PRE-STRESSING- Flexural-Introduction

- In the case of flexural analysis, the concrete and steel dimensions, as well as the magnitude and line of action of the effective pre-stress force, are usually known.
- If the loads are given, one may wish to find the resulting stresses and compare these against a set of permissible values.
- Alternately, if permissible stresses are known, then one may calculate maximum loads that could be carried without exceeding those stresses.
- For known material strengths, the member capacity can be calculated and safety against collapse determined for any loading.
- **In contrast**, in flexural design, permissible stresses and material strengths are known, the loads to be resisted are specified, and the engineer must determine concrete and steel dimensions as well as the magnitude and line of action of the pre-stressing force.
- The analysis of pre-stressed flexural members is by far the simpler task.

PRE-STRESSING- Flexural-Introduction

- In design, a trial member chosen on the basis of approximate calculation is analyzed to check its adequacy and then refined.
- In this way, the designer converges on the solution that is “best” in some sense.
- Both analysis and design of pre-stressed concrete may require the consideration of several load stages as follows:
- **1.** Initial pre-stress, immediately after transfer, when P_i alone may act on the concrete.
- **2.** Initial pre-stress plus self-weight of the member.
- **3**. Initial pre-stress plus full dead load.
- **4.** Effective pre-stress, P_e, after losses, plus service loads consisting of full dead and expected live loads.
- **5.** Ultimate load, when the expected service loads are increased by load factors, and the member is at incipient failure.

PRE-STRESSING- Partial loss of pre-stress

- the jacking tension initially applied to the tendon, is reduced at once to what is termed the initial prestress force $\mathbf{P_i}$.
- A part of this loss in jacking tension, that due to friction between a post-tensioned tendon and its encasing duct, actually occurs before the transfer of pre-stress force to the concrete. The remainder due to elastic shortening of the concrete and due to slip at post-tensioning anchorages as the wedges take hold, occurs immediately upon transfer.
- Additional losses occur over an extended period, because of concrete shrinkage and creep, and because of relaxation of stress in the steel tendon.
- As a result the pre-stress force is reduced from $\mathbf{P_i}$ to its final or effective value, $\mathbf{P_e}$ after all significant time-dependent losses have taken place.
- The values of greatest interest to the designer are the initial pre-stress $\mathbf{P_i}$ and the effective pre-stress **Pe**.
- It is convenient to express the relation between these values in terms of an effectiveness ratio R, defined such that

PRE-STRESSING- Partial loss of pre-stress

- $\mathbf{P_e = R \times P_i}$
- Another way of writing ratio of time dependent losses to initial pre-stress force is as follows.
- $\frac{\boldsymbol{P_i - Pe}}{\boldsymbol{P_i}} = \mathbf{1 - R}$

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

- Following notation will be used in further discussion:

  P_i = initial prestress force just after transfer without time-dependent losses.

  P_e = effective prestress force after all the short and long-term losses.

  R = effectiveness ratio = P_e / P_i.

  e = eccentricity of prestressing force from the centroid at a particular section.

  M_g = bending moment due to self weight.

  M_d = bending moment due to imposed dead load.

  M_l = bending moment due to service live load.

C_1 = distance of top fiber from the centroid.

C_2 = distance of bottom fiber from the centroid.

S_1 = elastic section modulus with respect to top fibers.

S_2 = elastic section modulus with respect to bottom fibers.

f_1 = stress at the top fiber.

f_2 = stress at the bottom fiber.

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

- There are four distinct stages of loading for a prestressed member as follows:

Stage 1:

- Only initial prestress force (P_i) along with self-weight of member are acting. The concrete strength at transfer of prestress is to be considered.

Stage 2:

- This is a stage that may come any time during the life of a structure when no external load is acting but full losses have already occurred. The prestress force becomes P_e and full concrete strength is available.

Stage 3:

- This is a stage when full service dead load and live load are acting along with self-weight and effective prestress force.

