

PRESTRESSED CONCRETE-INTRODUCTION

- Prestressing of a member means introduction of permanent stresses in the member, before the application of design loads, of a calculated magnitude and designed distribution such that these stresses neutralize / balance the stresses due to external loading up to a desired level.
- The compressive strength of concrete is high and tensile strength of concrete is low and unreliable.
- The basic purpose of prestressing of concrete structures is to reduce or eliminate the tensile stresses by presence of a pre-compression in the member.
- The process of prestressing improves overall performance of reinforced concrete structures.

PRESTRESSED CONCRETE-INTRODUCTION

- Prestressing is applied by pulling high strength steel wires or strands (a group of which placed together is called tendon) using hydraulic jacks and then anchoring these at the ends by special anchorage devices or by bond between tendons and concrete (Fig. 19.1).

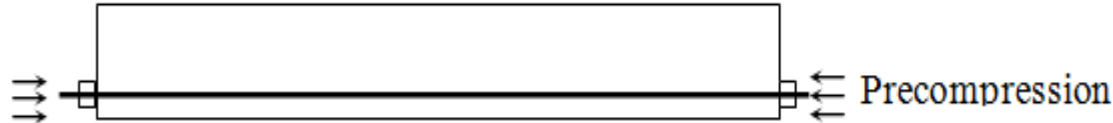


Fig. 19.1. Prestressing of Beams.

- When the steel tries to regain its original length due to elasticity, it exerts compressive forces on the adjoining concrete.
- Due to this compressive force P , shown in Fig. 19.2, a negative bending moment is produced in the beam equal to $P \times e$.

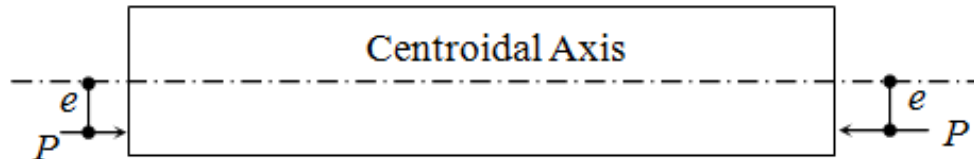


Fig. 19.2. Prestressing Force and Its Eccentricity.

PRESTRESSED CONCRETE-INTRODUCTION

- The beam is curved up, tension is developed at the top and compression is developed at the bottom.
- The member is well set to carry loads producing positive bending moment as opposite stresses are already present in the member.
- Positive moment due to load will produce tension at the bottom and compression at the top.
- As the load is gradually increased the compression at the bottom reduces, becomes zero and tension is then developed.
- However, this tension will be significantly lesser than for similar beam with same loading but without prestressing.

PRESTRESSED CONCRETE-INTRODUCTION

- High strength steel may not efficiently be used in ordinary reinforced concrete structures, as it will produce excessive cracking.
- Modulus of elasticity of high-strength steel is almost the same as that of ordinary steel.
- Very high strains are produced in high strength steel close to its yield strength and hence cause wide and excessive cracking in adjoining concrete.
- The cracking reduces moment of inertia, increases deflection, reduces shear strength and causes corrosion of steel reinforcement.
- The appearance of the members is also disturbed due to cracking and structure above the member under consideration may also be damaged due to large deflections

PRESTRESSED CONCRETE-INTRODUCTION

- Use of high strength concrete in ordinary reinforced concrete structures is not much beneficial as only a smaller part of the section on compression side is subjected to maximum compressive stresses.
- In greater part of the section, compressive strength is not fully utilized and the tensile strength of concrete is neglected altogether after the first cracking.
- In prestressed concrete members, larger portion of the cross-section is subjected to compressive stresses and high strength of concrete is effectively utilized.

PRESTRESSED CONCRETE- PRESTRESSING STEEL

- Ordinary steels, such as Grade 280 steel, are ineffective in providing effective prestressing.
- Grade 280 steel is used to prestress a beam and is stretched to a stress of $0.94f_y = 263\text{MPa}$.
- The corresponding steel strain is $\varepsilon_s = 263/200000 = 0.00132$.
- In case the steel bars are anchored, there will be some slip in the anchorage and some elastic shortening of the member will take place reducing the strain by approximately 0.0007.
- Steel strain just after transfer will be approximately $0.00132 - 0.0007 = 0.00062$ and about half of the prestressing force will be lost almost suddenly.

PRESTRESSED CONCRETE- PRESTRESSING STEEL

- Long term strain in concrete also occurs due to shrinkage and concrete, which may be up to 0.0008.
- Clearly all of the prestress will be lost for such a member after one year or so.
- Relaxation in steel stress may also take place with time .
- Compared with this, is high-strength steels are used, the losses in terms of strains remain nearly the same and very high effective strains and stresses are left within the member.
- The prestressing steel may be in the form of smaller diameter wires, relatively larger diameter alloy steel bars or in the form of strands

PRESTRESSED CONCRETE- PRESTRESSING STEEL

- The term strand is used for a group of wires helically wound. A seven-wire strand is shown in Fig. 19.3.
- The term tendon is used for one group of reinforcement consisting of a single wire, a strand or a group of strands used together.



Fig. 19.3. A Seven Wire Strand.

PRESTRESSED CONCRETE- PRESTRESSING STEEL

- Various types of available prestressing steel are given in Table 19.1 and the following standards are used to check the prestressing steel requirements:
- **ASTM A421** – Standard specification for uncoated stress-relieved steel wire for prestressed concrete.
- **ASTM A416** – Standard specification for steel strand (uncoated seven wire stress-relieved) for prestressed concrete.
- **ASTM A722** – Standard specification for uncoated high-strength steel bar for prestressed concrete.

PRESTRESSED CONCRETE- PRESTRESSING STEEL

Table 19.1. Standard Metric Prestressing Steels.

Type	Nominal Diameter (mm)	Nominal Area (mm ²)	Nominal Weight (kg/m)
Seven Wire Strand (Grade 1725)	6.35	23.22	0.0182
	7.94	37.42	0.294
	9.53	51.61	0.405
	11.11	69.68	0.548
	12.70	92.90	0.730
	15.24	139.35	1.094
Seven Wire Strand (Grade 1860)	9.53	54.84	0.432
	11.11	74.19	0.582
	12.70	98.71	0.775
	15.24	140.00	1.102
Prestressing Wire (Grades 1620, 1655, 1725)	4.88	18.7	0.146
	4.98	19.4	0.149
	6.35	32	0.253
	7.1	39	0.298
Prestressing Bars, Plain (Grades 1035, 1100)	19	284	2.23
	22	387	3.04
	25	503	3.97
	29	639	5.03
	32	794	6.21
	35	955	7.52
Prestressing Bars, Deformed (Grades 1035, 1100)	15	181	1.46
	20	271	2.22
	26	548	4.48
	32	806	6.54
	36	1019	8.28

PRESTRESSED CONCRETE- PRESTRESSING STEEL

- The typical stress-strain curves of Fig. 19.4 show that the prestressing steels do not have a well-defined yield and the yielding occurs gradually.

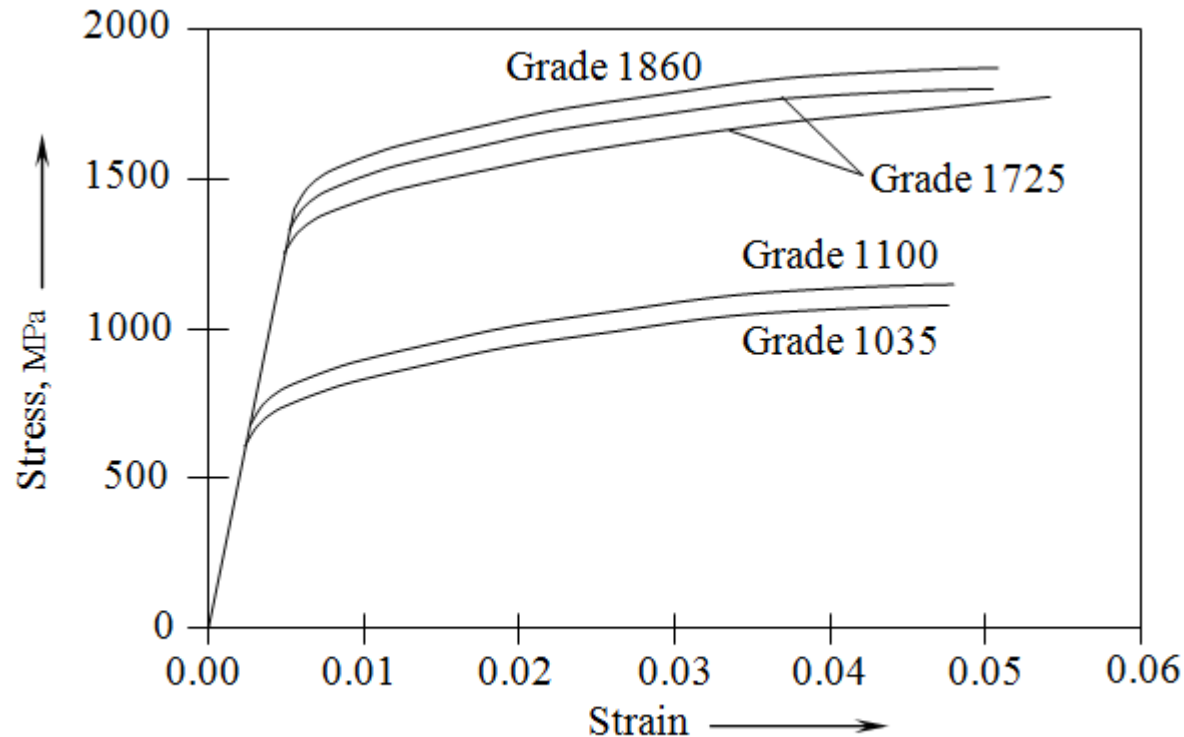


Fig. 19.4. Typical Stress-Strain Curves for Prestressing Steels.

PRESTRESSED CONCRETE- PRESTRESSING STEEL

- Equivalent yield strengths (f_{py}) are defined as the stresses corresponding to 1% strain for strands and wires and corresponding to 0.7% strain for alloy steel bars.
- The grades of prestressing steels are defined with respect to the ultimate strengths (f_{pu}) in MPa.
- Prestressing steels having significantly less ductility, the failure strain may be between 0.05 and 0.06 having a minimum requirement of 3.5%.
- Compared with this a Grade 420 reinforcing bar may fail at a strain of 13% or more.
- The field stress (f_p) is approximately $0.9f_{pu}$ but should be determined exactly. Modulus of elasticity (E_p) for prestressing steel is approximately as follows:

Strands	197,000 MPa
Prestressing wires	200,000 MPa
Alloy steel bars	207,000 MPa

PRESTRESSED CONCRETE- MAXIMUM PERMISSIBLE PRESTRESSING STEEL STRESSES

- Table 19.2 gives the ACI maximum steel stresses at various stages of loading for the prestressed members.

Table 19.2. ACI Maximum Permissible Prestressing Steel Stresses.

S.No.	Description	Maximum Stress Level
1.	Stress due to prestressing steel jacking force.	It should not exceed a stress smaller of $0.80f_{pu}$ and $0.94f_{py}$.
2.	At anchorage devices for post-tensioning tendons, immediately after anchorage.	$0.70f_{pu}$

PRESTRESSED CONCRETE- MAXIMUM PERMISSIBLE PRESTRESSING STEEL STRESSES

- These high stresses can easily be measured by either using the hydraulic jacking pressure and area of piston or by using steel strains and elongations.
- The AASHTO stress limits for prestressing tendons are given in Table 19.3

Table 19.3. AASHTO Maximum Permissible Prestressing Steel Stresses.

Loading Stage	Tendon Type		
	Stress Relieved Strand and Plain High-Strength Bars	Low Relaxation Strand	Deformed High-Strength Bars
At Jacking (f_{pj}):			
i) Pre-tensioning	$0.72 f_{pu}$	$0.78 f_{pu}$	–
ii) Post-tensioning	$0.76 f_{pu}$	$0.80 f_{pu}$	$0.75 f_{pu}$
After Transfer (f_{pt}):			
i) Pre-tensioning	$0.70 f_{pu}$	$0.74 f_{pu}$	–
ii) Post-tensioning, general	$0.70 f_{pu}$	$0.70 f_{pu}$	$0.66 f_{pu}$
iii) Post-tensioning at anchorages and couplers immediately after anchor set	$0.70 f_{pu}$	$0.74 f_{pu}$	$0.66 f_{pu}$
At Service Limit State (f_{pe}):			
i) After losses	$0.80 f_{py}$	$0.80 f_{py}$	$0.80 f_{py}$

PRESTRESSED CONCRETE- CONCRETE FOR PRESTRESSED CONSTRUCTION

- Higher strength concrete, having $f_c' \geq 35$ MPa, is usually used for prestressed members for quick and efficient construction with lesser loss of prestressing force due to elastic shortening, creep and shrinkage.
- The advantages of high strength concrete in prestressed construction are as follows:
 1. With larger compressive strength of concrete, its modulus of elasticity is increased reducing the elastic shortening due to prestress force. Further, long term deformations due to creep and shrinkage are also reduced. Hence, by the use of high strength concrete, the prestress losses are significantly reduced increasing the efficiency of such construction.
 2. The concrete gains high early strength and hence the prestress may be applied to the concrete earlier. The speed of construction is increased when high strength concrete is used.

