Plain & Reinforced Concrete-1

Sixth Term
Civil Engineering

CE-314

Lecture # 14

Flexural Analysis and Design of Beams

(Ultimate Strength Design of Beams)

DESIGN OF DOUBLY REINFORCED SECTIONS

- ***Data:***
  i) Loads and span length
  ii) f_c', f_y and E_s
  iii) Architectural depth restriction
- ***Required:***
  i) Beam cross-sectional dimensions such as b, d, d' and h
  ii) Areas of tension and compression steels, A_s and A_s'
  iii) Detailing

- Calculate all types of loads acting on the structure and hence calculate the total factored load.
- Calculate the total maximum factored moment, M_u, acting on the member. Also decide the tentative cross-sectional dimensions of the member.
- Calculate minimum effective depth required for singly reinforced section.
- If the available effective depth is lesser than this depth, design the member as a doubly reinforced section.

$$d_{min} = \sqrt{\frac{M_u}{0.205 f_c' b}}$$

- Calculate the maximum moment that can be resisted by the tension-steel according to the maximum steel ratio for singly reinforced section for an extreme tensile strain of 0.005.

- $\rho \;=\; \rho_{max} \;=\; 0.85 \times 0.375 \beta 1 \dfrac{f_c'}{f_y}$
- $A_{s1} = \rho_{max}\, bd$
- $a \;=\; \beta_1 \text{ x } 3/8\, d$
- $M_1 = \phi_b M_n \;=\; \phi_b\, A_{s1}\, f_y\, (d - a / 2)$

- In the above expressions, $M1$ is the moment capacity provided by the maximum compression in the concrete and A_{s1} is the area of tension steel to balance the maximum compression developed in the concrete according to the code allowed value of extreme tensile strain.
- Calculate the excess moment (M_2) that must be resisted by the moment capacity provided by the compression in the compression steel.

$$M_2 = M_u - M_1$$

- Assume that the compression steel is yielding ($f_s' = f_y$, to be checked later). Calculate the amount of compression steel for the above assumption.

$$A_s' = \frac{M_2}{\phi_b f_y (d - d')}$$

- The total amount of tension steel may be calculated by adding an extra amount of tension steel equal to the above required compression steel in to the already calculated tension steel.

$$A_{s2} = A_s'$$

$$A_s = A_{s1} + A_{s2}$$

- Calculate the tensile steel ratio and check for yielding of compression steel by any method.

$$\rho = \frac{A_s}{bd} \qquad \bar{\rho}_{cy} = 0.85\,\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \left(\frac{600}{600 - f_y} \right) + \rho'$$

- If $\rho \geq \bar{\rho}_{cy}$, compression steel is yielding, as assumed before and amount of compression steel = A_s'
- If the steel ratio (ρ) is lesser than (compression steel is not yielding), stress in compression steel is determined from the strain compatibility.
- When we compensate the internal compressive force in steel as in the next step, the depth of rectangular stress block (a) remains the same as in step 4.

$$f_s' = 600\,\frac{a - \beta_1 d}{a}$$

- If the compression steel is not yielding, compression steel area (A_s') is increased to compensate the reduction in compression steel stress to get the required compressive force.

$$A_s'_{,\ \text{revised}} = A_s'_{,\ \text{trial}} \ \frac{f_y}{f_s'}$$

Example 4.2:

Design a simply supported rectangular beam having a span of 6m, subjected to a total factored load including the self weight of 70 kN/m. The depth of the beam should not exceed its span over 12 and its width is to be 300 mm. Use C–20 concrete and Grade 420 steel. Design the beam for the following two options selecting SI bars:

i) $\rho - \rho' \times \; = 0.5\, \rho_b$

ii) Area of compression steel is minimum

Note: Keeping maximum tension steel ratio, the amount of compression steel will be the minimum.

Solution:

- ℓ = 6 m
- w_u = 70 kN/m
- b = 300 mm
- Assume the depth of centroid of compression steel from the extreme compression face to be:
  d' = 58 mm
- f_y = 420 MPa
- f_c' = 20 MPa
- h_{max} = ℓ / 12 = 6000 / 12 = 500 mm
- h_{min} for deflection control = ℓ / 16 = 6000 / 16 = 375 mm

- h_{sel} = 500 mm
- d = h_{sel} – 75 = 425 mm
- M_u = 70 x 6^2 / 8 = 315 kN-m
- d_{min} for singly reinforced section

$$= \sqrt{\frac{M_u}{0.205 f_c' b}} = \sqrt{\frac{315 \times 10^6}{0.205 \times 20 \times 300}} = 506 \text{ mm}$$

- $d \leq d_{min}$, design as doubly reinforced section.

Case (i)

$$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y}\left(\frac{600}{600 + f_y}\right)$$

$$= 0.85^2\,\frac{20}{420}\left(\frac{600}{600+420}\right) = 0.0202$$

$$\rho - \rho' \times \frac{f_s'}{f_y} = 0.5\,\rho_b = 0.0101$$

$$A_s - A_s' \times \frac{f_s'}{f_y} = (\rho - \rho' \times \frac{f_s'}{f_y})\,bd$$

$$= 0.0101 \times 300 \times 425 = 1288 \text{ mm}^2$$

- $\rho - \rho' \times \dfrac{f'_s}{f_y} < \rho b \Rightarrow \rho < \rho b + \rho' \times \dfrac{f'_s}{f_y}$

  $\Rightarrow$ tension steel is yielding with sufficient margin, $\phi_b = 0.9$.