Stage 4:

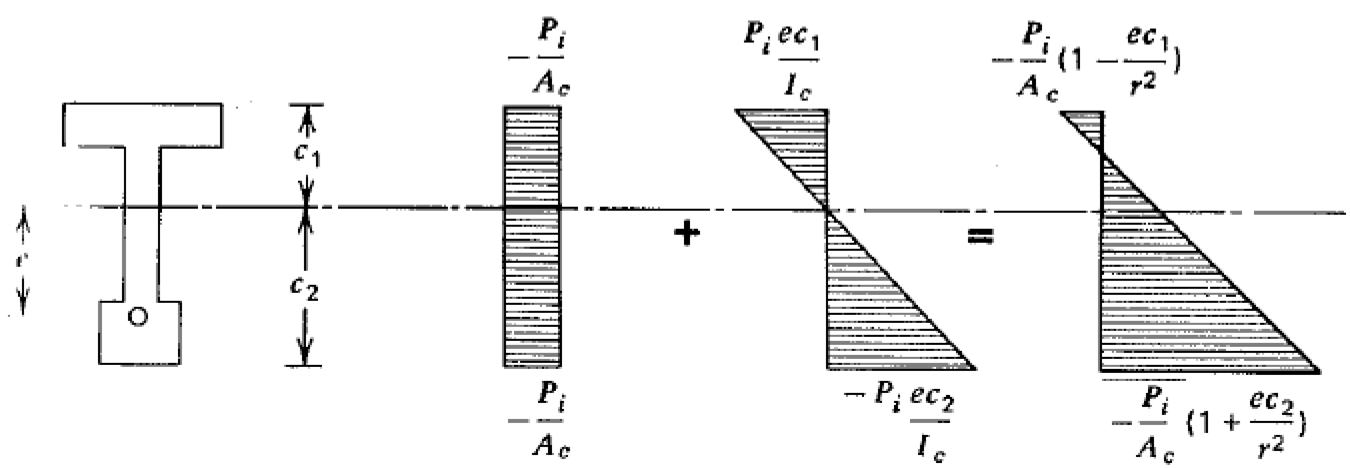
- This stage represents the maximum expected overload stage. The stresses in this stage go to the inelastic range.

PRE-STRESSING- Elastic Stresses

- As long as the beam remains uncracked, and both steel and concrete are stressed within their elastic ranges, then concrete stresses can be found using the equations of mechanics, based on linear elastic behavior.

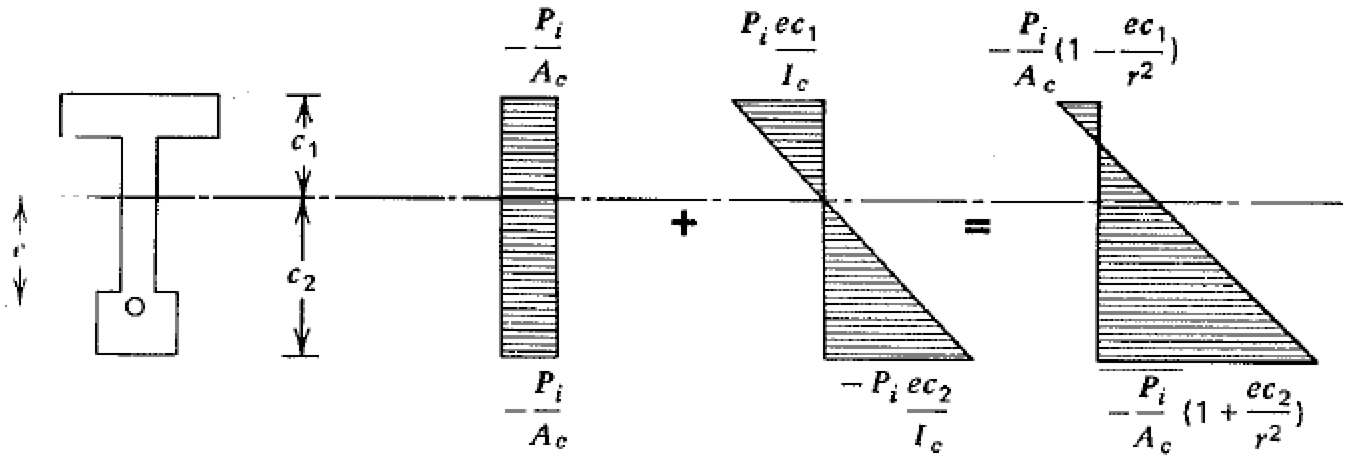
PRE-STRESSING- Elastic Stresses

- If the member is subjected only to the initial pre-stressing force P_i .
- The concrete f_1 at the top face of the member and f_2 at the bottom face can be found by superimposing axial and bending effects.