PRESTRESSED CONCRETE- CONCRETE FOR PRESTRESSED CONSTRUCTION

3. Bearing strength is better for high strength concrete. This makes it easy to transfer the prestress forces at the anchorages, which may require lesser contact area of anchorage fittings.
 4. The bond between steel and concrete is improved when high strength concrete is used.
 5. Higher strengths are easy to achieve in case of prestressed precast construction in factories where batching, mixing, placing and curing is carried out under controlled conditions.
- Following notation is used to specify strength of concrete for prestressed construction:
 - f_c' = the specified minimum 28-day cylinder compressive strength of concrete.
 - f_{ci}' = the minimum cylinder compressive strength of concrete at the time of initial transfer of prestressing force.

PRESTRESSED CONCRETE- ACI PERMISSIBLE CONCRETE STRESSES

- The allowable stresses in concrete are different at various stages of loading and for different categories of flexural members.
- Prestressed flexural members are classified into three classes depending upon their behavior.

Class U Members

- These members behave as uncracked members and computed extreme fiber tensile stress at service loads (f_t) is lesser than or equal to .
- Prestressed two-way slab systems are also considered in this category.

Class T Members

- These are transition members between uncracked and cracked cases.
- The computed extreme fiber stress is within the limits: $< f_t \leq$.
- For this class, the stresses at service loads are allowed to be computed using the uncracked section as for class U.
- Deflections are calculated by bilinear curve for cracked section.

PRESTRESSED CONCRETE- ACI PERMISSIBLE CONCRETE STRESSES

Class C Members

- These are cracked members whose behavior at service loads must be studied using cracked section.
- The computed extreme fiber stress, f_t , is greater than .
- Deflections are to be calculated by bilinear curve for cracked section.
- Side skin reinforcement may be required as in ACI 10.6.7.
- A good reference for analysis on the basis of cracked section is “Analysis of Cracked Prestressed Concrete Sections: A Practical Approach,” PCI Journal, V.43, No.4, Jul-Aug, 1998

PRESTRESSED CONCRETE- ACI PERMISSIBLE CONCRETE STRESSES

- Concrete stresses immediately after transfer of prestressing force before the time-dependent losses should not be more than the following limits:
 - a) Extreme fiber stress in compression, except as in (b). $0.60f'_c$
 - b) Extreme fiber stress in compression at ends of simply supported members. $0.70\sqrt{f'_c}$
 - c) Extreme fiber stress in tension, except as in (d). If this limit is exceeded, additional bonded reinforcement is to be provided in tension zone. $0.25\sqrt{f'_c}$
 - d) Extreme fiber stress in tension at ends of simply supported members. If this limit is exceeded, additional bonded reinforcement is to be provided in tension zone. $0.5\sqrt{f'_c}$

PRESTRESSED CONCRETE- ACI PERMISSIBLE CONCRETE STRESSES

- If computed tensile stresses are greater than the limits in (c) and (d) above, bonded additional non-prestressed or prestressed reinforcement is to be provided to resist the total tensile force in concrete computed using uncracked section properties.
- The stress in steel may be taken equal to lesser of $0.6f_y$ and 210 MPa (not a code requirement).

PRESTRESSED CONCRETE- ACI PERMISSIBLE CONCRETE STRESSES

- For classes U and T members, stresses in concrete at service loads based on uncracked section properties after all the prestress losses should not exceed the following limits:
 - a) Extreme fiber compressive stress due to prestress plus sustained load. $0.45f'_c$
 - b) Extreme fiber compressive stress due to prestress plus total load. $0.60f'_c$
 - c) Extreme fiber tensile stress in precompressed zone due to prestress plus total load should be as under:
 - i) U-type member $0.62\sqrt{f'_c}$
 - ii) T-type member $\sqrt{f'_c}$
- The sustained load in the above expressions includes dead and live load only in cases where its duration is sufficiently large to cause significant time-dependent deformations.

PRESTRESSED CONCRETE- ACI PERMISSIBLE CONCRETE STRESSES

- The AASHTO-1994 stress limits are as under:

Before Losses For Fully Prestressed Components

a) Compressive Stresses:

i) In pre-tensioned components.

$$0.60 f'_{ci}$$

ii) In post-tensioned components.

$$0.55 f'_{ci}$$

b) Tensile Stresses:

i) In areas other than the precompressed tensile zones and without bonded auxiliary reinforcement.

$$0.25\sqrt{f'_{ci}}$$

$$\leq 1.38 \text{ MPa}$$

ii) In areas with bonded reinforcement that is sufficient to resist 120% of the tension force in the cracked concrete computed on the basis of an uncracked section.

$$0.58\sqrt{f'_{ci}}$$

PRESTRESSED CONCRETE- ACI PERMISSIBLE CONCRETE STRESSES

After Losses For Fully Prestressed Components

- a) Compressive Stresses:
- i) Due to permanent loads. $0.45 f'_c$
 - ii) Due to permanent and transient loads and during shipping and handling. $0.60 f'_c$
- b) Tensile Stresses in Precompressed Tensile Zone Assuming Uncracked Sections:
- i) For components with bonded prestressing tendons other than piles. $0.50 \sqrt{f'_c}$
 - ii) For components subjected to severe corrosive conditions. $0.25 \sqrt{f'_c}$
 - iii) For components with unbonded prestressing tendons. Zero tension

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

- Class C members are usually designed by using the strength design method at the ultimate limit state.
- Class U and T members are basically designed at the service load limit state by checking stresses at various stages of loading and the main objective is usually to improve the performance of the members at service loads.
- However, these are also checked for the strength limit state against overloads.
- The deflections and cracking under service loads must also be investigated.
- As a simplification, gross area may be considered in place of uncracked transformed sectional area.

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

- Following notation will be used in further discussion:

P_i = initial prestress force just after transfer without time-dependent losses.

P_e = effective prestress force after all the short and long-term losses.

R = effectiveness ratio = P_e / P_i .

e = eccentricity of prestressing force from the centroid at a particular section.

M_g = bending moment due to self weight.

M_d = bending moment due to imposed dead load.

M_l = bending moment due to service live load.

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

$C_1 =$ distance of top fiber from the centroid.

$C_2 =$ distance of bottom fiber from the centroid.

$S_1 =$ elastic section modulus with respect to top fibers.

$S_2 =$ elastic section modulus with respect to bottom fibers.

$f_1 =$ stress at the top fiber.

$f_2 =$ stress at the bottom fiber.

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

- There are four distinct stages of loading for a prestressed member as follows:

Stage 1:

- Only initial prestress force (P_i) along with self-weight of member are acting. The concrete strength at transfer of prestress is to be considered.

Stage 2:

- This is a stage that may come any time during the life of a structure when no external load is acting but full losses have already occurred. The prestress force becomes P_e and full concrete strength is available.

Stage 3:

- This is a stage when full service dead load and live load are acting along with self-weight and effective prestress force.

Stage 4:

- This stage represents the maximum expected overload stage. The stresses in this stage go to the inelastic range.

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

Stresses At Stage-1

(a) Only P_i Is Acting:

- The prestress force (P_i) acting at an eccentricity (e) from the centroid of the section is equivalent to a concentric force P_i and a moment $P_i \times e$.
- The force P_i produces a uniform compression and the moment $P_i \times e$ produces tension at the top and compression at the bottom for beam of Fig. 19.5.

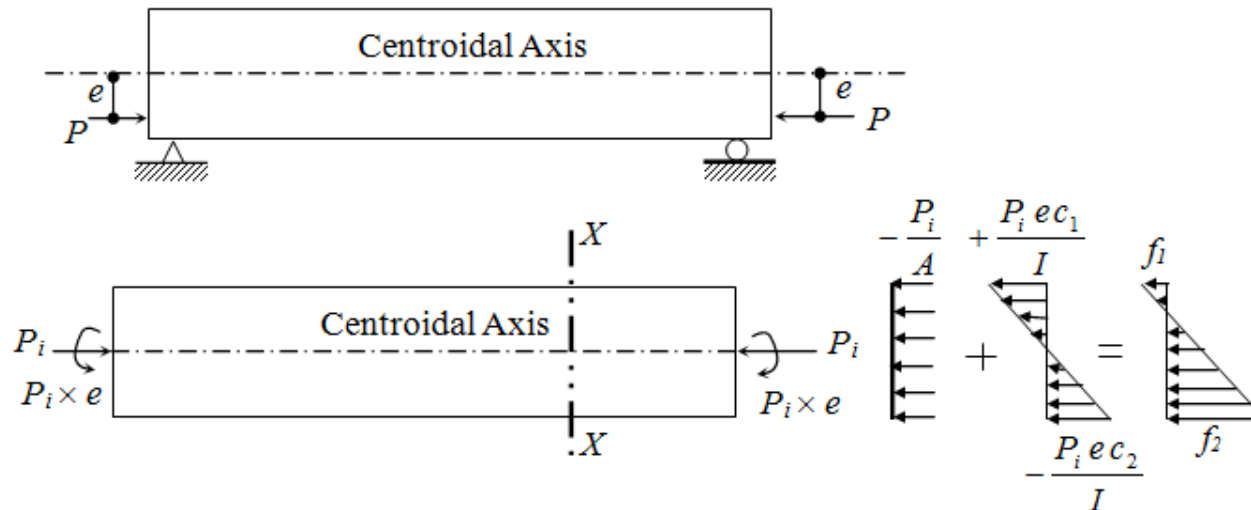


Fig. 19.5. Stress Diagrams for Stage-1a of Loading a Prestressed Beam.

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

- The result may be smaller tension at the top and larger compression at the bottom.

$$f_1 = -\frac{P_i}{A} + \frac{P_i e c_1}{A r^2} = -\frac{P_i}{A} \left(1 - \frac{e c_1}{r^2} \right)$$

$$f_2 = -\frac{P_i}{A} + \frac{P_i e c_2}{A r^2} = -\frac{P_i}{A} \left(1 - \frac{e c_2}{r^2} \right)$$

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

(b) Prestress P_i And Self Weight Are Acting:

- The self-weight moment modifies the above stress diagram and is given in Fig. 19.6.

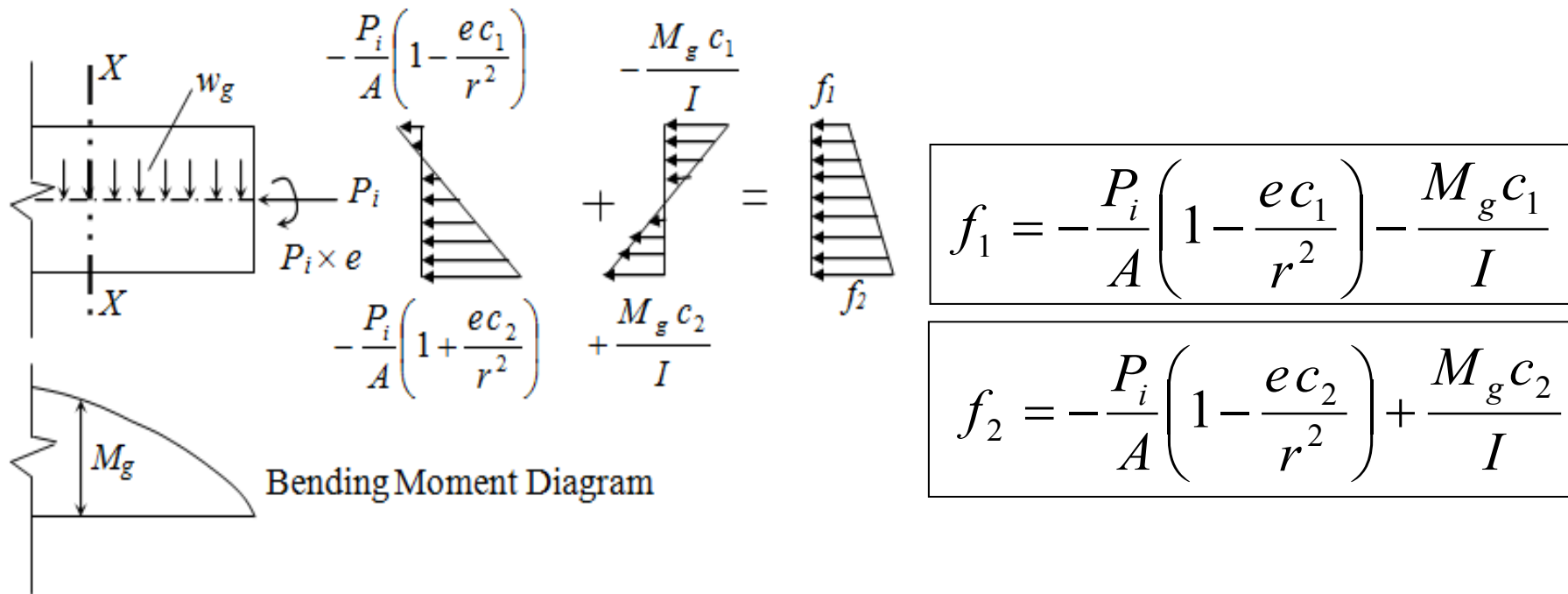


Fig. 19.6. Stress Diagrams for Stage-1b of Loading a Prestressed Beam.