- $a = \dfrac{(A_s - A'_s \dfrac{f'_s}{f_y}) f_y}{0.85\, f'_c\, b} = \dfrac{1288 \times 420}{0.85 \times 20 \times 300} = 106$ mm
- $c = a / \beta_1 = 106 / 0.85 = 125$ mm
- $\varepsilon_y = f_y / E_s = 420 / 200{,}000 = 0.0021$

$$\varepsilon_s' = 0.003\,\frac{c - d'}{c} = 0.003\,\frac{125 - 58}{125}$$

$$= 0.00161$$

$< \varepsilon_y \Rightarrow$ compression steel is not yielding

$$f_s' = 0.00161 \times 200{,}000 = 322 \text{ MPa}$$

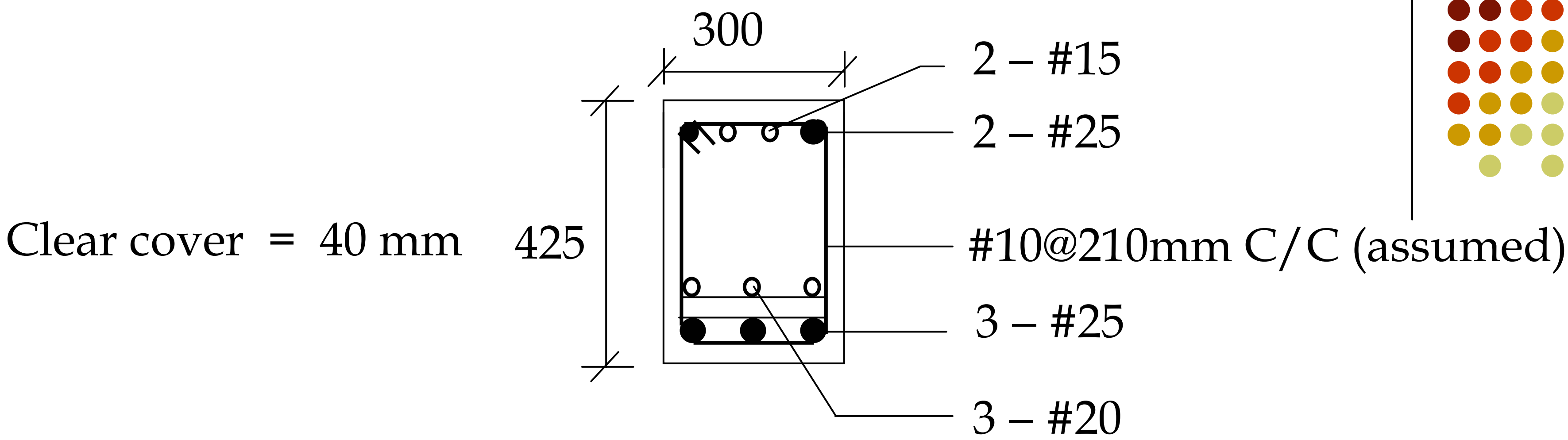
$$A_s - A_s' \times \frac{f_s'}{f_y} = 1288$$

