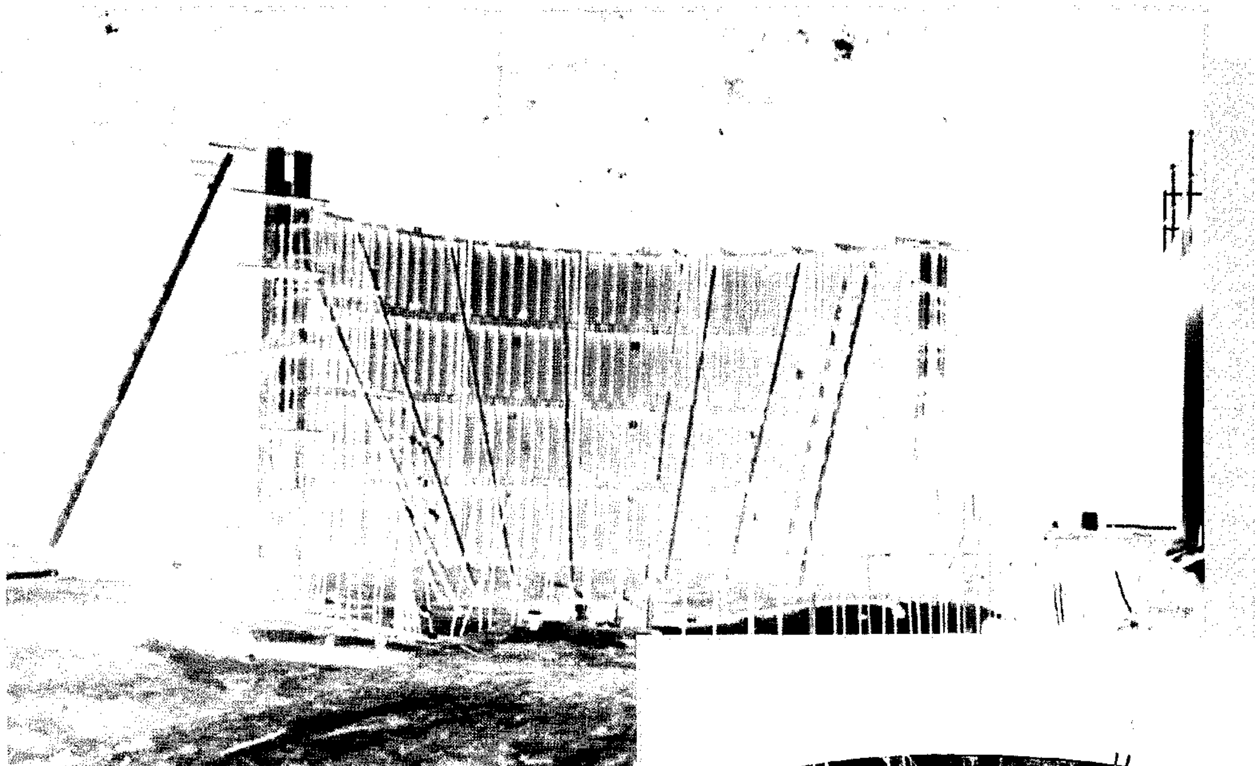
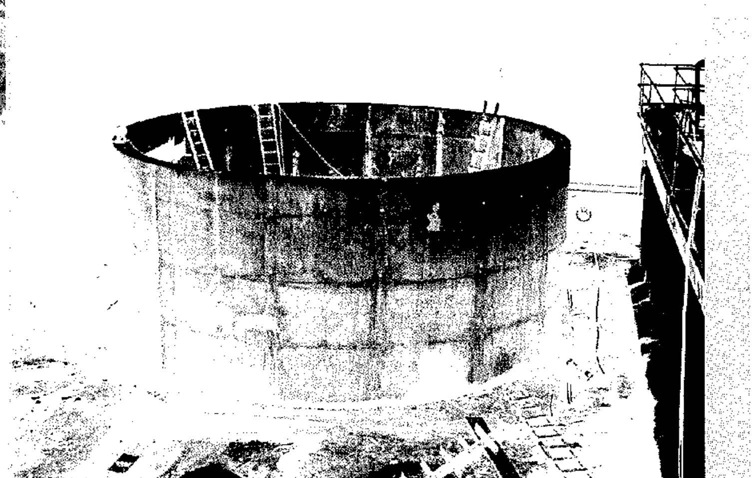
PORTLAND CEMENT ASSOCIATION

CIRCULAR CONCRETE TANKS WITHOUT PRESTRESSING





The first edition of this publication was produced over a half century ago. The theory used at that time for the structural analysis is still valid and utilized herein. This edition, which updates this publication to include the latest standards and codes, was written by August W. Domel, Jr., Senior Structural Engineer, Portland Cement Association, and Anand B. Gogate, Anand Gogate Consulting Engineers, Worthington, Ohio.

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Circular Concrete Tanks Without Prestressing

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An organization of cement manufacturers to improve and extend the uses of portland cement and concrete through market development, engineering, research, education, and public affairs work.

Contents

1 *Introduction*

Conventionally reinforced (non-prestressed) circular concrete tanks have been used extensively in municipal and industrial facilities for several decades. The design of these structures requires that attention be given not only to strength requirements, but to serviceability requirements as well. A properly designed tank must be able to withstand the applied loads without cracks that would permit leakage. The goal of providing a structurally sound tank that will not leak is achieved by providing the proper amount and distribution of reinforcement, the proper spacing and detailing of construction joints, and the use of quality concrete placed using proper construction practices.

A thorough review of the latest report by ACI Committee 350 entitled *Environmental Engineering Concrete Structures*[1] is essential in understanding the design of tanks. The latest edition (1983) of that document recommends that, unless noted otherwise, the structural design should conform to *Building Code Requirements for Reinforced Concrete (ACI 318-89) (Revised 1992)*[2]. Therefore, a working knowledge of ACI 318 is also necessary.

The topics discussed in this publication are:

- Loading Conditions (Section 2)
- Design Methods (Section 3)
- Wall Thickness (Section 4)
- Reinforcement (Section 5)
- Crack Control (Section 6)
- Design of Tank Walls (Sections 7 through 13)
- Design of Roof Slabs (Sections 14 through 16)
- Effect of Variation in Wall Thickness (Section 17)
- Temperature Variation in Tank Walls (Section 18)
- Design of Base Slabs (Section 19)

A detailed design example is given in Section 20. Also, at the end of the publication is a list of references pertaining to the design and analysis of tanks.

2 *Loading Conditions*

A tank must be designed to withstand the loads that it will be subjected to during many years of use. But it is equally important to consider loads during construction. An example of some of the loading conditions that must be considered for a partially buried tank is shown in Fig. 1. The tank must be designed and detailed to withstand the forces from each of these loading conditions. The tank may also be subjected to uplift forces from hydrostatic pressure on the bottom of the slab when the tank is empty. Therefore, it is important for the design engineer to determine all possible loading conditions on the structure. According to ACI 350, the full effects of the soil loads and water pressure must be designed for without the benefit of resistance of the loads which could minimize the effects of each other.

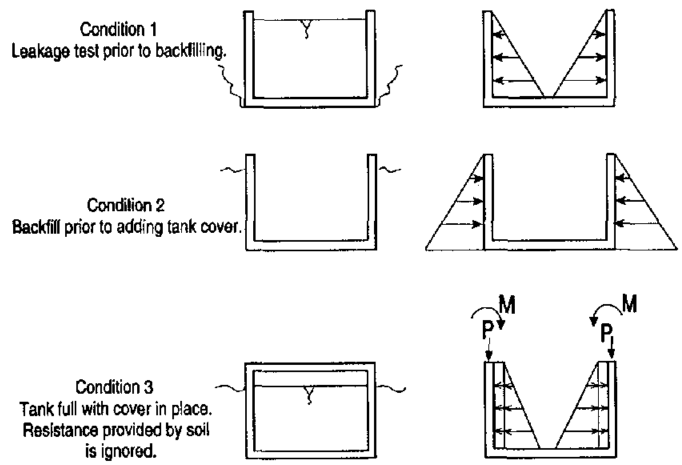


Figure 1—Possible loading conditions for a partially buried tank

3 *Design Methods*

Two approaches currently exist for the design of reinforced concrete members:

(1) Strength Design, and (2) Allowable Stress Design (referred to in Building Code Requirements for Reinforced Concrete (ACI 318-89) (Revised 1992) Appendix A, as the Alternate Design method).

The Strength Design method became the commonly adopted procedure for conventional buildings after the 1963 revision to the ACI Building Code, and constitutes the basic procedure of design in the present ACI Building Code (ACI 318-89) with the Alternate Design method in an appendix (Appendix A).

Until recently, the use of strength design for municipal and other facilities was considered inappropriate due to the lack of reliable assessment of crack widths at service loads. The advances in this area of knowledge in the last two decades has led to the acceptance of the strength design method for municipal liquid retaining structures in general and circular concrete tanks in particular. The latest ACI Committee 350 report recommends procedures for the use of both Allowable Stress Design, and Strength Design for liquid retaining structures. The new recommendations by Committee 350 for strength design essentially suggest inflated load factors to control service load crack widths to fall in the range of 0.004 in. to 0.008 in., as suggested in a 1968 paper by Gogate.[3]

Service state analysis of reinforced concrete structures should include computations of crack widths and their long term effects on the structure in terms of its stability and functional performance. The present state-of-the-art of reinforced concrete design leads to computations which are, at best, a modified form of elastic analysis of the composite reinforced steel/concrete system. Due to the well known effects of creep, shrinkage, volume changes, and temperature, all analyses of this type, in terms of computed stresses, are indices of performance of the structure and should not be construed to have any more significance than that.

The following discussion describes the alterations in the design methods of ACI 318 provided by ACI 350.

Strength Design—The load combinations to determine the required strength, U, are given in Section 9.2 of ACI 318-89. ACI 350 requires the following two modifications to that section.

Modification 1—The load factor to be used for lateral liquid pressure, F, is taken as 1.7 rather than the value of 1.4 specified in ACI 318. This value of 1.7 may be overconservative for some tanks, since they are filled to the top only during leak testing or because of accidental overflow. Since leak testing usually occurs only once and since most tanks are equipped with overflow pipes, some designers have considered using the load factor of 1.4 in an attempt to reduce the amount of required steel which results in less shrinkage restraint. However, this publication suggests that tank designs meet ACI 350 and therefore, recommends the use of a load factor of 1.7.

Modification 2—The members must be designed to meet the required strength, U, under ACI 318-89. ACI 350 requires that the value of U be increased by using a multiplier called the sanitary coefficient. The sanitary coefficient will increase the design loads to provide a more conservative design with less cracking. The increased required strength is given by:

Required strength = Sanitary coefficient × U

where the sanitary coefficient equals:

1.3 for flexure

1.65 for direct tension

1.3 for shear beyond that of the capacity provided by the concrete

Working Stress Design—ACI 350 recommends that this alternative design method be in accordance with ACI 318. ACI 350 implies in its document that the maximum allowable stress for Grade 60 reinforcing steel is 30 ksi. This is considerably larger than the 24 ksi allowed in Appendix A of ACI 318-89.

ACI 350 recommends the allowable stress in hoop tension for Grade 60 reinforcing steel as 20 ksi and for Grade 40 reinforcing steel as 14 ksi.

4 Wall Thickness

Typically, in the design of reinforced concrete members, the tensile strength of concrete is ignored. Any significant cracking in a liquid containing tank is unacceptable. For this reason, it must be assured that the stress in the concrete from ring tension is kept at a minimum to prevent excessive cracking. Neither ACI 350 or ACI 318 provide guidelines for the tension carrying capacity for this condition. The allowable tensile strength of concrete is usually between 7% and 12% of the compressive strength. A value of 10% of the concrete strength will be used in this publication.

According to ACI 350, reinforced concrete walls 10 ft high or taller, which are in contact with liquid, shall have a minimum thickness of 12 in.

As concrete dries and loses moisture, it contracts in size. This contraction (drying shrinkage), if constrained, will produce tensile stresses that may exceed the capacity of the concrete and cause cracking. Figure 2(a) illustrates a block of concrete with a bar as shown, but otherwise unrestrained. The height of the block is chosen as 1 ft, since tension in a circular ring of a tank wall is computed for that height. The dimension marked, t, corresponds to the wall thickness. The steel area is A_s and the steel percentage is ρ.

If the bar is left out as in Fig. 2(b) (which is obviously out of scale), shrinkage will shorten the 1-in. long block a distance of C, which denotes the shrinkage per unit length. The presence of the steel bar prevents some of the shortening of the concrete, so the difference in length of the block in Fig. 2(b) and Fig. 2(c) is a distance xC, in which x is an unknown quantity.

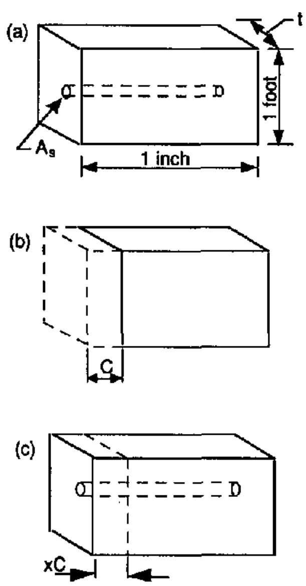


Figure 2—Shrinkage in a concrete section

Compared with (b), the concrete in (c) is elongated by a distance xC from its unstressed condition, so the concrete stress is:

$$f_{cs} = xCE_c$$

Compared with (a), the steel in (c) is shortened by a distance (1 – x) C from its unstressed condition, so the steel stress is:

$$f_{ss} = (1 - x)\, CE_s$$

Considering equilibrium, the total tension in the concrete must equal the total compression in the steel, so $\rho f_{ss} = f_{cs}$. The stresses derived from these equations are:

$$f_{ss} = CE_s \frac{1}{1 + n\rho} \text{(compression)}$$

$$f_{cs} = CE_s \frac{\rho}{1 + n\rho} \text{(tension)}$$

The concrete stress due to ring tension, T, is practically equal to $T/A_c (1 + n\rho)$, when n is the ratio E_s/E_c and the combined concrete tensile stress equals:

$$f_c = \frac{CE_sA_s + T}{A_c + nA_s} \qquad (1)$$

This formula will be used to investigate ring stresses in circular walls.

The usual procedure in tank design is to provide horizontal steel, A_s, for all the ring tension at a certain allowable stress, f_s, as though designing for a cracked section. After determining $A_s = T/f_s$, the concrete tensile stress in the uncracked section due to combined ring tension and shrinkage is checked by inserting the value of A_s in Equation 1. Setting $A_c = 12t$ (t in in.), and solving for t gives:

$$t = \frac{CE_s + f_s - nf_c}{12 f_c f_s} \times T \qquad (2)$$

This formula may be used to estimate the wall thickness. The value of C, coefficient of shrinkage for reinforced concrete, is in the range of 0.0002 to 0.0004. The value of C for plain concrete ranges from 0.0003 to 0.0008. The shrinkage coefficient for plain concrete was used to derive Equation 2 which would require a value of C between 0.0003 and 0.0008. However, this equation has traditionally used the value of 0.0003, the average value for reinforced concrete, with success. For illustration, assuming the shrinkage coefficient, C, of concrete as 0.0003, the allowable concrete stress as $0.1 \times 4000 = 400$ psi, (for $f'_c = 4000$ psi) and the stress in the steel as 18,000 psi:

$$t = \frac{0.0003 \times 29 \times 10^6 + 18{,}000 - 8 \times 400}{12 \times 400 \times 18{,}000} \times T$$

$$= \frac{8700 + 18{,}000 - 3200}{86{,}400{,}000} \times T = 0.0003T$$

5 *Reinforcement*

The amount, size, and spacing of reinforcing bars has a great effect on the extent of cracking. The amount of reinforcement provided must be sufficient for strength and serviceability including temperature and shrinkage effects. The amount of temperature and shrinkage reinforcement is dependent on the length between construction joints as shown in Fig. 3. Figure 3 is based on the assumption that the wall segment will be able to complete shrinkage movement without being restrained at the ends by adjacent sections. The designer should provide proper details to ensure that cracking will occur at joints and that joints are properly leakproofed. According to ACI 350, concrete sections that are 24 in. thick or thicker can have the minimum temperature and shrinkage reinforcement at each face, based on a 12 in. thickness.

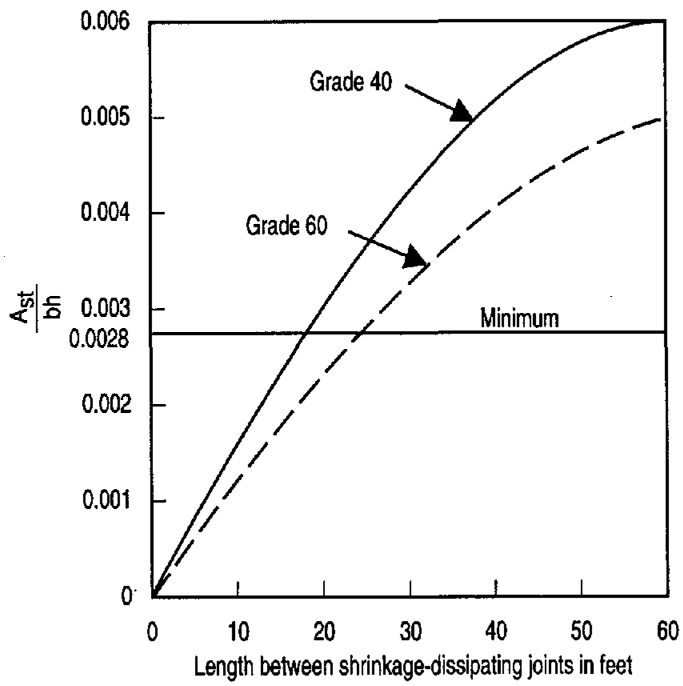


Figure 3—Minimum temperature and shrinkage reinforcement ratio (ACI 350)

The size of reinforcing bars should be chosen recognizing that cracking can be better controlled by using a larger number of small diameter bars rather than fewer larger diameter bars. The size of reinforcing bars, according to ACI 350, should preferably not exceed #11. Spacing of reinforcing bars should be limited to a maximum of 12 in., and the minimum concrete cover for reinforcement in the tank wall should be at least 2 in.