(a)

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

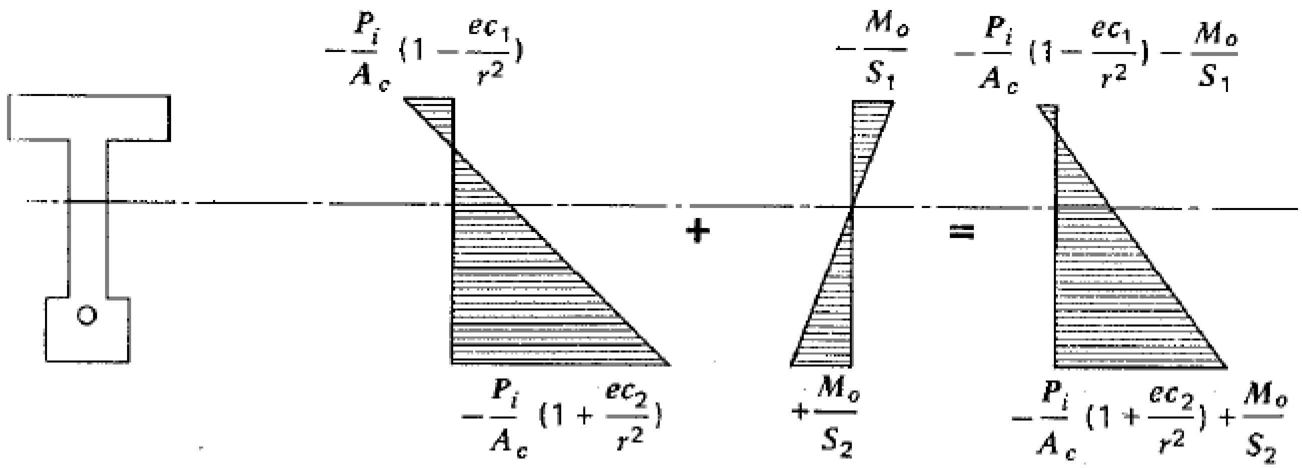


(a)

$$f_1 = -\frac{P_i}{A_c} + \frac{P_i e c_1}{I_c} \qquad f_1 = -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right)$$

$$f_2 = -\frac{P_i}{A_c} - \frac{P_i e c_2}{I_c} \qquad f_2 = -\frac{P_i}{A_c}\left(1 + \frac{ec_2}{r^2}\right)$$

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE



(b)

$$f_1 = -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right) - \frac{M_o}{S_1}$$