Multiplying with f_y, we get,

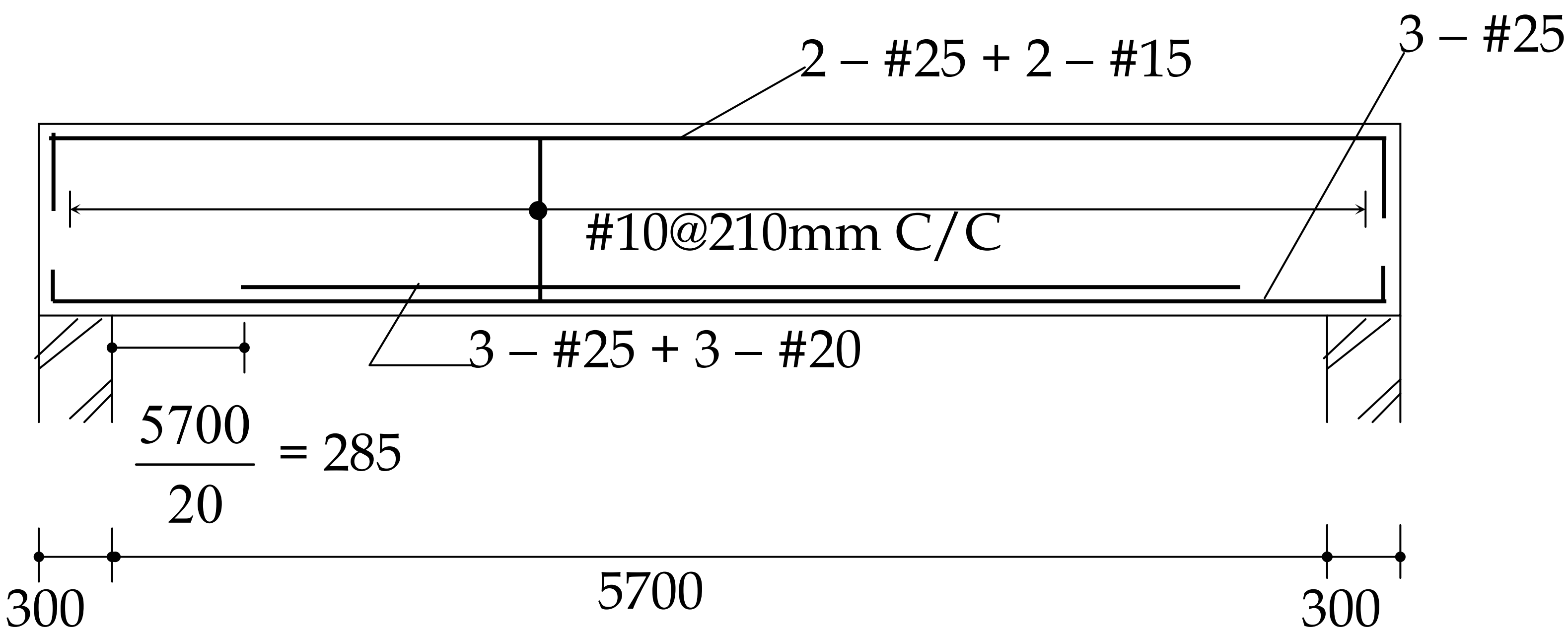
$$A_s f_y - A_s' f_s' = 1288 f_y$$

$$\Rightarrow A_s' f_s' = (A_s - 1288) f_y$$

- $M_u = \phi_b M_n = \phi_b(A_s - A_s' \times) f_y (d - a / 2)$
  $+ \phi_b A_s' f_s' (d - d')$
  $315 \times 10^6 = (0.9)(1288)(420)(425 - 106 / 2)$
  $+ (0.9)(A_s - 1288)(420)(425 - 58)$
  $138{,}726 A_s = 3.1257 \times 108$
- $A_s = 2253 \text{ mm}^2$ [3-#25 + 3-#20]
- $A_s' = (A_s - 1288) f_y / f_s'$
  $= (2253 - 1288) \times 420 / 322$
  $= 1259 \text{ mm}^2$ [2-#25 + 2-#15]



a) Mid-Span Cross Section



Case (ii)

- $\dfrac{d'}{d} = 58 / 425 = 0.1365 > 0.132,$

compression steel will not be yielding

<u>Moment Capacity As Singly Reinforced Section</u>

$$\rho = \rho_{max} = 0.85 \times 0.375\beta_1 \frac{f'_c}{f_y}$$

$$= 0.85 \times 0.85 \times 0.375 \times (20 / 420) = 0.0129$$

- $A_{s1} = \rho_{max}\, bd = 0.0129 \times 300 \times 425 = 1645 \text{ mm}^2$
- $a = \beta_1 \text{ x } 3/8\, d = 0.85 \times 0.375 \times 425 = 135 \text{ mm}$

- $M_1 = \phi_b M_n = \phi_b A_{s1} f_y (d - a / 2)$

  $= (0.9)(1645)(420)(425 - 135 / 2) / 10^6$

  $= 222.3$ kN-m

<u>Design For Balance Moment Capacity</u>

- $M_2 = M_u - M_1$

  $= 315.0 - 222.3 = 92.7$ kN-m

Assuming compression steel to be yielding,

- $$A_s' = \frac{M_2}{\phi_b f_y (d - d')} = \frac{92.7 \times 10^6}{(0.9)(420)(425 - 58)}$$

  $= 668\ \text{mm}^2$

$A_{s2} = A_s' = 668 \text{ mm}^2$

$A_s = A_{s1} + A_{s2}$

$= 1645 + 668 = 2313 \text{ mm}^2$

[3–#25 + 3–#20]

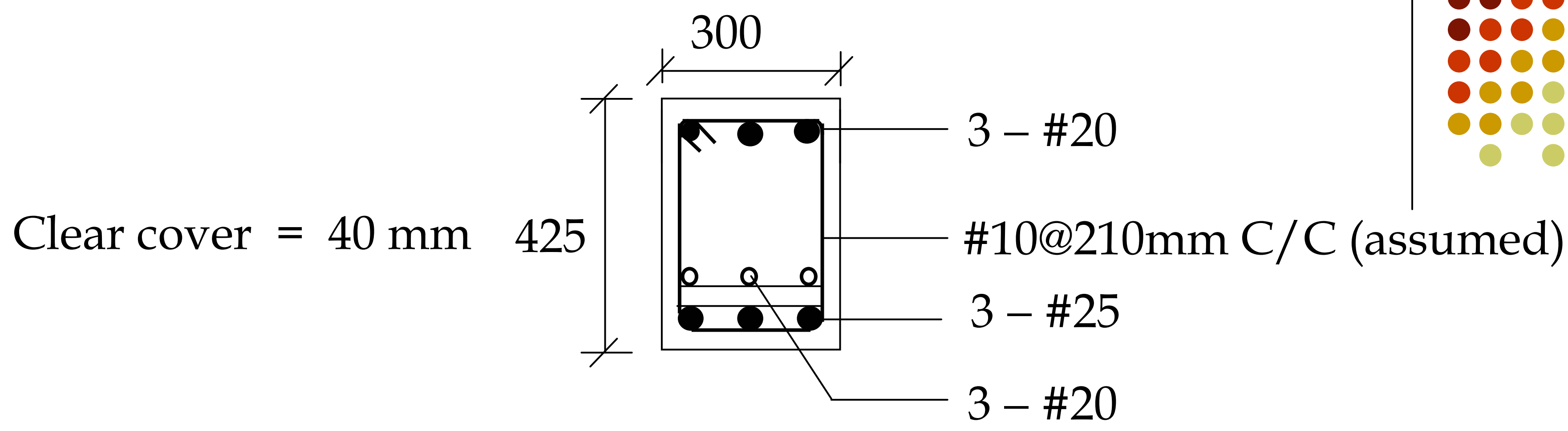
$a = 135$ mm (as calculated before)