The wall thickness should be sufficient to keep the concrete from cracking. If the concrete does crack, the ring steel must be able to carry all the ring tension alone. This can be achieved by the procedure stated in Section 2.

Low steel stresses actually tend to make the concrete crack because the lower the allowable steel stress, the greater the area of steel provided to carry the tensile force. This results in higher concrete stresses due to shrinkage. If $A_s = T/f_s$ is inserted into Equation 1, the stress in the concrete is given as:

$$f_c = \frac{CE_s + f_s}{A_c f_s + nT} \times T$$

For illustration, use the following data:

$T = 24{,}000$ lb

$n = 8$

$E_s = 29 \times 10^6$ psi

$C = 0.0003$

$A_c = 12 \times 10 = 120$ in.^{2}

f_s	10,000	12,000	14,000	16,000	18,000	20,000	Infinity*
f_c	322	304	291	281	272	266	200

*When f_s = infinity, $A_s = 0$ and $f_c = T/A_c$.

If the allowable steel stress is reduced from 20,000 psi to 10,000 psi, the concrete stress is actually increased from 266 psi to 322 psi. From this point of view, it is desirable to use a higher allowable steel stress so that less steel is used, resulting in less restraint shrinkage and smaller tensile stresses in the concrete.

Reinforcement splices should conform to the requirements of ACI 318. The required length of the splice is a function of many factors. The length depends on the class of splice required, the clear cover, the clear distance between adjacent bars, and the size of the bar. Other factors affecting splice length include: the type of concrete used (lightweight or normal weight), bar coating, if used, and the amount of fresh concrete cast below the bar. Chapter 12 of ACI 318-89 should be referred to in determining splice lengths.

In circular tanks, the location of horizontal splices should be staggered. Splices should be staggered horizontally by not less than one lap length or 3 ft and should not coincide in vertical arrays more frequently than every third bar (see Fig. 4).

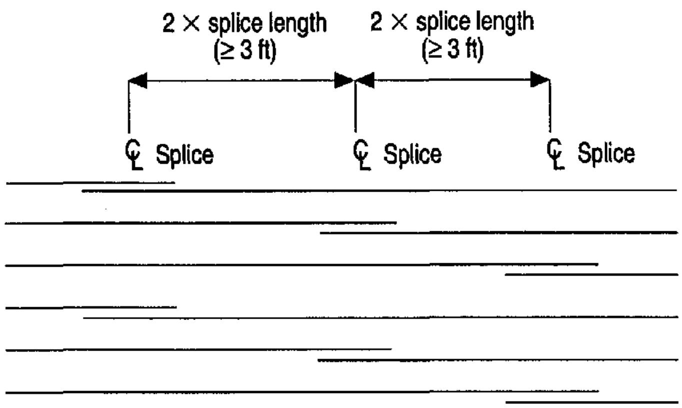


Figure 4—Staggering of ring bar splices

6 Crack Control

Crack widths must be minimized in tank walls to prevent leakage and corrosion of reinforcement. A criterion for flexural crack width is provided in ACI 318-89 (10.6.4). This limitation is based on the Gergely-Lutz expression for crack width and is as follows:

$$z = f_s \sqrt[3]{d_c A} \qquad (3)$$

where,

- z = quantity limiting distribution of flexural reinforcement.
- f_s = calculated stress in reinforcement at service loads, ksi.
- d_c = thickness of concrete cover measured from extreme tension fiber to center of bar located closest thereto, in.
- A = effective tension area of concrete surrounding the flexural tension reinforcement having the same centroid as that reinforcement, divided by the number of bars, sq in.

The determination of d_c and A are shown in Fig. 5. In ACI 350, the cover is taken equal to 2.0 in. for any cover greater than 2.0. Rearranging Equation 3 and solving for the maximum bar spacing for a given value of z gives:

$$\text{max spacing} = \frac{z^3}{2 \times d_c^2 \times f_s^3}$$

ACI 318-89 does not allow z to exceed 175 kips/in. for interior exposure and 145 kips/in. for exterior exposure. These values of z correspond to crack widths of 0.016 in. and 0.013 in., respectively. ACI 350 has stricter requirements than ACI 318, since cracking is typically of greater consequence in liquid

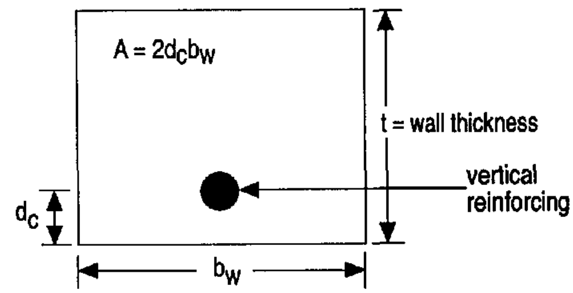


Figure 5—Diagram to determine effective tension area of concrete for calculation of z

retaining structures. The limiting value of z specified in ACI 350 is 115 kips/in. For severe environmental exposures, the quantity z should not exceed 95 kips/in. Note that the z factor is valid only for one-way flexural members and is not directly applicable to hoop tension.

Joints in the circular tank walls, will allow dissipation of temperature and shrinkage stresses and thereby reduce cracking. As discussed previously, the amount of temperature and shrinkage reinforcement is a function of the distance between shrinkage-dissipating joints. Therefore, it is prudent to limit the size of concrete placement. Maximum length of wall placed at one time will usually not exceed 60 ft, with 30 ft to 50 ft being more common. Note that water stops should be used in all joints to prevent the possibility of leakage. The cracking from temperature and shrinkage will be a function of the base restraint. A sliding wall has no base fixity and this will have less restraint than tanks with fixed bases. Tanks with fixed bases tend to develop shrinkage cracks just above the slab.

7 Wall with Fixed Base and Free Top-Triangular Load

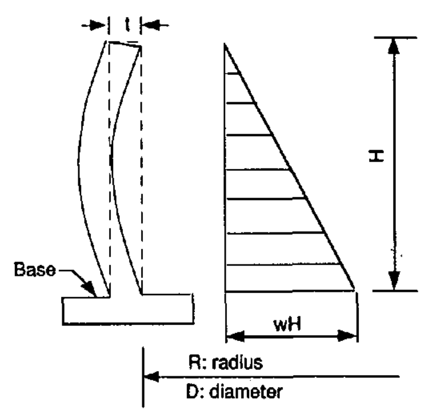


Figure 6—Wall with fixed base and free top-triangular load

This section will discuss the analysis of a tank wall assuming that the top of the wall is free to displace and rotate, and the base is prevented from movement. In practice, it would be rare that a base would be fixed against rotation and such an assumption could lead to an improperly designed wall. Therefore, the calculations in this section are for illustrative purposes only.

The numerical values listed below will be used for design calculations in this and subsequent sections.

Height, H = 20.0 ft
Diameter to inside of wall, D = 54.0 ft
Weight of liquid, w = 62.5 lbs per cu ft
Shrinkage coefficient, c = 0.0003
Modulus of elasticity of steel, $E_s = 29 \times 10^6$ psi
Specified compressive strength of concrete, $f'_c = 4000$ psi
Specified yield strength of reinforcement, $f_y = 60{,}000$ psi
Ratio of moduli of elasticity, n = 8

For a wall with a fixed base and a free top, as shown in Fig. 6, the coefficient to determine the ring tension, moments, and shears in the tank wall are shown in Tables A-1, A-2, and A-12 (Note that table numbers preceded by the letter A are located in the Appendix). The appropriate values to be used for the given dimension of a tank are determined by finding the value of H^2/Dt. This term is a common factor involved in all values of ring tension, moment, and shear and is therefore a convenient characteristic to use in the tables provided. The value of H^2/Dt with the thickness of the tank, t, estimated as 10 in. is:

$$H^2/(Dt) = (20)^2/(54 \times 10/12) = 8.89, \text{ use } 9.0$$

The ring tension per foot of height is computed by multiplying w_uHR by the coefficients in Table A-1 with the value of $H^2/Dt = 9$. As discussed in Section 2 of this text, w_u, for ring tension is determined as follows:

$$w_u = \text{sanitary coefficient} \times (1.7 \times \text{Lateral Force}) = 1.65 \times (1.7 \times 62.5) = 175.3 \text{ lbs per cu ft}$$

Therefore,

$$w_uHR = 175.3 \times 20 \times 54/2 = 94{,}662 \text{ lbs per cu ft}$$

This is the factored ring tension that would exist at the base if it could slide freely. Since the base cannot move freely, this value must be adjusted by the coefficients taken from Table A-1 and shown in Table 1.

Note that point 0.0H denotes the top of the tank and point 1.0H denotes the base of the tank.

Table 1—Ring Tension in Tank for Wall with Fixed Base and Free Top

Point	Coefficient From Table A-1	Ring Tension (lbs/ft)
0.0H	-0.011	-1041
0.1H	+0.101	+9561
0.2H	+0.213	+20,163
0.3H	+0.329	+31,144
0.4H	+0.440	+41,651
0.5H	+0.538	+50,928
0.6H	+0.591	+55,945
0.7H	+0.559	+52,916
0.8H	+0.410	+38,811
0.9H	+0.165	+15,619
1.0H	0	0

A plus sign denotes tension, so there is compression at the top, but it is negligible. The ring tension is zero at the base since it is assumed that the base has no radial displacement. Figure 7 compares the ring tension for a tank with a fixed base to a tank with a free sliding base.

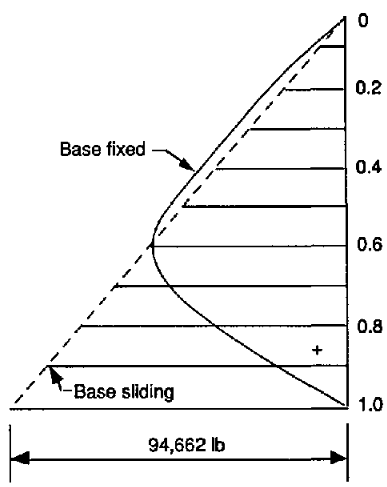


Figure 7—Ring tension in tank for wall with fixed base and free top–triangular load

The amount of ring steel required is given by:

$$A_s = \text{maximum ring tension}/(0.9 \times f_y) = 55{,}945/(0.9 \times 60{,}000) = 1.04 \text{ sq in. per ft}$$

Use #6 bars spaced at 10 in. o.c. in two curtains (A_s = 1.06 sq in. per ft) at this location. The reinforcement for ring tension elsewhere in the height of the wall is determined in a similar manner though it may not be economically prudent to change the bar sizes and spacing.

The maximum tensile stress in the concrete under service loads and including the effects of shrinkage is:

$$f'_c = CE_sA_s + T_{max\,(unfactored)}/(A_c + nA_s)$$

$= [(0.0003 \times 29 \times 10^6 \times 1.06) + 55{,}945/(1.65 \times 1.7)]/(10 \times 12 + 8 \times 1.06)$
$= (9222 + 19{,}945)/(120 + 8.5)$
$= 227.0$ psi

Since 400 psi (0.1 × 4000) is considered acceptable, the 10 in. wall thickness is sufficient.

The moments in vertical wall strips that are considered as one foot wide are computed by multiplying w_uH^3 by the coefficients from Table A-2. The value of w_u for flexure is:

w_u = Sanitary coefficient × (1.7 × Lateral force)
= (1.3 × 1.7 × 62.5) = 138.1 lbs per cu ft

Therefore,

$wH^3 = 138.1 \times (20)^3 = 1{,}104{,}800$ ft-lb/ft

The resulting moments along the height are shown in Table 2. These moments are plotted in Fig. 8 with negative numbers denoting tension on the inside face.

Table 2—Bending Moments for Tank Wall with Fixed Base and Free Top

Point	Coefficient From Table A-2	Moment (ft-lb)
0.0H	0	0
0.1H	0	0
0.2H	0	0
0.3H	+0.0002	+221
0.4H	+0.0006	+663
0.5H	+0.0012	+1326
0.6H	+0.0024	+2652
0.7H	+0.0034	+3756
0.8H	+0.0029	+3204
0.9H	-0.0017	-1878
1.0H	-0.0134	-14,804

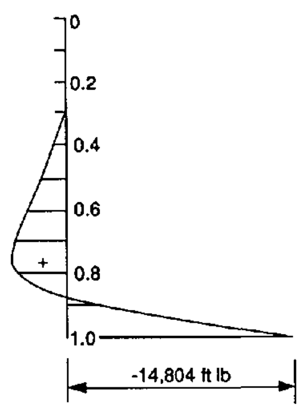


Figure 8—Bending moments for tank wall with fixed base and free top–triangular load

The tension on the inside face exists for a distance of approximately 0.12 × 20 = 2.4 ft above the base. Reinforcing bars will be required to extend from the base to 2.4 ft plus the proper development length above the base.

The required amount of reinforcing on the inside face for the maximum moment of -14,804 ft-kips is:

$M_u/(\phi f'_c bd^2) = -14{,}804 \times 12/(0.9 \times 4000 \times 12 \times (7.5)^2)$
$= 0.0730$
where $d = 10 - 2(\text{cover}) - 1.0/2 = 7.5$

From standard design aid in Appendix A:

$\omega = 0.0765$
$A_s = \omega bd f'_c/f_y = 0.0765 \times 12 \times 7.5 \times 4/60 = 0.459$ in.^{2}
Use #5 bars at 8 in. ($A_s = 0.465$ in.^{2})

These bars are only needed on the inside face near the bottom of the wall and temperature and shrinkage reinforcement will be required for the remainder. The required vertical reinforcement for the outside of the wall for a maximum moment of 3,756 ft-lbs is:

$M_u/(\phi f'_c bd^2) = 3756 \times 12/(0.9 \times 4000 \times 12 \times (7.5)^2)$
$= 0.0185$

From standard design aid in Appendix A:

$\omega = 0.0187$
$A_s = \omega bd f'_c/f_y = 0.0187 \times 12 \times 7.5 \times 4/60 = 0.112$ in.^{2}
Use #5 bars at maximum allowable spacing of 12 in.
($A_s = 0.31$ in.^{2})

The shear capacity of a 10 in. wall with $f'_c = 4000$ psi is:

$V_c = 2\sqrt{f'_c} \times b_w d$
$= 2\sqrt{4000} \times 12 \times 7.5$
$= 11{,}384$ kips
$\phi V_c = 0.85 \times 11{,}384 = 9676$ kips

The applied shear is given by multiplying w_uH^2 by the coefficient of 0.166 from Table A-12. The value w_u is determined using a sanitary coefficient of 1.0 if V_u is less than V_c.

w_u = sanitary coefficient × (1.7 × Lateral Force)
= 1.0 × (1.7 × 62.5)
= 106.3 lbs per cu ft

Therefore,

$w_uH^2 = 106.3 \times (20)^2 = 42{,}520$

The resulting shear is:

$V_u = 0.166 \times 42{,}520 = 7058 \text{ kips} < 9676 \text{ kips}$

8 Wall with Hinged Base and Free Top-Triangular Load

The design in the previous section was based on the assumption that the base of the tank is fixed. Though

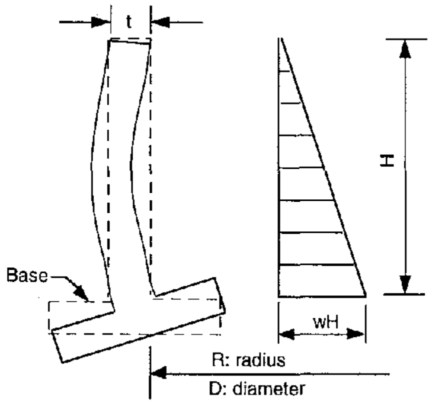


Figure 9—Wall with hinged base and free top–triangular load

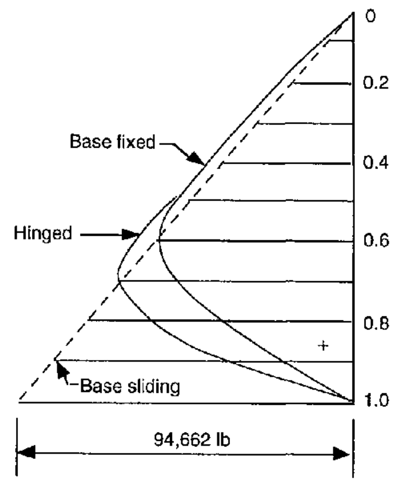


Figure 10—Ring tension for tank wall with hinged base and free top– triangular load

it is difficult to predict the behavior of the subgrade and its effect upon restraint at the base, it is more reasonable to assume that the base is hinged rather than fixed, which results in a more conservative design.

The design example in this section will use the same numerical values from the previous section with $H^2/(Dt) = 9$ and $w_u = 175.3$ lbs per cu ft for ring tension. The ring tension is determined by multiplying w_uHR by the coefficients taken from Table A-5. The ring tension along the height of the tank is shown in Table 3 ($w_uHR = 94{,}662$ lbs per cu ft).

Table 3—Ring Tension For Wall with Hinged Base and Free Top–Triangular Load

Point	Coefficient From Table A-5	Ring Tension (lbs/ft)
0.0H	-0.012	-1136
0.1H	+0.096	+9088
0.2H	+0.204	+19,311
0.3H	+0.318	+30,103
0.4H	+0.436	+41,273
0.5H	+0.558	+52,821
0.6H	+0.663	+62,761
0.7H	+0.713	+67,494
0.8H	+0.649	+61,436
0.9H	+0.409	+38,717
1.0H	0	0

Figure 10 compares ring tension for tank bases that are fixed, free, and hinged. In the upper half of the wall, the base condition has little effect on the value of ring tension. In the bottom half of the wall, the difference between the hinged and fixed base becomes increasingly larger. Maximum ring tension for a hinged base is 67,494 lbs while that for a fixed base is 55,945 lbs. Therefore, the hinged base condition, which is probably more realistic, gives a ring tension 21% greater than the same loading condition with a fixed base.

The amount of ring steel required is given by:

A_s = maximum ring tension/$(0.9 \times f_y)$
$= 67{,}494/(0.9 \times f_y)$
$= 67{,}494/(0.9 \times 60{,}000)$
$= 1.25$ sq in. per ft

Therefore, at 0.7H, use #6 bars spaced at 8 in. o.c. in two curtains ($A_s = 1.32$ sq in. per ft). The reinforcement for ring tension elsewhere in the height of the wall is determined in a similar manner.