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

Stresses At Stage-2

- The shapes of stress diagrams are the same as those of stage-1b but the ordinates are changed due to replacement of P_i with P_e . The top and bottom stresses are given as under:

$$f_1 = -\frac{P_e}{A} \left(1 - \frac{ec_1}{r^2} \right) - \frac{M_g c_1}{I}$$

$$f_2 = -\frac{P_e}{A} \left(1 - \frac{ec_2}{r^2} \right) + \frac{M_g c_2}{I}$$

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

Stresses At Stage-3

- The service load moments further modify the stress diagrams by producing larger tensile stresses at the bottom and compressive stresses at the top, as shown in Fig. 19.7.

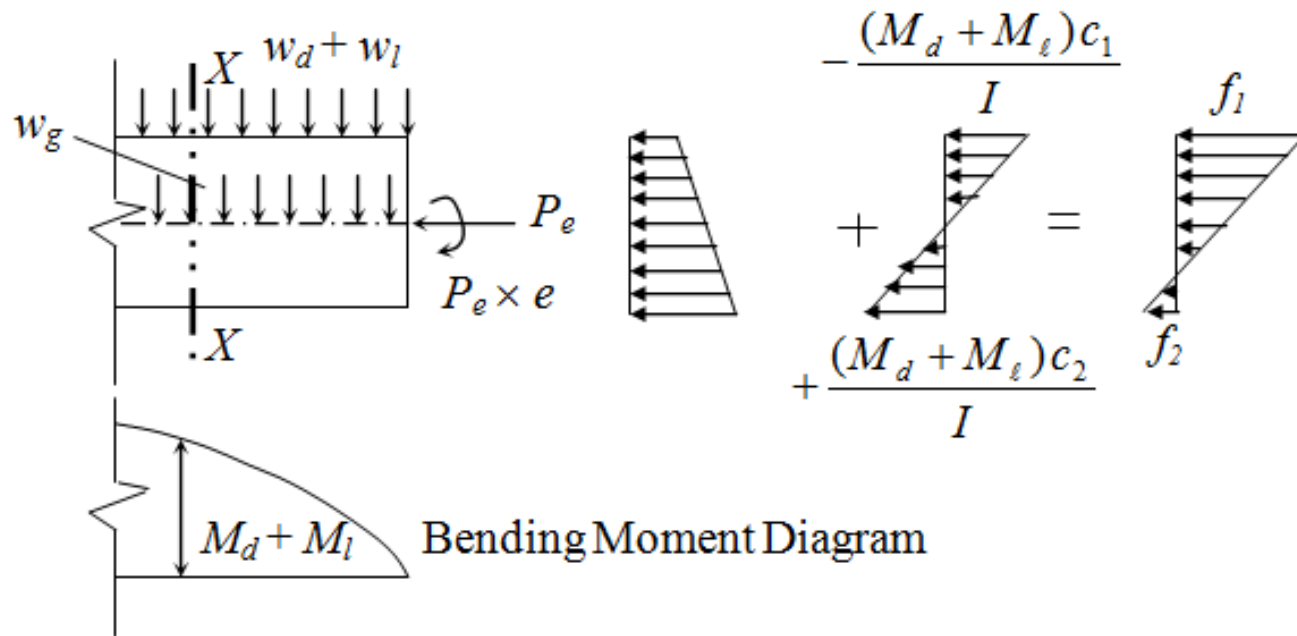


Fig. 19.7. Stress Diagrams for Stage-3 of Loading a Prestressed Beam.

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

- The resultant stresses at this stage of loading are given by the following expressions:

$$f_1 = -\frac{P_e}{A} \left(1 - \frac{ec_1}{r^2} \right) - \frac{c_1}{I} (M_g + M_d + M_\ell)$$

$$f_2 = -\frac{P_e}{A} \left(1 - \frac{ec_2}{r^2} \right) + \frac{c_2}{I} (M_g + M_d + M_\ell)$$

- For a well-designed member, all of the above stresses must satisfy the ACI limits

PRESTRESSED CONCRETE- FLEXURAL ANALYSIS CONSIDERING SERVICE LOAD LIMIT STATE

- Depending upon the relative magnitudes of prestress force, self weight and service loads, the final stress diagram in stage-3 may have one situation out of : a) tension at the bottom b) compression at the bottom c) zero stress at the bottom (Fig. 19.8).

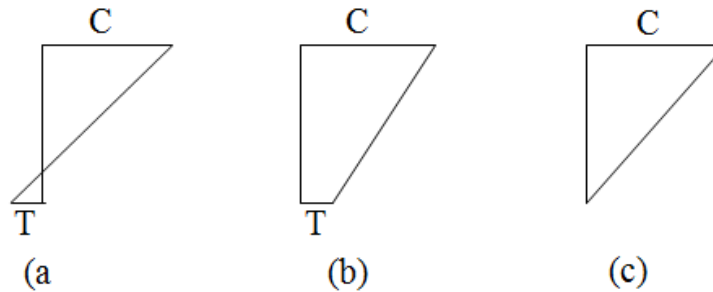


Fig. 19.8. Final Stress Diagrams at Stage-3.

- The general formula for stress may be written if prestress force is considered tension positive, the distance e and c are considered positive if these indicate fibers below the centroidal axes and if sagging moments are taken as positive.

$$f = \frac{P}{A} + \frac{P \times e \times c}{I} + \frac{M \times c}{I}$$

PRESTRESSED CONCRETE- KERN / CORE OF SECTION

- This is that inner portion of a section within which if the prestressing force is applied, no tension is developed anywhere in the section.
- The limiting points of this region are located by the distances y_1 and y_2 in Fig. 19.9.
- To find the upper kern point, the bottom tensile stress due to prestress force alone may be equated to zero and the prestress may be applied at negative eccentricity $-y_1$.

$$f_2 = -\frac{P}{A} \left(1 - \frac{e c_2}{r^2} \right) = 0$$

$$1 - \frac{(-y_1) c_2}{r^2} = 0$$

$$y_1 = \frac{r^2}{c_2}$$

$$\text{Similarly, } y_2 = \frac{r^2}{c_1}$$

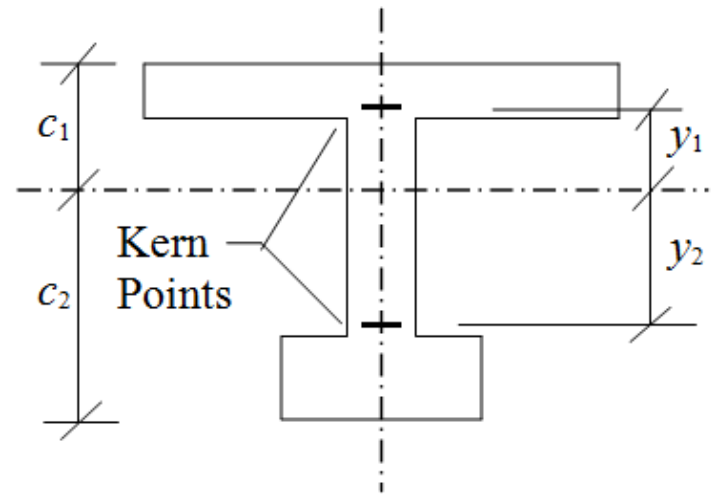


Fig. 19.9. Kern Points of an I-section.

PRESTRESSED CONCRETE- METHODS OF PRESTRESSING

- This is that inner portion of a section within which if the prestressing force is applied, no tension is developed anywhere in the section.
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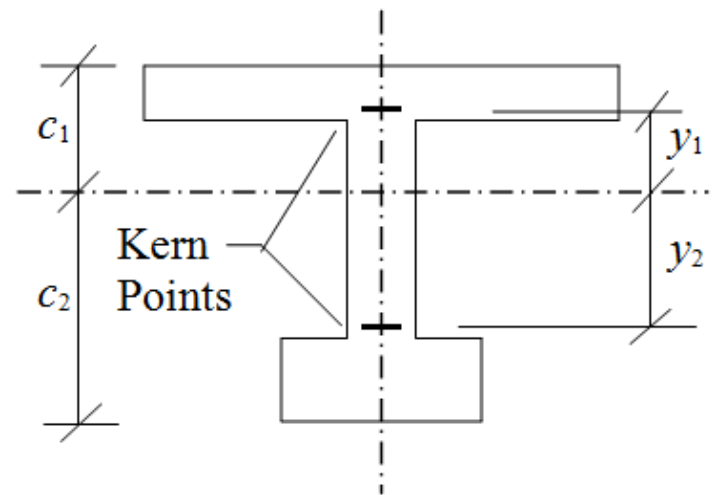


Fig. 19.9. Kern Points of an I-section.

PRESTRESSED CONCRETE- METHODS OF PRESTRESSING

Pre-Tensioning

- In this method, the prestressing tendons passing through the formwork are stretched between abutments before the placing of concrete (Fig.19.10).
- The concrete is then poured and allowed to harden.
- After sufficient gain of concrete strength, the tendons are cut / released at the ends.
- The prestress force is transferred to the concrete by bond between steel tendons and hardened concrete.

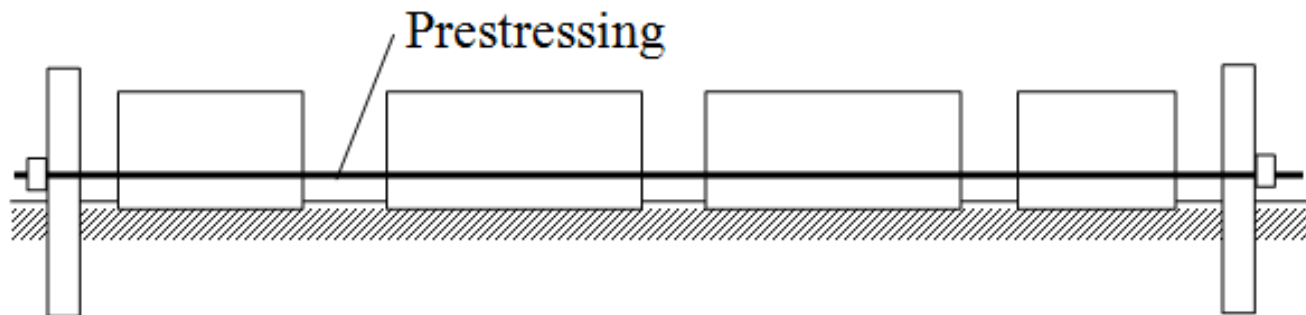


Fig. 19.10. Prestressing Bed for Pre-Tensioning.

PRESTRESSED CONCRETE- METHODS OF PRESTRESSING

- This method is suitable to maintain a constant eccentricity throughout the length of the member.
- In case of industrial applications, long casting beds can be made to accommodate many precast prestressed units.
- The tendons are stretched throughout the entire length of the bed and individual units are cast in portions of this total length.
- This method is very well suited for mass production.

PRESTRESSED CONCRETE- METHODS OF PRESTRESSING

Post-Tensioning

- In case of post-tensioning, the steel tendon is stretched and anchored against the member after the concrete has gain sufficient part of its strength (Fig. 19.11).
- Plain concrete beams or beams with some ordinary reinforcement are cast with embedded plastic or metal tubes forming ducts for the tendons to be placed later.

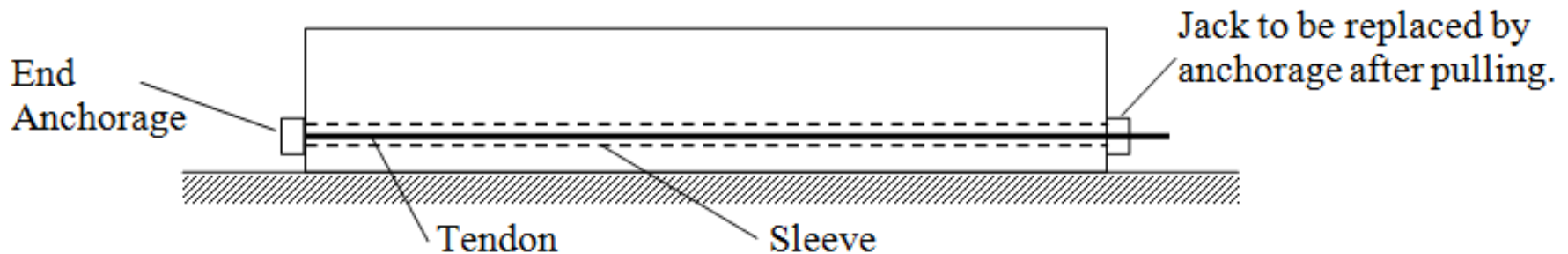


Fig. 19.11. Beam Subjected to Post-Tensioning.

