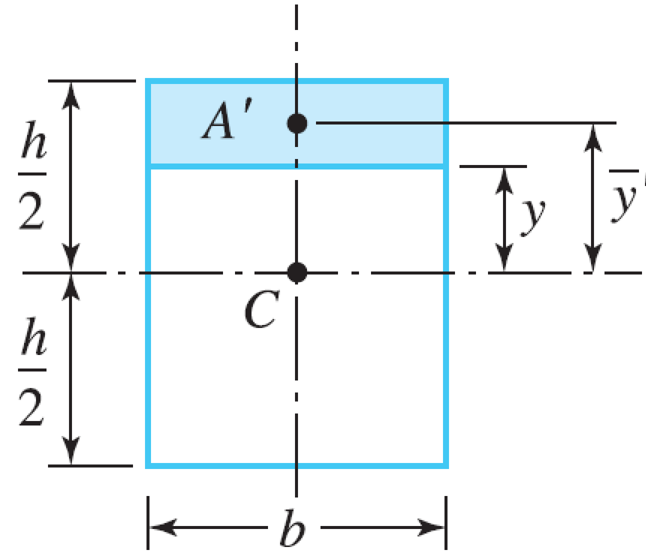
$$\Rightarrow \quad \tau_x = \tau_y$$



- It is concluded that when a beam is subjected to transversal loading, both horizontal and vertical shearing stresses, numerically equal in magnitude arise in the beam
- The vertical shearing stresses are of such magnitude that their resultant at any cross-section is exactly equal to the shearing force ***V*** at that same section

Horizontal Shear Stresses Distribution over Cross-Section:

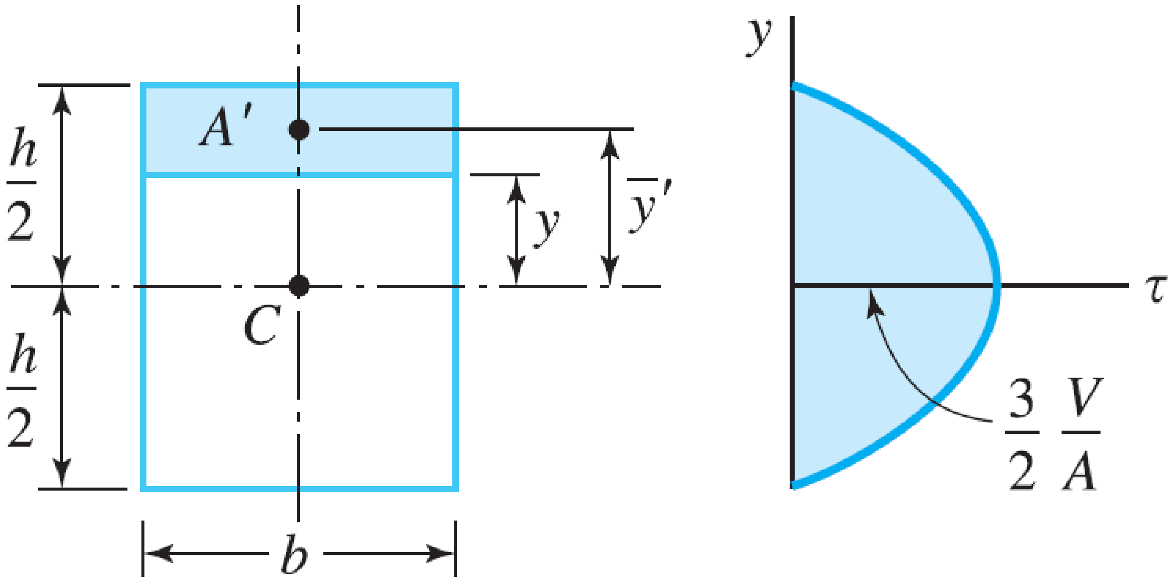
Let assume a beam of rectangular section ***bxh***, the shear stress at a layer of distance ***y*** from the **N.A.** due to shear force ,***V*** at that section is given as following



$$\tau = \frac{V}{Ib}A'\bar{Y} = \frac{V}{Ib}\left[b\left(\frac{h}{2}-y\right)\right]\left[y+\frac{1}{2}\left(\frac{h}{2}-y\right)\right] = \frac{V}{2I}\left[\frac{h^2}{4}-y^2\right]$$

- The equation shows that shear stress is parabolically distributed across the depth of the section.
- The shear stress are maximum at N.A, where y is equal to zero.

$$\tau = \frac{V}{2I}\left[\frac{h^2}{4}-0\right] = \frac{Vh^2}{2(bh^3/12)} = \frac{3}{2}\frac{V}{A}$$

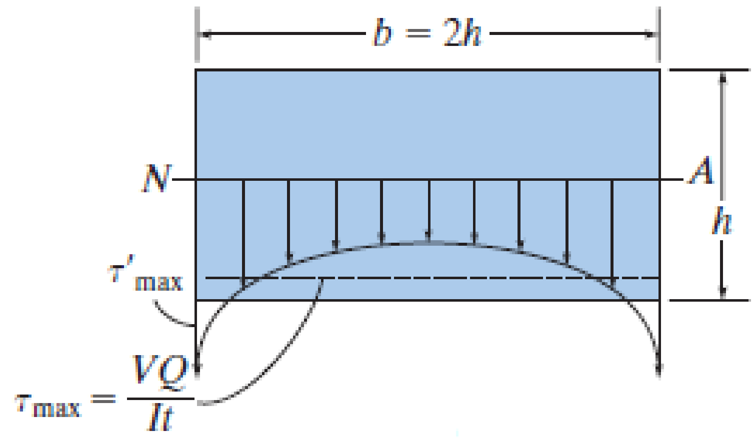


Typical Parabolic Horizontal Distribution Profile

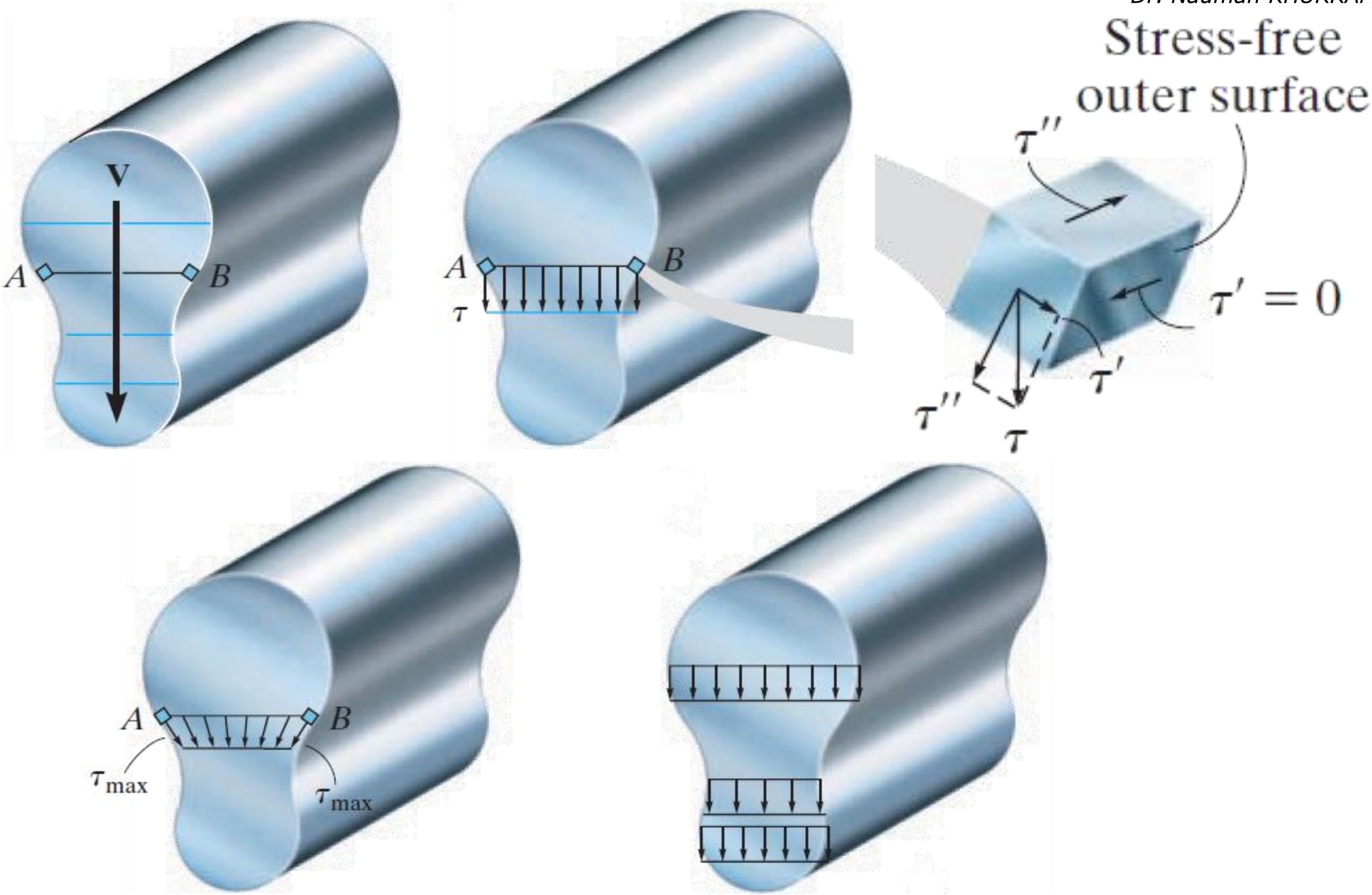
Failure at Neutral Surface due to Maximum Horizontal Stress

LIMITATION OF SHEAR FORMULA

- The shear equation shows that (**τ_{max}**) Maximum Shear Stress are 50% more than the applied shear (***V/A***), which is due to the wrongly assuming the uniform stress distribution along the width of section.