The maximum tensile stress in the concrete under service loads and including the effects of shrinkage is:

$f_c' = CE_sA_s + T_{max\ (unfactored)}/(A_c + nA_s)$
$= [(0.0003 \times 29 \times 10^6 \times 1.32) + 67{,}494/(1.65 \times 1.7)]/(10 \times 12 + 8 \times 1.32)$
$= (11{,}484 + 24{,}062)/(120 + 10.6)$
$= 272$ psi

Since 400 psi is considered acceptable, the 10 in. wall thickness is sufficient.

The moments in the vertical wall strips that are considered as one foot wide are computed by multiplying w_uH^3 by the coefficients from Table A-7. The value of w_uH^3 for flexure was calculated in the previous section as 1,104,800 ft-lb/ft. The resulting moments along the height are shown in Table 4. These moments as well as the moments for a fixed base condition are shown in Fig. 11. The actual condition of restraint of a wall footing is somewhere between fixed and hinged, but probably closer to hinged. Com-

Table 4—Moments in Wall with Hinged Base and Free Top–Triangular Load

Point	Coefficient From Table A-7	Moment (ft-lb)
0.0H	0	0
0.1H	0	0
0.2H	0	0
0.3H	-0.0002	-221
0.4H	0	0
0.5H	+0.0005	+552
0.6H	+0.0016	+1768
0.7H	+0.0032	+3535
0.8H	+0.0050	+5524
0.9H	+0.0050	+5524
1.0H	0	0

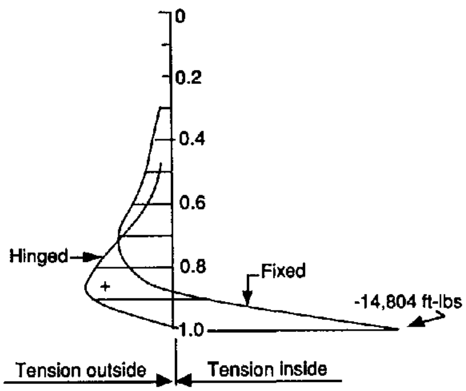


Figure 11—Moments in wall with hinged base and free top–triangular load

parisons of the two different base fixity conditions show that for the exterior face, the hinged condition provides a conservative although not wasteful design. Note that depending on the fixity of the base, reinforcing may be required to resist moment on the interior face at the lower portion of the wall.

The required vertical reinforcement for the outside face of the wall for a maximum moment of 5,524 ft-lb is:

$$M_u/(\phi f'_c bd^2) = 5524 \times 12/(0.9 \times 4000 \times 12 \times (7.5)^2) = 0.0273$$

From standard design aid in Appendix A:

$\omega = 0.0278$

$A_s = 0.0278 \times 12 \times 7.5 \times 4/60 = 0.167$ in.^{2}

$\rho = 0.167/(12 \times 7.5) = 0.00189$

$\rho_{min} = 200/f_y = 0.0033 > 0.00189$

Use #5 bars at the maximum allowable spacing of 12 in. ($A_s = 0.31$ in.^{2}, $\rho = 0.0035$).

The shear capacity of a 10 in. wall with $f'_c = 4000$ psi was previously calculated to be 9676 kips. The applied shear is given by multiplying $w_u H^2$ (previously determined to be 42,520) by the coefficient of 0.092. The resulting shear is:

$V_u = 0.092 \times 42{,}520 = 3912$ kips < 9676 kips

9 *Wall with Hinged Base and Free Top-Trapezoidal Load*

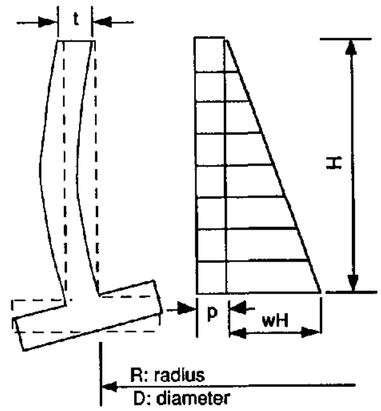


Figure 12—Wall with hinged base and free top–trapezoidal load

Under certain loading conditions, the tank may be subjected to a uniform loading along the height of the wall. For example, this loading condition may occur from vapor pressure developed in a closed tank. The overall loading condition for the combination of the vapor pressure and fluid pressure results in a loading with a trapezoidal distribution as shown in Fig. 12.

In this section, the design procedure for trapezoidal loading is illustrated. The data used in the previous designs will also be used in this section with the addition of a vapor pressure of 420 lbs per sq ft. Because of the additional load from the vapor pressure, the wall thickness will be increased to 15 in. For a wall thickness of 15 in.:

$$\frac{H^2}{Dt} = \frac{20^2}{54 \times 15/12} = 5.9, \text{ use } 6.0$$

The value of the ring tension from the fluid pressure is computed by multiplying $w_u HR$ by the coefficients in Table A-5 with the value of $H^2/Dt = 6$. As previously determined, the value of $w_u HR$ is equal to 94,662 lbs per cu ft. The value of the ring tension is shown in Table 5.

Table 5—Ring Tension in Wall with Hinged Base and Free Top from Fluid Pressure

Point	Coefficient From Table A-5	Ring Tension (lbs/ft)
0.0H	-0.011	-1041
0.1H	+0.103	+9750
0.2H	+0.223	+21,110
0.3H	+0.343	+32,469
0.4H	+0.463	+43,829
0.5H	+0.566	+53,579
0.6H	+0.639	+60,489
0.7H	+0.643	+60,868
0.8H	+0.547	+51,780
0.9H	+0.327	+30,954
1.0H	0	0

The value of the ring tension from the vapor pressure is computed by multiplying p_uR by the coefficients in Table A-6 with a value of $H^2/Dt = 6$. The value of p_u is determined as follows:

p_u = sanitary coefficient × (1.7 × Lateral Force)
= 1.65 × (1.7 × 420) = 1178.1 psf
Therefore, pR = 1178.1 × 27 = 31,809 lb per ft

The values of the ring tension are shown in Table 6.

Table 6—Ring Tension in Wall with Hinged Base and Free Top Tank from Vapor Pressure

Point	Coefficient From Table A-6	Ring Tension (lbs/ft)
0.0H	+0.989	+31,459
0.1H	+1.003	+31,904
0.2H	+1.023	+32,541
0.3H	+1.043	+33,176
0.4H	+1.063	+33,813
0.5H	+1.066	+33,908
0.6H	+1.039	+33,050
0.7H	+0.943	+29,996
0.8H	+0.747	+23,761
0.9H	+0.427	+13,582
1.0H	0	0

The values of the combined ring tension from both the fluid and vapor pressure are shown in Table 7.

Table 7—Combined Ring Tension in Wall from Fluid and Vapor Pressures

Point	Ring Tension from Fluid Presuure	Ring Tension from Vapor Pressure	Total Ring Tension
0.0H	-1041	+31,459	+30,418
0.1H	+9750	+31,904	+41,654
0.2H	+21,110	+32,541	+53,651
0.3H	+32,469	+33,176	+65,645
0.4H	+43,829	+33,813	+77,642
0.5H	+53,579	+33,908	+87,487
0.6H	+60,489	+33,050	+93,539
0.7H	+60,868	+29,996	+90,864
0.8H	+51,780	+23,761	+75,541
0.9H	+30,954	+13,582	+44,536
1.0H	0	0	0

The total ring tensions are plotted in Fig. 13 together with the ring tensions that would exist if the base could slide freely. The maximum tension for a hinged base condition is 93,539 lbs per ft and occurs at 0.6H. Above 0.6H, there is not much different in the ring tension if the base is either hinged or free sliding. Below 0.6H, ring tension for a hinged base decreases rapidly until it becomes zero at the base. Actually, the condition at the base will be somewhere between hinged and free sliding, so it is inadvisable to design the ring bars below point 0.6H for a hinged base. This condition will be discussed in greater detail in Section 11. The amount of steel required is given by:

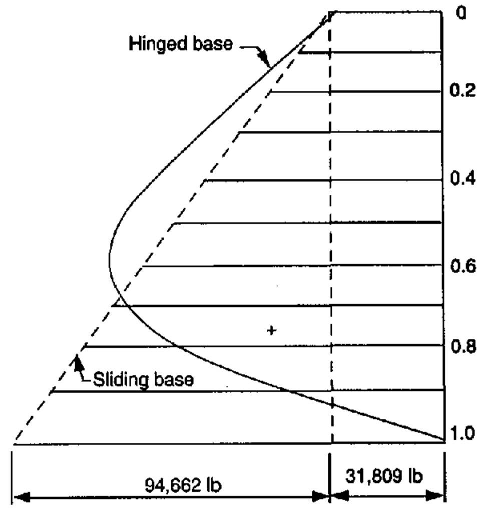


Figure 13—Combined ring tension in wall from fluid and vapor pressures

A_s = maximum ring tension/(0.9 × f_y)
= 93,539/(0.9 × 60,000)
= 1.73 sq in.
Use #6 bars spaced at 6 in. o.c. in two curtains (A_s = 1.76 sq in. per ft) at this location.

The maximum tensile stress in the concrete under service loads and including the effects of shrinkage is:

$f'_c = CE_sA_s + T_{max\ (unfactored)}/(A_c + nA_s)$
= [(0.0003 × 29 × 10^6 × 1.73) + 93,539/(1.65 × 1.7)]/(15 × 12 + 8 × 1.73)
= (15,051 + 33,347)/(180 + 13.8)
= 249.7 psi

Since 400 psi is considered allowable, the 15 in. wall thickness is sufficient.

The moments in vertical wall strips that are considered as one foot wide are determined on the basis of the coefficients taken from Table A-7 for $H^2/Dt = 6$. The coefficients from the table are multiplied by:

$w_uH^3 + pH^2 = 1{,}104{,}800 + 371{,}280 = 1{,}476{,}080$

where $w_uH^3 = 1{,}104{,}800$ (see Section 7)

and pH^2 = sanitary coefficient $\times$ (1.7 $\times$ Lateral Force) $\times$ H^2

$= 1.3 \times (1.7 \times 420) \times (20)^2$

$= 371{,}280$ ft lb per ft

The resulting moments along the height are shown in Table 8.

Table 8—Bending Moments for Wall with Hinged Base and Free Top–Trapezoidal Load

Point	Coefficient From Table A-7	Moment (ft-lb)
0.0H	0	0
0.1H	0	0
0.2H	0	0
0.3H	+0.0002	+295
0.4H	+0.0008	+1181
0.5H	+0.0019	+2805
0.6H	+0.0039	+5757
0.7H	+0.0062	+9152
0.8H	+0.0078	+11,513
0.9H	+0.0068	+10,037
1.0H	0	0

The moments are plotted in Fig. 14. The required vertical reinforcement for the wall with a maximum moment of 11,513 ft-lbs is:

$M_u/(\phi f'_c bd^2) = 11{,}513 \times 12/(0.9 \times 4000 \times 12 \times (12.5)^2)$
$= 0.0205$

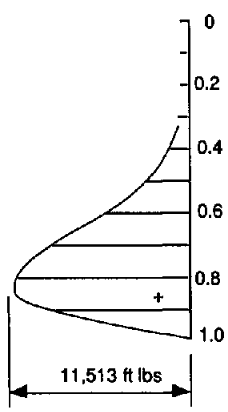


Figure 14—Bending moments for wall with hinged base and free top–trapezoidal load

From standard design aid:

$\omega = 0.0207$

$A_s = \omega bd f'_c/f_y = 0.0207 \times 12 \times 12.5 \times 4/60 = 0.207$

$\rho = 0.207/(12 \times 12.5) = 0.00138$

$\rho_{min} = 200/f_y = 0.0033 > 0.00138$

Use #6 bars at maximum allowable spacing of 10 in. ($A_s = 0.31$ in.^{2}, $\rho = 0.0035$).

The shear capacity of a 15 in. wall with $f'_c = 4000$ psi is:

$V_c = 2\sqrt{f'_c} \times b_w d$

$= 2\sqrt{4000} \times 12 \times 12.5$

$= 18{,}974$ kips

$\phi V_c = 0.85 \times 18{,}974 = 16{,}128$ kips

The applied shear at the base of the tank using the coefficient of 0.110 taken from Table A-12 for $H^2/Dt = 6$ equals:

$V_u = 0.110 \times (w_uH^2 + p_uH)$

$= 0.110 \times (1.7 \times 62.4 \times (20)^2 + 1.7 \times 420 \times 20)$

$= 0.110 \times (42{,}432 + 14{,}280)$

$= 6238 \text{ lbs} < 16{,}128 \text{ lbs}$

10 *Wall with Shear Applied at Top*

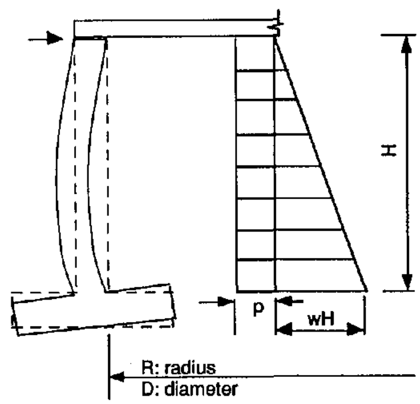


Figure 15—Wall with shear applied at top

As shown in Fig. 15, the presence of a slab on the top of the tank may prevent free movement at that location. The previous designs discussed were performed assuming that the top of the tank was free to displace. When displacement is prevented, the top cannot expand and the ring tension will be zero at that location. In the design of Section 9, with the top free to expand, the ring tension is 30,418. To prevent displacement, a shear must be added at the top sufficient to eliminate this ring tension.

Ring tension due to a shear, V, at the top is computed by using coefficients in Table A-8 for $H^2/Dt = 6$. The applicable coefficient equals -9.02 VR/H per ft at the top. The shear force required at the top of the tank to produce zero ring tension is:

$$-9.02 \times \frac{VR}{H} = -30{,}418$$

Therefore:

$$V = \frac{-30{,}418}{-9.02} \times \frac{H}{R}$$

$$V = 3372.3 \times \frac{20}{27} = 2498 \text{ lbs per ft}$$

To determine the ring tension, multiply coefficients in Table A-8 by VR/H = 2498 × 27/20 = 3372.3 lbs per ft. The results are shown in Table 9.

Table 9—Ring Tension in Wall with Shear Applied at Top

Point	Coefficient From Table A-8	Ring Tension (lbs/ft)
0.0H	-9.02	-30,418
0.1H	-5.17	-17,335
0.2H	-2.27	-7655
0.3H	-0.50	-1686
0.4H	+0.34	+1147
0.5H	+0.59	+1990
0.6H	+0.53	+1787
0.7H	+0.35	+1180
0.8H	+0.17	+573
0.9H	+0.01	+34
1.0H	0	0

The factored shear used to modify ring tension is 2498 lbs per ft. The sanitary coefficient for ring tension is 1.65, the coefficient for bending moments is 1.3. Therefore, the factored shear to determine bending moments is 2498 × 1.3/1.65 = 1968. The bending moments are determined by multiplying the coefficients of Table A-9 by VH = 1968 × 27 = 53,136 ft-lbs per ft. The results are shown in Table 10.

Table 10—Bending Moments in Wall with Shear Applied at Top

Point	Coefficient From Table A-9	Moment (ft-lb)
0.0H	0	0
0.1H	+0.062	+3294
0.2H	+0.070	+3720
0.3H	+0.056	+2976
0.4H	+0.036	+1913
0.5H	+0.018	+956
0.6H	+0.006	+319
0.7H	0	0
0.8H	-0.003	-159
0.9H	-0.005	-266
1.0H	-0.006	-319

The ring tensions and moments are plotted in Fig. 16. Note that the values in the lower one-half of the wall are so small that they can be ignored.

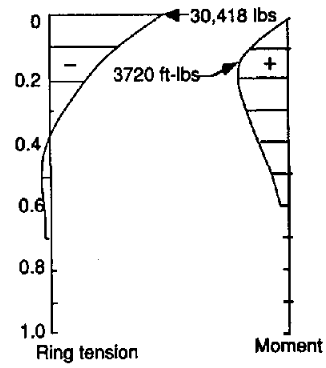


Figure 16—Ring tension and bending moments in wall with shear applied at top

Ring tensions and moments computed in this section are added to those in Section 9. The results of this addition are plotted in Fig. 17. It can be seen from this figure that the assumption of the top being free would be satisfactory. Consequently, the investigation made in this section may be omitted in most cases with the exception of tanks in which the ring tension is relatively large at the top and the wall is rigidly attached to the roof slab.

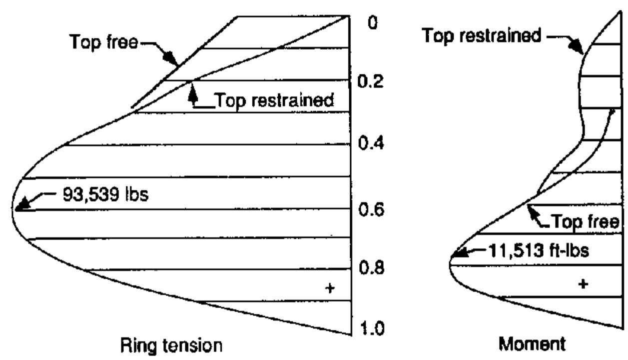


Figure 17—Ring tension and bending moments for trapezoidal load with roof in place

11 Wall with Shear Applied at Base

The shear developed at the base of the tank wall in the example of Section 9 is 6238 lbs per ft. This shear can only develop if the base of the tank is restrained against horizontal displacement. If the base were free to slide, the reaction at this location would be zero. Therefore, the shear at the base, not including the sanitary coefficient factor, will be somewhere between 0 and 6238 lbs.

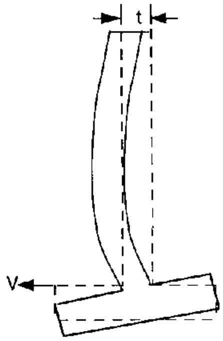


Figure 18—Wall with shear applied at base

It is difficult to ascertain the amount of shear force the base can resist without moving horizontally. Therefore, any value used will be nothing more than a reasonable estimate. For this example, an average value will be used as shown below:

For direct tension: Sanitary Coefficient × Average Shear
$1.65 \times 6238/2 = 5146$ lbs

For bending: Sanitary Coefficient × Average Shear
$1.3 \times 6238/2 = 4055$ lbs

The ring tension and moments will be obtained by superimposing two design conditions, one is the trapezoidal load with a hinged base as determined in Section 9, and the other for the shear of 5146 lbs for direct tension and 4055 lbs for bending applied outward at the base. The procedure for design for shear at the base will be demonstrated below.