PRESTRESSED CONCRETE- METHODS OF PRESTRESSING

- In this method, no or very less bond is developed between the prestressing steel and the concrete and hence the tendons are to be anchored at the ends of the member by mechanical anchorage devices.
- One end is gripped at the start of the tendon while the tendon is pulled at the other end by a hydraulic jack.
- Finally, anchorage device is also installed at the jacking end.
- The concrete must be strong against the end bearing at the anchorages.
- Sometimes, the conduit around the tendon is later on grouted by cement or mortar slurry.
- Post-tensioning can be applied to precast or cast-in-place members.

PRESTRESSED CONCRETE- ADVANTAGES OF PRESTRESSED CONCRETE

Prestressed concrete has the following advantages over ordinary reinforced concrete:

1. Due to prestressing entire cross-section is efficiently utilized to resist the applied loads and high-strength materials are effectively used in such construction. Smaller size members may be constructed to support the applied loads reducing the dead load. This reduction in dead load indirectly makes whole of the structure much more economical. Further, reduction in depth of beams and thickness of flat slabs reduces center-to-center height of stories and causes considerable saving in multi-story buildings.

PRESTRESSED CONCRETE- ADVANTAGES OF PRESTRESSED CONCRETE

2. Prestressed concrete members may be used for longer spans and for heavy loading, such as in case of bridges.
3. Due to prestressing, tension in the cross-section is either reduced or is completely eliminated. This means that the amount of cracking in such construction is less at service loads. Structures may also be designed to consist of completely uncracked members. This makes the structure more watertight. Further, corrosion of steel is reduced, which is especially beneficial in aggressive surroundings.
4. Prestressed structures require less maintenance cost and have longer life due to less cracking and use of high strength materials.

PRESTRESSED CONCRETE- ADVANTAGES OF PRESTRESSED CONCRETE

5. Due to pre-compression of almost the entire cross-section, shear strength is improved by reduction in diagonal tension. Also a component of force in curved tendons partly neutralizes the shear force and thus further increasing the shear strength. Hence, thin web sections may be used reducing the dead load and cost of the structure.
6. The quality of both the materials is tested during the prestressing stage.
7. Deflections under the service loads are significantly reduced. The prestressing produces negative moments in simply supported beams causing upward camber. The service loads first balance these upward deflections and then cause reduced downward deformations.
8. A crack-free prestressed member behaves as a homogeneous and elastic material satisfying the requirements for applicability of most of the formulas.

PRESTRESSED CONCRETE- DISADVANTAGES OF PRESTRESSED CONCRETE

1. Higher initial cost may be required for high strength concretes and steels. Special materials may be required for making these high strength concretes.
2. More sophisticated formwork is generally required increasing the cost.
3. Hydraulic jacks are required to apply the prestressing force. Further, jacking operation may be injurious if proper precautions against failure of tendons during pulling are not employed.
4. End anchorage devices and end beam plates are required.
5. Shrinkage and creep of concrete become much more important as these reduce the effective prestressing force

PRESTRESSED CONCRETE- DISADVANTAGES OF PRESTRESSED CONCRETE

6. Higher degree of quality control is needed in prestressed concrete construction. For example, if the prestressing force during jacking remains less than the required magnitude due to any reason, the result will be a drastic reduction in the overall strength of the structure.
7. Design is to be performed at different stages of loading. Failure of a prestressed member may even occur when the prestressing force is transferred to the members without the application of service loads.

PRESTRESSED CONCRETE- DISADVANTAGES OF PRESTRESSED CONCRETE

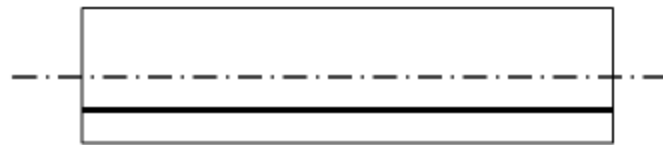
8. If a prestressed precast member is turned upside down, that may happen for symmetrical members, its behavior completely changes. In most cases, a prestressed member may fail just due to dead load during transportation and handling if the member is picked in such a way that dead loads produce negative moments or if the member is transported in upside-down position.

PRESTRESSED CONCRETE- DIFFERENCE BETWEEN BEHAVIOR OF PRESTRESSED AND ORDINARY REINFORCED CONCRETES

- The external bending moment in a beam is resisted by a couple formed by internal tensile force in steel and compressive force in concrete.
- In ordinary RC beam, the increase in lever arm by the increase of loading is lesser and the resisting moment basically increases by the increase in steel tensile force and compression in the concrete.
- The situation is reverse in case of prestressed concrete beam where the forces in steel and concrete remain nearly constant and the resisting moment increases by the increase in lever arm.
- After the development of tensile stresses in concrete due to increase of load on prestressed beam, its behavior is almost similar to ordinary reinforced concrete beams.

PRESTRESSED CONCRETE- TENDON PROFILES

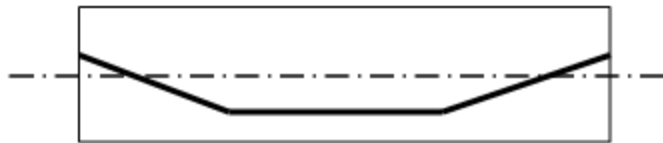
- The path of prestressing tendon along the length of prestressed member is called **tendon profile** and it is decided based on the shape of bending moment diagram (Fig. 19.12).
- The tendon profile may be straight, parabolic, draped or harped, as shown in Fig. 19.12.



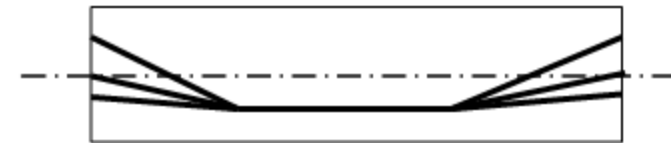
(a) Straight Tendon or Tendon at Constant



(b) Parabolic Tendon



(c) Draped Tendon



(d) Harped Tendon

Fig. 19.12. Various Types of Tendon Profiles.

PRESTRESSED CONCRETE- TENDON PROFILES

- Positive moment requires prestressing force below the section centroid, negative moment requires it above the centroid and desired eccentricity of prestressing force increases with the increase of applied bending moment.
- Curved tendon profiles approximately following the shape of bending moment diagram are better and are easy to be placed in case of post-tensioning.
- However, in case of pre-tensioning especially for large prestressing beds having a number of units to be cast together, straight tendons are easy to be placed.

PRESTRESSED CONCRETE- TENDON PROFILES

- In case of an inclined or curved tendon, the prestress force may be resolved into a horizontal and a vertical component (Fig. 19.13).
- The vertical component of the prestress force, $P \sin \theta$, throughout the length of member gives the amount of upward force directly resisting the applied load.
- if straight or curved tendons are used, the moment ($P \times e$) at a particular section may be considered as a resisting moment.
- These upward forces or resisting moments are called **equivalent loads**

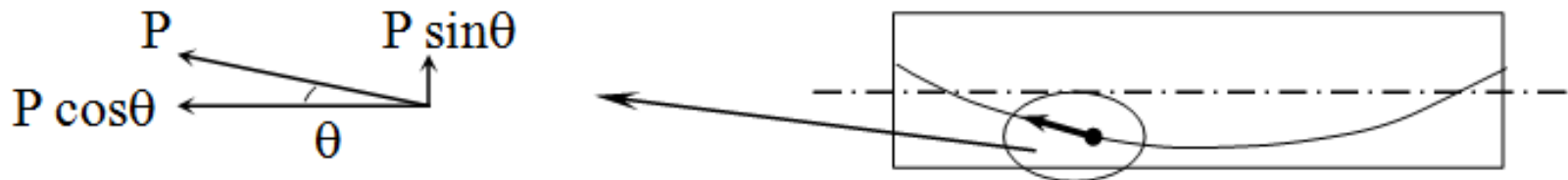


Fig. 19.13. Components of Prestress Force in Curved Tendons.

PRESTRESSED CONCRETE- METHODS OF ANALYSIS AND DESIGN

- Three different approaches are used to analyze and design prestressed concrete members.

Stress Control Method

- The first one is to check the stresses at service load stage against the Code limits and is termed as ***Stress Control Method***.
- This method is considered under the serviceability limit states of the strength design method and elastic behavior is considered for the determination of stresses.

Load Balancing Method

- The second method is to consider prestressing as equivalent loads and is called ***Load Balancing Method***.

PRESTRESSED CONCRETE- METHODS OF ANALYSIS AND DESIGN

Strength Limit State

- The third approach is to consider the behavior at collapse just like ordinary reinforced concrete to evaluate safety against collapse and is known as design under ***Strength Limit State***.
- Strength method gives no information about cracking or deflections at service loads, whereas the first two methods do not give the ultimate safety factor

PRESTRESSED CONCRETE- LOSS OF PRESTRESS

Jacking force(P_j)

- is the force required to pull the tendon to required stress level.

Initial prestress force (P_i)

- is the force after transfer and is equal to jacking force reduced due to the instantaneous shortening of the concrete.

Effective prestress force (P_e)

- is the amount of prestress force left after instantaneous and time dependent losses.

Effectiveness ratio (R)

- is defined as the ratio of effective prestress to the initial prestress.

$$R = P_e / P_i$$

PRESTRESSED CONCRETE- LOSS OF PRESTRESS

- A usually conservative value of $R = 0.85$ may be used in place of detailed actual calculations.
- A better estimate of losses may be made by ***Time-Step Method*** where losses are calculated for the required time by dividing it into a number of intervals.
- For each time interval, losses are estimated with respect to starting and ending of the interval.
- This approach reduces the inter-dependence of various losses on one another

PRESTRESSED CONCRETE- LOSS OF PRESTRESS

- Total loss of prestress may be divided into two categories as under:

Instantaneous Losses:

- These are the losses due to anchorage set, friction between the tendon and concrete and elastic shortening of concrete.
- Following notation may be used to represent these losses:

Δf_{PF} = loss due to friction, MPa

Δf_{PA} = loss due to anchorage set, MPa

Δf_{PE} = loss due to elastic shortening, MPa

PRESTRESSED CONCRETE- LOSS OF PRESTRESS

b) Time Dependent Losses:

- These are the losses due to creep and shrinkage of concrete and relaxation of steel and are denoted by the following symbols:

Δf_{PSR} = loss due to shrinkage of concrete, MPa

Δf_{PCR} = loss due to creep of concrete, MPa

Δf_{PR} = loss due to relaxation of steel, MPa

- Total prestress loss (Δf_{PT}) for the two methods of prestressing may be written as under:

i) *Pre-tensioned members:* $\Delta f_{PT} = \Delta f_{PES} + \Delta f_{PSR} + \Delta f_{PCR} + \Delta f_{PR}$

ii) *Post-tensioned members:* $\Delta f_{PT} = \Delta f_{PF} + \Delta f_{PA} + \Delta f_{PES} + \Delta f_{PSR} + \Delta f_{PCR} + \Delta f_{PR}$

PRESTRESSED CONCRETE- Example 19.1

A simply supported beam has a span of 15m and has a symmetrical I-shaped cross-section. Following data is available for the beam:

Beam has constant eccentricity throughout

$$S_1 = S_2 = 47186 \times 10^3 \text{ mm}^3$$

$$f_c' = 40 \text{ MPa}$$

$$A_c = 221125 \text{ mm}^2$$

$$f_{ci}' = 29 \text{ MPa}$$

$$I = 2,241,319 \times 10^4 \text{ mm}^4 \text{ Class U – member}$$

$$c_1 = c_2 = 475 \text{ mm}$$

$$M_d + M_l = 703.1 \text{ kN-m}$$

$$P_i = 1626 \text{ kN}$$

$$e = 291.5 \text{ mm}$$

$$R = 0.85$$

Calculate concrete stresses at transfer stage for the mid-span and at the ends and at service load stage for the simply supported ends. Check these values against the ACI limits.