$$f_2 = -\frac{P_i}{A_c}\left(1 + \frac{ec_2}{r^2}\right) + \frac{M_o}{S_2}$$

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

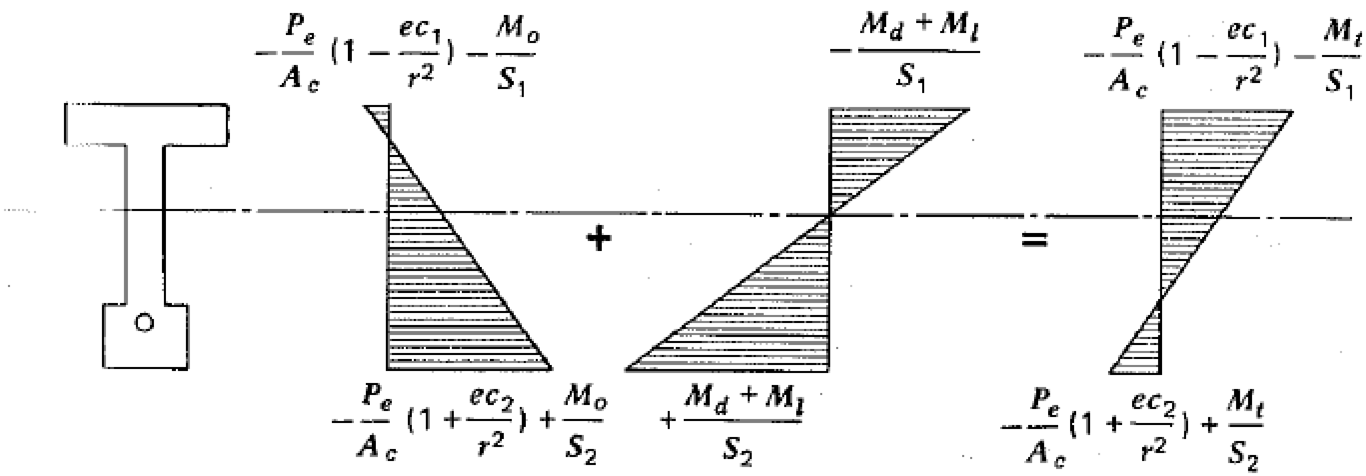


FIGURE 3.2 Elastic stresses in an uncracked prestressed beam. (*a*) Effect of initial prestress. (*b*) Effect of initial prestress plus self-weight. (*c*) Effect of final prestress plus full service load.

$$f_1 = -\frac{P_e}{A_c}\left(1 - \frac{ec_1}{r^2}\right) - \frac{M_t}{S_1}$$

$$f_2 = -\frac{P_e}{A_c}\left(1 + \frac{ec_2}{r^2}\right) + \frac{M_t}{S_2}$$

$$M_t = M_o + M_d + M_l$$

PRESTRESSED CONCRETE- CROSS SECTION KERN OR CORE

- When the pre-stressing force, acting alone, causes no tension in the cross section, it is said to be acting within the kern or the core of the cross section.
- In the limiting cases, triangular stress distributions will result from application of the pre-stress force, with zero concrete stress at the top or the bottom of the member.
- The kern limit dimensions can be found from Eqs. (3.4a) and (3.4b).
- To find the lower kern dimension, the concrete stress at the top surface is set equal to zero as illustrated in Fig.

PRESTRESSED CONCRETE- CROSS SECTION KERN OR CORE

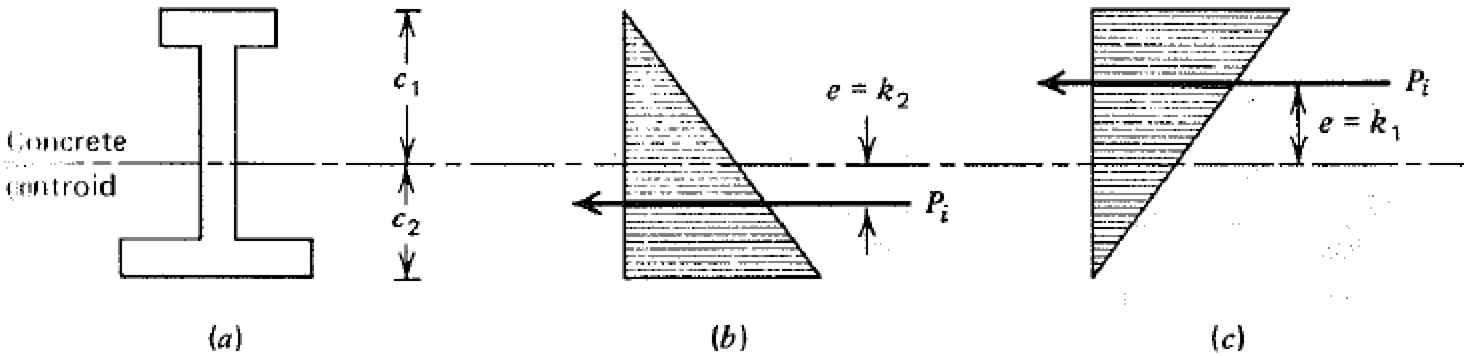


FIGURE 3.3 Stress distributions for prestress force applied at kern limits. (*a*) Cross section. (*b*) Lower kern limit. (*c*) Upper kern limit.