To determine ring tension, multiply coefficients from Table A-8 by $VR/H = 5146 \times 27/20 = 6947$ lbs per ft. These values, including the effects from Section 9, are shown in Table 11 using values for $H^2/Dt = 6.0$.

Table 11—Ring Tension for Wall with Trapezoidal Load and Shear Applied at Base

Point	Coefficient from Table A-8	Ring Tension for Shear Force	Ring Tension from Section 9	Total Ring Tension
0.0H	0	0	+30,418	+30,418
0.1H	-0.01	-69	+41,654	+41,585
0.2H	-0.17	-1181	+53,651	+52,470
0.3H	-0.35	-2431	+65,645	+63,214
0.4H	-0.53	-3682	+77,642	+73,960
0.5H	-0.59	-4099	+87,487	+83,388
0.6H	-0.34	-2362	+93,539	+91,177
0.7H	+0.50	+3474	+90,864	+94,338
0.8H	+2.27	+15,770	+75,541	+91,311
0.9H	+5.17	+35,916	+44,536	+80,452
1.0H	+9.02	+62,662	0	+62,662

The bending moment is calculated by multiplying the coefficients from Table A-9 by $VH = 4055 \times 20 = 81{,}100$ ft-lbs per ft. These values including the effects from Section 9 are shown in Table 12 using a value of $H^2/Dt = 6.0$.

Table 12—Bending Moments for Wall with Trapezoidal Load and Shear Applied at Base

Point	Coefficient from Table A-18	Moment from Shear Force	Moment from Section 9	Total Moment
0.0H	0.006	+487	0	+487
0.1H	+0.005	+406	0	+406
0.2H	+0.003	+243	0	+243
0.3H	0	0	+295	+295
0.4H	-0.006	-487	+1181	+694
0.5H	-0.018	-1460	+2805	+1345
0.6H	-0.036	-2920	+5757	+2837
0.7H	-0.056	-4542	+9152	+4610
0.8H	-0.070	-5677	+11,513	+5836
0.9H	-0.062	-5028	+10,037	+5009
1.0H	0	0	0	0

It makes a considerable difference whether the base is fully or partially restrained for horizontal displacement, as shown in Fig. 19. The effects of the movement of the base, though difficult to calculate, cannot be ignored. But, it is often possible to omit the investigation in this section and still obtain a satisfactory solution. A possible solution is to use the solution from the regular ring tension for a hinged base from the top of the tank down to the point of maximum tension. The maximum tension is then used from this location to the base as shown in Fig. 19. The difference between the moment curves is considerable and using the larger values obtained from the hinged base are recommended.

Another possible solution is to use the average of the results from that of a restrained base (Section 9 results) and that of free sliding base. These results are shown in Figure 19(b). This method is much quicker and gives results as reasonable as the previous method.

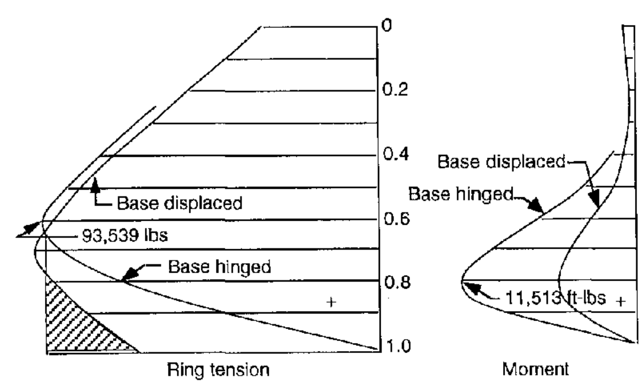


Figure 19a—Bending moments and ring tension for wall with trapezoidal load and shear applied at base

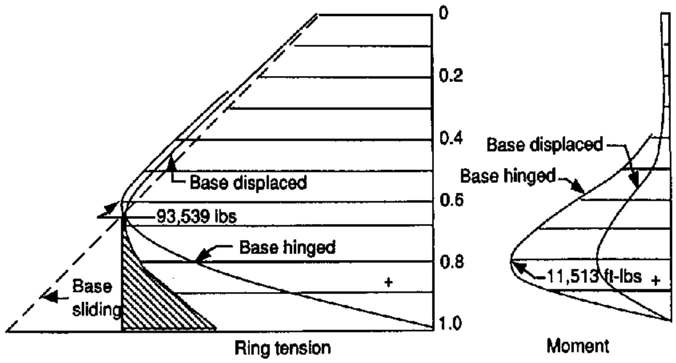


Figure 19b—Bending moments and ring tension for wall with trapezoidal load and shear applied at base (averaging method)

12 *Wall with Moment Applied at Top*

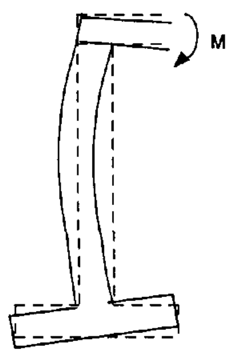


Figure 20—Wall with moment applied at top

When the top of the wall and the roof slab are made continuous, as shown in Fig. 20, the deflection of the roof slab will rotate the top of the wall. This rotation will induce a moment at the top of the wall.

The cover assumed for the tank design in this section will be a 12 in. thick reinforced concrete slab with a center support. It will also be assumed that the factored moment (excluding the sanitary coefficient) with fixed edges is -12,500 ft-lbs per ft. Since the tank roof and wall are integral, a portion of this moment will be transferred to the tank wall.

The procedure used to determine the amount of moment transferred from the roof to the wall is similar to moment distribution of continuous frames. The data in Tables A-15 and A-16 are stiffnesses which denote moments required to impart a unit rotation at the edge of the wall and the slab.

The moment required to rotate the tangent at the edge through a given angle is proportional to the following relative stiffness factors.

For the wall (Table A-15 for $H^2/Dt = 6$):

$0.0783t^3/H = 0.783 \times 15^3/20 = 132$

For the slab (Table A-16 for $c/D = 0.15$ where c is the column capital diameter = 8 ft):

$0.332t^3/R = 0.332 \times 12^3/27 = 21$

The distribution factors are:

For the wall: $\dfrac{132}{132+21} = 0.86$

For the slab: $\dfrac{21}{132+21} = 0.14$

The factored moment of -12,500 ft-lbs per ft will tend to rotate the fixed joint as shown in Fig. 21. When the artificial restraint is removed, the rotation of the joint will induce additional moments in the wall. The sum of the induced moment and the original fixed end moments are the final moments. The moments must be equal and opposite as shown in Fig. 21. Calculations may be arranged in accordance with the usual moment distribution procedure.

	Wall	Slab
Distribution factor	0.86	0.14
Fixed end moment	0	-12,500
Distributed moment	10,750	1750
Final moment	10,750	-10,750

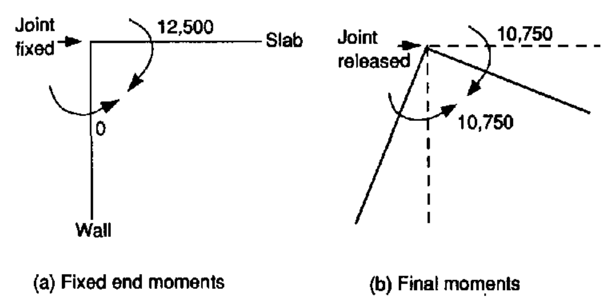


Figure 21—Rotation of slab-wall joint

The ring tension caused by the moment applied at the top is determined by multiplying the coefficients of Table A-10 by MR/H^2. The value of MR/H^2 is determined as follows:

$$MR/H^2 = (\text{sanitary coefficient} \times 10{,}750) \times 27/20^2$$
$$= 1.65 \times 10{,}750 \times 27/20^2$$
$$= 1197.3 \text{ lbs per ft}$$

The ring tension along the height of the tank is shown in Table 13 and Fig. 22.

Table 13—Ring Tension for Wall with Moment Applied at Top

Point	Coefficient From Table A-10	Ring Tension (lbs/ft)
0.0H	0	0
0.1H	+11.41	+13,661
0.2H	+13.08	+15,661
0.3H	+10.28	+12,308
0.4H	+6.54	+7830
0.5H	+3.34	+3999
0.6H	+1.21	+1449
0.7H	-0.05	-60
0.8H	-0.59	-706
0.9H	-0.86	-1030
1.0H	-1.04	-1245

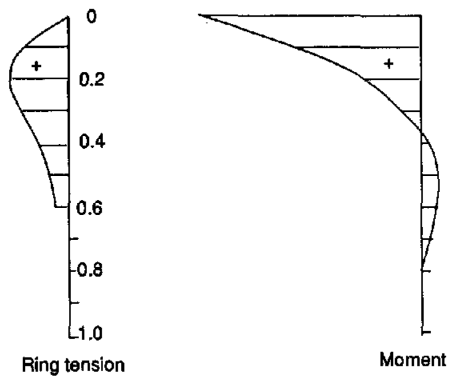


Figure 22—Ring tension and bending moments for wall with moment applied at top

The moments in the vertical wall strips that are considered as one foot wide are determined on the basis of the coefficients from Table A-11 multiplied by M where:

$M = 1.3 \times 10{,}750 = 13{,}975$

The resulting moments along the height are shown in Table 14 and Fig. 22.

Table 14—Bending Moments for Wall with Moment Applied at Top

Point	Coefficient From Table A-11	Moment (ft-lb)
0.0H	+1.00	+13,975
0.1H	+0.572	+7994
0.2H	+0.252	+3522
0.3H	+0.057	+797
0.4H	-0.037	-517
0.5H	-0.065	-908
0.6H	-0.058	-811
0.7H	-0.040	-559
0.8H	-0.018	-252
0.9H	-0.005	-70
1.0H	0	0

The ring tension and the moments determined in this section are added to those in Section 9 as shown in Tables 15 and 16.

Table 15—Combined Ring Tension from Trapezoidal Load and Moment Applied at Top

Point	Ring Tension from Section 9	Ring Tension from Moment	Total Ring Tension
0.0H	+30,418	0	+30,418
0.1H	+41,654	+13,661	+55,315
0.2H	+53,651	+15,661	+69,312
0.3H	+65,645	+12,308	+77,953
0.4H	+77,642	+7830	+85,472
0.5H	+87,487	+3999	+91,486
0.6H	+93,539	+1449	+94,988
0.7H	+90,864	-60	+90,804
0.8H	+75,541	-706	+74,835
0.9H	+44,536	-1030	+43,506
1.0H	0	-1245	-1245

Table 16—Combined Bending moments for Trapezoidal Load and Moment Applied at Top

Point	Moment from Section 9	Moment from this Section	Total Moment
0.0H	0	+13,975	+13,975
0.1H	0	+7994	+7994
0.2H	0	+3522	+3522
0.3H	+295	+797	+1092
0.4H	+1181	-517	+664
0.5H	+2805	-908	+1897
0.6H	+5757	-811	+4946
0.7H	+9152	-559	+8593
0.8H	+11,513	-252	+11,161
0.9H	+10,037	-70	+9967
1.0H	0	0	0

The effect of adding the moment at the top of the wall is shown in Fig. 23. The moment will increase both the ring tension and bending moments at the top of the wall.

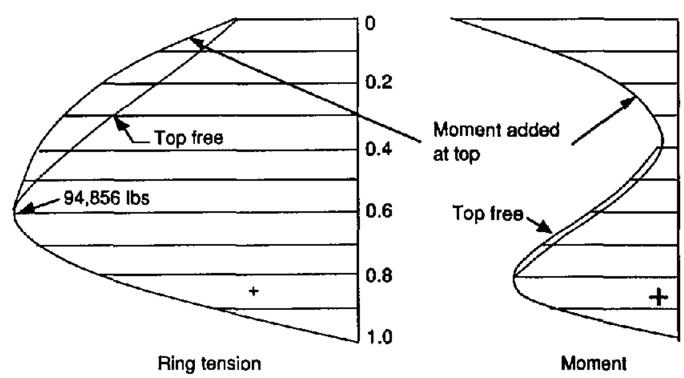


Figure 23—Combined ring tension and bending moments for trapezoidal load and moment applied at top

13 Wall with Moment Applied at Base

In the previous sections, the wall has been assumed to rest on a footing, not continuous with the bottom slab of the tank. In many cases, the base slab and tank wall

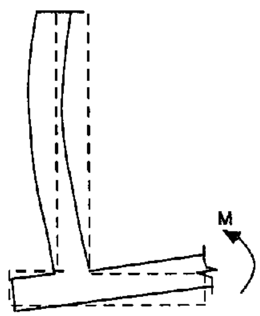


Figure 24—Wall with moment applied at base

are one integral unit. Because of this continuity, a portion of the bending moments that may be present in the base slab will be transferred to the tank wall.

For this section it is assumed that the factored moment (not including the sanitary coefficient) at the fixed edge of the base slab is 38,000 ft-lbs per ft. A triangular load on the wall of w = 62.5 lbs per cu ft will be used for the liquid pressure.

The moment at the base of the wall is first computed on the assumption that the base is fixed, and a correction is then made for rotation of the base of the wall caused by the continuity between the slab and the wall. The fixed end moment at the base of the wall for the liquid pressure is determined for the triangular loading with the coefficient from Table A-2 for $H^2/Dt = 6$. The moment is equal to:

$$\begin{aligned}\text{Base moment} &= -0.0187 \times wH^3 \\ &= -0.0187 \times (1.3 \times 1.7 \times 62.5) \times 20^3 \\ &= -20{,}664 \text{ ft-lbs per ft}\end{aligned}$$

As long as the base of the wall is artificially fixed against any rotation, it is subject to two moments. One moment is due to the outward pressure of the liquid, the other is due to the moment at the edge of the base slab. When the artificial restraint is removed, the joint will rotate and the moments will be redistributed. Calculation of the final moments may be arranged in accordance with the usual moment distribution procedure.

Moment Distribution Procedure

	Wall	Slab
Distribution factor (use values from Section 12)	0.86	0.14
Fixed end moment		(1.3 × 38,000) =
	-20,664	-49,400
Induced moment	60,255	9808
Final moment	39,591	-39,591

The ring tension and bending moments throughout the height of the wall are investigated in two steps. First, the base is assumed fixed, and second, a moment equal to the induced moment is applied at the base. The results are then combined to obtain the actual base moment.

The moments in the vertical wall strips for the first part of the analysis are computed by multiplying w_uH^3 by the coefficients from Table A-2. The value of w_uH^3 was previously calculated in Section 7 as 1,104,800 ft-lbs/ft. The resulting moments along the height for an H^2/Dt value of 6 are shown in Table 17. The moments along the height of the wall for an applied moment at the base of 60,255 ft-lbs are shown in Table 18. The results are considered in Table 19.

Table 17—Moments in Tank from Liquid Pressure

Point	Coefficient From Table A-2	Moment (ft-lb)
0.0H	0	0
0.1H	+0.0001	+110
0.2H	+0.0003	+331
0.3H	+0.0008	+884
0.4H	+0.0019	+2099
0.5H	+0.0032	+3535
0.6H	+0.0046	+5082
0.7H	+0.0051	+5634
0.8H	+0.0029	+3204
0.9H	-0.0041	-4530
1.0H	-0.0187	-20,664

Table 18—Moments in Tank from Applied Moment at Base

Point	Coefficient From Table A-11	Moment (ft-lb)
0.0H	0	0
0.1H	-0.005	-301
0.2H	-0.018	-1085
0.3H	-0.040	-2410
0.4H	-0.058	-3495
0.5H	-0.065	-3917
0.6H	-0.037	-2229
0.7H	+0.057	+3435
0.8H	+0.252	+15,184
0.9H	+0.572	+34,466
1.0H	+1.000	+60,255

The resulting ring tension along the height of the wall for the triangular load, with w_uHR of 94,662 lbs per cu ft (see Section 7) is given in Table 20. For ring tension, the sanitary coefficient is 1.65 whereas for flexure it is 1.3. Therefore, the induced moment, M, at the base of the wall to determine ring tension is $(1.65/1.3) \times 60{,}255 = 76{,}478$ ft-lbs and $MR/H^2 = 5162$. The ring tension along the height of the wall for an applied moment of 76,478 ft-lbs is shown in Table 21. The results are combined in Table 22.

Table 19—Combined Bending Moments from Liquid Pressure and Applied Moment at Base

Point	Moments from Liquid Pressure	Moments from Base Slab	Total Moments
0.0H	0	0	0
0.1H	+110	-301	-191
0.2H	+331	-1085	-754
0.3H	+884	-2410	-1526
0.4H	+2099	-3495	-1396
0.5H	+3535	-3917	-382
0.6H	+5082	-2229	+2853
0.7H	+5634	+3435	+9069
0.8H	+3204	+15,184	+18,388
0.9H	-4530	+34,466	+29,936
1.0H	-20,664	+60,255	+39,591

Table 20—Ring Tension from Liquid Pressure

Point	Coefficient From Table A-1	Ring Tension (lbs/ft)
0.0H	+0.018	+1704
0.1H	+0.119	+11,265
0.2H	+0.234	+22,151
0.3H	+0.344	+32,564
0.4H	+0.441	+41,746
0.5H	+0.504	+47,710
0.6H	+0.514	+48,656
0.7H	+0.447	+42,314
0.8H	+0.301	+28,493
0.9H	+0.112	+10,602
1.0H	0	0

Table 21—Ring Tension from Applied Moment at Base

Point	Coefficient From Table A-10	Ring Tension (lbs/ft)
0.0H	-1.04	-5368
0.1H	-0.86	-4439
0.2H	-0.59	-3046
0.3H	-0.05	-258
0.4H	+1.21	+6246
0.5H	+3.34	+17,241
0.6H	+6.54	+33,759
0.7H	+10.28	+53,065
0.8H	+13.08	+67,519
0.9H	+11.41	+58,898
1.0H	0	0

Table 22—Combined Ring Tension Liquid Pressure and Applied Moment at Base

Point	Ring Tension from Liquid Pressure	Ring Tension from Base Moment	Total Ring Tension
0.0H	+1704	-5368	-3664
0.1H	+11,265	-4439	+6826
0.2H	+22,151	-3046	+19,105
0.3H	+32,564	-258	+32,306
0.4H	+41,746	+6246	+47,992
0.5H	+47,710	+17,241	+64,951
0.6H	+48,656	+33,759	+82,415
0.7H	+42,314	+53,065	+95,379
0.8H	+28,493	+67,519	+96,012
0.9H	+10,602	+58,898	+69,500
1.0H	0	0	0

Ring tension and moments for fixed base and for the actual base condition are plotted in Fig. 25.