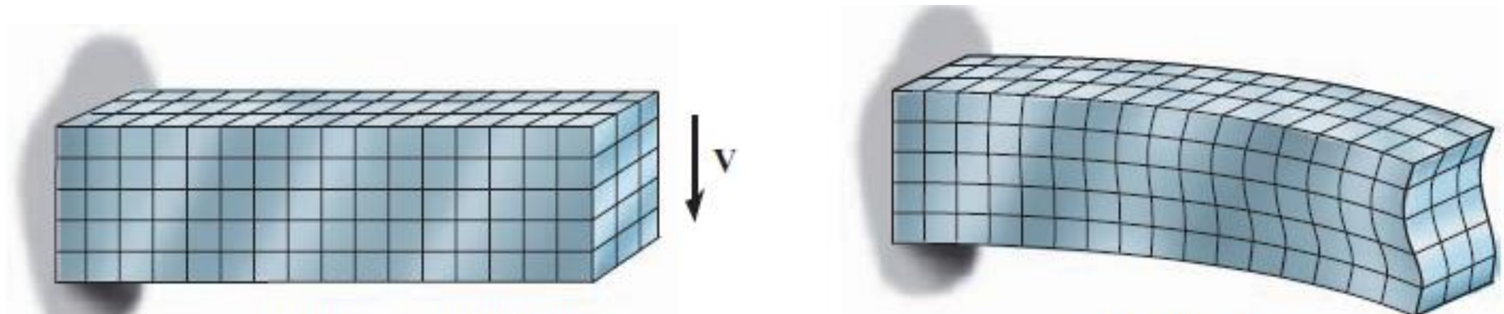
- The shear formula does not give accurate results when applied to members having cross sections that are short or flat, or at points where the cross section suddenly changes.
- This difference of the stress value is negligible (*i.e.*, 0.8%) if ***b < h/4***. For the flatter section this difference is even very large at the end.
- Shear formula also should not be applied across a section that intersects the boundary of the member at an angle other than 90°.



However, equation can be used to calculate the $(\tau_{max})_{avg}$ average value of shear stress at the neutral axes and layers which are perpendicular to the width.

Important points

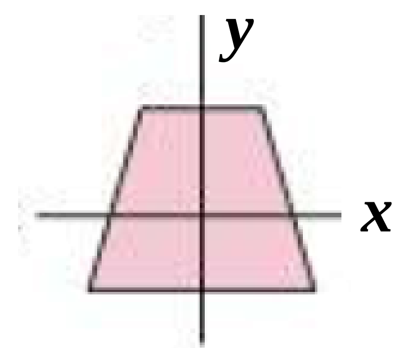
- Shear forces in beams cause *nonlinear shear-strain distributions over the cross section, causing it to warp.*



(a) Before deformation (b) After deformation

- Due to the complementary property of shear stress, the shear stress developed in a beam acts over the cross section of the beam and along its longitudinal planes.

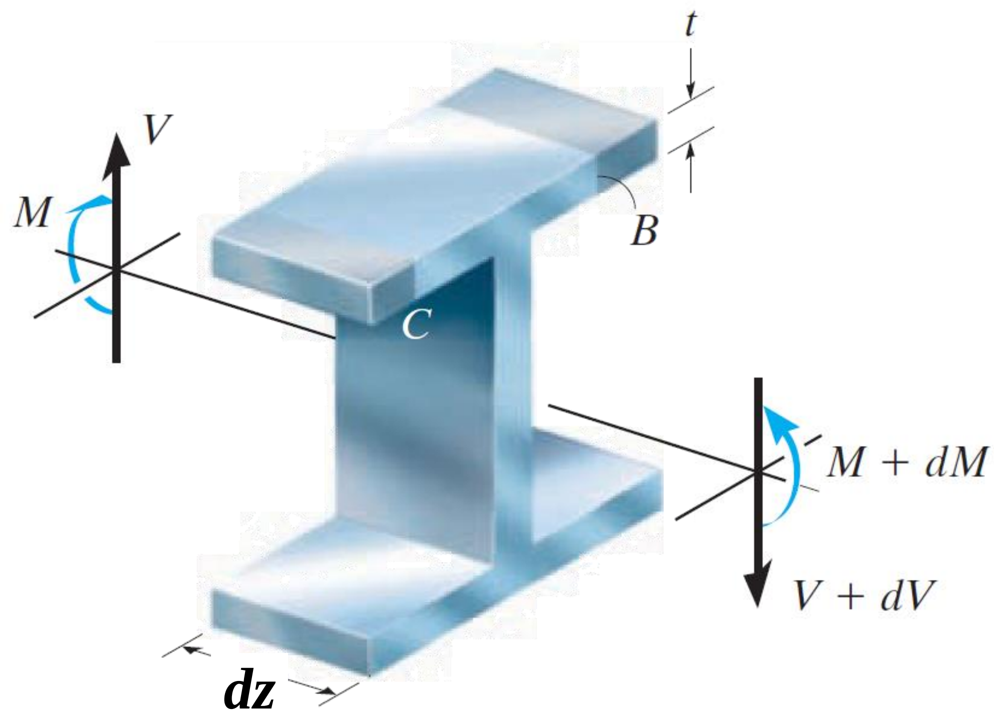
- Static moment of Inertia (***Q***) is maximum at N.A (***y_i = 0***) but ***τ_{avg}*** may not maximum as it also depend on the thickness