PRESTRESSED CONCRETE- Example 19.1

Solution:

For class-U members:

$$f_{ci} = -0.6 f'_{ci} = -17.4 \text{ MPa}$$

$$f_{ti} = 0.25 \sqrt{f'_{ci}} = 1.35 \text{ MPa at mid-span}$$

$$= 0.5 \sqrt{f'_{ci}} = 2.69 \text{ MPa at ends}$$

$$f_{cs} = -0.60 f'_c = -24.0 \text{ MPa}$$

$$f_{ts} = 0.62 \sqrt{f'_c} = 3.92 \text{ MPa}$$

PRESTRESSED CONCRETE- Example 19.1

Mid-Span Section At Transfer

This corresponds to the loading stage-1b as defined earlier.

$$r^2 = I / A_c = 2,241,319 \times 10^4 / 221,125 = 101,360 \text{ mm}^2$$

$$\begin{aligned} f_1 &= -\frac{P_i}{A_c} \left(1 - \frac{ec_1}{r^2} \right) - \frac{M_g c_1}{I} \\ &= -\frac{1626 \times 1000}{221,125} \left(1 - \frac{291.5 \times 475}{101,360} \right) - \frac{147 \times 10^6 \times 475}{2,241,319 \times 10^4} \\ &= -0.424 \text{ MPa} \quad (\text{compared with allowed tension of } 1.35 \text{ MPa}) \end{aligned}$$

$$\begin{aligned} f_2 &= -\frac{P_i}{A_c} \left(1 + \frac{ec_2}{r^2} \right) + \frac{M_g c_2}{I} \\ &= -\frac{1626 \times 1000}{221,125} \left(1 + \frac{291.5 \times 475}{101,360} \right) + \frac{147 \times 10^6 \times 475}{2,241,319 \times 10^4} \\ &= -14.28 \text{ MPa} \quad (\text{compared with allowed compression of } 17.40 \text{ MPa}) \end{aligned}$$

PRESTRESSED CONCRETE- Example 19.1

End Section At Transfer

$$\begin{aligned}f_1 &= -\frac{P_i}{A_c} \left(1 - \frac{ec_1}{r^2}\right) \\&= -\frac{1626 \times 1000}{221,125} \left(1 - \frac{291.5 \times 475}{101,360}\right) \\&= 2.69 \text{ MPa} \quad (\text{allowed value is the same} - \text{OK})\end{aligned}$$

$$\begin{aligned}f_2 &= -\frac{P_i}{A_c} \left(1 + \frac{ec_2}{r^2}\right) \\&= -\frac{1626 \times 1000}{221,125} \left(1 + \frac{291.5 \times 475}{101,360}\right) \\&= -17.40 \text{ MPa} \quad (\text{allowed value is the same} - \text{OK})\end{aligned}$$

PRESTRESSED CONCRETE- Example 19.1

Mid-Span Section At Service Loads

$$\begin{aligned}f_1 &= -\frac{P_e}{A_c} \left(1 - \frac{ec_1}{r^2}\right) - \frac{c_1}{I} (M_g + M_d + M_\ell) \\&= -\frac{1626 \times 0.85 \times 1000}{221,125} \left(1 - \frac{291.5 \times 475}{101,360}\right) - \frac{850.1 \times 10^6 \times 475}{2,241,319 \times 10^4} \\&= -15.73 \text{ MPa} \quad (\text{compared with allowed compression of } 25.00 \text{ MPa}) \\f_2 &= -\frac{P_e}{A_c} \left(1 + \frac{ec_2}{r^2}\right) + \frac{c_2}{I} (M_g + M_d + M_\ell) \\&= -\frac{1626 \times 0.85 \times 1000}{221,125} \left(1 + \frac{291.5 \times 475}{101,360}\right) + \frac{850.1 \times 10^6 \times 475}{2,241,319 \times 10^4} \\&= 3.23 \text{ MPa} \quad (\text{compared with allowed tension of } 3.92 \text{ MPa})\end{aligned}$$

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS(Beams With Variable Eccentricity)

- In case of variable eccentricity, the eccentricity of tendon from the section centroid varies along the length.
- More eccentricity is provided at the sections having larger moments.
- This eccentricity is gradually reduced towards the lesser moment sections.
- In such cases, the applied moment and the moment due to prestress force are both maximum at a single section, for the transfer and the service load stages, and hence design is only required for this single location.

Let

Face-1 = top fibers of the section

Face-2 = bottom fibers of the section

f_{ci} = permissible concrete compressive stress at transfer stage

f_{ti} = permissible concrete tensile stress at transfer stage

f_{cs} = permissible concrete compressive stress at service load stage

f_{ts} = permissible concrete tensile stress at service load stage

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

P_i = initial prestress force just after transfer without time-dependent losses.

P_e = effective prestress force after all the short and long-term losses.

R = effectiveness ratio = P_e / P_i .

e = eccentricity of prestressing force from the centroid at a particular section.

M_g = bending moment due to self-weight at critical section.

M_d = bending moment due to imposed dead load at critical section.

M_l = bending moment due to service live load at critical section.

h = total depth of the section.

C_1 = distance of top fiber from the centroid.

C_2 = distance of bottom fiber from the centroid.

S_1 = elastic section modulus with respect to top fibers.

S_2 = elastic section modulus with respect to bottom fibers.

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

Δf_1 = reduction in tensile stress at top face due to time dependent losses.

Δf_2 = reduction in compressive stress at bottom face due to time dependent losses.

f_{1r} = range of stress available at top for which service load moment may safely be applied.

f_{2r} = range of stress available at bottom for which service load moment may safely be applied.

A_c = concrete cross-sectional area.

f_{cci} = uniform stress in concrete section if initial prestress force (P_i) is applied at the concrete centroid, equal to P_i / A_c .

f_{cce} = uniform stress in concrete section if effective prestress force (P_e) is applied at the concrete centroid, equal to P_e / A_c .

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

Stage 1a:

- Only initial prestress force (P_i) is applied at an eccentricity 'e', which creates a negative moment ($P_i \times e$) producing tension on the top and compression at the bottom.

$$\text{Top tension} = -P_i / A_c + \frac{P_i \times e}{S_1}$$

$$\text{Bottom compression} = -P_i / A_c - \frac{P_i \times e}{S_2}$$

The magnitudes of these stresses, due to moments only, may be written as under:

$$\text{Top tension} = \frac{P_i \times e}{S_1} \quad \text{Bottom compression} = \frac{P_i \times e}{S_2}$$

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

Stage 1b:

- Just after the initial prestress force is applied, the self-weight starts acting producing a positive moment.
- This reduces the top tension and the bottom compression.
- At this stage, the tensile stress must be lesser than or equal to its corresponding maximum allowed value, f_{ti} , and the compressive stress must be lesser than or equal to its maximum value, f_{ci} .
- These limits are shown in Fig. 19.16 by arrows.

$$\text{Top tension} = -P_i / A_c + \frac{P_i \times e}{S_1} - \frac{M_g}{S_1} \quad (\text{tension positive})$$

$$\text{Bottom compression} = -P_i / A_c - \frac{P_i \times e}{S_2} + \frac{M_g}{S_2} \quad (\text{tension positive})$$

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

- The magnitudes of these stresses, due to moments only, may be written as under:

$$\text{Top tension} = \frac{P_i \times e}{S_1} - \frac{M_g}{S_1}$$

$$\text{Bottom compression} = \frac{P_i \times e}{S_2} - \frac{M_g}{S_2}$$

$$\text{Compressive stress at concrete centroid} = f_{cci} = P_i / A_c$$

- A conservative approximation for further calculations is that the extreme fiber stresses at this stage become just equal to f_{ti} and f_{ci} .

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

Stage 2:

- This is an imaginary stage in loading after all the time dependent prestress losses have fully taken place, but the service load is not yet applied.
- The prestress reduces to P_e causing a corresponding reduction at the concrete centroid.
- These losses are to be calculated on stresses due to P_i or on stresses of stage-1a, which correspond to the ACI stress limits, increased by the dead load stresses (ACI limits are imposed on stresses of stage-1b).
- The top tension reduces by some magnitude Δf_1 and the bottom compression reduces by some magnitude Δf_2 .

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

Compressive stress at concrete centroid $= f_{cce} = P_e / A_c$

Loss in prestress force $= P_i - P_e = P_i - R \times P_i = (1 - R)P_i$

$$\Delta f_1 = (1 - R) (f_{ti} + M_g / S_1) \quad (\text{I})$$

$$\Delta f_2 = (1 - R) (-f_{ci} + M_g / S_2) \quad (\text{II})$$

- It is to be noted that the value of stress f_{ci} is negative making $-f_{ci}$ positive in the above equation.

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

Stage 3:

- At this stage, the maximum service dead and live loads are applied.
- The resulting positive moment produces tension at the bottom and compression at the top.
- The concrete centroidal stress still remains at the level of f_{cce} .

$$\text{Top tension} = \frac{M_d + M_\ell}{S_1} - \frac{P_e \times e}{S_1} + \frac{M_g}{S_1} \quad (\text{only due to moments}) \quad (\text{III})$$

$$\text{Bottom compression} = \frac{M_d + M_\ell}{S_2} - \frac{P_e \times e}{S_2} + \frac{M_g}{S_2} \quad (\text{only due to moments}) \quad (\text{IV})$$

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

- The above compressive stress must be lesser than the limit f_{cs} and the tensile stress must be lesser than the limit f_{ts} .
- Again, the value of stress f_{cs} is negative making the term $-f_{cs}$ positive in magnitude.
- From the stress diagram of Fig. 19.16, we get,

$$\begin{aligned} f_{1r} &= f_{ti} - \Delta f_1 - f_{cs} \\ &= f_{ti} - (1 - R) f_{ti} - (1 - R) M_g / S_1 - f_{cs} && \text{(Using Eq. I)} \\ &= R f_{ti} - (1 - R) M_g / S_1 - f_{cs} && \text{(V)} \end{aligned}$$

$$\begin{aligned} \text{and } f_{2r} &= f_{ts} - f_{ci} - \Delta f_2 \\ &= f_{ts} - f_{ci} + (1 - R) f_{ci} - (1 - R) M_g / S_2 && \text{(Using Eq. II)} \\ &= f_{ts} - R f_{ci} - (1 - R) M_g / S_2 && \text{(VI)} \end{aligned}$$

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

Now the required section moduli may be found as follows:

$$S \geq \frac{\text{Service load bending moment}}{\text{Available stress range}} \quad (\text{VII})$$

$$S_1 \geq \frac{M_d + M_\ell}{f_{1r}}$$
$$\geq \frac{M_d + M_\ell}{R f_{ti} - (1 - R) \frac{M_g}{S_1} - f_{cs}}$$

$$S_1 (R f_{ti} - f_{cs}) - (1 - R) M_g \geq M_d + M_\ell$$

$$S_1 (R f_{ti} - f_{cs}) \geq (1 - R) M_g + M_d + M_\ell$$

$$S_1 \geq \frac{(1 - R) M_g + M_d + M_\ell}{R f_{ti} - f_{cs}} \quad (\text{VIII})$$

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

Similarly,

$$\begin{aligned}
 S_2 &\geq \frac{M_d + M_\ell}{f_{2r}} \\
 &\geq \frac{M_d + M_\ell}{f_{ts} - R f_{ci} - (1 - R) \frac{M_g}{S_2}} \\
 S_2 (f_{ts} - R f_{ci}) - (1 - R) M_g &\geq M_d + M_\ell \\
 S_2 (f_{ts} - R f_{ci}) &\geq (1 - R) M_g + M_d + M_\ell \\
 S_2 &\geq \frac{(1 - R) M_g + M_d + M_\ell}{f_{ts} - R f_{ci}} \quad \text{(IX)}
 \end{aligned}$$

Further the section should be such that

$$\begin{aligned}
 I_c = S_1 c_1 = S_2 c_2 \\
 \frac{c_1}{c_2} = \frac{S_2}{S_1} \quad \text{or} \quad \frac{c_1}{h} = \frac{S_2}{S_1 + S_2} \quad \text{(X)}
 \end{aligned}$$

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

- Considering the stress diagram at stage 1b (Fig. 19.17) after prestress transfer, it is observed that the Δ s ABC and DEF are similar and the following is obtained:

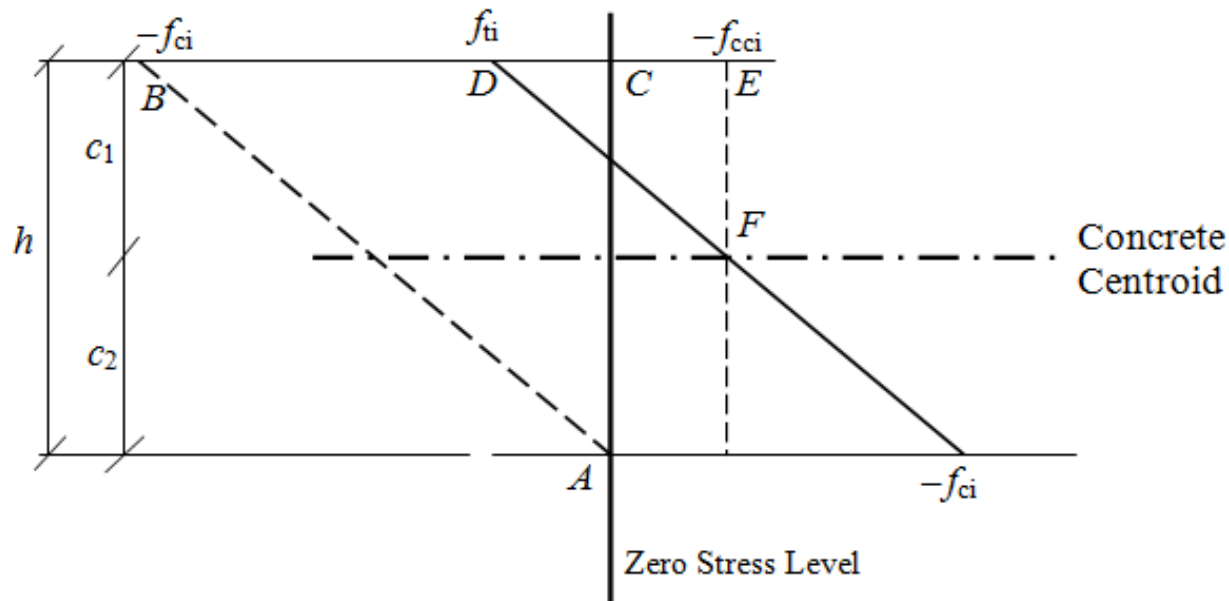


Fig. 19.17. Stress Diagram at Stage 1-b.