$$f_s' = 600\,\frac{a - \beta_1 d'}{a} = 600\,\frac{135 - 0.85 \times 58}{135}$$

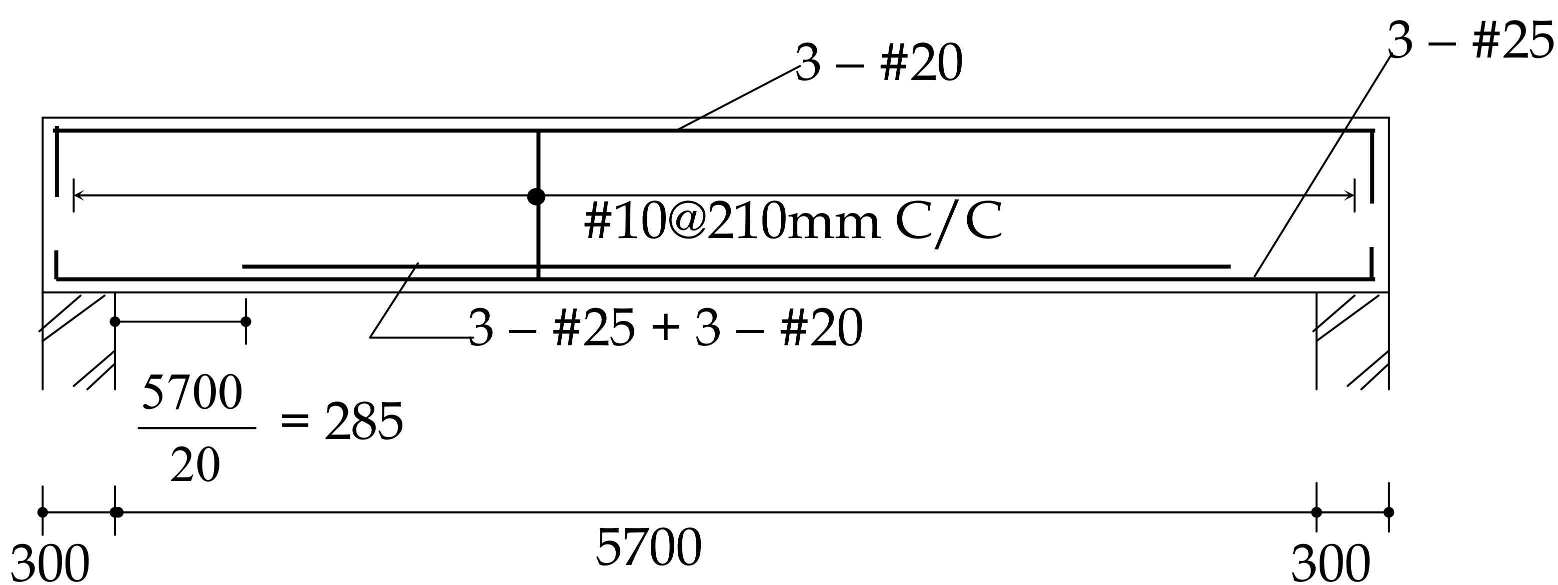
$= 380.9$ MPa $< f_y$, compr. steel not yielding

$$A_s'{}_{,\,\text{revised}} = A_s'{}_{,\,\text{trial}}\,\frac{f_y}{f_s'}$$

$= 668 \times 420 / 380.9 = 737 \text{ mm}^2$ [3–#20]



a) Mid-Span Cross Section



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T & L Beams

- With the exception of pre-cast systems, reinforced concrete floors, roofs, decks, etc., are almost always monolith.
- Beam stirrups and bent bars extend up into the slab.
- It is evident, therefore, that a part of the slab will act with the upper part of the beam to resist longitudinal compression.
- The slab forms the beam flange, while a part of the beam projecting below the slab forms what is called the “web” or “stem”.