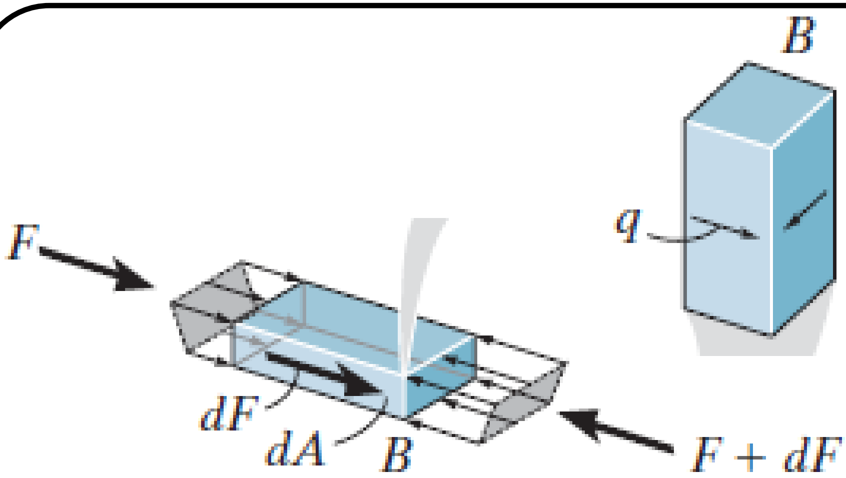


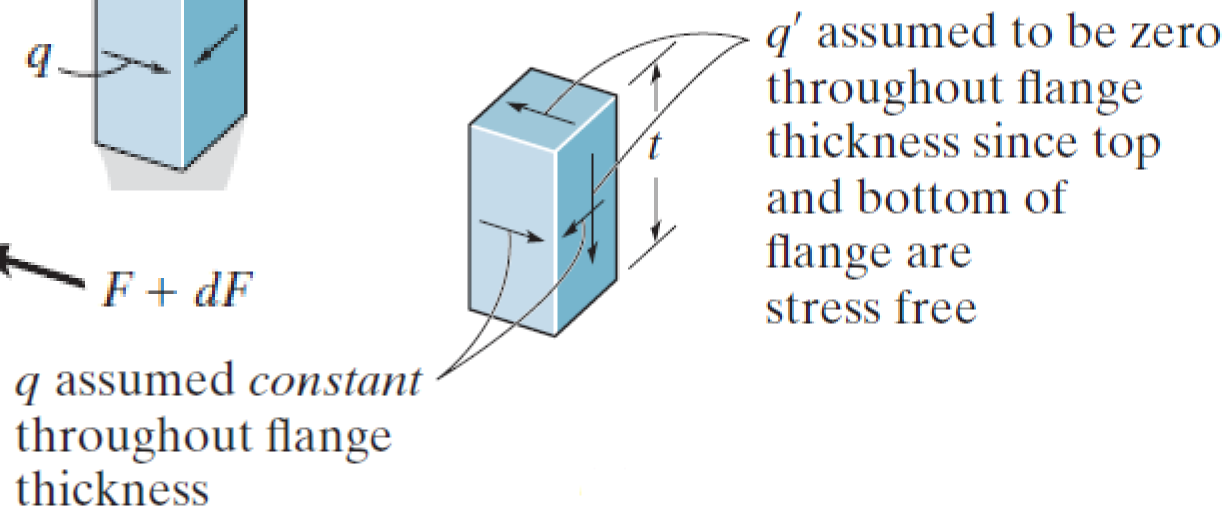
Shear Stresses in I-section Beam

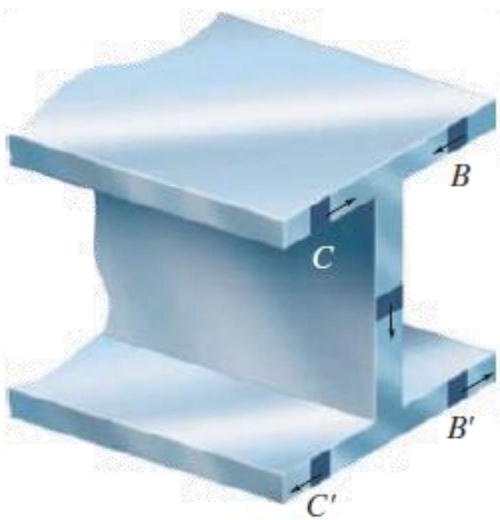
Let consider a differential segment (***dz***) of cross-section has two axis of symmetry. Flanges and web are assumed of uniform thickness. A force ***dF*** must act on the longitudinal section in order to balance the normal forces $F_1 = F$ and $F_2 = F + dF$ created by the moments $M_1 = M$ and $M_2 = M + dM$ respectively.

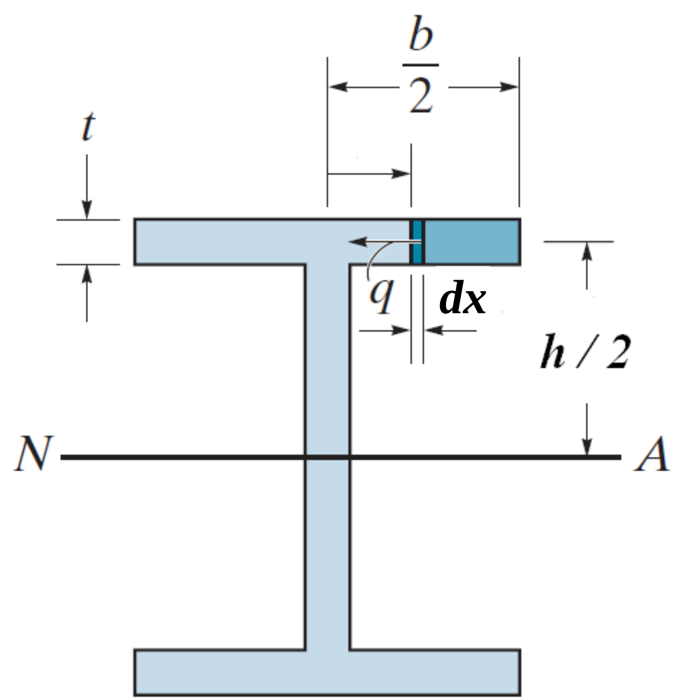
If corner elements **B** and **C** of each segment are removed the Transversal components of stress , ***T*** (or Shear Flow, ***q***) act on the cross section at cutting plane.











Considering a small portion of flange of width ***dx*** the magnitude of longitudinal shear in flange may be computed by shear formula

$$\tau = \frac{V}{Ib}A'\bar{y} = \frac{V}{Ib}\int y\,dA$$

$$\tau = \frac{V}{It}\int_0^{b/2}\frac{h}{2}(t\,dx) = \frac{Vh}{2I}\int_0^{b/2}dx$$

$$\tau = \frac{Vh}{2I}\left|X\right|_0^{b/2}$$

Eqn. shows that shear stress varies linearly from the free end

At z = 0, $\tau = 0$

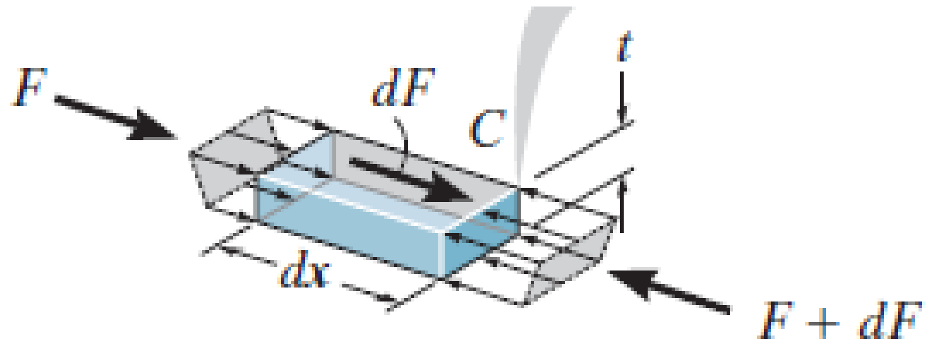
At z = b/2,

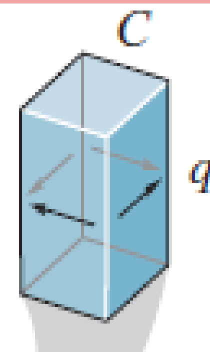
$$\tau = \frac{Vh}{2I}\left(\frac{b}{2}\right) = \frac{Vbh}{4I}$$

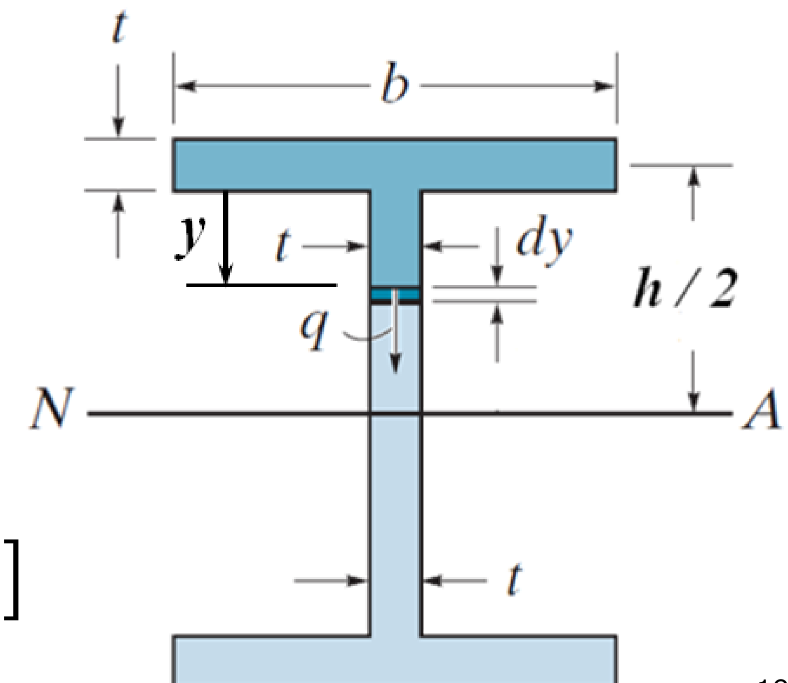
$$q = \tau \times t = \frac{Vbht}{4I}$$

Since cross section is symmetrical about the y-axis the shear stress in adjacent flange also increase linearly from zero at free end edge.