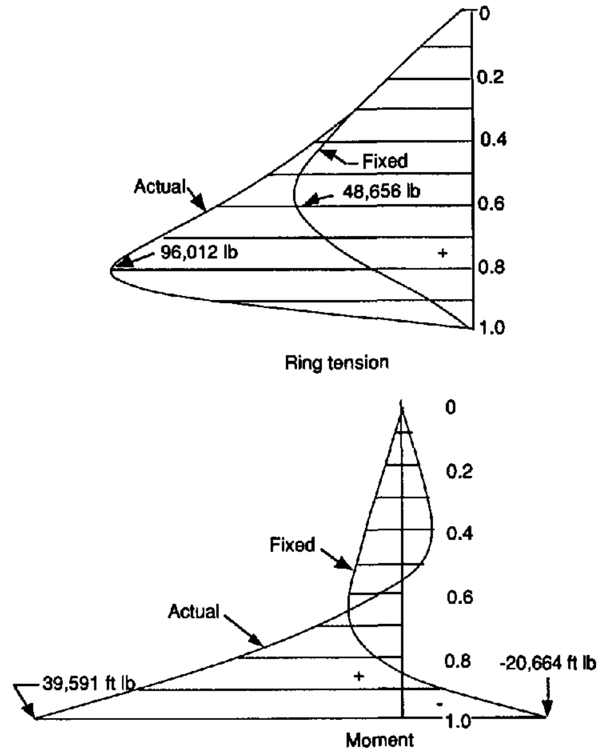


Figure 25—Combined ring tension and bending moments for liquid pressure and applied moment at base

The maximum ring tension is 48,656 if the base is fixed and is 96,012 for the actual base condition. The moment at the base is changed from -20,660 to +39,591. It is clear that continuity between wall and bottom slab materially affect both ring tension and moments and, if applicable, must be considered in design.

Shear at the base of the wall when the base is fixed may be computed as the sum of the products of coefficients taken from Table A-12 multiplied by w_uH^2 $(1.0 \times 1.7 \times 62.5 \times 20^2) = 42{,}500$ lbs per ft and M/H $(60{,}255/(1.3 \times 20)) = 2318$ lbs per ft.

When the base is fixed:

$0.197 \times wH^2 = 0.197 \times 42{,}500 = +8373$ lbs

Effect of M at base:

$-4.49 \times M/H = -4.49 \times 2318 = \underline{-10{,}408 \text{ lbs}}$

-2035 lbs

14 *Roof Slab Without Center Support*

Conventionally reinforced flat plate tank roofs without any interior supports will have limited span lengths

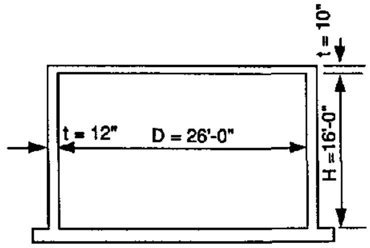


Figure 26—Roof slab without center support

and are feasible only for small diameter tanks. This type of roof is typically used for tanks with diameters less than 30 ft.

ACI 350 does not provide guidelines for the designs of slabs and ACI 318 must be consulted. ACI 318-89 provides a minimum slab thickness for both one-way (Section 9.5.2.1) and two-way slabs (Section 9.5.3.2). Neither of these two sections can be directly applied to a circular roof slab without interior supports. When the roof slab is continuous with the wall, a suggested approach is to choose a minimum slab thickness between that of a one-way slab ($\ell/28$) and a two-way slab ($\ell/33$), where ℓ is the span length. In the case of a simply supported slab, a minimum slab thickness of $\ell/20$ should be appropriate. If deflection control is critical or ponding of the roof is a possibility, the designer should perform a more detailed analysis to determine the deflection.

The dimensions of the roof slab to be designed are shown in Fig. 26. The roof will be designed for a live load of 100 psf and a superimposed dead load of 300 psf. The required strength, U, will not be multiplied by the sanitary coefficient. If crack control and corrosion of the roof slab are a concern, the designer may consider using the sanitary coefficients. The required strength, U, is:

$$U = 1.4 \times (300 + 150 \times 10/12) + 1.7 \times 100$$
$$= 765 \text{ psf}$$

For the wall, $H^2/Dt = 16^2/(26 \times 1) = 9.8$, say, 10. From Table A-15, for $H^2/Dt = 10$, the relative stiffness of the wall is $1.010 \times t^3/H = 1.010 \times 12^3/16 = 109$. The relative stiffness of a circular plate without any interior supports (Table A-16) is $0.104t^3/R = 0.104 \times 10^3/13 = 8.0$. The distribution factors are:

$$\text{wall} = \frac{109}{109 + 8} = 0.93$$

$$\text{slab} = \frac{8}{109 + 8} = 0.07$$

When the slab is fixed at the edge, the edge moment may be computed by multiplying pR^2 by the coefficient from Table A-14 at point 1.00R: $-0.125 \times pR^2 = -0.125 \times 765 \times 13^2 = -16{,}161$ ft-lbs per ft of periphery.

The procedure for determining the final moments at the edge has been previously illustrated. The fixed end moments are shown in Fig. 27. The final moments which are also shown in Fig. 27 are computed by moment distribution.

	Wall	Slab
Distribution factor	0.93	0.07
Fixed end moment	0	16,161
Distributed moment	15,030	1131
Final moment	15,030	15,030

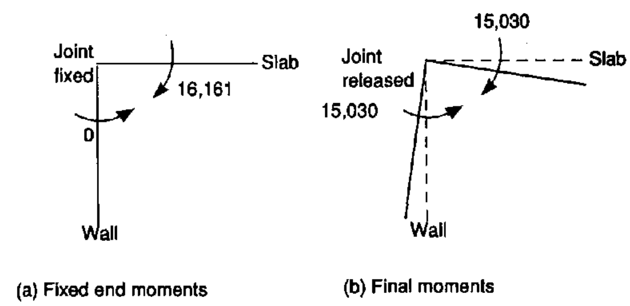


Figure 27—Rotation of slab wall joint

It is seen that a large moment is induced in the top of the wall. It has been shown previously how to determine ring tension and moments in a wall caused by a moment at the top of the tank wall (Section 12). Only design of the slab will be discussed in this section.

The shear capacity in a 10 in. thick slab with f'_c = 4000 psi is:

$$V_c = 2\sqrt{f'_c} \times b_w d$$
$$= 2 \times \sqrt{4000} \times 12 \times 8.5 = 12{,}902 \text{ lbs}$$

$$\phi V_c = 0.85 \times 12{,}902 = 10{,}967 \text{ lbs}$$

The applied shear is:

$$V = \frac{\text{Area} \times \text{Load}}{\text{Perimeter}} = \frac{\pi R^2 \times p}{2\pi R} = \frac{Rp}{2}$$
$$= 13 \times 765/2 = 4973 < 10{,}967 \text{ lbs per ft, O.K.}$$

For illustration, consider a tank in which there is a joint at the top of the wall causing the slab to be hinged. The moments in the hinged slab may be computed by determining the moments in a fixed slab (Table A-14) and adding to them the moments in a slab in which the end moment of $0.125pR^2$ ft-lbs per ft is applied. These combined results will give the results at a hinged support. The most convenient way to do this is to add 0.125 to all the coefficients in Table A-14, both the radial and tangential moments, and then to multiply the modified coefficients by pR^2. Note that the coefficients for radial moments at the edge become zero by the addition of 0.125, and the tangential moment becomes 0.100. These are values for a slab hinged at the edge.

In the design problem of this section, the roof is continuous with the tank wall and the induced moment is 1131 ft-lbs per ft. Therefore, the final moment coefficients are those for a fixed edge (Table A-14) to each of which must be added a quantity equal to $1131/pR^2 = 1131/(765 \times 13^2) = 0.009$. These new coefficients are multiplied by $pR^2 = 765 \times 13^2 = 129{,}285$ ft-lbs per ft. The results are shown in Table 23, Table 24 and Fig. 28 with 0.0R denoting the center, and Point 1.0R, the edge of the slab. Note that these moments are for a one-foot wide slab across the tank roof. Since the reinforcing will be placed radially, the design width for reinforcing will not be one-foot wide but will decrease as the center of the tank is approached. For this reason, the moments shown in Fig. 28 include the radial moment per section. These are obtained by multiplying the original moments by the fraction indicating its distance from the center. For illustration at 0.5R, the radial moment per segment is equal to: $4396 \times 0.5 = 2198$.

Table 23—Radial Moments for Roof Slab Without Center Support

Point	Coefficient from Table A-14	Add 0.009	Revised Coefficient	Radial Moment	Radial Moment per Segment*
0.0R	+0.075	+0.009	+0.084	+10,860	0
0.1R	+0.073	+0.009	+0.082	+10,601	+1060
0.2R	+0.067	+0.009	+0.076	+9826	+1965
0.3R	+0.057	+0.009	+0.066	+8533	+2560
0.4R	+0.043	+0.009	+0.052	+6723	+2689
0.5R	+0.025	+0.009	+0.034	+4396	+2198
0.6R	+0.003	+0.009	+0.012	+1551	+931
0.7R	-0.023	+0.009	-0.014	-1810	-1267
0.8R	-0.053	+0.009	-0.044	-5689	-4551
0.9R	-0.087	+0.009	-0.078	-10,084	-9076
1.0R	-0.125	+0.009	-0.116	-14,997	-14,997

*1 foot wide at outside edge

Table 24—Tangential Moments for Roof Slab Without Center Support

Point	Coefficient from Table A-14	Add 0.009	Revised Coefficient	Tangential Moment
0.0R	+0.075	+0.009	+0.084	+10,860
0.1R	+0.074	+0.009	+0.083	+10,731
0.2R	+0.071	+0.009	+0.080	+10,343
0.3R	+0.066	+0.009	+0.075	+9696
0.4R	+0.059	+0.009	+0.068	+8791
0.5R	+0.050	+0.009	+0.059	+7628
0.6R	+0.039	+0.009	+0.048	+6206
0.7R	+0.026	+0.009	+0.035	+4525
0.8R	+0.011	+0.009	+0.020	+2586
0.9R	-0.006	+0.009	+0.003	+388
1.0R	-0.025	+0.009	-0.016	-2069

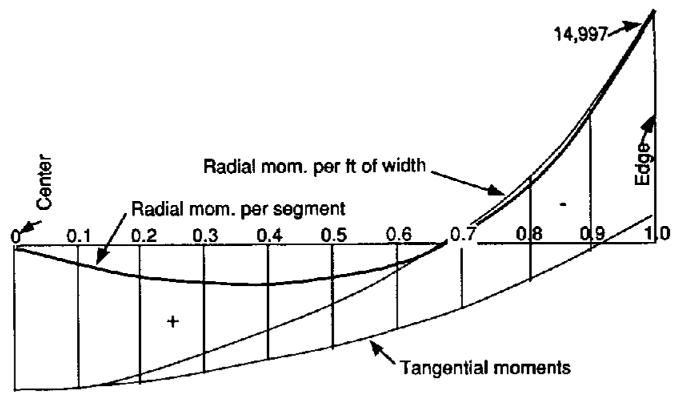


Figure 28—Radial and tangential moments for roof slab without center support

The maximum negative radial moment is 14,997 ft-lbs per ft. The required amount of reinforcing on the top of the slab at that location is:

$M_u/(\phi f'_c bd^2) = 14{,}997 \times 12/(0.9 \times 4000 \times 12 \times (8.5)^2)$
$= 0.0577$

For standard design aid in Appendix A:

$\omega = 0.060$

$A_s = \omega bd f'_c/f_y = 0.060 \times 12 \times 8.5 \times 4/60 = 0.408$ in.^{2}

Use #5 bars spaced 9 in. o.c. ($A_s = 0.413$ in.^{2}) in the top slab at the wall.

Total number of bars required is $2\pi R/s = 2\pi \times 13 \times 12/9 = 109$ bars. For simplicity, these bars will be used for the entire length of the negative moments. Therefore, the length of these 109 bars will be 0.35R + development length.

The largest positive moment is located at approximately Point 0.4R and has a value of 6723 ft-lbs per ft. The length of the concentric circle through 0.4R is $2\pi(0.4R) = 2\pi \times 0.4 \times 13 = 32.7$ ft.

The required amount of reinforcing on the bottom of the slab at this location is:

$M_u/(\phi f'_c bd^2) = 6723 \times 12/(0.9 \times 4000 \times 12 \times 8.5^2)$
$= 0.0258$

From standard design aid in Appendix A:

$\omega = 0.026$

$A_s = \omega bd f'_c/f_y = 0.026 \times 12 \times 8.5 \times 4/60$
$= 0.177$

Use #5 bars ($A_s = 0.31$ in.^{2})

Figure 29 shows one arrangement with eight radial bars in each quadrant. Sixteen bars are required for the slab and are bent as shown. Note that there are only two layers where the bars cross at the center in Fig. 29 and only four types of bent bars are required.

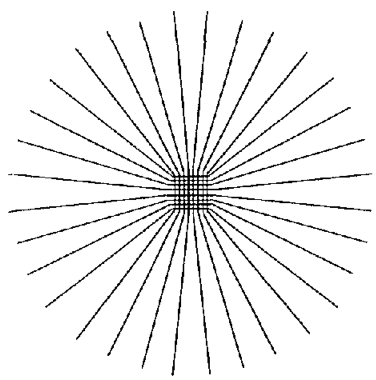


Figure 29—Radial reinforcement at center of roof slab without center support

Ring bars are proportioned to fit the tangential curve in Fig. 28. The maximum area of steel is required near the center. The required amount of reinforcing steel at the bottom near the center is:

$M_u/(\phi f'_c bd^2) = 10{,}860 \times 12/(0.9 \times 4000 \times 12 \times (8.5)^2)$
$= 0.0418$

From standard design aid in Appendix A:

$\omega = 0.0428$

$A_s = \omega bd f'_c/f_y = 0.0428 \times 12 \times 8.5 \times 4/60 = 0.291$ in.^{2}

$\rho = 0.291/(12 \times 8.5) = 0.0029$

$\rho_{min} = 200/f_y = 0.0033 > 0.0029$

Use #5 bars on 10 in. o.c. ($A_s = 0.31$ in.^{2}, $\rho = 0.0037$)

Since the required area of the bars for tangential moments decreases gradually toward Point 0.9R, the #5 bars at 12 in. o.c. can be used for all the top circular reinforcing. Between 0.9R and 1.0R the bars will be placed in the top of the slab.

This design utilized radial and circular reinforcement. It is also common to use a rectangular layout for the reinforcing.

15 *Roof Slab with Center Support*

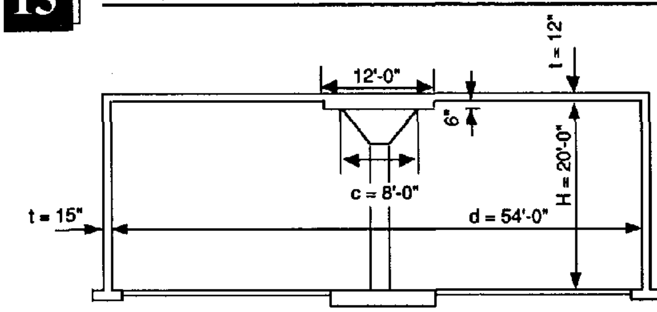


Figure 30—Roof slab with center support

The tank dimensions for the design of the roof slab in this section are shown in Fig. 30. The roof slab will be designed for a live load of 100 psf and a superimposed dead load of 300 psf. The required strength, U, is:

$U = 1.4 \times (300 + 150 \times 12/12) + 1.7 \times 100$
$= 800$ psf

Data are presented in Tables A-17, A-18, and A-19 for slabs with a center support for the following ratios of column capital to wall diameter, c/D = 0.05, 0.15, 0.20, and 0.25. The tables are for fixed and hinged edge as well as for moment applied at the edge.

The general procedure in this section is the same as in the previous section. First the edge of the roof slab is considered fixed and the fixed end moments are computed. Then the moments at the edge are distributed and adjustments are made for the change in the edge moment.

The values in Tables A-17, A-18, and A-19 are based on a uniform slab thickness. The presence of a drop panel will have some effect, but it is believed that the change is relatively small especially since the ratio of panel area to total slab area is typically very small.

The relative stiffness factors for a roof slab and wall with the dimensions used here were previously calculated in Section 13. The relative stiffness factors are 0.86 for the wall and 0.14 for the slab.

The radial fixed end moment equals the coefficient of -0.0490 from Table A-17 (for c/D = 8/54 = 0.15 at Point 1.0R) multiplied by pR^2. For p = 800 psf, the fixed end moment is $-0.0490 \times 800 \times 27^2 = -28{,}577$ ft-lbs per ft. The final edge moment for which the slab is designed is $-28{,}577 \times (1 - 0.14) = 24{,}576$ ft-lbs per ft.

The procedure is to design the slab for a fixed edge moment of -28,577 and then add the effect of a moment 28,577 – 24,576 = 4001 ft-lbs applied at the edge, but first, shearing stresses must be investigated.

The column load is determined by multiplying coefficients taken from Table A-13 by pR^2.