PRESTRESSED CONCRETE- SERVICEABILITY DESIGN OF PRESTRESSED MEMBERS

$$\frac{c_1}{h} = \frac{DE}{BC} = \frac{f_{ti} - f_{cci}}{f_{ti} - f_{ci}}$$

$$f_{ti} - f_{cci} = \frac{c_1}{h} (f_{ti} - f_{ci})$$

$$f_{cci} = f_{ti} - \frac{c_1}{h} (f_{ti} - f_{ci}) \quad \text{(XI)}$$

Then, $P_i = A_c \times \text{magnitude of } f_{cci}$ (XII)

Let, $e_m =$ required eccentricity at maximum moment section

The top stress due to initial prestress is given by:

$$\begin{aligned} f_{\text{top}} &= -\frac{P_i}{A_c} + \frac{P_i \times e_m}{S_1} \\ &= f_{cci} + \frac{P_i \times e_m}{S_1} \end{aligned}$$

However, this stress must not exceed $f_{ti} + M_g / S_1$, and for this maximum value we have:

$$f_{ti} + M_g / S_1 = f_{cci} + \frac{P_i \times e_m}{S_1}$$

$$\frac{P_i \times e_m}{S_1} = (f_{ti} - f_{cci}) + M_g / S_1$$

$$e_m = (f_{ti} - f_{cci}) \frac{S_1}{P_i} + \frac{M_g}{P_i} \quad \text{(XIII)}$$

PRESTRESSED CONCRETE- Selection Of Trial Section

- The cross-sectional dimensions must be selected such that the equations VIII, IX and X are satisfied.
- When the required values of S_1 and S_2 are sufficiently close, asymmetrical section with section modulus S equal to larger of S_1 and S_2 may be used.

a) Rectangular Section :

- The depth is first selected satisfying the deflection and other code requirements and then width is calculated by using the expression

$$S = bh^2 / 6.$$

PRESTRESSED CONCRETE- Selection Of Trial Section

b) *Symmetrical I-Section :*

- The depth is again selected as a first step satisfying the deflection and other code requirements.
- The thickness of flanges and web may then be taken as follows (see Fig. 19.18):

$$t \approx 0.15 h \quad (I)$$

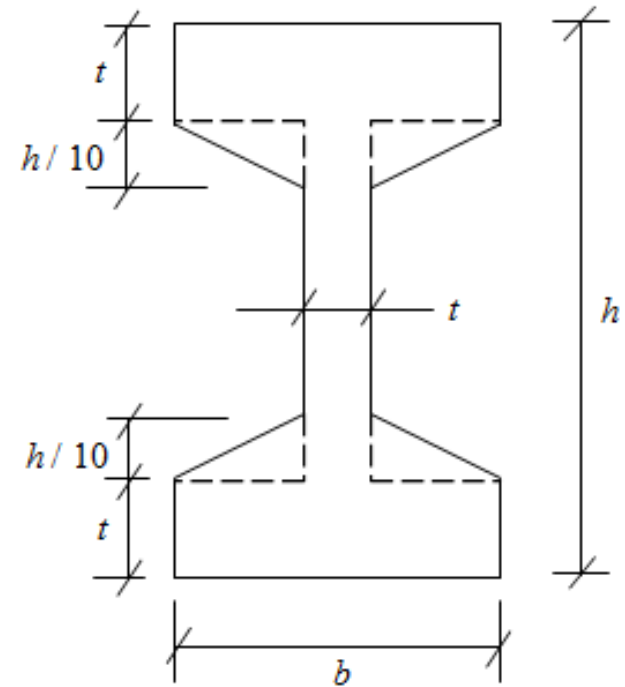


Fig. 19.18. A Typical Symmetrical I-Section.

PRESTRESSED CONCRETE- Selection Of Trial Section

The moment of inertia of beam, approximately considering the sloping portion of the flanges, is:

$$I = \frac{bh^3}{12} - \frac{(b-t)(h-h/15-2t)^3}{12}$$

$$S = \frac{I}{h/2} = \frac{bh^2}{6} - \frac{b(0.93h-2t)^3}{6h} + \frac{t(0.93h-2t)^3}{6h}$$

$$= \frac{1}{6h} [bh^3 - b(0.93h-2t)^3 + t(0.93h-2t)^3]$$

$$6Sh - t(0.93h-2t)^3 = b[h^3 - (0.93h-2t)^3]$$

$$b = \frac{6Sh - t(0.93h-2t)^3}{h^3 - (0.93h-2t)^3} \quad (\text{II})$$

$$\text{Approximate self weight} = 0.007 h^2 \text{ (N/m)}$$

PRESTRESSED CONCRETE- Depth Of Prestressed Beams

- i. The recommended depth of prestressed beams and slabs may be taken equal to approximately 70 % of that for the non-prestressed beams and slabs, respectively, using Grade 300 steel.

$$\text{Minimum depth for simply supported beams} = \frac{\ell}{20} \times 0.70 = \ell / 28$$

Usual depth for simply supported beams

$$\text{for variable eccentricity sections} = \frac{\ell}{12} \times 0.70 = \ell / 17$$

Usual depth for simply supported beams

$$\text{for constant eccentricity sections} = \ell / 16$$

- ii. For bridges, the depth of girders ranges from $\ell / 25$ to $\ell / 15$.
- iii. As a thumb rule,

$$h \text{ (mm)} \cong 33\sqrt{M_{\max}} \text{ to } 40\sqrt{M_{\max}} \text{ for variable eccentricity sections}$$

$$h \text{ (mm)} \cong 36\sqrt{M_{\max}} \text{ to } 44\sqrt{M_{\max}} \text{ for constant eccentricity sections}$$

where, M_{\max} = the maximum bending moment in kN-m
= $M_d + M_e$

PRESTRESSED CONCRETE- Example 19.2

- Design a simply supported prestressed beam of 15m span carrying a superimposed dead load of 10 kN/m and service live load of 15 kN/m. Normal weight concrete of $f_c' = 40$ MPa and $f_{ci}' = 29$ MPa and seven wire steel strands of Grade 1725 are to be used with $f_{py} = 1550$ MPa. The section is to be of symmetrical I-shape and the eccentricity of prestress force will be varied along the span according to the bending moment diagram. The losses after transfer are estimated to be 15%. The member is required to be of class-U.

PRESTRESSED CONCRETE- Example 19.2

Solution:

$$\ell = 15 \text{ m}$$

$$w_D = 10 \text{ kN/m}$$

$$w_L = 15 \text{ kN/m}$$

$$f_c' = 40 \text{ MPa}$$

$$f_{ci}' = 29 \text{ MPa}$$

Class U – member

$$f_{pu} = 1725 \text{ MPa}$$

$$f_{py} = 1550 \text{ MPa}$$

$$R = 0.85 \text{ for } 15\% \text{ time dependent losses}$$

$$M_D + M_L = \frac{1}{8} \times 25 \times 15^2 = 703.1 \text{ kN-m}$$

$$h \approx 33 \sqrt{M_{\max}} = 33 \sqrt{703.1} = 875 \text{ mm}$$

Also, $h \approx \ell / 17 = 15 \times 1000 / 17 = 882 \text{ mm}$

$$\therefore h = 875 \text{ mm}$$

$$\begin{aligned} \text{Approximate self weight} &= 0.006 h^2 \text{ to } 0.007 h^2 \text{ N/m} \\ &= 0.007 \times 875^2 / 1000 = 5.36 \text{ kN/m} \end{aligned}$$

$$M_g = \frac{1}{8} \times 5.36 \times 15^2 = 150.8 \text{ kN-m}$$

PRESTRESSED CONCRETE- Example 19.2

For class U member, the stress limits are:

$$f_{ci} = -0.6f'_{ci} = -0.6 \times 29 = -17.4 \text{ MPa}$$

$$f_{ti} = 0.25\sqrt{f'_{ci}} = 0.25\sqrt{29} = 1.35 \text{ MPa}$$

$$f_{cs} = -0.60f'_c = -0.60 \times 40 = -24.0 \text{ MPa}$$

$$f_{ts} = 0.62\sqrt{f'_c} = 0.62\sqrt{40} = 3.92 \text{ MPa}$$

$$\begin{aligned} S_1 &\geq \frac{(1-R)M_g + M_d + M_\ell}{Rf_{ti} - f_{cs}} \\ &= \frac{[(1-0.85) \times 150.8 + 703.1] \times 10^6}{0.85 \times 1.35 - (-24.0)} = 28859 \times 10^3 \text{ mm}^3 \end{aligned}$$

$$\begin{aligned} S_2 &\geq \frac{(1-R)M_g + M_d + M_\ell}{f_{ts} - Rf_{ci}} \\ &= \frac{[(1-0.85) \times 150.8 + 703.1] \times 10^6}{3.92 - 0.85 \times (-17.4)} = 38788 \times 10^3 \text{ mm}^3 \end{aligned}$$

PRESTRESSED CONCRETE- Example 19.2

For symmetrical section, $S = 38788 \times 10^3 \text{ mm}^3$

$$t \approx 0.15 h = 0.15 \times 875 = 130 \text{ mm}$$

$$0.93 h - 2 t = 0.93 \times 875 - 2 \times 130 = 554 \text{ mm}$$

$$b = \frac{6 S h - t (0.93 h - 2 t)^3}{h^3 - (0.93 h - 2 t)^3}$$
$$= \frac{6 \times 38788 \times 10^3 \times 875 - 130 \times 554^3}{875^3 - 554^3} = 363 \text{ mm} \approx 370 \text{ mm}$$

$$b / h = 370 / 875 = 0.423 \quad (\text{between } 0.2 \text{ and } 0.6) \quad \text{OK}$$

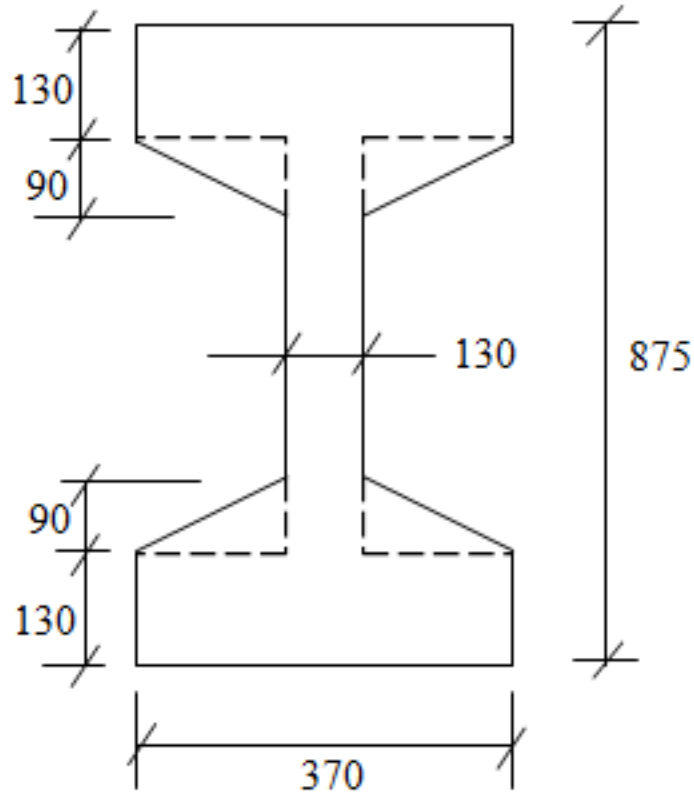
The cross-sectional details are given in Fig. 19.19.

$$A_c = 2 \times 370 \times 130 + 615 \times 130 + 4 \times 0.5 \times 90 \times 120 = 197750 \text{ mm}^2$$

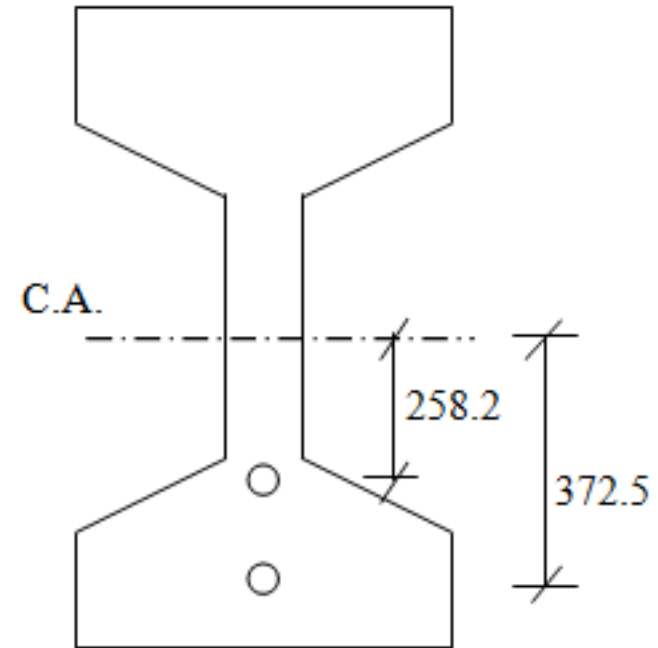
$$\text{Self weight} = 0.198 \times 2400 \text{ kg /m} = 4.66 \text{ kN/m}$$

(15.11 % lesser than the assumed value, usually it should not be more by 10 %)

PRESTRESSED CONCRETE- Example 19.2



a) Cross-Sectional Dimensions



b) Eccentricity of Tendons

Fig. 19.19. Symmetrical I-Section for Example 9.2.