Considering an small portion of width ***dy*** through the web at a distance ***y*** from the junction of flanges and web. In the evaluation of ***Q*** (static moment of area) total area must be considered above the cut off point.







$$A'\bar{y} = (bt)\frac{h}{2} + (ty)\left(\frac{h}{2} - \frac{y}{2}\right)$$

$$A'\bar{y} = \frac{t}{2}\left[bh + y(h-y)\right]$$

$$\tau = \frac{V}{It}A'\bar{y} = \frac{V}{2I}\left[bh + y(h-y)\right]$$

At $y = 0$,

$$\tau = \frac{V}{2I}\left[bh + 0\right] = \frac{Vbh}{2I}$$

$$\Rightarrow \qquad q = \tau \times t = \frac{Vbht}{2I}$$

At $y = b/2$,

$$\tau = \frac{V}{2I}\left[bh + \frac{h}{2}(h - \frac{h}{2})\right] = \frac{V}{2I}\left[bh + \frac{h}{4}\right]$$

$$\tau = \frac{Vbh}{2I}\left[1 + \frac{h}{4b}\right]$$

$$\Rightarrow \qquad q = \tau \times t = \frac{Vbht}{2I}\left[1 + \frac{h}{4b}\right]$$

- Note that shear stress, ***τ*** (or shear flow ***q***) varies parabolically throughout the depth of the web, attaining the maximum value at the neutral axes.
- In the flanges shear stress is parallel to ***z***-axis and contribute nothing (negligibly) to the total force on the section parallel to ***y***-axes.
- At the junction of the web and flanges, shear stress in the web is twice the shear stress in the flanges.

Shear stress in flanges $\tau_f = \frac{Vbh}{2I}$

Shear stress in web $\tau_w = \frac{Vbh}{4I}$

Consider a unit length of beam along the line of junction.

For longitudinal equilibrium

$$2\left[\tau_f \times (t \times 1)\right] = \tau_w \times (t \times 1) \quad \Rightarrow \quad \tau_w = 2\tau_f$$

If flanges and web are of different thicknesses ***t_f*** and ***t_w***, respectively then equilibrium condition at junction will be

$$2\left[\tau_f \times (t_f \times 1)\right] = \tau_w \times (t_w \times 1)$$

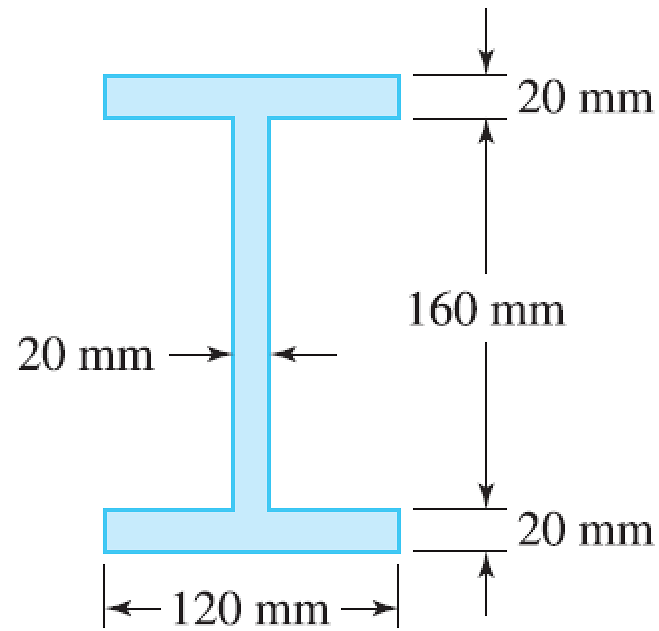
$$\frac{\tau_w}{\tau_f} = \frac{2t_w}{t_f}$$

It means, for the flanges and web of an I-section the sum of shearing force per unit length for the components meeting at the junction is zero.

$$\Sigma \tau_t = 0$$

Where, τ is the shear stress in any junction and t is the thickness of the element.

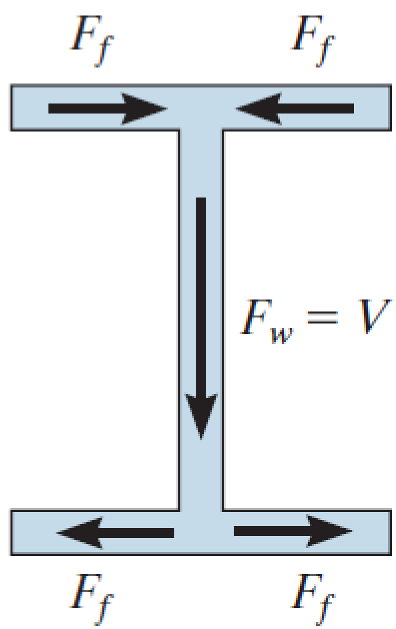
Prob 5.62 (Mech. of Material by Andrew Pytel)
The vertical shear force acting on the I-section shown is 100 kN. Compute
(a) the maximum shear stress acting on the section;
(b) the percentage of the shear force carried by the web.