$$f_1 = -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right) = 0$$

$$1 - \frac{k_2 c_1}{r^2} = 0$$

$$k_2 = \frac{r^2}{c_1}$$

PRESTRESSED CONCRETE- CROSS SECTION KERN OR CORE

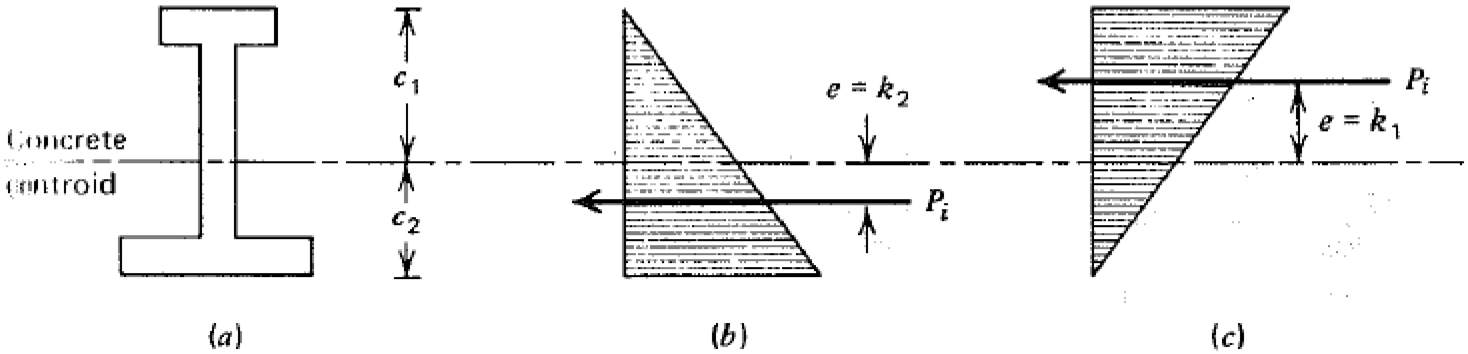


FIGURE 3.3 Stress distributions for prestress force applied at kern limits. (*a*) Cross section. (*b*) Lower kern limit. (*c*) Upper kern limit.

Similarly, the upper kern limit is found by setting the expression for the concrete stress at the bottom surface equal to zero, from which

$$k_1 = -\frac{r^2}{c_2} \tag{3.8b}$$

the minus sign confirming that the limit dimension is measured upward from the concrete centroid.