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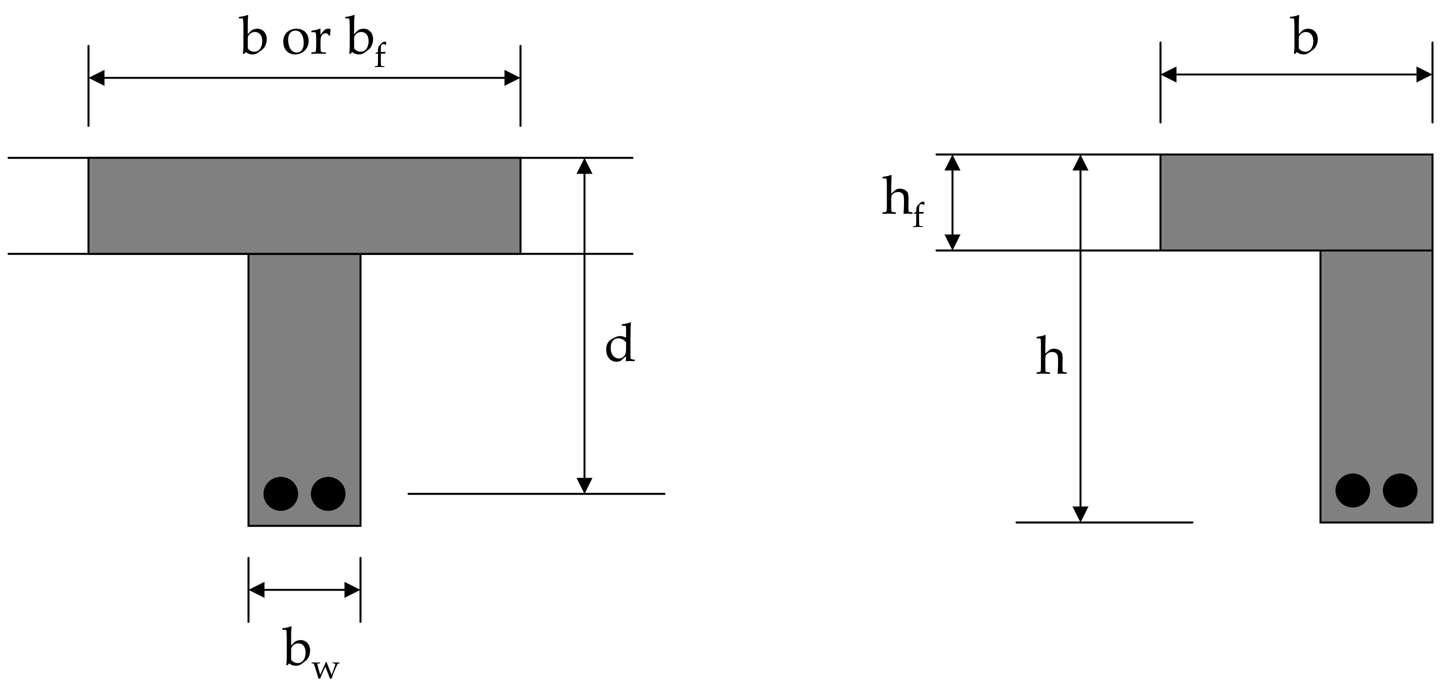
T & L Beams (contd…)



b = Effective width

b_w = width of web/rib/stem

h_f = Thickness of flange

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Effective With of T & L Beams

T-Beams

Effective width (b) will be minimum of the following:

1. L/4
2. $16h_f + b_w$
3. b_w + ½ x (clear spacing of beams (S_i) on both sides)
   = c/c spacing for beams at regular interval

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Effective With of T & L Beams

L-Beams

Effective width (b) will be minimum of the following:

1. $L/12$
2. $6h_f + b_w$
3. $b_w + S_c/2$ on one side

Where S_c is the clear distance to the next web.

Note: Only above discussion is different for isolated (pre-cast) T or L beam. Other discussion is same (analysis and design formula).

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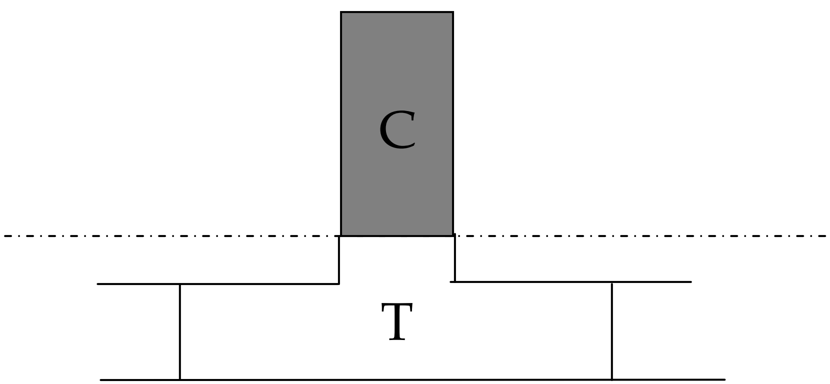
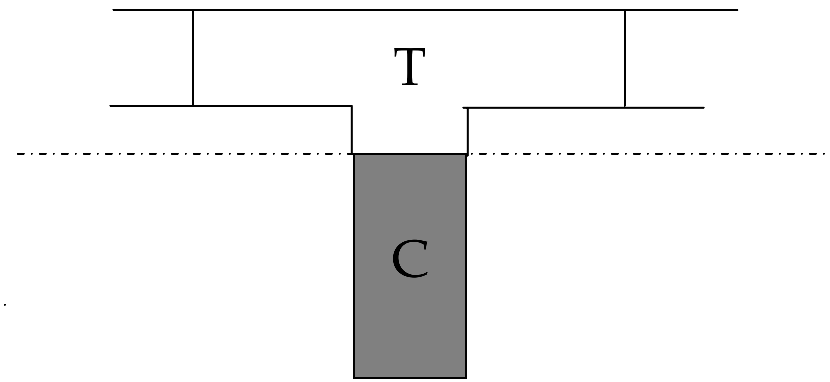
Flexural Behavior