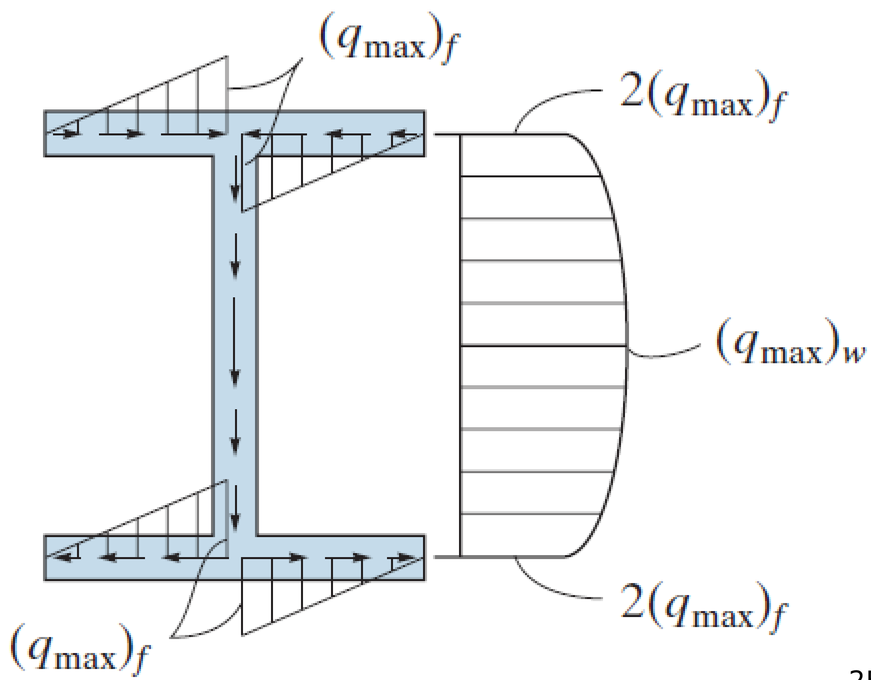


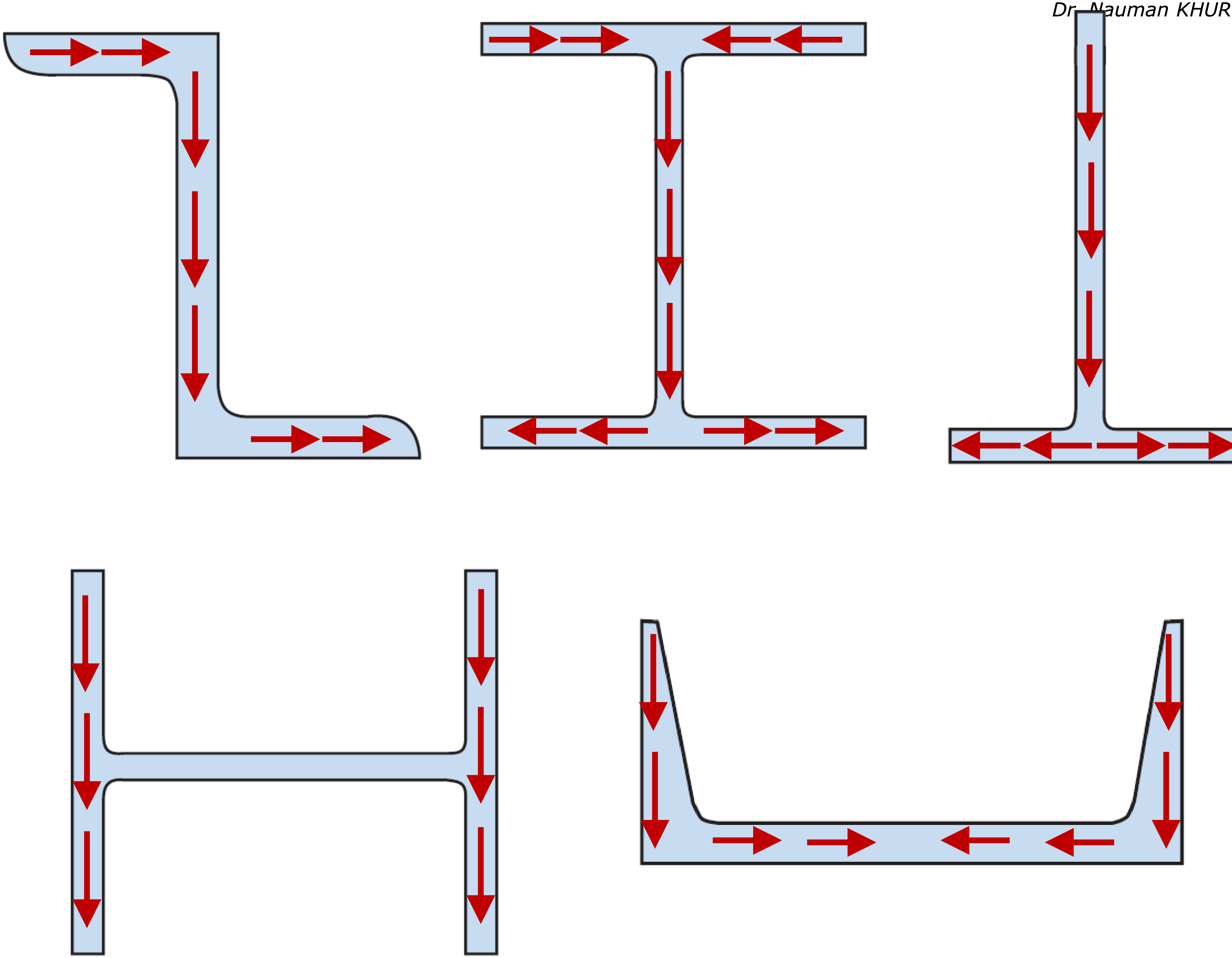
CHARACTERIOSTICS OF SHEAR FLOW

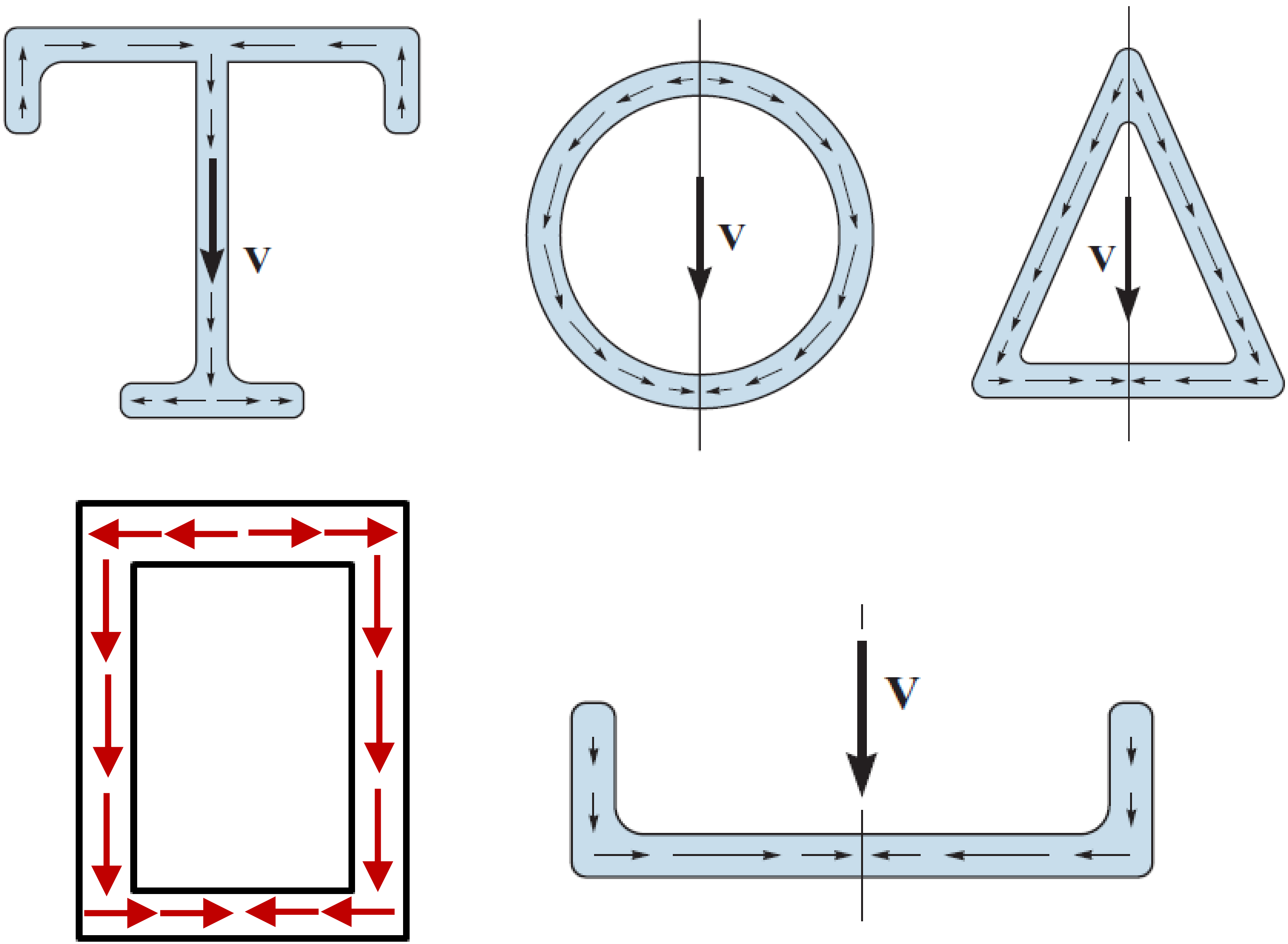
1. Shear flow in the part of the element parallel to the resisting shear (***V***) is always in the direction to the resisting shear force (***V***) at any section (or opposite to the applied shear, ***P***).
2. Shear flow occurs in one direction through the thin wall of the open section
3. At the junction of elements, incoming flow is equal to the outgoing flow.
4. The value of shear flow is zero at the free tips of the element and more shear flow is generated as more area added moving towards the Neutral Axes
5. Shear flow is assumed to be generated from the one side of N.A and I assumed to be absorbed at other end.
6. The amount of shear flow (***q***) is proportional to the “First Moment (***Q =Ay***) of all the areas added up to the point under consideration.

8. For the straight element perpendicular to the load, shear flow (***q***) varies linearly as ***Q*** (first moment of area) increases linearly with constant moment arm.
9. For the element parallel to the load, ***Q*** moment of area and moment arm both increase and hence shear flow (***q***) varies parabolically.
10. Those elements for which the contribution is insignificant in providing the *Moment of Inertia* are assumed to develop no shear Flow.