PRESTRESSED CONCRETE- PROBLEM

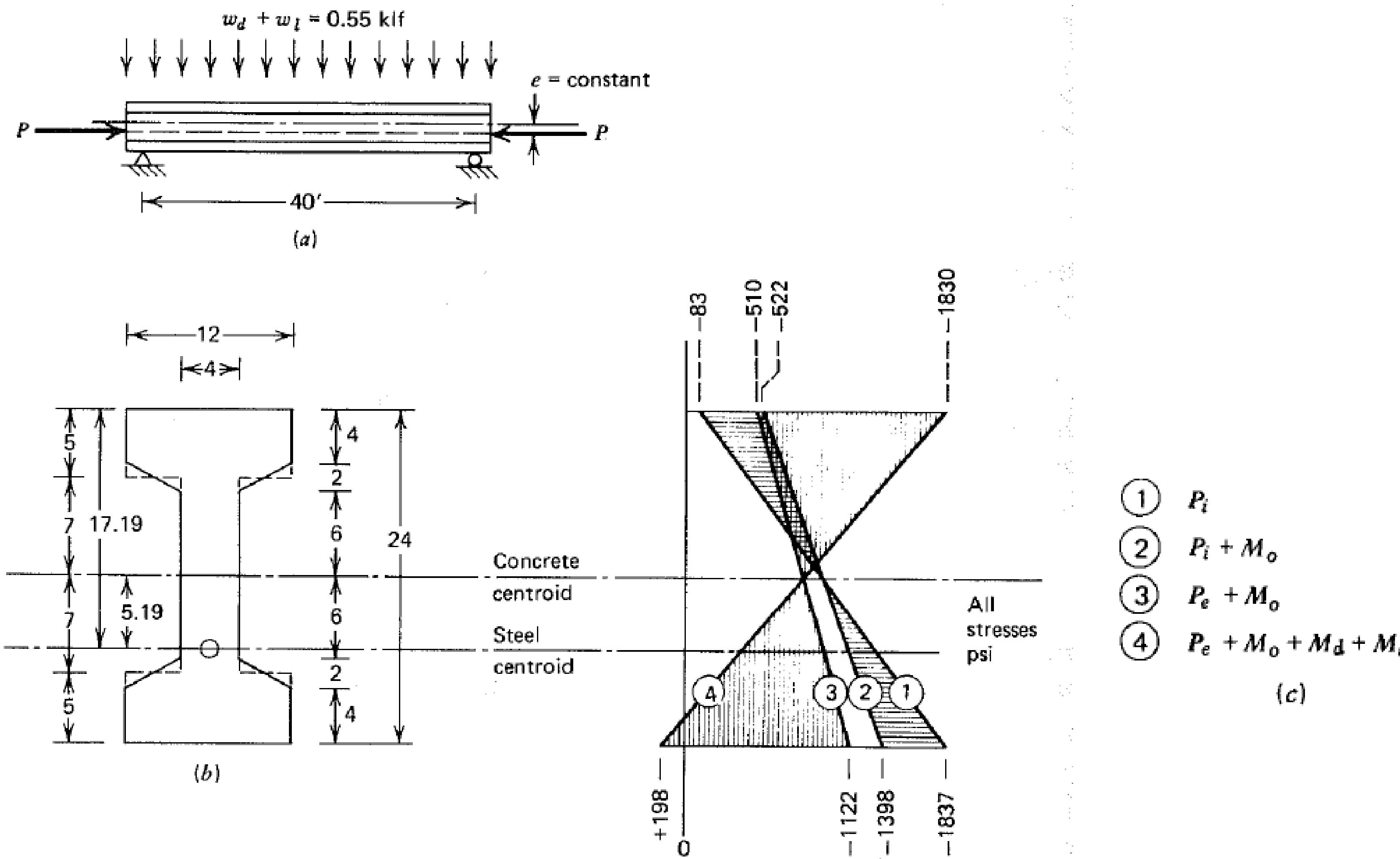
- The simply supported I-beam shown in cross section and elevation in Fig. 3.4 is to carry a uniformly distributed service dead and live load totaling 0.55 kips/ft over the 40-ft span, in addition to its own weight. Normal concrete having density of 150 lb/ft^3 will be used.
- The beam will be pre-tensioned using multiple seven-wire strands; eccentricity is constant and equal to 5.19 in. The prestress force F; immediately after transfer (after elastic shortening loss) is 169 kips. Time-dependent losses due to shrinkage, creep, and relaxation total 15 percent of the initial prestress force. Find the concrete flexural stresses at midspan and support sections under initial and final conditions. (Load = 8.02 kN/m, span = 12.19 m, density = 24 kN/m3, e = 132 mm, F; = 752 kN.)

PRESTRESSED CONCRETE- PROBLEM

- For pre tensioned beams using stranded cables, the difference between section properties based on the gross and transformed section is usually small.
- Accordingly, all calculations will be based on properties of the gross concrete section. Average flange thickness will be used, as shown in Fig. 3.4b.
- For that section

Moment of inertia	$I_c = 12{,}000 \text{ in.}^4$ $(4.99 \times 10^9 \text{ mm}^4)$
Concrete area	$A_c = 176 \text{ in.}^2$ $(114 \times 10^3 \text{ mm}^2)$
Section modulus	$S_1 = S_2 = 1{,}000 \text{ in.}^3$ $(16.4 \times 10^6 \text{ mm}^3)$
Radius of gyration	$r^2 = I_c/A_c = 68.2 \text{ in.}^2$ $(44 \times 10^3 \text{ mm}^3)$