For a fixed edge:

$1.007 \times pR^2 = 1.007 \times 800 \times 27^2 = 587{,}282$ lbs

Effect of moment at edge:

$9.29M = 9.29 \times 4001 = 37{,}169$

Total Column Load = 624,451 lbs

The radius of the critical section for shear around the column capital is $4 \times 12 + 18 - 1.5 = 64.5$ in. = 5.38 ft. The length of this section is $2\pi \times 64.5 = 405$ in. Load on the area within this section is $800 \times \pi \times 5.38^2$ = 72,745 lbs. The shear at the face of the column capital is (624,451 – 72,745)/1000 = 552 kips. The shear capacity of a 12 in. thick slab with a 6 in. thick drop panel is:

$$V_c = 2\sqrt{f'_c} \times b_w d$$
$$= 2\sqrt{4000} \times 405 \times 16.5/1000 = 845 > 552 \text{ kips, O.K.}$$

The radius of the critical section for shear around the drop panel is $6 \times 12 + 12 - 1.5 = 82.5$ in. = 6.88 ft. Length of this section is $2\pi \times 82.5 = 518$ in. The load on the area within this section is $800 \times \pi \times 6.88^2$ = 118,964. The shear edge of the drop panel is (624,451 – 118,964)/1000 = 505 kips. The shear capacity of a 12 in. thick slab is:

$$V_c = 2\sqrt{f'_c} \times b_w d$$
$$= 2\sqrt{4000} \times 518 \times 10.5/1000 = 688 > 505 \text{ kips}$$

Shear at the edge of the tank wall is:

$$V = \pi pR^2 - \text{column load}$$
$$= (\pi \times 800 \times 27^2) - 624{,}451 = 1{,}207{,}726 \text{ lbs}$$
$$= 1207 \text{ kips}$$

The circumference of the tank is $2 \times \pi \times 27 \times 12$ = 2036 in. The shear capacity of a 12 in. thick slab is:

$$V_c = 2\sqrt{f'_c} \times b_w d$$
$$= 2\sqrt{4000} \times 2036 \times 10.5/1000 = 2704$$
$$\phi V_c = 0.85 \times 2704 = 2298 > 1207 \text{ kips}$$

The radial moments are computed by selecting coefficients for c/D = 0.15 from Tables A-17 and A-19, and multiplying them by $pR^2 = 800 \times 27^2$ = 583,200 ft-lbs per ft (for fixed edge), and by M = 4001 ft-lbs per ft (for moment edge). These moments are shown in Table 25 and in Fig. 31. The maximum negative moment at the center occurs at the edge of the column capital.

Table 25—Radial Moments for Roof Slab with Center Support

Point	Coefficient for Fixed Conditions	Radial Moment for Fixed Edge	Coefficient for Moment at Edge	Radial Moment for Moment at Edge	Total Radial Moment	Radial Moment per Segment
0.15R	-0.1089	-63,510	-1.594	-6378	-69,888	-10,483
0.2R	-0.0521	-30,385	-0.930	-3721	-34,106	-6821
0.25R	-0.0200	-11,664	-0.545	-2181	-13,845	-3461
0.3R	+0.0002	+117	-0.280	-1120	-1003	-301
0.4R	+0.0220	+12,830	+0.078	+312	+13,142	+5257
0.5R	+0.0293	+17,088	+0.323	+1292	+18,380	+9190
0.6R	+0.0269	+15,688	+0.510	+2041	+17,729	+10,637
0.7R	+0.0169	+9856	+0.663	+2653	+12,509	+8756
0.8R	+0.0006	+350	+0.790	+3161	+3511	+2809
0.9R	-0.0216	-12,597	+0.900	+3601	-8996	-8096
1.0R	-0.0490	-28,577	+1.000	+4001	-24,576	-24,576

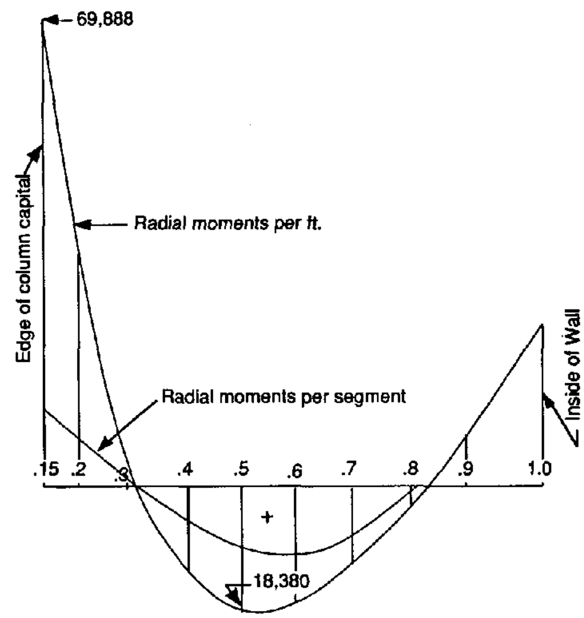


Figure 31—Radial moments for roof slab with center support

The theoretical moment across the section around the column capital is larger than the moment that actually exists. It should be remembered that the moment coefficients in this section are computed for a slab that is assumed to be fixed at the edge of the capital. Actually, the edge is not fixed, but has some rotation and a reduction in the theoretical moment will occur.

The problem of determining the actual moment at the capital is similar to that which exists in regular flat slab design. As a matter of fact, the region around the center column in the tank slab is stressed very much as in ordinary flat slab floor construction, so that the design should be practically identical in the column region of both types of structures.

A rigorous analysis of two-way slabs with circular capitals was presented by Nichols in 1914[4]. The expression derived by Nichols for the sum of the total positive and negative moment, M_o, is given as:

$$M_o = \frac{WL}{8}\left(1 - \frac{2c}{3L}\right)^2$$

where L = span length, center-to-center of column

c = diameter of column capital

W = total panel load

The ACI 318 codes have not required that the slab be designed for the full theoretical value of M_o. In fact, even though the equation for determining the design for M_o has changed over the years, it is consistently about 25% lower than the value of M_o from the rigorous analysis. In view of this discussion, it seems reasonable to also use a 25% reduction in the theoretical moments around the center columns of the tank slab. The reduction will be used here for radial moments at the capital only. The other moments in the slab are not large enough to consider a reduction.

For the slab shown in Fig. 30, the moment at the edge of the capital will then be taken as (1 – 0.25) × 69,888 = 52,416 ft-lbs per ft. The required amount of reinforcement at the top of the slab at this location is:

$$M_u/(\phi f'_c bd^2) = 52{,}416 \times 12/(0.9 \times 4000 \times 12 \times (16.5)^2) = 0.0535$$

From standard design aid in Appendix A:

$\omega = 0.055$

$A_s = \omega bd f'_c/f_y = 0.055 \times 12 \times 16.5 \times 4/60 = 0.726$ in.^{2}/ft

Total $A_s = 8 \times \pi \times 0.726 = 18.2$ in.^{2}

Use 28-#8 bars (A_s = 22.1 in.^{2}) as shown in Fig. 32.

Across the edge of the drop panel the moment is approximately 34,106 ft-lbs per ft (at point 6/27 = 0.22, use 0.2). Using the 25% reduction, the moment to be designed for is 34,106 × (1 – 0.25) = 25,580 ft-lbs per ft. The required amount of reinforcement to the top of the slab at this location is:

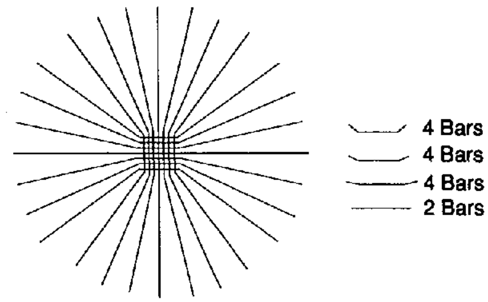


Figure 32—Radial reinforcement for roof slab with center support

$$M_u/(\phi f'_c bd^2) = 25{,}580 \times 12/(0.9 \times 4000 \times 12 \times (10.5)^2) = 0.0644$$

From standard design aid in Appendix A:

$\omega = 0.067$

$A_s = \omega bd f'_c/f_y = 0.067 \times 12 \times 10.5 \times 4/60 = 0.563$ in.^{2}/ft

Total $A_s = 12 \times \pi \times 0.563 = 21.2$ in.^{2} The 28-#8 bars (A_s = 22.1 in.^{2}) will be adequate.

The maximum positive moment per segment occurs at Point 0.6R as indicated in Fig. 31. The moment at this point is 17,729 ft-lbs per ft. The required amount of reinforcing at the bottom of the slab at this location is:

$$M_u/(\phi f'_c bd^2) = 17{,}729 \times 12/(0.9 \times 4000 \times 12 \times (10.5)^2) = 0.0447$$

From standard design aid in Appendix A:

$\omega = 0.046$

$A_s = \omega bd f'_c/f_y = 0.046 \times 12 \times 10.5 \times 4/60 = 0.39$ in.^{2}/ft

Total $A_s = 2 \times \pi \times 0.6 \times 27 \times 0.39 = 39.7$ in.^{2}

Use 128-#5 bars (39.7 in.^{2}).

The spacing at this location will be:

spacing = $2 \times \pi \times 0.6 \times 27 \times 12/128 = 9.5$ in.

The maximum negative moment at the inside face of the wall is 24,576 ft-lbs per ft. The required amount of reinforcing at the top of the slab at this location is:

$$M_u/(\phi f'_c bd^2) = 24{,}576 \times 12/(0.9 \times 4000 \times 12 \times (10.5)^2) = 0.0619$$

From standard design aid in Appendix A:

w = 0.065

$A_s = \omega bd f'_c/f_y = 0.65 \times 12 \times 10.5 \times 4/60 = 0.546$ in.^{2}/ft

Total $A_s = 2 \times \pi \times 27 \times 0.546 = 92.6$ in.^{2}

Use 212-#6 bars (93.3 in.^{2}).

The spacing at this location will be:

spacing = 2 × π × 27 × 12/212 = 9.6 in.

The tangential moments are computed by selecting coefficients for c/D = 0.15 from Tables A-17 and A-19 and multiplying them by $pR^2 = 800 \times (27)^2 = 583{,}200$ ft-lbs per ft (for fixed edge), and by M = 4001 ft-lbs per ft (for moment edge). The resulting tangential moments are shown in Table 26 and Fig. 33.

Table 26—Tangential Moments for Roof Slab with Center Support

Point	Coefficient for Fixed Conditions	Tangential Moment for Fixed Edge	Coefficient for Moment at Edge	Tangential Moment for Applied Edge Moment	Total Tangential Moment
0.15R	-0.0218	-12,714	-0.319	-1276	-13,990
0.2R	-0.0284	-16,563	-0.472	-1888	-18,451
0.25R	-0.0243	-14,172	-0.463	-1852	-16,024
0.3R	-0.0177	-10,323	-0.404	-1616	-11,939
0.4R	-0.0051	-2,974	-0.251	-1004	-3978
0.5R	+0.0031	+1808	-0.100	-400	+1408
0.6R	+0.0080	+4666	+0.035	+140	+4806
0.7R	+0.0086	+5016	+0.157	+628	+5644
0.8R	+0.0057	+3324	+0.263	+1052	+4376
0.9R	-0.0006	-350	+0.363	+1452	+1102
1.0R	-0.0098	-5715	+0.451	+1804	-3911

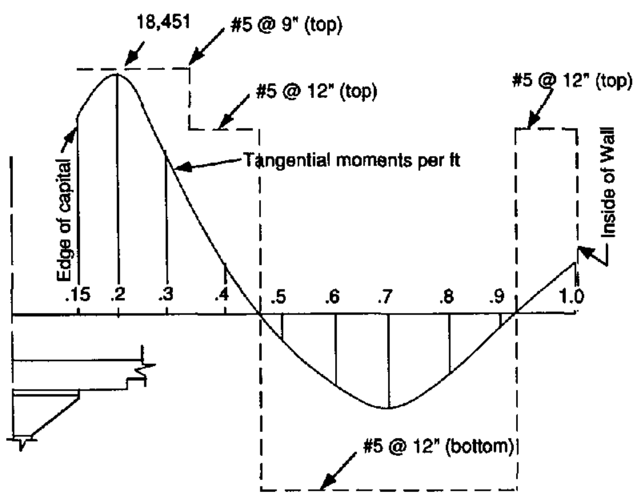


Figure 33—Tangential moments for roof slab with center support

Within the drop panel, the effective depth of the slab is 16.5 in. instead of the 10.5 in. that is beyond the drop panel. If the moments in that region are reduced in the ratio of 10.5/16.5, it is seen that the critical moment for design occurs just beyond the edge of the drop panel. The moment at the edge the drop panel is taken equal to 18,451 which is slightly larger than the moment at that location. The required circular reinforcing at this location is:

$M_u/(\phi f'_c bd^2) = 18{,}451 \times 12/(0.9 \times 4000 \times 12 \times (10.5)^2)$
$= 0.046$

From standard design aid in Appendix A:

$\omega = 0.0474$

$A_s = \omega bd f'_c/f_y = 0.0474 \times 12 \times 10.5 \times 4/60 = 0.40$ in.^{2}/ft

Use #5 bars spaced at 9 in. ($A_s = 0.41$ in.^{2}).

In the remainder of the slab, #5 bars at 12 in. will be adequate ($M_u = 14{,}350$). As indicated in Fig. 32, some of the bars are in the bottom of the slab depending on the sign of the tangential moments.

This design utilized radial and circular reinforcement. It is also common to use a rectangular layout for the reinforcing.

16 *Roof Slab with Multiple Interior Supports*

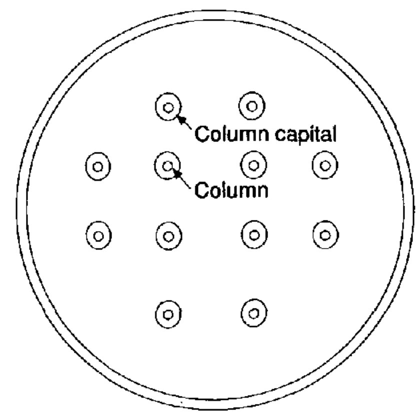


Figure 34—Roof slab with multiple interior supports

Figure 34 illustrates a column layout for a roof slab that has multiple interior supports. In the two previous sections (roof slabs with one or no interior supports), the roof slabs are designed as circular plates. The reinforcing bars for these types of slabs were placed both radially and in a circular pattern. In contrast, when multiple interior supports are used, the reinforcing bars are placed in two perpendicular directions as is typically done for flat slabs in buildings. Design examples for flat slabs are widely available.[5,6,7] For this reason, this publication will only briefly discuss the design of flat slabs.

The analysis of a flat slab system consists of two steps. The first step is to determine the factored moments at critical sections (usually at midspan and at the supports). The second step is to distribute the moments transversely across the slab.

ACI 318-89 provides two methods to perform the first step. These methods are the *Equivalent Frame Method* and the *Direct Design Method*. Both of these methods will be discussed below.

Equivalent Frame Method—This method provides representation in two dimensions of a three-dimensional system by defining flexural stiffnesses which reflect the torsional rotation possible in the three-dimensional system. The equivalent frame consists of the horizontal slab, the columns above and below the slab, and the portion of the structure that provides moment transfer between the slab and column (called a torsional member). The three parts of the equivalent frame are shown in Fig. 35.

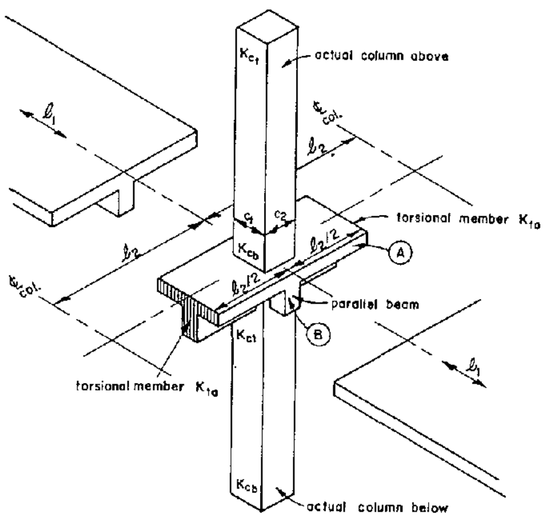


Figure 35—Equivalent frame

Once the equivalent frame is established, the carryover and stiffness factors are determined. With these factors and the fixed end moments, the moment distribution is performed. This method of determining the critical moments is tedious and time consuming, and is best suited for computer use or where geometry is irregular, preventing use of the simple Direct Design Method discussed below.

Direct Design Method—The moments at critical sections are determined with much less labor under this method than by the Equivalent Frame Method. The moments are determined at midspan and at column faces by multiplying coefficients by the total factored design moment, M_o, which equals:

$$M_o = w_u \ell_2 \ell_n^2/8$$

where w_u = factored load per unit area

ℓ_2 = length of span transverse to direction of analysis, measured center-to-center of supports

ℓ_n = length of clear span in direction of analysis.

Once this moment is determined, it is multiplied by the coefficients from ACI 318-89 Section 13.6.3.3, which are reproduced in Table 27.

Table 27—Distribution of Static Moments

Location	Percentage of M_o
Interior negative factored moment	70%
Positive factored moment	50%
Exterior negative factored moment	30%

Since the Direct Design Method is based on many assumptions, its use is allowed only if the structure conforms to certain limitations. These limitations are:

- There shall be a minimum of three continuous spans in each direction.
- Panels shall be rectangular with a ratio of longer to shorter span center-to-center of supports within a panel not greater than two.
- Successive span lengths center-to-center of supports in each direction shall not differ by more than one-third the longer span.
- Columns may be offset a maximum of 10% of the span (in direction of offset) from either axis between centerlines of successive columns.
- All loads shall be due to gravity only and uniformly distributed over an entire panel.
- Live load shall not exceed three times the dead load.

Once the factored moments are determined by either the *Direct Design Method* or the *Equivalent Frame Method*, the second step of laterally distributing the moments across the slab must be performed. The width of slab centered at a column line and extending to half-way between adjacent column lines is called the design strip. The design strip is divided into column and middle strips. The column strip is defined as having a width equal to one-half the transverse or longitudinal span, whichever is smaller. The remainder of the design strip is composed of two half-middle strips. These strips are determined as shown in Fig. 36.

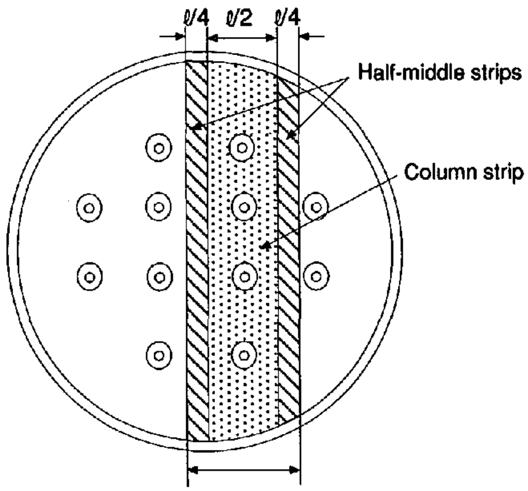


Figure 36—Design strip of roof slab

The percentages of moments carried by the column and the middle strips are shown in Table 28.