V
V
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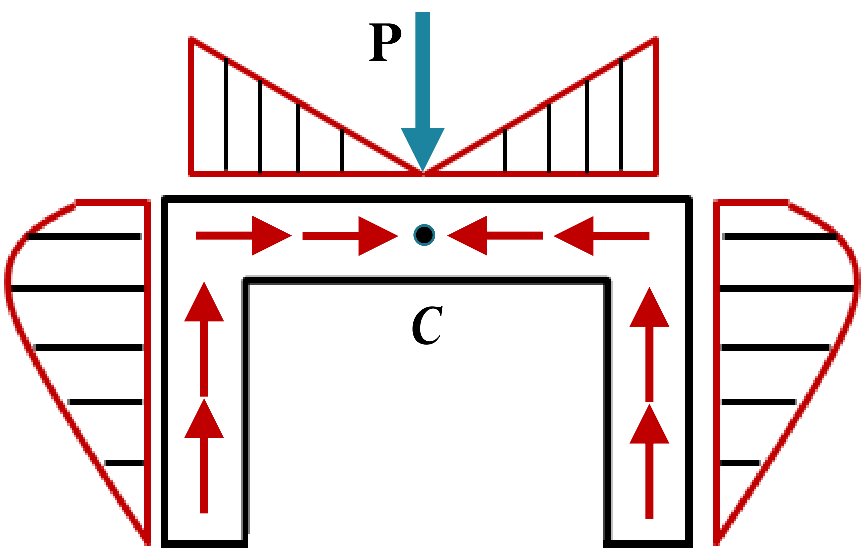
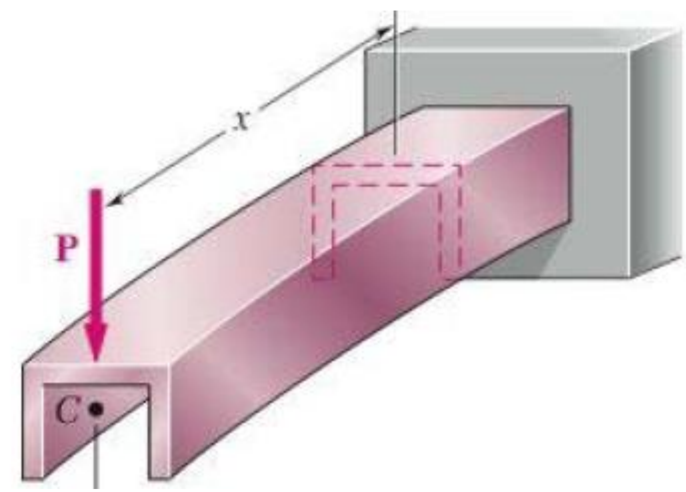
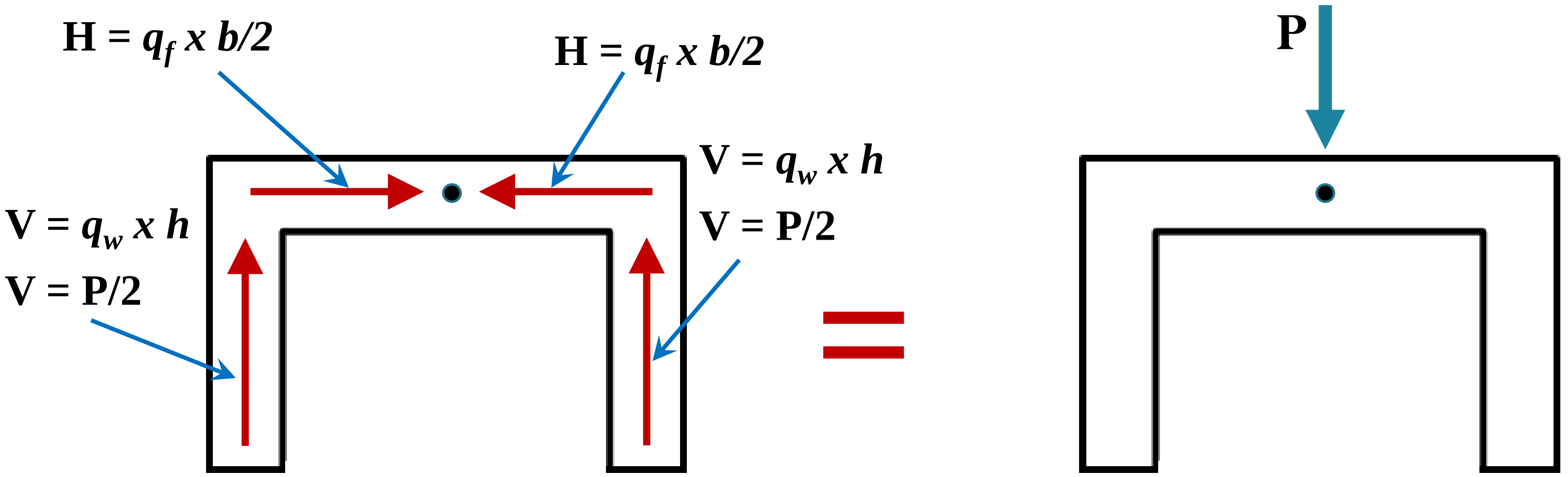
Thin Section With One Axes of Symmetry

- If the applied force is acting parallel to the axis of symmetry then there will be pure bending in the beam section without any twisting.
- At any section bending and shear stresses due to Bending Moment (***M***) Shear Force (***V***) can be calculated by the flexure and shear equation, respectively.







- However, if load is applied perpendicular to the axes of symmetry (or any axes other than parallel to the plane of symmetry) through the centroid then beam will bend along with the twisting.
- Bending stresses due to Bending Moment (***M***) still can be calculated by the flexure equation as beam is still bending about the ***Principal Axes*** and ***Neutral Axes*** (N.A.) will coincide with that axes.

- But ***Shear Equation*** cannot be used to calculate the shear stresses as this equation is derived for a member with vertical plane of symmetry (***i.e.,*** load is acting along the plane of symmetry).

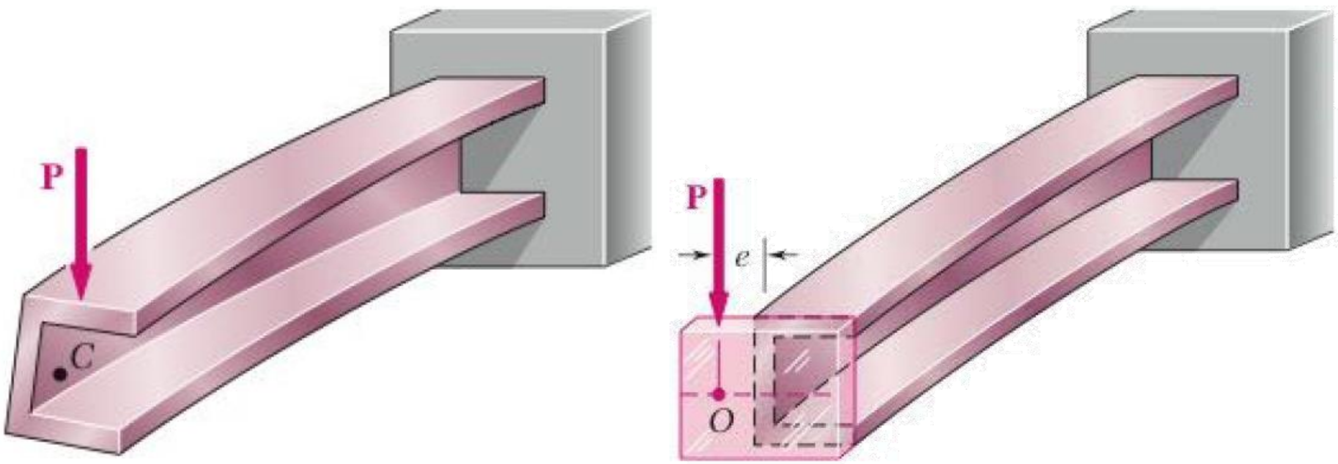
Shear Center

Shear center is the point in (or outside) the cross-section of beam through which the plane of loading must pass so that beam will bend without twisting.

All type of the section which undergo to the bending other than the Pure Bending are subjected to the shear stresses.

These stresses create the internal shearing force whose resultant must be equal, opposite and collinear to the external shear force. Otherwise, the bending occurs along with twisting.

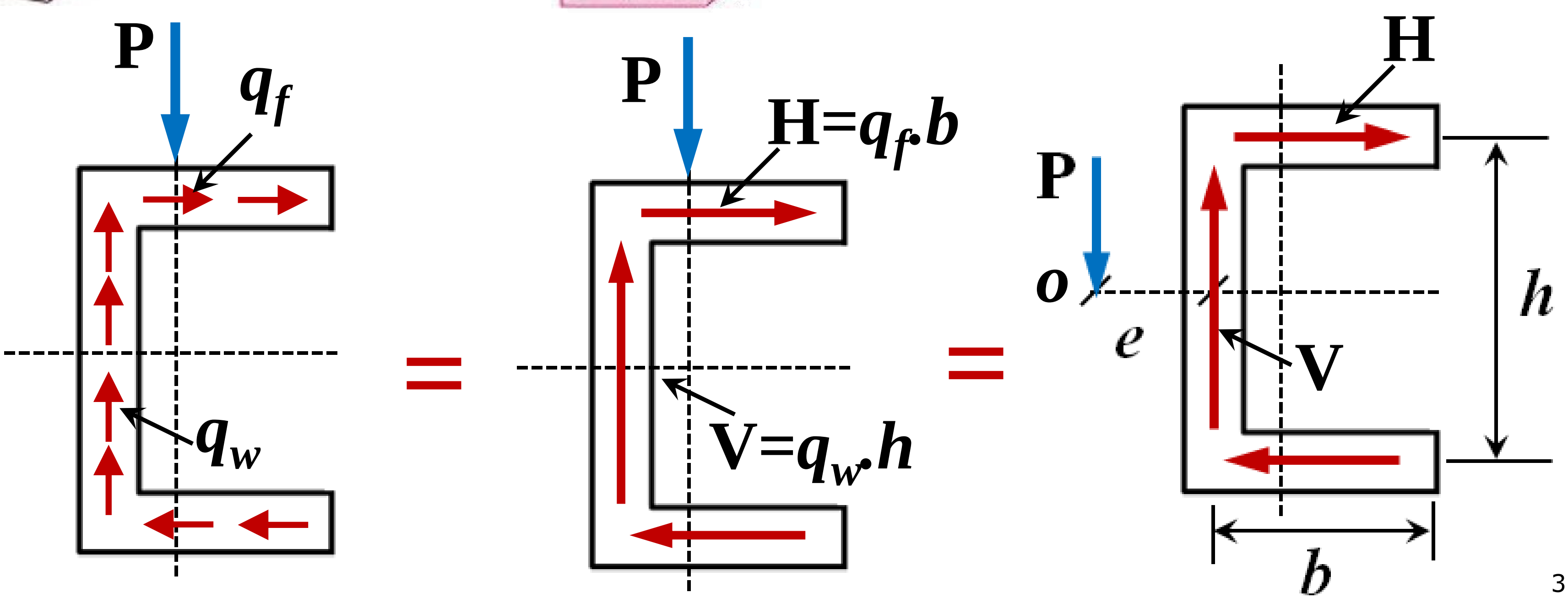
- Bending without twisting occurs only when the resultant of shear force passes through Shear Center (also called the center of twist or flexure center.