Case-I: Flange is in Tension





-ve Moment

In both of the above cases beam can be designed as rectangular beam

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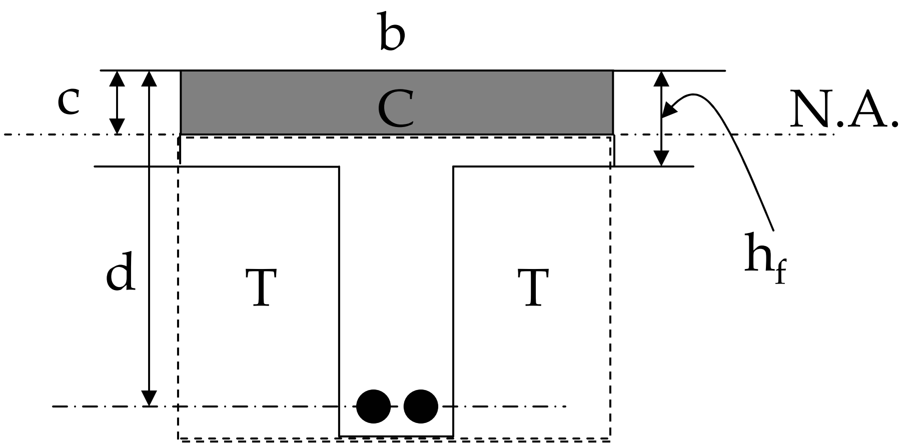
Case-II: Flange is in Compression and N.A. lies with in Flange

Beam can be designed as a rectangular beam of total width “b” and effective depth “d”.



$$c \le h_f$$

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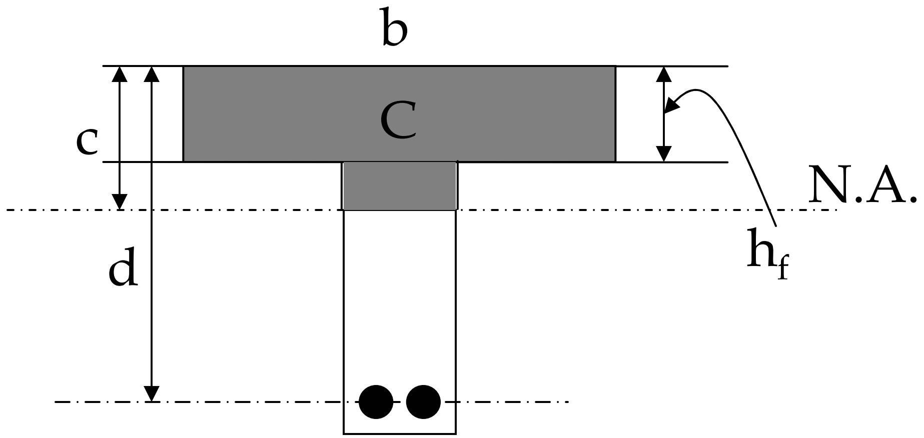
Flexural Behavior (contd…)

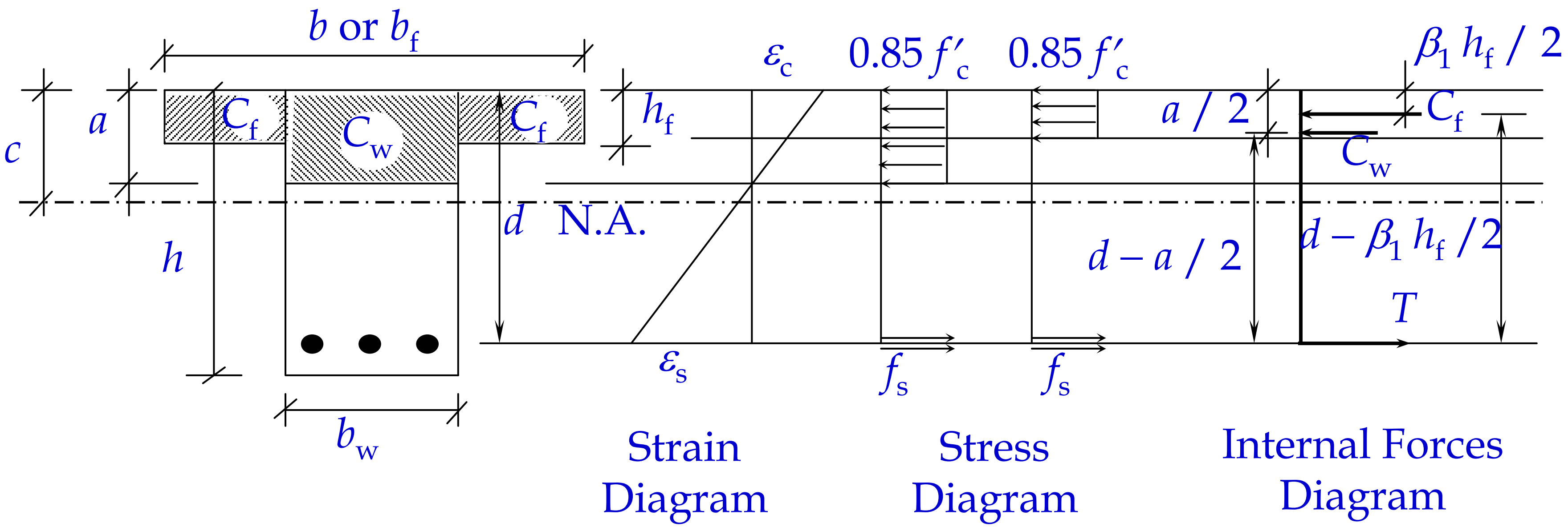
Case-III: Flange is in Compression and N.A. lies out of the Flange

Beam has to be designed as a T-Beam.