PRESTRESSED CONCRETE- Example 19.2

$$f_{cci} = f_{ti} - \frac{c_1}{h} (f_{ti} - f_{ci})$$

$$= 1.35 - 0.5 (1.35 + 17.4) = - 8.025 \text{ MPa}$$

$$P_i = A_c \times |f_{cci}| = 197750 \times 8.025 / 1000 = 1586.9 \text{ kN}$$

Allowable steel stress just after transfer, f_{ps} = lesser of $0.74f_{pu}$ and $0.82f_{py}$
= lesser of 1276.5 MPa and 1271 MPa
= 1271 MPa

$$A_{ps} = \frac{P_i}{f_{ps}} = \frac{1586.9 \times 1000}{1271} = 1249 \text{ mm}^2$$

Area of 12.70 mm diameter strand = 92.90 mm²

$$\text{Number of strands, } N_s = \frac{A_{ps}}{92.90} = \frac{1249}{92.90} = 13.44 \quad \text{say } \underline{14 \text{ strands}}$$

$$\text{Initial force in each strand} = \frac{P_i}{N_s} = \frac{1586.9}{14} = \mathbf{113.35 \text{ kN}}$$

PRESTRESSED CONCRETE- Example 19.2

For the calculation of the required maximum eccentricity, we need the actual values of S_1 and M_g .

$$I = \frac{370 \times 875^3}{12} - \frac{(875 - 130 \times 2 - 30 \times 2)^3 \times 240}{12}$$

$$= 1,723,685 \times 10^4 \text{ mm}^4$$

$$S = 39,399 \times 10^3 \text{ mm}^3$$

$$M_g = \frac{1}{8} \times 4.66 \times 15^2 = 131.1 \text{ kN-m}$$

$$e_m = (f_{ti} - f_{cci}) \frac{S_1}{P_i} + \frac{M_g}{P_i}$$

$$= (1.35 + 8.025) \frac{39,399 \times 10^3}{1586.9 \times 10^3} + \frac{131.1 \times 10^6}{1586.9 \times 10^3}$$

$$= 315.4 \text{ mm}$$

< 372.5 mm, the eccentricity up to center of the flange OK

The details of the eccentricity are shown in Fig. 19.19(b). After carrying out the design, the stresses at transfer and under service load stages must be checked by using the procedure explained earlier to make sure that these are within the ACI limits.

PRESTRESSED CONCRETE- Beams With Constant Eccentricity

- In this case, straight tendons are used, as shown in Fig. 19.20, and hence zero or lesser moment sections may be more critical at the transfer stage.
- This is because self-weight moment is also less at these sections, which if present is helpful in reducing the stresses.
- This means that stress diagrams are to be drawn at maximum moment section and at a section where self-load moment is the minimum for the various loading stages and all the stress limits are to be satisfied.
- Same nomenclature as already used for the variable eccentricity case and repeated below is used for the required derivations.

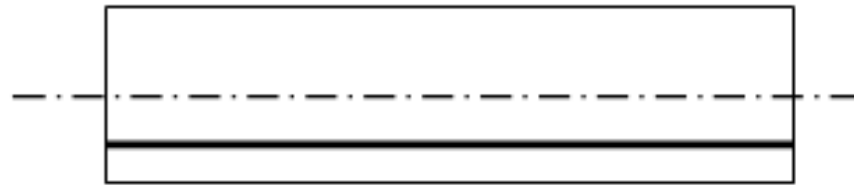


Fig. 19.20. Beams with Constant Eccentricity.

PRESTRESSED CONCRETE- Beams With Constant Eccentricity

Let

Face-1 = top fibers of the section

Face-2 = bottom fibers of the section

f_{ci} = permissible concrete compressive stress at transfer stage

f_{ti} = permissible concrete tensile stress at transfer stage

f_{cs} = permissible concrete compressive stress at service load stage

f_{ts} = permissible concrete tensile stress at service load stage

P_i = initial prestress force just after transfer without time-dependent losses.

P_e = effective prestress force after all the short and long-term losses.

R = effectiveness ratio = P_e / P_i .

e = eccentricity of prestressing force from the centroid at a particular section.

M_g = bending moment due to self-weight at critical section.

M_d = bending moment due to imposed dead load at critical section.

M_ℓ = bending moment due to service live load at critical section.

h = total depth of the section.

c_1 = distance of top fiber from the centroid.

PRESTRESSED CONCRETE- Beams With Constant Eccentricity

- c_2 = distance of bottom fiber from the centroid.
- S_1 = elastic section modulus with respect to top fibers.
- S_2 = elastic section modulus with respect to bottom fibers.
- Δf_1 = reduction in tensile stress at top face due to time dependent losses.
- Δf_2 = reduction in compressive stress at bottom face due to time dependent losses.
- f_{1r} = range of stress available at top for which service load moment may safely be applied.
- f_{2r} = range of stress available at bottom for which service load moment may safely be applied.
- A_c = concrete cross-sectional area.
- f_{cci} = uniform stress in concrete section if initial prestress force (P_i) is applied at the concrete centroid, equal to P_i / A_c .
- f_{cce} = uniform stress in concrete section if effective prestress force (P_e) is applied at the concrete centroid, equal to P_e / A_c .

PRESTRESSED CONCRETE- Beams With Constant Eccentricity

- The stress diagrams at various loading stages, as given in Fig. 19.21, may be drawn to derive the formulas

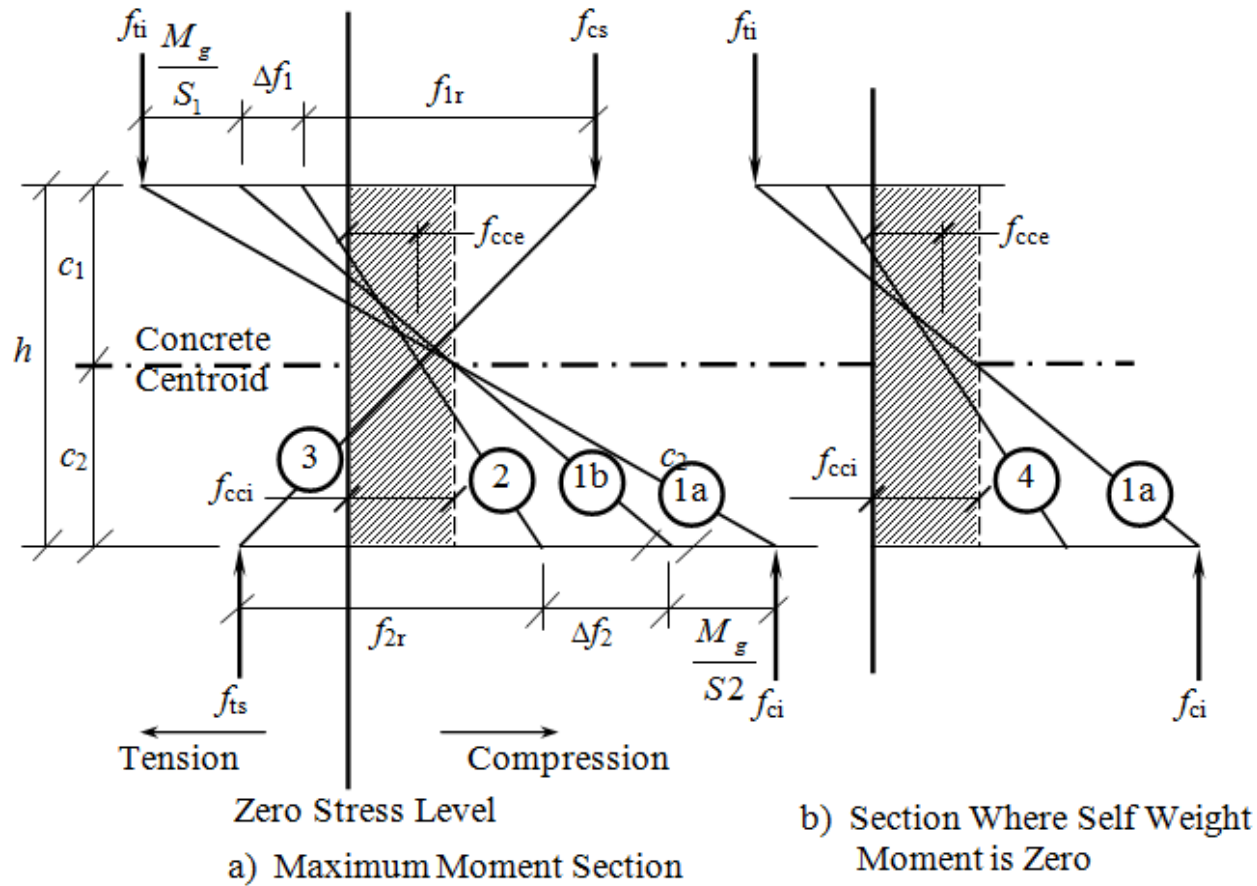


Fig. 19.21. Stress Diagrams at Various Loading Stages.

PRESTRESSED CONCRETE- Beams With Constant Eccentricity

Stages At Maximum Moment Section

Stage 1a:

- Only initial prestress force (P_i) is acting.

Stage 1b:

- The initial prestress force (P_i) and self-weight moment (M_g) are acting.

Stage 2:

- The effective prestress force (P_e) and self-weight moment (M_g) are acting.

Stage 3:

- The effective prestress force (P_e), self-weight moment (M_g), dead load moment (M_d) and live load moment (M_l) are acting.

PRESTRESSED CONCRETE- Beams With Constant Eccentricity

Stages At Zero Self-load Moment Section

Stage 1a:

- Only initial prestress force (P_i) is acting.

Stage 4:

- Only effective prestress force (P_e) is acting.

The time dependent losses Δf_1 and Δf_2 depending

$$\Delta f_1 = (1 - R) (f_{ti}) \quad (\text{I})$$

$$\Delta f_2 = (1 - R) (-f_{ci}) \quad (\text{II})$$

PRESTRESSED CONCRETE- Beams With Constant Eccentricity

The stress ranges available with respect to top and bottom edges for the service loads are as follows:

$$\begin{aligned}f_{1r} &= f_{ti} + (-f_{cs}) - \Delta f_1 - M_g / S_1 \\ &= f_{ti} - f_{cs} - f_{ti} + R f_{ti} - M_g / S_1 && \text{(Using Eq. I)} \\ &= R f_{ti} - f_{cs} - M_g / S_1 && \text{(V)}\end{aligned}$$

$$\begin{aligned}\text{and } f_{2r} &= f_{ts} + (-f_{ci}) - \Delta f_2 - M_g / S_2 \\ &= f_{ts} - f_{ci} + f_{ci} - R f_{ci} - M_g / S_2 && \text{(Using Eq. II)} \\ &= f_{ts} - R f_{ci} - M_g / S_2 && \text{(VI)}\end{aligned}$$

Hence the required section moduli are as under:

$$S \geq \frac{\text{Service load bending moment}}{\text{Available stress range}} \quad \text{(VII)}$$

$$\begin{aligned}S_1 &\geq \frac{M_d + M_\ell}{f_{1r}} \\ &\geq \frac{M_d + M_\ell}{R f_{ti} - \frac{M_g}{S_1} - f_{cs}}\end{aligned}$$

PRESTRESSED CONCRETE- Beams With Constant Eccentricity

$$S_1 (R f_{ti} - f_{cs}) - M_g \geq M_d + M_\ell$$

$$S_1 (R f_{ti} - f_{cs}) \geq M_g + M_d + M_\ell$$

$$S_1 \geq \frac{M_g + M_d + M_\ell}{R f_{ti} - f_{cs}} \quad \text{(VIII)}$$

Higher allowed value of f_{ti} is to be used in case of simply supported ends. The condition that $S_1 \geq \frac{(1-R)M_g + M_d + M_\ell}{R f_{ti} - f_{cs}}$ must also be checked using the smaller allowed value of f_{ti} .