$$\sum (M)_o = 0$$

$$V.e = H.h$$

$$\Rightarrow \quad e = \frac{H.h}{V}$$



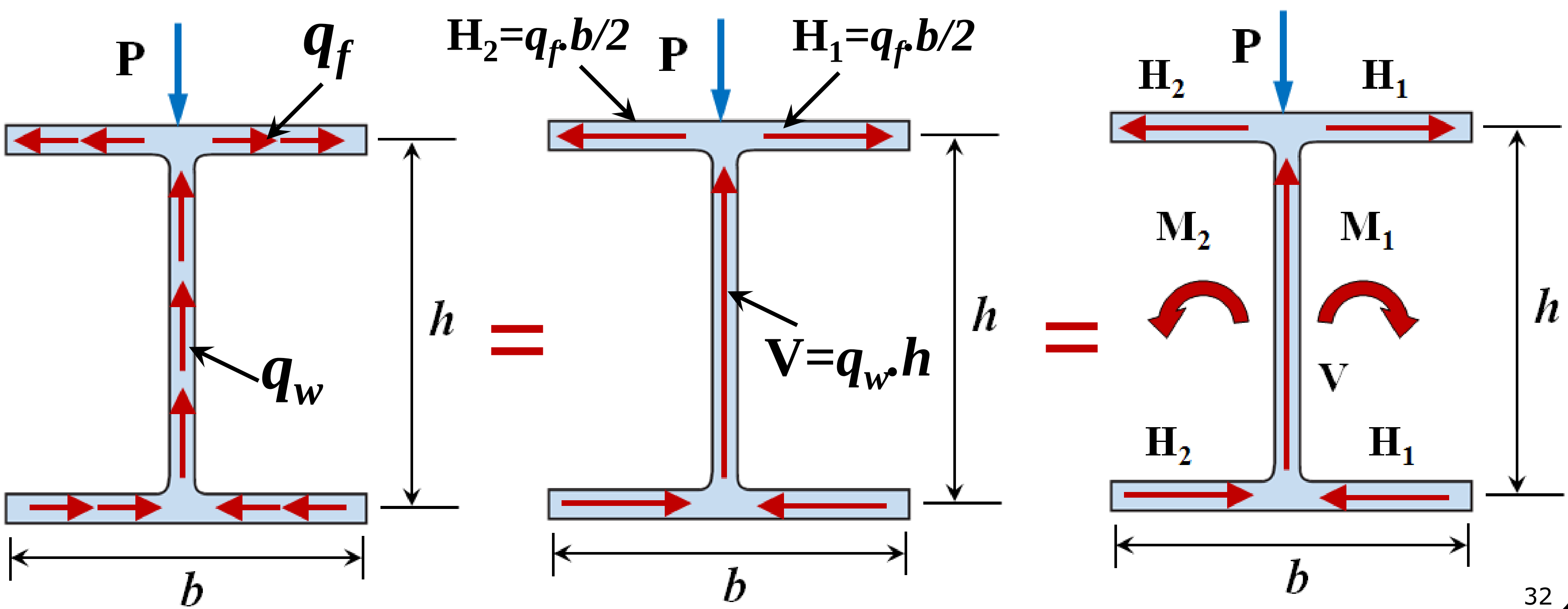
- Doubly symmetric section or singly symmetric section where load is applied parallel to the plane of symmetry are always subjected to the bending without twisting.

For I section shown in the Figure

$$\Sigma M = 0, \quad M_1 = M_2$$

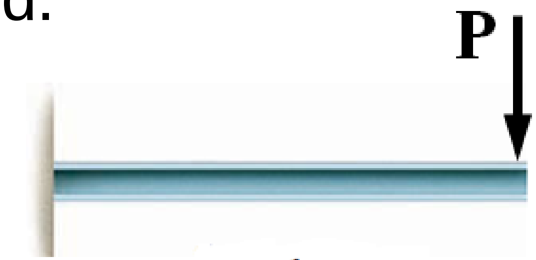
$$\Sigma F_x = 0, \quad H_1 = M_2$$

$$\Sigma F_y = 0, \quad P = V$$



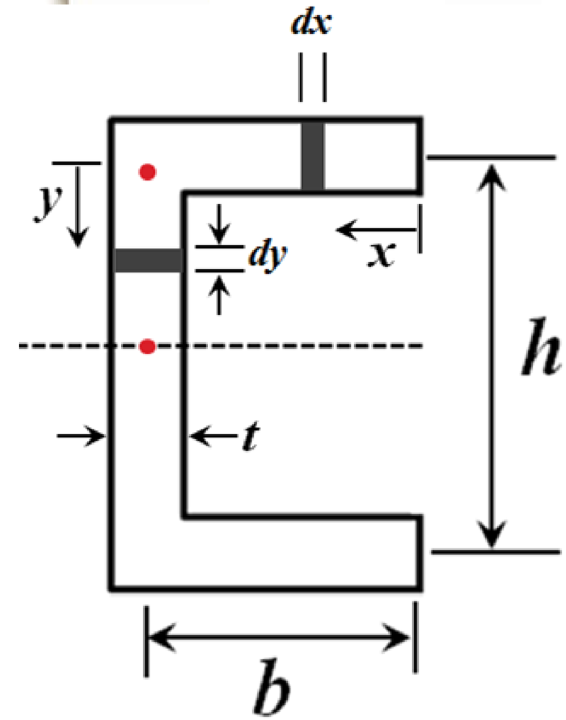
Shear Center of Channel Section

Consider a channel section (Singly Symmetric) of uniform thickness ***t***. let ***b*** is the total width and ***h*** is the center to center distance between flanges. Beam is supported at one end and shear force is applied at free end.



Stress in Flange

Considering a differential area in flange. ($dA = t.dx$)



$$\tau_f = \frac{V}{It}\int_0^b \bar{y}\,dA = \frac{V}{It}\int_0^b \frac{h}{2}\,t\,dx = \frac{V}{2I}\left|X\right|_0^b$$

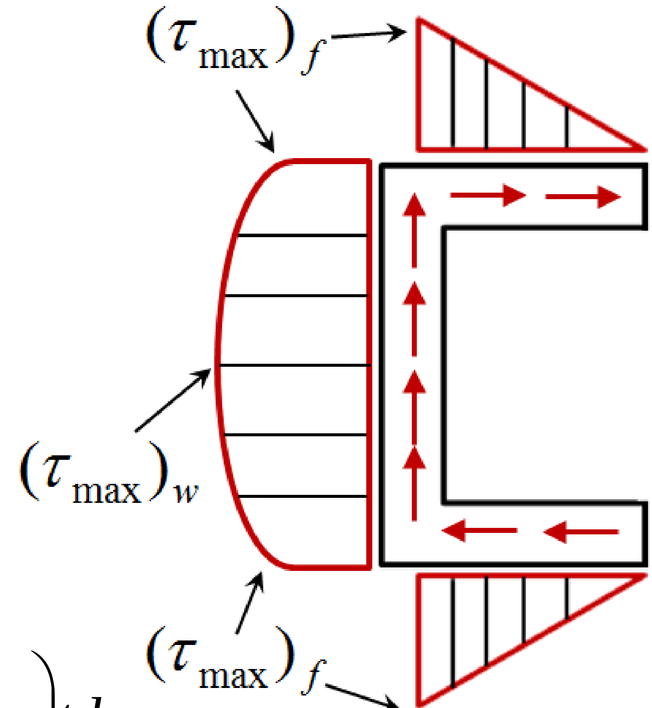
At x = 0, $\tau_f = 0$

At x = b, $\tau_f = \frac{Vhb}{It}$

Horizontal Force in Flange

$$H = \left(\tau_f\right)_{avg} \times flange\ \ Area$$

$$H = \frac{1}{2}\left(\frac{Vhb}{2I}\right) \times (b.t) = \frac{Vthb^2}{4I}$$



Stress in Web

Considering a differential area in web.
(*dA =t.dy*)

$$\tau_w = \left(\tau_f\right)_{max} + \frac{V}{It}\int_0^{h/2} \bar{y}dA = \left(\tau_f\right)_{max} + \frac{V}{It}\int_0^{h/2}\left(\frac{h}{2} - y\right)tdy$$

$$\tau_w = \left(\tau_f\right)_{max} + \frac{V}{I}\left|\frac{hy}{2} - \frac{y^2}{2}\right|_0^{h/2}$$