Separate expressions are to be developed for analysis and design.



$c > h_f$


b or b_f
c
a
h
C_f
C_w
C_f
h_f
d
N.A.
b_w
ε_c
ε_s
0.85 f′_c
f_s
0.85 f′_c
f_s
a / 2
d − a / 2
β_1 h_f / 2
C_f
C_w
d − β_1 h_f / 2
T
Strain
Diagram
Stress
Diagram
Internal Forces
Diagram

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C_w = Compression developed in the web = $0.85f_c'b_wa$

C_f = Compression developed in the overhanging flange

$= 0.85f_c'(b-b_w)\ \beta_1h_f$

C = Total Compression = $C_w + C_f$

T = Total Tension = $T_w + T_f$

T_w = Tension to balance C_w

T_f =Tension to balance C_f

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It is convenient to divide total tensile steel into two parts. **The first part, A_{sf}** represents the steel area which, when stressed to f_y, is required to balance the longitudinal compressive force in the overhanging portions of the flange that are stressed uniformly at $0.85f_c'$.

- The remaining steel area **$A_s - A_{sf}$**, at a stress f_y, is balanced by the compression in the rectangular portion web above the N.A.

$$T_f = A_{sf} f_y$$

$$T_w = A_{sw} f_s = \left(A_s - A_{sf}\right) f_y$$

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Flexural Behavior (contd…)

Majority of T and L beams are under-reinforced (tension controlled). Because of the large compressive concrete area provided by the flange. In addition, an upper limit can be established for the reinforcement ratio to ensure the yielding of steel.

For longitudinal equilibrium

$$\Sigma F = 0$$

$$T_f = C_f \qquad \& \qquad T_w = C_w$$

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$$T_f = C_f$$

$$A_{sf} f_y = 0.85 f_c' (b - b_w) \beta_1 h_f$$

$$\boxed{A_{sf} = 0.85 \beta_1 h_f \frac{f_c'}{f_y} (b - b_w)}$$

Only for case-III

$$T_w = C_w$$

$$(A_s - A_{sf}) f_y = 0.85 f_c' b_w a$$

$$\boxed{a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w}} \quad \text{and} \quad \boxed{c = \frac{a}{\beta_1}}$$

If $c > h_f$ N.A. is outside the flange

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Flexural Capacity

From Compression Side

$$M_n = M_{nf} + M_{nw}$$

$$M_n = C_f\left(d - \frac{\beta_1 h_f}{2}\right) + C_w\left(d - \frac{a}{2}\right)$$

$$\boxed{M_n = 0.85 f_c'(b - b_w)\beta_1 h_f\left(d - \frac{\beta_1 h_f}{2}\right) + 0.85 f_c' b_w a\left(d - \frac{a}{2}\right)}$$

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Flexural Capacity

From Tension Side

$$M_n = M_{nf} + M_{nw}$$

$$M_n = T_f\left(d - \frac{\beta_1 h_f}{2}\right) + T_w\left(d - \frac{a}{2}\right)$$

$$\boxed{M_n = A_{sf} f_y\left(d - \frac{\beta_1 h_f}{2}\right) + \left(A_s - A_{sf}\right) f_y\left(d - \frac{a}{2}\right)}$$

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Tension Controlled Failure of T-Beam

$$\rho_w = \frac{A_s}{b_w d}$$ = Total tension steel area (all steel will be in web)

$$\rho_f = \frac{A_{sf}}{b_w d}$$ = Steel ratio to balance the flange compressive force

ρ_b = Balanced steel ratio for the singly reinforced rectangular section

$(\rho_w)_{max}$ = Maximum steel ratio for T-Beam

ρ_{max} = Maximum steel ratio for the singly reinforced rectangular section

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For longitudinal equilibrium

$$T = C_w + C_f$$

$$T = C_w + T_f$$

$$A_s f_y = 0.85 f_c' b_w a_\ell + A_{sf} f_y$$

Divide by $f_y b_w d$

$$\frac{A_s}{b_w d} = 0.85 \frac{f_c'}{f_y} \times \frac{a_\ell}{d} + \frac{A_{sf}}{b_w d}$$

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For longitudinal equilibrium

For $\varepsilon_{st} = 0.005$

$$a_{\ell} = \beta_1 \frac{3}{8} \times d$$ **To ensure tension controlled section**

$$\Rightarrow \quad (\rho_w)_{max} = 0.85 \frac{f_c'}{f_y} \times \frac{3}{8}\beta_1 + \frac{A_{sf}}{b_w d}$$

$$= 0.85\beta_1 \times \frac{3}{8} \times \frac{f_c'}{f_y} + \frac{A_{sf}}{b_w d}$$