Table 28—Lateral Distribution of Moments for Roof Slabs

	Column strip	Middle strip
Negative moment at exterior column	100%	0%
Negative moment at interior column	75%	25%
Positive moments	60%	40%

Flat slabs must also meet serviceability requirements. The roof slab must be designed to have adequate stiffness to prevent deflection problems. For slabs with drop panels, the minimum thickness shall not be less than 4 in. (ACI 318-89, Section 9.5.3.2). The slab must also have a thickness no less than $\ell_n/36$ for a flat slab and $\ell_n/33$ for slabs without drop panels.

For complete details of flat slab design, refer to the publications referenced at the beginning of this section.

17 *Effect of Variation in Wall Thickness*

All tables and numerical examples in preceding sections are based on the assumption that the wall has uniform thickness from the top to the base. The effect of tapering the wall will now be discussed.

If ACI 350 recommendations are followed, reinforced concrete walls that are 10 ft high or taller, that are in contact with liquid, shall have a minimum thickness of 12 in. Therefore, it is unlikely that very large tapers will occur. In the examples in the preceding section, 15 in. is the thickness required for maximum ring tension which occurs approximately 0.6H below the top. As discussed in Section 11, the investigation for a shear applied at the base of the wall can be omitted by designing for the maximum ring tension from the location of this maximum ring tension to the base of the tank. Therefore, in the preceding examples, the tank wall can be tapered from 12 in. to 15 in. only for the upper one-half of the wall. The cross-sectional area of the wall will be reduced from $1.25 \times 20 = 25.0$ sq ft to $25 - 0.5 \times 10 \times 3/12 = 23.75$ sq ft. The reduction in cost from the reduced volume of concrete will probably not offset the added cost of forms for the tapered circular wall.

Gray[8] has presented data for wall sections that vary from a maximum at the base to zero at the top. For illustration, consider a wall with H = 20 ft, D= 54 ft, and t = 1.25. For this wall, Gray's data show that maximum ring tension is approximately eight percent greater for triangular than for rectangular wall sections, that is, when the sectional area is reduced from 25.0 sq ft to 12.5 sq ft. For the reduction of 1.25 sq ft in the foregoing paragraph, it may be estimated roughly that the increase in maximum ring tension will be $1.25 \times 8/12.5 = 0.8\%$. At any rate, the increase appears to be negligible.

Timoshenko[9] gives an example with H = 14 ft and D = 60 ft. The wall thickness is 14 in. in one case but varies from 14 in. to 3.5 in. in the other case. Moment and shear at the base are as follows:

	Moment, in.-lb	Shear, lb
Uniform thickness (14 in.):	13,960	564
Variable thickness (3.5 in. to 14 in.):	13,900	527

It is seen that the moment is practically unchanged and the shear is reduced by only 6.5 percent. The change will be even smaller when the taper is from 14 in. at mid-height to 12 in. at top. However, the taper will increase the ring tension and the decrease in wall width must be taken into account. In this case, for moment and shear the taper may be ignored, but under extreme circumstances it may be advisable to take it into account. This may be done approximately by inserting in H^2/Dt the value of t which exists at the point being investigated in the wall, or, in other words, to use values of H^2/Dt which vary from top to base.

18 *Temperature Variation in Tank Wall*

When the temperature on the exterior of a tank differs from that on the interior, a temperature gradient will exist in the wall. A wall of a circular tank cannot expand or contract freely causing temperature induced stresses. The magnitude of these stresses are difficult to predict. The analysis requires the solving of complex differential equations, complicated by the possibility of a nonlinear temperature gradient and the changing of geometric properties due to cracking in the wall. Determination of the stresses under these circumstances are beyond the scope of this publication and references suggested in the bibliography should be consulted. In this section, temperature induced stress calculations will be presented using the closed form solutions presented by Ghali and Elliot in Reference 10. These closed form solutions can be used only if the following conditions are satisfied:

- Tank has a value of $H^2/(Dt) > 2.9$ (i.e. a deep tank).
- Temperature varies linearly through the thickness of the wall.
- The wall section remains uncracked.

The temperature gradient in the wall will produce a hoop force, N_ϕ, and a circumferential moment, M_ϕ, which, when combined, may produce vertical cracking if the tensile strength of the concrete is exceeded. The temperature differential may also produce horizontal cracking. For a complete discussion of this type of cracking, see Reference 10. Table 29 shows the closed form solutions for various base support conditions.

The following numerical values will be used to calculate the stresses induced by a temperature gradient. It should be noted that the notation used in this section is different than that of other sections in this publication.

Table 29—Closed Form Solution for Temperature Stresses in Circular Wall That Cause Vertical Cracking

Bottom Fixed & Top Free	$N_\phi = -E\alpha[\frac{h}{2}(T_o + T_i)Z_1 + \frac{(1+\mu)}{2r\beta^2}(T_o - T_i)\bar{Z}_3]$ $M_\phi = -E\alpha[\frac{\mu r\beta^2 h^3}{12(1-\mu^2)}(T_o + T_i)Z_3 + \frac{h^2}{12(1-\mu)}(T_o - T_i)(1 - \mu\bar{Z}_1)]$
Bottom Hinged & Top Free	$N_\phi = -E\alpha[\frac{h}{2}(T_o + T_i)Z_4 + \frac{(1+\mu)}{2r\beta^2}(T_o - T_i)(Z_3 + \bar{Z}_3 - Z_4)]$ $M_\phi = -E\alpha[\frac{-\mu r\beta^2 h^3}{12(1-\mu^2)}(T_o + T_i)Z_2 + \frac{h^2}{12(1-\mu)}(T_o - T_i)[1 - \mu(Z_1 - Z_2 + \bar{Z}_1)]]$
Bottom Free & Top Free	$N_\phi = -E\alpha\frac{(1+\mu)}{2r\beta^2}(T_o - T_i)(Z_3 + \bar{Z}_3)$ $M_\phi = -\frac{E\alpha h^2}{12(1-\mu)}(T_o - T_i)(1 - \mu Z_1 - \mu\bar{Z}_1)$

where $\beta = [3(1-\mu^2)]^{1/4}/\sqrt{rh}$ and values of Z and Z will be provided later in this section.

Temperature on external face, $T_o = 30°$ F
Temperature on internal face, $T_i = 0°$ F
Height of tank, L = 20 ft
Radius, r = 27 ft
Poisson's ratio, $\mu = 0.16$
Coefficient of thermal expansion, $\alpha = 0.0000056$
Wall thickness, h = 10 in.
Modulus of elasticity of concrete, $E_c = 3605$ ksi
Bottom of tank is assumed hinged.

Based on the numerical values provided, the following variables are determined to calculate the stresses at the top of the tank ($x = L$ and $\bar{x} = L - x = 0$).

$$\beta = [3(1-\mu^2)]^{1/4}/\sqrt{rh}$$
$$= [3(1-(0.16)^2)]^{1/4}/\sqrt{(12 \times 27 \times 10)} = 0.02297$$

$\beta x = 0.02297 \times 20 \times 12 = 5.513$

$\beta\bar{x} = 0$

$e^{-\beta x} = 0.00405$

$e^{-\beta\bar{x}} = 1$

$\cos(\beta x) = 0.7179$

$(\beta\bar{x}) = 1$

$\sin(\beta x) = -0.6961$

$\sin(\beta\bar{x}) = 0$

$Z_1 = e^{-\beta x}(\cos\beta x + \sin\beta x) = 0.00405 \times (0.7179 - 0.6961)$
$= 0.000088$

$Z_2 = e^{-\beta x}(\sin\beta x) = 0.00405 \times (-0.6961) = -0.002819$

$Z_3 = e^{-\beta x}(\cos\beta x - \sin\beta x) = 0.00405 \times (0.7179 + 0.6961)$
$= 0.005721$

$Z_4 = e^{-\beta x}(\cos\beta x) = 0.00405 \times (0.7179) = 0.002908$

$\bar{Z}_1 = e^{-\beta\bar{x}}(\cos\beta\bar{x} + \sin\beta\bar{x}) = 1$

$\bar{Z}_2 = e^{-\beta\bar{x}}(\sin\beta\bar{x}) = 0$

$\bar{Z}_3 = e^{-\beta\bar{x}}(\cos\beta\bar{x} - \sin\beta\bar{x}) = 1$

$\bar{Z}_4 = e^{-\beta\bar{x}}(\cos\beta\bar{x}) = 1$

Substitution of these values into the closed form solution for a wall with a free top and hinged base gives the following:

$$N_\phi = -E_a\left[\frac{h}{2}(T_o + T_i)Z_4 + \frac{(1+\mu)}{2r\beta^2}(T_o - T_i)(Z_3 + \bar{Z}_3 - Z_4)\right]$$

$$= (-3605) \times (0.0000056) \times [\frac{10}{2} \times (30 - 0) \times (0.002908)$$

$$+ \frac{(1+0.16) \times 30}{(2 \times 27 \times 12 \times (0.02297)2)} \times (0.005721 + 1 - 0.002908)$$

$$= (-3605) \times (0.0000056) \times [0.4362 + 102.1]$$

$$= -2.07 \text{ kips/in.}$$

$$M_\phi = -E_a[-\mu r\beta^2 h^3 (T_o + T_i) Z_2/((1-\mu^2) \times 12)$$

$$+ h^2 (T_o - T_i)(1 - \mu(Z_1 - Z_2 + \bar{Z}_1))/((1-\mu) \times 12)$$

$$= (-3605) \times (0.0000056)](-0.16) \times (27 \times 12) \times (0.02297)2$$

$$\times (10)^3 \times (30) \times (-0.002819)/((1 - (0.16)^2) \times 12)$$

$$+ (10)^2 \times (30) \times (1 - 0.16(0.000088 + 0.002819 + 1))$$

$$/((1 - 0.16) \times 12)$$

$$= (-3605) \times (0.0000056) \times (0.197) + 249.861)$$

$$= -5.05 \text{ kips-in./in.}$$

The maximum stress on the interior face at this location is determined from the following equation and as shown in Fig. 37.

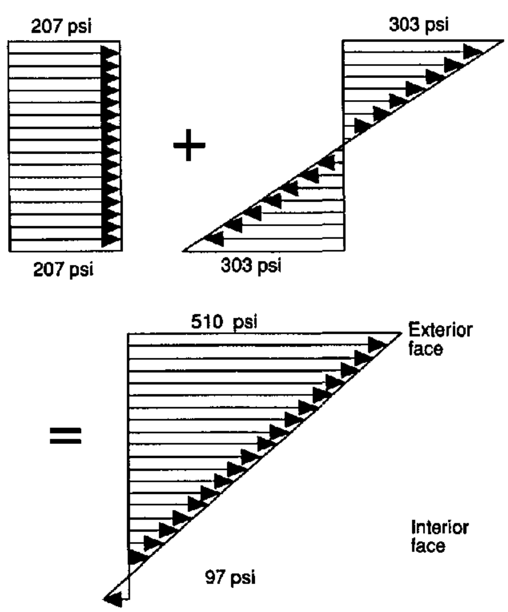


Figure 37—Stress diagrams for temperature variation in tank wall

$$\sigma_{max} = \frac{N}{A} \pm \frac{M}{S}$$

where

$N_1 = 2.07$ k/in.

$A = 1 \text{ in.} \times (1 \times 10) = 10 \text{ in.}^2$

$M = 5.05$ kips in./in.

$S = bd^2/6 = (1 \text{ in.} \times (10)^2/6) = 16.67 \text{ in.}^3$

$\sigma_{max} = (-2.07/10) - (-5.05/16.67) = 0.097 \text{ ksi} = 97 \text{ psi}$

Table 30 shows the resultant stresses occurring on the inside face of the tank. These stresses can cause vertical cracking if the tensile strength of the concrete is exceeded.

Table 30—Stresses on Inside Face of Tank Wall

Location	N_ϕ (k/in.)	M_ϕ (k-in./in.)	Stress on Inside face (psi)
1.0L (top)	-2.06	-5.05	97.0
0.9L	-0.41	-5.26	274.8
0.8L	0.29	-5.63	366.6
0.7L	0.45	-5.92	399.7
0.6L	0.42	-6.08	406.3
0.5L	0.40	-6.12	406.9
0.4L	0.44	-6.03	406.4
0.3L	0.45	-5.82	393.9
0.2L	0.14	-5.82	343.2
0.1L	-0.88	-5.12	219.2
0.0L (bottom)	-3.04	-5.05	0

+ denotes tension

Observations of existing tanks have shown that tanks above ground are more vulnerable to problems from temperature variations than those below ground. In some cases, the previous calculations can be avoided by increasing the horizontal ring steel by 10% to 20% beyond that required for the critical load case. Also, it should be noted that if strength design is being used, load factors must be used to increase the service loads from temperature effects.

19 *Base Slab Design*

When the bottom of the tank is below the water table, the loading on the slab from hydrostatic pressure must be investigated. If the upward pressure exceeds the dead load of the tank floor, there may be a danger of heaving unless the floor is constructed as a structural slab with loading directed upward rather than downward. There are several methods to determine the required base slab reinforcing. These methods include finite element analysis, complex mathematical solutions,[11,12] and approximate methods. A rigorous treatment of a slab on an elastic foundation is beyond the scope of this publication, so the discussion on the design of the bottom slab will be based on what is believed to be a reasonable estimate. The sanitary coefficients were not used for the base slab design since it is not clear in ACI 350 whether this would be appropriate. The designer must use engineering judgment to determine if the following analysis and the absence of the sanitary coefficients are suitable for the specific project. The following discussion will investigate three different base design conditions.

The first condition occurs when there is no unbalanced hydrostatic force present. The discussion that follows is only applicable for tanks with small diameters. The loading on the slab equals the load on the roof, weight of the roof, and weight of the wall. The factored loads, excluding the sanitary coefficients are:

Roof: Live Load $= 1.7 \times w_L \times$ Tributary Area

$= 1.7 \times 100 \times \pi \times (13)^2$

$= 90{,}258$ lbs

Dead Load $= 1.4 \times w_D \times$ Tributary Area

$= 1.4 \times (10/12 \times 150) \times \pi \times (13)^2$

$= 92{,}913$ lbs

Wall: Dead Load $= 1.4 \times$ weight per radial foot $\times$ Circumference

$= 1.4 \times (12/12 \times 150 \times 16) \times 2 \times \pi \times 13$

$= 274{,}450$ lbs

Total Load = 90,258 + 92,913 + 274,450 = 457,621 lbs

The loading of 457,621 lbs is assumed to be distributed uniformly over the subgrade for small diameter tanks, giving an upward reaction on the bottom slab of:

$p = 457{,}621/(\pi \times 13^2) = 862$ psf

This load is applied upward on the slab and the same type of procedure as used in Section 14 is followed to determine the reinforcing. Note that the base area can be increased by extending the slab beyond the wall, thereby reducing the soil stress. This also provides a place to develop the slabs reinforcing and a work platform.

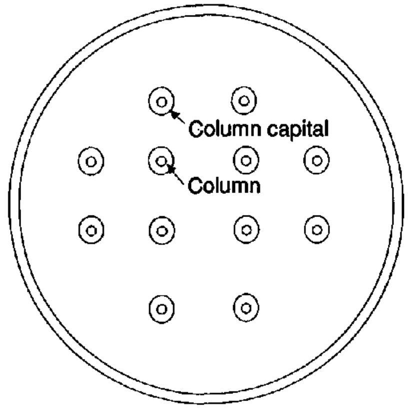
The second condition occurs when there is unbalanced hydrostatic pressure present. In this case, the upward force on the slab is equal to the hydrostatic pressure minus the pressure from the weight of the roof, wall, and slab as shown below:

$$\rho = \gamma_w \times \text{height of hydrostatic pressure} - (w_{roof} + w_{wall} + w_{slab})/(\text{area})$$

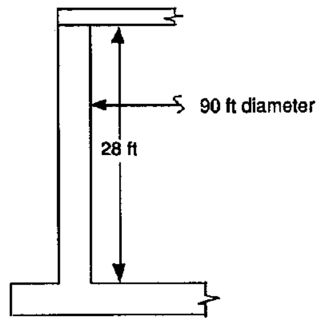
It should be noted that the two loading conditions presented above will only occur in small diameter tanks. In large diameter tanks, it is not realistic to assume that the weight of the wall and the roof will be uniformly distributed over the area of the slab.

The third condition is for tanks which do not have a small diameter. For this condition, the portion of the slab near the wall is designed similar to a cantilever wall base. The reinforcement in the remainder of the slab is designed for temperature and shrinkage effects as well as any other loading conditions that might occur.

20 *Design Example*



Plan View of Circular Tank



Section through Circular Tank

Design Information:

Internal fluid pressure = 65.0 lb/ft^3

External soil pressure = 90.0 lb/ft^3

External soil surcharge = 3 ft (use 90 lbs per ft)

f'_c = 4000 psi

f_y = 60,000 psi

E_s = 29 × 10^6 psi

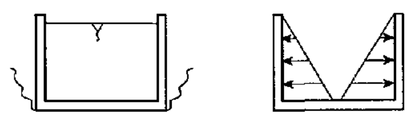
n = 8

Concrete roof is hinged connected to wall.

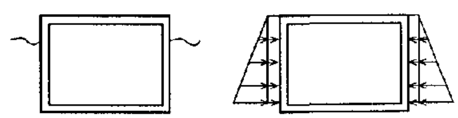
Roof is in place prior to backfilling.

I. Loading Conditions

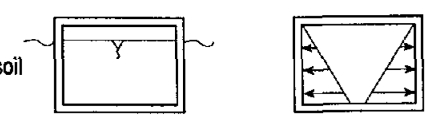
Loading Condition No. 1
Leakage test prior to backfilling.



Loading Condition No. 2
Empty tank with backfill present.



Loading Condition No. 3
Full Tank. Resistance provided by soil is ignored.



II. Estimate Tank Wall Thickness

Tank wall thickness is estimated by limiting the ring tension stress in the wall to 10% of f'_c. Initially, assume a wall thickness of 16 in. This gives a value of $H^2/(Dt) = (28)^2/(90 \times 16/12) = 6.5$.