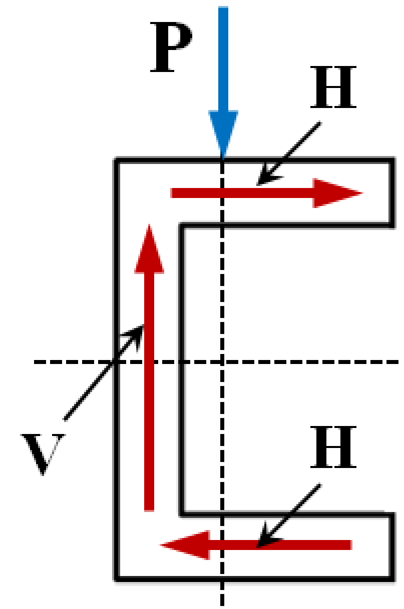
At y = 0, $\tau_w = (\tau_f)_{max}$

At y = h/2, $\tau_w = \left(\tau_f\right)_{max} + \frac{V}{I}\left(\frac{h^2}{2} - \frac{h^2}{8}\right) = \frac{Vhb}{2I} + \frac{Vh^2}{8I} = \frac{Vhb}{2I}\left(1 + \frac{h}{4b}\right)$

Vertical Force in Web

$$V = (\tau_w)_{avg} \times Web\ Area = \left[(\tau_{\max})_f + \frac{2}{3} \cdot \frac{Vh^2}{8I} \right]$$

$$V = \left(\frac{Vhb}{2I} + \frac{Vh^2}{12I} \right) \times h.t$$

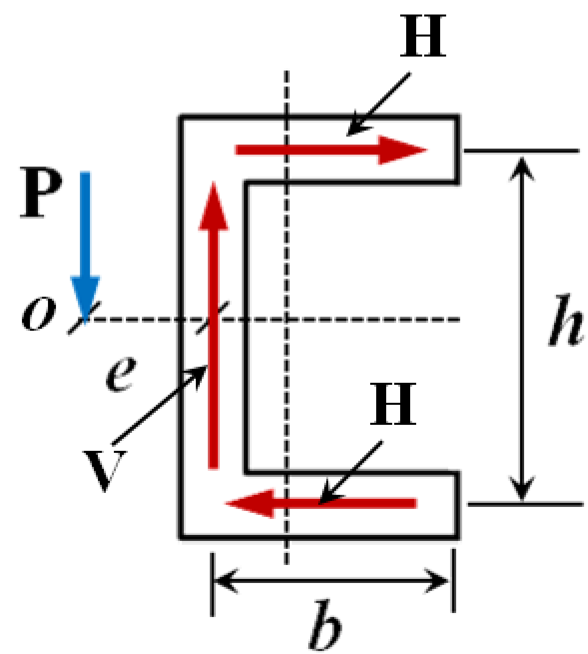


$\Sigma F_x = 0$, H = H

$\Sigma F_y = 0$, P = V

$\Sigma M = 0$, Vxe = Hx*h*

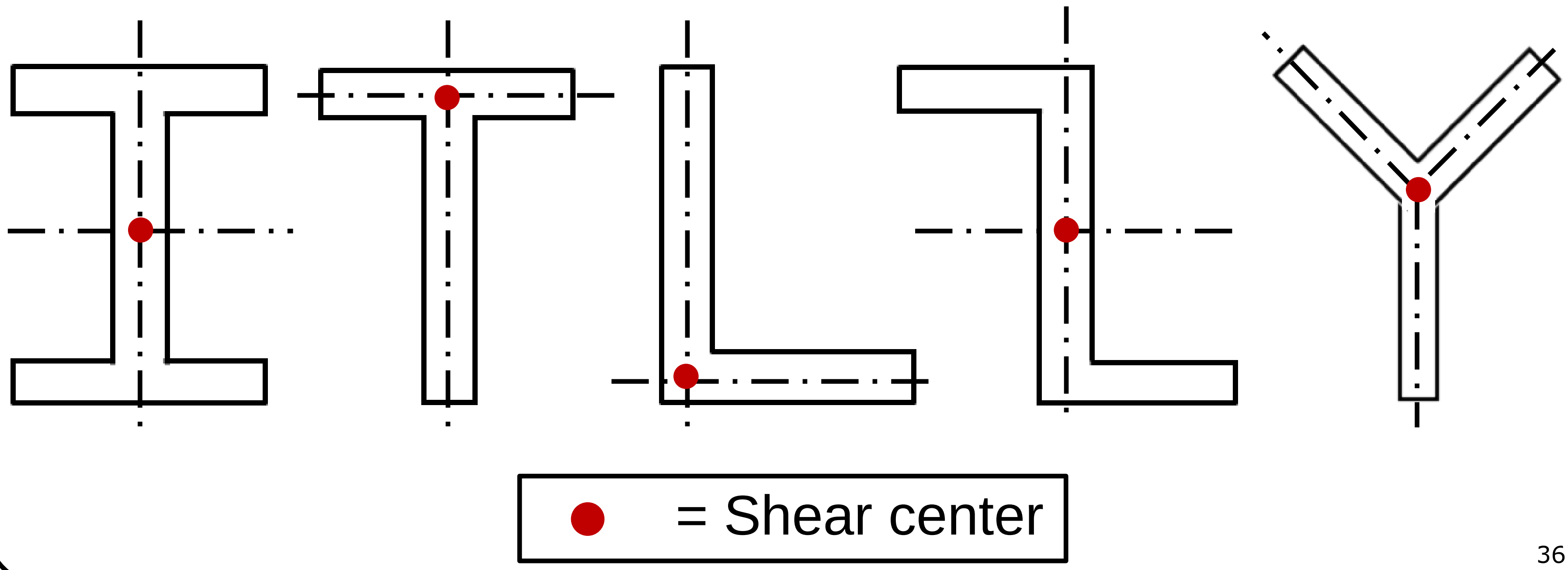
$$e = \frac{Vhtb^2}{4I} \times \frac{h}{V} = \frac{h^2 b^2 t}{4I}$$



The point **"*O*"** is the shear Center. It is the point through which resultant shear force must pass for bending to occur without twisting

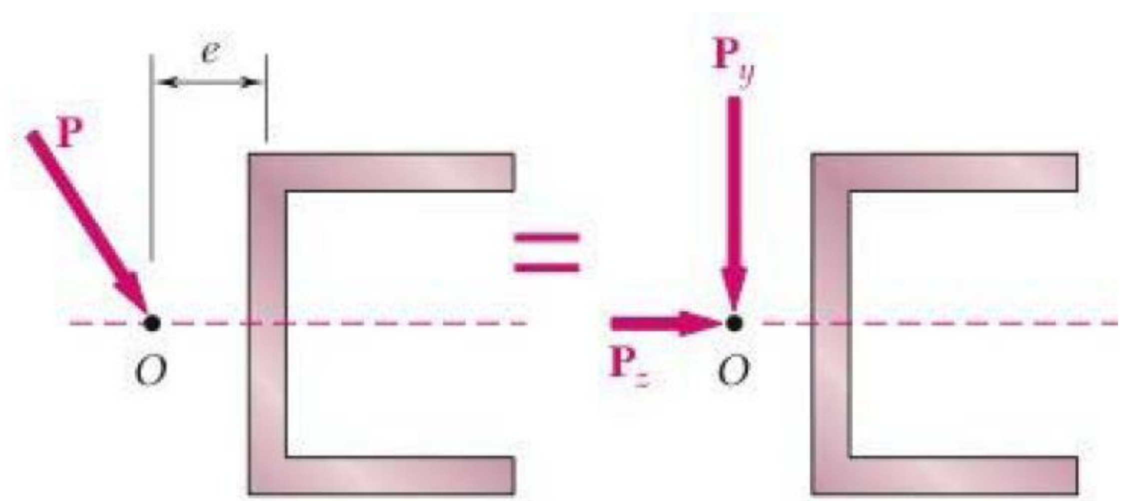
CHARACTERISTICS OF SHEAR CENTER

1. Shear center always lies on the axes of symmetry.
2. If two axes of symmetry exist for a section, then shear center is the intersection of these two axes.
3. If center-lines of all the elements of section intersect at a single point, this is the shear center
4. Shear center of Z-section lies at its centroid



5. The location of shear center is only a function of geometry of cross-section and does not depend upon the applied load.

6. The member will also be free of twist if an oblique (inclined) load **P** is applied at the shear center. In such case applied load **P** can be resolved into two components $\mathbf{P}_x$ and $\mathbf{P}_y$ to calculate pure bending stress.



Assignment Problem

Book: Mechanics of Materials 2nd Edition

By Andrew Pytel & Jaan Kiusalaas

Shear Flow and Shear Center

Problem 11.1, 11.3, 11.5, 11.7, 11.8, 11.9, 11.11
11.12, 11.14, 11.15, 11.18, 11.19

Submission time = 2 weeks