$$\boxed{(\rho_w)_{max} = \rho_{max} + \rho_f}$$

If $\rho_w \leq (\rho_w)_{max}$ Tension controlled section

If $\rho_w > (\rho_w)_{max}$ Transition or Compression controlled section

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Capacity Analysis

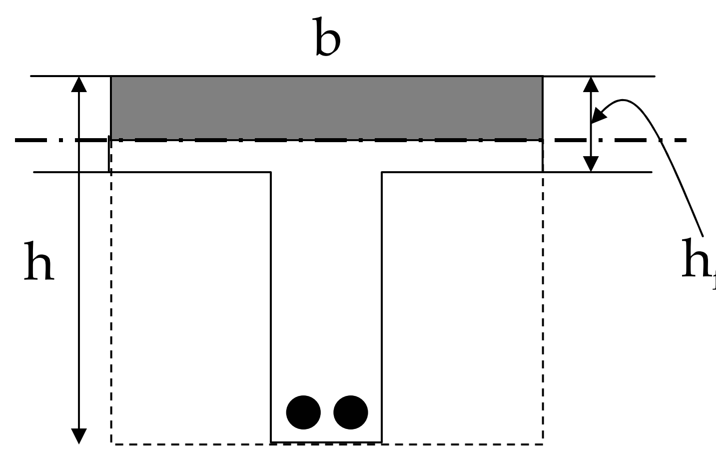
Known: $b, d, b_w, h_f, L, f_c', f_y, A_s$

Required: $\phi_b M_n$

Step # 1: Check whether the slab is in tension or not, if Yes, analyse as a rectangular section.

Step # 2: Assume the N.A. to be with in the flange and calculate the "a" and "c"

$$a = \frac{A_s f_y}{0.85 f_c' b} \Rightarrow c = \frac{a}{\beta_1}$$



If **c ≤ h_f** Our assumption is correct and beam will be designed as rectangular beam. (b x h)

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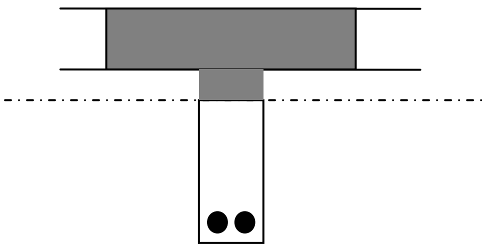
Capacity Analysis (contd…)



If $c > h_f$ Beam is to be designed as T-Beam

Step # 3

$$A_{sf} = 0.85\beta_1 h_f \frac{f_c'}{f_y}(b - b_w)$$

Step # 4

$$A_{sw} = A_s - A_{sf}$$

Step # 5 Revise "a"

$$a = \frac{A_{sw} f_y}{0.85 f_c' b_w}$$

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Capacity Analysis (contd…)

Step # 6 Calculate ε_s and ϕ factor

If

$$a \le \beta_1 \frac{3}{8} d \Rightarrow \phi = 0.9$$

Step # 7

$$\boxed{\phi_b M_n = \phi_b A_{sw} f_y \left(d - \frac{a}{2} \right) + \phi_b A_{sf} f_y \left(d - \frac{\beta_1 h_f}{2} \right)}$$

For Under-reinforced section

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Example:

A **T-Beam** has the following data:

$b = 800$ mm $\qquad b_w = 350$ mm $\qquad h_f = 125$ mm

$f_c' = 20$ MPa $\qquad f_y = 420$ MPa $\qquad d = 450$ mm

Determine the flexural strength for the following two cases if the slab is in compression.

- $A_s = 3900\ mm^2$
- $A_s = 3000\ mm^2$

Solution:

- ***Case (i)***

Considering the N.A. within the flange,

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3900 \times 420}{0.85 \times 20 \times 800} = 120.5mm$$

$c = a / \beta_1 = 141.7$ mm $> h_f$, N.A. lies outside the flange.

$A_{sf} = 0.85\, \beta_1\, h_f\, (f_c'/f_y)\, (b - b_w)$

$= 0.85 \times 0.85 \times 125 \times 20/420 \times (800 - 350)$

$= 1935$ mm^2

$A_{sw} = A_s - A_{sf} = 3900 - 1935 = 1965$ mm^2

$$A_{sw} = A_s - A_{sf} = 3900 - 1935 = 1965 \text{ mm}^2$$

$$a = \frac{A_{sw} f_y}{0.85\, f_c'\, b_w} = \frac{1965 \times 420}{0.85 \times 20 \times 350} = 138.7 mm$$

- $a_\ell = \beta_1 (3/8)\, d = 0.85 \times 0.375 \times 450 = 143.4$ mm
- $a \le a_\ell$ the limiting tensile strain is produced in the steel and $\phi_b = 0.9$
- $\phi_b M_{nt} = \phi_b A_{sw} f_y (d - a/2) + \phi_b A_{sf} f_y (d - \beta_1 h_f / 2)$

$$= [0.9 \times 1965 \times 420 (450 - 138.7/2)$$

$$+ 0.9 \times 1935 \times 420 (450 - 0.85 \times 125/2)] / 10^6$$

$$= 573.0 \text{ kN-m}$$

Case (ii)

Considering the N.A. within the flange,

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3000 \times 420}{0.85 \times 20 \times 800} = 92.6mm$$

$c \;=\; a / \beta_1 \;=\; 109 \text{ mm} \;<\; h_f$

$\therefore$ N.A. lies within the flange and the T-beam behaves like a rectangular beam of dimension $b \times d$.

$\phi_b M_n \;=\; \phi_b A_s f_y (d - a / 2)$

$= [0.9 \times 3000 \times 420 (450 - 92.6/2)] / 10^6$

$= 457.8$ kN-m

Concluded