Similarly,

$$\begin{aligned} S_2 &\geq \frac{M_d + M_\ell}{f_{2r}} \\ &\geq \frac{M_d + M_\ell}{f_{ts} - R f_{ci} - \frac{M_g}{S_2}} \end{aligned}$$

$$S_2 (f_{ts} - R f_{ci}) - M_g \geq M_d + M_\ell$$

$$S_2 (f_{ts} - R f_{ci}) \geq M_g + M_d + M_\ell$$

$$S_2 \geq \frac{(M_g + M_d + M_\ell)}{f_{ts} - R f_{ci}} \quad \text{(IX)}$$

PRESTRESSED CONCRETE- Beams With Constant Eccentricity

As already seen for the variable eccentricity case, we have,

$$I_c = S_1 c_1 = S_2 c_2$$
$$\frac{c_1}{c_2} = \frac{S_2}{S_1} \quad \text{or} \quad \frac{c_1}{h} = \frac{S_2}{S_1 + S_2} \quad (\text{X})$$

Also, considering the stress diagram at stage 1b of Fig. 19.17 (after prestress transfer), it is observed that the Δ s ABC and DEF are similar and the following is obtained:

$$\frac{c_1}{h} = \frac{DE}{BC} = \frac{f_{ti} - f_{cci}}{f_{ti} - f_{ci}}$$
$$f_{ti} - f_{cci} = \frac{c_1}{h} (f_{ti} - f_{ci})$$
$$f_{cci} = f_{ti} - \frac{c_1}{h} (f_{ti} - f_{ci}) \quad (\text{XI})$$

$$\text{Then, } P_i = A_c \times \text{magnitude of } f_{cci} \quad (\text{XII})$$

$$\text{Let, } e = \text{required constant eccentricity}$$

PRESTRESSED CONCRETE- Beams With Constant Eccentricity

The top stress due to initial prestress is given by:

$$f_{\text{top}} = -\frac{P_i}{A_c} + \frac{P_i \times e}{S_1} = f_{\text{cci}} + \frac{P_i \times e}{S_1}$$

However, this stress must not exceed f_{ti} in case of zero dead load moment section. For this limiting case, we have,

$$f_{\text{top}} = f_{\text{ti}}$$

$$f_{\text{cci}} + \frac{P_i \times e}{S_1} = f_{\text{ti}}$$

$$\frac{P_i \times e}{S_1} = (f_{\text{ti}} - f_{\text{cci}})$$

$$e = (f_{\text{ti}} - f_{\text{cci}}) \frac{S_1}{P_i} \quad \text{(XIII)}$$

PRESTRESSED CONCRETE- Example 19.3

Design a simply supported prestressed beam of 15m span carrying a superimposed dead load of 10 kN/m and service live load of 15 kN/m. Normal weight concrete of $f_c' = 40$ MPa and $f_{ci}' = 29$ MPa and seven wire steel strands of Grade 1725 are to be used with $f_{py} = 1550$ MPa. The section is to be of symmetrical I-shape and the eccentricity of prestress force will be kept constant along the span. The losses after transfer are estimated to be 15%. The member is required to be of class-U.

PRESTRESSED CONCRETE- Example 19.3

Solution:

$$\ell = 15 \text{ m}$$

$$w_D = 10 \text{ kN/m}$$

$$w_L = 15 \text{ kN/m}$$

$$f_c' = 40 \text{ MPa}$$

$$f_{ci}' = 29 \text{ MPa}$$

Class U – member

$$f_{pu} = 1725 \text{ MPa}$$

$$f_{py} = 1550 \text{ MPa}$$

$$R = 0.85 \text{ for 15\% time dependent losses}$$

$$M_D + M_L = \frac{1}{8} \times 25 \times 15^2 = 703.1 \text{ kN-m}$$

$$h \approx 36 \sqrt{M_{\max}} = 36 \sqrt{703.1} = 956 \text{ mm}$$

$$\text{so, } h \approx \ell / 16 = 15 \times 1000 / 16 = 937.5 \text{ mm}$$

$$\therefore h = 950 \text{ mm}$$

$$\begin{aligned} \text{Approximate self weight} &= 0.007 h^2 \text{ N/m} \\ &= 0.007 \times 950^2 / 1000 = 6.32 \text{ kN/m} \end{aligned}$$

$$M_g = \frac{1}{8} \times 6.32 \times 15^2 = 177.7 \text{ kN-m}$$

PRESTRESSED CONCRETE- Example 19.3

For class U member, the stress limits are:

$$f_{ci} = -0.6f'_{ci} = -0.6 \times 29 = -17.4 \text{ MPa}$$

$$f_{ti} = 0.25\sqrt{f'_{ci}} = 0.25\sqrt{29} = 1.35 \text{ MPa}$$

For ends of the simply supported member, $f_{ti} = 0.5\sqrt{f'_{ci}} = 2.69 \text{ MPa}$

$$f_{cs} = -0.60f'_c = -0.60 \times 40 = -24.0 \text{ MPa}$$

$$f_{ts} = 0.62\sqrt{f'_c} = 0.62\sqrt{40} = 3.92 \text{ MPa}$$

At the ends,

$$S_1 \geq \frac{M_g + M_d + M_\ell}{R f_{ti} - f_{cs}}$$
$$= \frac{[177.7 + 703.1] \times 10^6}{0.85 \times 2.69 - (-24.0)} = 33,508 \times 10^3 \text{ mm}^3$$

PRESTRESSED CONCRETE- Example 19.3

At the center,

$$\begin{aligned} S_1 &\geq \frac{(1-R)M_g + M_d + M_\ell}{Rf_{ti} - f_{cs}} \\ &= \frac{[(1-0.85) \times 177.7 + 703.1] \times 10^6}{0.85 \times 1.35 - (-24.0)} = 29,508 \times 10^3 \text{ mm}^3 \end{aligned}$$

$$\begin{aligned} S_2 &\geq \frac{M_g + M_d + M_\ell}{f_{ts} - Rf_{ci}} \\ &= \frac{[177.7 + 703.1] \times 10^6}{3.92 - 0.85 \times (-17.4)} = 47,076 \times 10^3 \text{ mm}^3 \end{aligned}$$

For a symmetrical section, $S = 47,076 \times 10^3 \text{ mm}^3$

$$t \approx 0.15 h = 0.15 \times 950 = 142.5 \text{ mm} \quad (\text{say } 140 \text{ mm})$$

$$0.93 h - 2t = 0.93 \times 950 - 2 \times 140 = 603.5 \text{ mm}$$

$$\begin{aligned} b &= \frac{6Sh - t(0.93h - 2t)^3}{h^3 - (0.93h - 2t)^3} \\ &= \frac{6 \times 47,076 \times 10^3 \times 950 - 140 \times 603.5^3}{950^3 - 603.5^3} = 373 \text{ mm} \approx 375 \text{ mm} \end{aligned}$$

$$b/h = 375 / 950 = 0.395$$

(between 0.2 and 0.6)

OK

PRESTRESSED CONCRETE- Example 19.3

The cross-sectional details are given in Fig. 19.22.

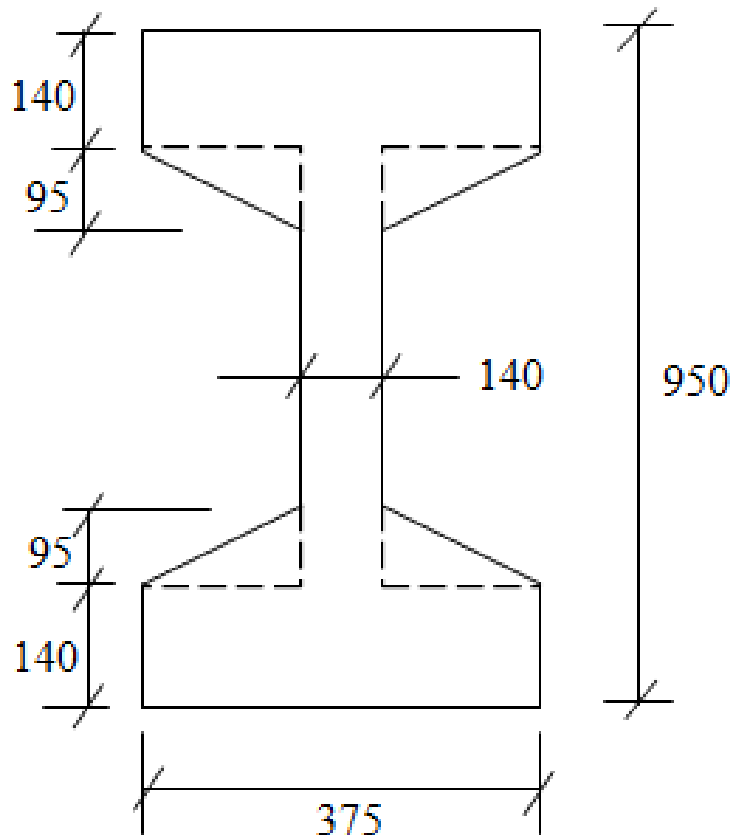
$$\begin{aligned} A_c &= 2 \times 375 \times 140 + 670 \times 140 + 4 \times 0.5 \times 95 \times 117.5 \\ &= 221,125 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Self weight} &= 0.221 \times 2400 \text{ kg/m} \\ &= 5.21 \text{ kN/m vs } 6.32 \text{ kN/m assumed} \end{aligned} \quad \text{OK}$$

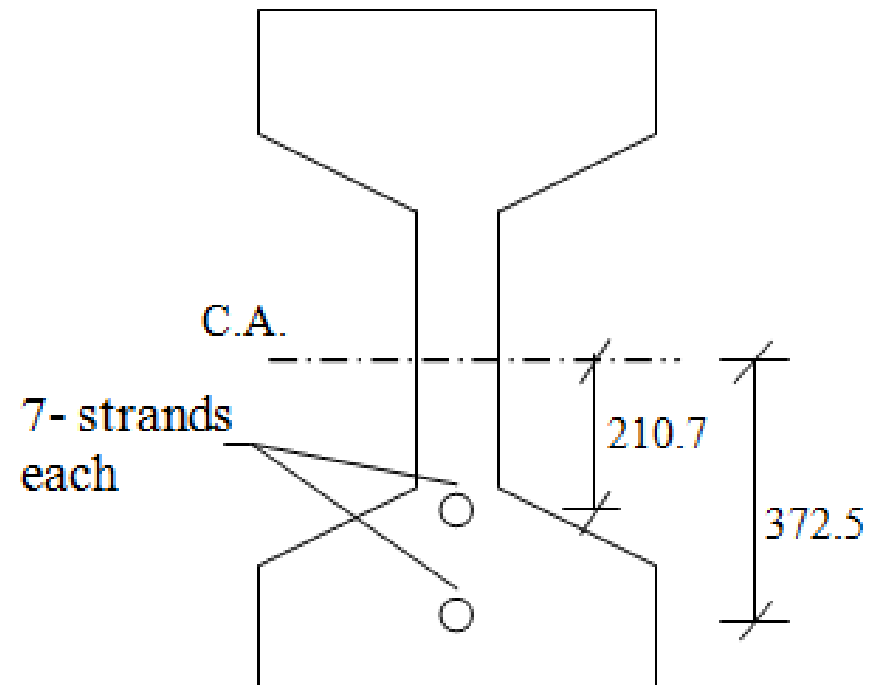
$$\begin{aligned} f_{cci} &= f_{ti} - \frac{c_1}{h} (f_{ti} - f_{ci}) \\ &= 2.69 - 0.5 (2.69 + 17.4) = -7.355 \text{ MPa} \end{aligned}$$

$$P_i = A_c \times |f_{cci}| = 221,125 \times 7.355 / 1000 = 1626 \text{ kN}$$

PRESTRESSED CONCRETE- Example 19.3



a) Cross-Sectional Dimensions



b) Eccentricity of Tendons

Fig. 19.22. Symmetrical I-Section for Example 19.3.

PRESTRESSED CONCRETE- Example 19.3

Allowable steel stress just after transfer, f_{ps} = lesser of $0.74f_{pu}$ and $0.82f_{py}$
= lesser of 1276.5 MPa and 1271 MPa
= 1271 MPa

$$A_{ps} = \frac{P_i}{f_{ps}} = \frac{1626 \times 1000}{1271} = 1280 \text{ mm}^2$$

Area of 12.70 mm diameter strand = 92.90 mm²

$$\text{Number of strands, } N_s = \frac{A_{ps}}{92.90} = \frac{1280}{92.90} = 13.78 \quad \text{say } \underline{14 \text{ strands}}$$

$$\text{Initial force in each strand} = \frac{P_i}{N_s} = \frac{1626}{14} = \mathbf{116.14 \text{ kN}}$$

PRESTRESSED CONCRETE- Example 19.3

For the calculation of the required maximum eccentricity, we need the actual value of S_1 .

$$I = \frac{375 \times 950^3}{12} - \frac{(950 - 140 \times 2 - 95/3 \times 2)^3 \times 235}{12} = 2,241,319 \times 10^4 \text{ mm}^4$$

$$S_1 = 47,201 \times 10^3 \text{ mm}^3$$

$$e = (f_{ti} - f_{cci}) \frac{S_1}{P_i}$$

$$= (2.69 + 7.355) \frac{47,201 \times 10^3}{1626 \times 10^3} = 291.6 \text{ mm}$$

< 372.5 mm, the eccentricity up to center of the flange OK

The details of the eccentricity are shown in Fig. 19.22(b). After carrying out the design, the stresses at transfer and under service load stages must be checked by using the procedure explained earlier to make sure that these are within the ACI limits.