PRESTRESSED CONCRETE- PROBLEM



PRESTRESSED CONCRETE- PROBLEM

Stresses in the concrete resulting from the initial prestress force of 169 kips may be found by Eq. (3.4). At the top and bottom surfaces, respectively, these stresses are

$$f_1 = -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right) = -\frac{169{,}000}{176}\left(1 - \frac{5.19 \times 12}{68.2}\right) = -83 \text{ psi}$$

$$f_2 = -\frac{P_i}{A_c}\left(1 + \frac{ec_2}{r^2}\right) = -\frac{169{,}000}{176}\left(1 + \frac{5.19 \times 12}{68.2}\right) = -1837 \text{ psi}$$

$$w_o = \frac{176}{144} \times 0.150 = 0.183 \text{ kips/ft}$$

At midspan the corresponding moment is

$$M_o = \frac{1}{8} \times 0.183 \times 40^2 = 36.6 \text{ ft-kips}$$

This moment produces top and bottom concrete stresses at midspan of

$$f_1 = -\frac{M_o}{S_1} = -\frac{36.6 \times 12{,}000}{1{,}000} = -439 \text{ psi}$$

$$f_2 = +\frac{M_o}{S_2} = +\frac{36.6 \times 12{,}000}{1{,}000} = +439 \text{ psi}$$

PRESTRESSED CONCRETE- PROBLEM

The combined effect of initial prestress and self-weight is found by superposition.

$$f_1 = -83 \quad - 439 = -522 \text{ psi } (-3.6 \text{ MPa})$$

$$f_2 = -1{,}837 + 439 = -1398 \text{ psi } (-9.6 \text{ MPa})$$

as shown by distribution (2).

Time-dependent losses are 15 percent of P_i. Accordingly, the effectiveness ratio

$$R = \frac{P_e}{P_i} = 0.85$$

and the effective prestress force after all losses is

$$P_e = 0.85 \times 169 = 144 \text{ kips}$$

PRESTRESSED CONCRETE- PROBLEM

Top and bottom concrete stresses due to P_e are

$$f_1 = 0.85 \times (-83) \quad = -71 \text{ psi}$$

$$f_2 = 0.85 \times (-1{,}837) = -1{,}561 \text{ psi}$$

Flexural stresses due to self-weight must be superimposed as before. The resulting midspan stresses due to P_e and self-weight are

$$f_1 = -71 \quad -439 = -510 \text{ psi } (-3.5 \text{ MPa})$$

$$f_2 = -1{,}561 + 439 = -1{,}122 \text{ psi } (-7.7 \text{ MPa})$$

as given by distribution (3) in Fig. 3.4*c*.

The midspan moment due to superimposed dead and live load is

$$M_d + M_l = \frac{1}{8} \times 0.55 \times 40^2 = 110 \text{ ft-kips}$$

and the corresponding concrete stresses are

$$f_1 = -\frac{110 \times 12{,}000}{1{,}000} = -1320 \text{ psi}$$

$$f_2 = +\frac{110 \times 12{,}000}{1{,}000} = +1320 \text{ psi}$$

PRESTRESSED CONCRETE- PROBLEM

Then, combining effective prestress force with moments due to self-weight and superimposed load, the stresses produced are

$$f_1 = -510 \quad - 1{,}320 = -1{,}830 \text{ psi } (-12.6 \text{ MPa})$$

$$f_2 = -1{,}122 + 1{,}320 = +198 \text{ psi } (+1.4 \text{ MPa})$$

as shown by distribution (4). In Fig. 3.4*c*, the stress change resulting from the member self-weight is shown by horizontal shading, and that resulting from superimposed dead and live loads is shown by vertical shading.

At the support sections, the transverse loads cause no flexural stresses, and concrete stresses are those resulting from prestress alone. The initial values of −83 and −1,837 psi at the top and bottom surfaces gradually reduce to −71 and −1,561 psi, respectively, as time-dependent losses occur.