Maximum ring tension is determined by multiplying the largest coefficient for ring tension (Table A-5) for $H^2/(Dt) = 6.5$ by w_uHR.

Largest ring tension coefficient = 0.657

$$w_u = \text{sanitary coefficient} \times 1.7 \times w$$
$$= 1.65 \times 1.7 \times 65.0 = 182.33 \text{ lbs/ft}^3$$

This gives a maximum ring tension of:

$$T_{max} = 0.657 \times w_u \times H \times R$$
$$= 0.657 \times 182.33 \times 28 \times 45$$
$$= 150{,}936 \text{ lbs}$$

The required reinforcement to resist this tensile force is:

$$\text{Area} = T_{max}/(0.9 \times F_y)$$
$$= 150{,}936/(0.9 \times 60{,}000)$$
$$= 2.80 \text{ in.}^2$$

Use #9 bars E.F. at 8 in. ($A_s = 3.0$ in.^{2})

The maximum tensile stress in the concrete due to ring tension and shrinkage is:

$$f_c = \frac{CE_sA_s + T_{max}\text{ (unfactored)}}{A_c + nA_s}$$

where T_{max} (unfactored) = 150,936/(1.7 × 1.65) = 53,810

$$f_c = \frac{0.0003 \times 29 \times 10^6 \times 3.0 + 53{,}810}{12 \times 16 + 8 \times 3.0}$$

$$= \frac{26{,}100 + 53{,}810}{216}$$

$$= 370 \text{ psi} < 400 \text{ psi}, \quad \text{O.K.}$$

III. Analysis for Loading Condition No. 1

During construction, prior to backfilling, the tank will be checked for leaks. It will be assumed that the tank cover has not yet been constructed at the time of the leakage test. The following procedure will be utilized for this loading condition.

- Determine ring forces and bending moments from internal fluid pressure.
- Add effects of possibility of outward movement of the base of the wall.

For a wall with a hinged base and a free top subjected to a triangular load, the ring tension is calculated by multiplying the coefficients from Table A-5 by w_uHR as shown in Table E-1.

Table E-1—Ring Tension in Tank Wall for Loading Condition No. 1

Point	Coefficient for $H^2/(Dt) = 6.5$ (see Table A-5)	Ring Force (+ denotes tension)
0.0H	-0.012	-2757
0.1H	+0.101	+23,203
0.2H	+0.219	+50,311
0.3H	+0.338	+77,649
0.4H	+0.458	+105,216
0.5H	+0.565	+129,797
0.6H	+0.645	+148,176
0.7H	+0.657	+150,932
0.8H	+0.566	+130,027
0.9H	+0.342	+78,568
1.0H	0	0

Note: $w_uHR = 1.7 \times 1.65 \times 65 \times 28 \times 45 = 229{,}730$

The bending moments for the same loading condition are determined by multiplying the coefficients taken from Table A-7 by w_uH^3 as shown in Table E-2.

Table E-2—Bending Moments in Tank Wall for Loading Condition No. 1

Point	Bending Moment Coefficient for $H^2/(Dt) = 6.5$ (see Table A-7)	Bending Moment (+ denotes tension on interior face)
0.0H	0.0000	0
0.1H	0.0000	0
0.2H	0.0000	0
0.3H	+0.0001	315
0.4H	+0.0006	+1892
0.5H	+0.0016	+5045
0.6H	+0.0034	+10,722
0.7H	+0.0056	+17,659
0.8H	+0.0073	+23,020
0.9H	+0.0065	+20,497
1.0H	0.0000	0

Note: $w_uH^3 = 1.7 \times 1.3 \times 65 \times (28)^3 = 3{,}153{,}405$

In the analysis for a free top and hinged base, it is assumed that the base is restrained from lateral displacement. As discussed in Section 11, the base slab may not be able to provide complete restraint. It was also noted, and will also be utilized for this design, that this complex analysis could be omitted by using the maximum ring tension to design the entire bottom portion of the wall, and ignoring the decrease in moments that result from the displacement of the base.

The resulting ring force and bending moments are shown in Fig. E-1.

IV. Analysis for Loading Condition No. 2

While in service, it is possible that the tank will not be filled with liquid. Under this loading condition, soil pressure will be acting inward on the tank wall. The following procedure will be utilized for this loading condition.

- Determine ring compression and bending moments from external soil pressure.
- Add effects of lateral restraint provided by roof slab.

For a wall with a hinged base and free top subjected to a trapezoidal load, the ring compression is calculated by multiplying the coefficients taken from Tables A-5 and A-6, by w_uHR and pR respectively. These results are provided in Table E-3. Note that the sanitary coefficient for ring tension of 1.65 was used even though the ring forces are compressive since a sanitary coefficient for this condition is not provided in ACI 350.

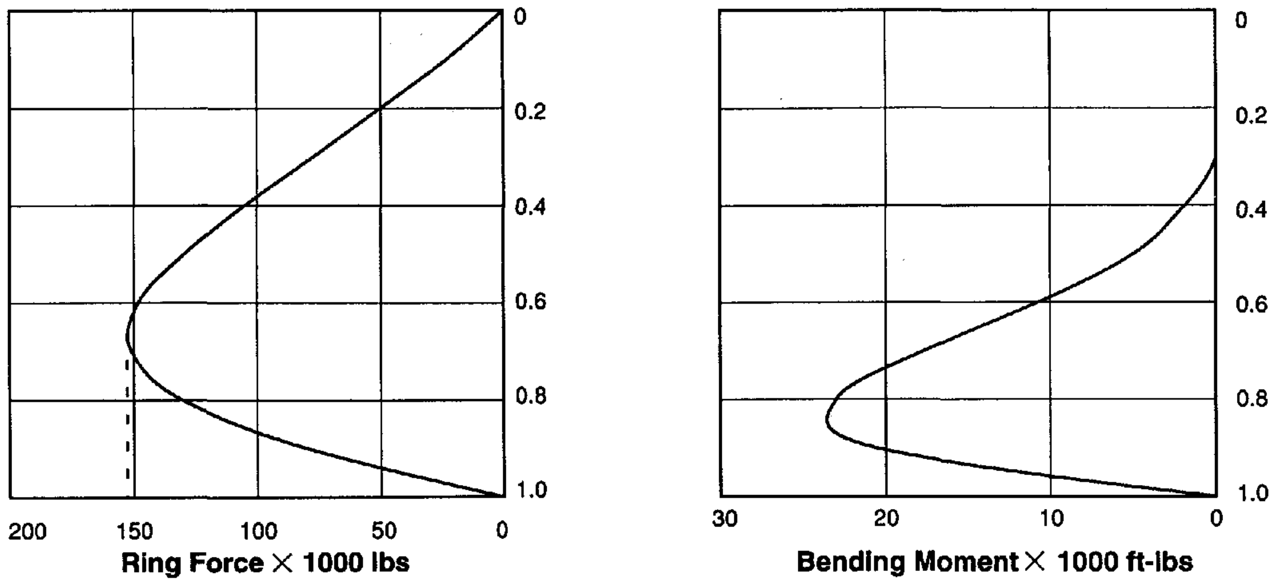


Figure E-1—Ring forces and bending moments in tank wall for loading condition no. 1

Table E-3—Ring Force in Tank Wall for Trapezoidal Loading

Point	Ring Force Coefficient from Table A-5 for Triangular Load $H^2/(Dt) = 6.5$	Ring Force Coefficient from Table A-6 for Rectangular Load $H^2/(Dt) = 6.5$	Ring Force for Triangular Loading	Ring Force for Rectangular Loading	Total Ring Force (+denotes tension)
0.0H	-0.012	+0.988	3816	-33,672	29,856
0.1H	+0.101	+1.001	-32,127	-34,113	66,240
0.2H	+0.219	+1.019	-69,660	-34,728	104,388
0.3H	+0.338	+1.038	-107,514	-35,376	142,890
0.4H	+0.458	+1.058	-145,683	+36,657	181,740
0.5H	+0.565	+1.065	-179,718	-36,294	216,012
0.6H	+0.645	+1.045	-205,167	-35,613	240,780
0.7H	+0.657	+0.957	-208,583	-32,616	241,599
0.8H	+0.566	+0.766	-180,036	-26,106	206,142
0.9H	+0.342	+0.442	-108,786	-15,063	123,849
1.0H	0	0	0	0	0

Note: $w_uHR = (1.7 \times 1.65 \times (-90) \times 28 \times 45) = -318{,}087$

$p_uR = (1.7 \times 1.65 \times (-90 \times 3) \times 45) = -34{,}080$

For a wall with a hinged base and free top subjected to a trapezoidal load, the bending moments are calculated by multiplying the coefficients taken from Table A-7 by $(w_uH^3 + p_uH^2)$. These results are shown in Table E-4.

The results for this loading condition were obtained considering that the top of the tank is free to displace laterally. In actuality, the concrete roof slab will prevent lateral movement at the top of the wall and will result in changes in the ring forces and bending moments. When the top of the tank is free to displace laterally, the ring force is 29,856 lbs in compression. To prevent displacement, a shear force acting in an opposite direction of the soil loads must be added to reduce the ring force to zero.

Ring tension due to a shear, V, at the top of the wall is computed as discussed in Section 10. The shear force required at the top of the tank to produce zero ring force is:

$$-9.37 \times \frac{VR}{H} = 29{,}856$$

$$\therefore V = -1983 \text{ lbs per ft}$$

Table E-4—Bending Moments in Tank Wall for Trapezoidal Load

Point	Bending Moment Coefficient from Table A-7 for $H^2/(Dt) = 6.5$	Bending Moment (+denotes tension on interior frame)
0.0H	0.0000	0
0.1H	0.0000	0
0.2H	0.0000	0
0.3H	+0.0001	483
0.4H	+0.0006	-2901
0.5H	+0.0016	-7743
0.6H	+0.0034	-16,437
0.7H	+0.0056	-27,072
0.8H	+0.0073	-35,289
0.9H	+0.0065	-31,422
1.0H	0.0000	0

Note: $(w_uH^3 + pH^2) = 1.7 \times 1.3 \times ((-90) \times 28^3) + (3 \times (-90) \times 28^2)) = -4{,}834{,}065$

The change in the ring force is determined by multiplying coefficients taken from Table A-8 by VR/H. These results are shown in Table E-5.

Table E-5—Ring Force for Loading Condition No. 2

Point	Ring Force Coefficient from Table A-8 for $H^2/(Dt) = 6.5$	Ring Force from V applied at top of wall	Ring Force from Trapezoidal Load	Total Ring Force (+denotes tension)
0.0H	-9.37	+29,856	29,856	0
0.1H	-5.22	+16,632	66,240	-49,608
0.2H	-2.17	+6915	104,388	-97,473
0.3H	-0.38	+1212	142,890	-141,678
0.4H	+0.41	-1305	181,740	-183,042
0.5H	+0.61	-1944	216,012	-217,956
0.6H	+0.51	-1626	240,780	-242,406
0.7H	+0.32	-1020	241,599	-242,619
0.8H	+0.15	-477	206,142	-206,619
0.9H	+0.01	-33	123,849	-123,882
1.0H	0	0	0	0

Note: $VR/H = -1983 \times 45/28 = -318$

Bending moments due to a shear, V, at the top is calculated in a similar manner except that the sanitary coefficient is different for bending moments. Therefore, the additional top force must be reduced proportionally. The revised moments are shown in Table E-6. The resulting ring forces and bending moments are shown in Fig. E-2.

Table E-6—Bending Moments for Loading Condition No. 2

Point	Bending Moment Coefficient from Table A-9 for $H^2/(Dt) = 6.5$	Bending Moment from V applied at top of wall	Bending Moment from Trapezoidal Load	Total Bending Moment (+denotes tension)
0.0H	0.000	0	0	0
0.1H	+0.061	-2667	0	-2667
0.2H	+0.067	-2931	0	-2931
0.3H	+0.052	-2274	483	-1791
0.4H	+0.032	-1401	-2901	-4302
0.5H	+0.015	-657	-7743	-8391
0.6H	+0.004	-174	-16,437	-16,611
0.7H	-0.001	+45	-27,072	-27,027
0.8H	-0.003	+132	-35,289	-35,157
0.9H	-0.004	+174	-31,422	-31,248
1.0H	-0.005	+219	0	+219

Note: $VH = \left(\frac{1.3}{1.65}\right) \times -1983 \times 28 = -43{,}740$

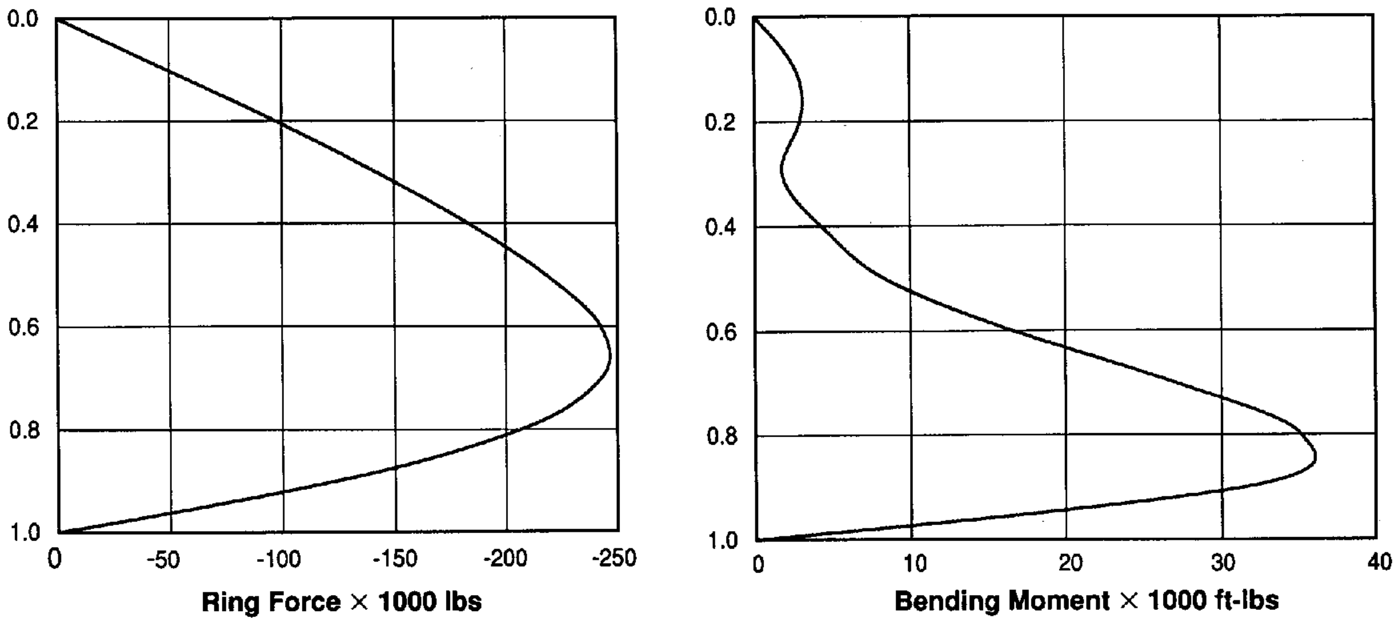


Figure E-2—Ring force and bending moments for load condition no. 2

V. Analysis for Loading Condition No. 3

This loading condition occurs while the tank is in use. The tank roof is in place, the tank is full with liquid, and the backfilling is completed. According to ACI 350, the resistance of the soil must not be taken into account as a resisting force.

The following procedures will be utilized for this loading condition:

- Determine ring forces and bending moments from internal fluid pressure. (This has previously been completed in the calculations for Loading Condition No. 1.)
- Add effects of possibility of movement of the base of the wall. (This has previously been completed in the calculations for Loading Condition No. 1.)
- Add effects of lateral restraint of top of tank provided by roof slab.

The first two steps have been completed in the calculations for Loading Condition No. 1, the third step, accounting for the effects of restraint at the top must be added to those results. To prevent lateral displacement, a shear force must be added at the top of the tank as previously discussed.

$$-9.37 \times \frac{VR}{H} = 2757$$

$$\therefore V = -183.08 \text{ lbs per ft}$$

The change in the ring force is determined by multiplying coefficients taken from Table A-8 by VR/H. These results are shown in Tables E-7 and E-8.

Table E-7—Ring Tension for Load Condition No. 3

Point	Ring Force Coefficient from Table A-9 for $H^2/(Dt) = 6.5$	Ring Force from V applied at top of wall	Ring Force from Triangular Load	Total Ring Force (+denotes tension)
0.0H	-9.37	+2757	-2757	0
0.1H	-5.22	+1536	+23,203	+24,739
0.2H	-2.17	+638	+50,311	+50,949
0.3H	-0.38	+112	+77,649	+77,761
0.4H	+0.41	-121	+105,216	+105,095
0.5H	+0.61	-179	+129,797	+129,618
0.6H	+0.51	-150	+148,176	+148,026
0.7H	+0.32	-94	+150,932	+150,838
0.8H	+0.15	-44	+130,027	+129,983
0.9H	+0.01	-3	+78,568	+78,565
1.0H	0	0	0	0

Note: $VR/H = -183.08 \times 45/28 = -294.2$

The bending moments due to the restraining force at the top is calculated in a similar manner.

Table E-8—Bending Moments for Loading Condition No. 3

Point	Bending Moment Coefficient from Table A-9 for $H^2/Dt = 6.5$	Bending Moment from V applied at top of wall	Bending Moment from Triangular Load	Total Bending Moment
0.0H	0.000	0	0	0
0.1H	+0.061	-246	0	-246
0.2H	+0.067	-271	0	-271
0.3H	+0.052	-210	+315	+105
0.4H	+0.032	-129	+1892	+1763
0.5H	+0.015	-61	+5045	+4985
0.6H	+0.004	-16	+10,722	+10,706
0.7H	-0.001	+4	+17,659	+17,663
0.8H	-0.003	+12	+23,020	+23,032
0.9H	-0.004	+16	+20,497	+20,513
1.0H	-0.005	+20	0	+20

Note: $VH = \frac{13}{1.65} \times -183.08 \times 28 = -4039$

The change in ring forces and bending moments from restraint of the roof are relatively small compared to those from the lateral forces and therefore, the graphs will practically be the same as those for Loading Condition No. 1.

VI. Design of Ring Steel

Figure E-3 shows the distribution of maximum ring tension and compression along the height of the wall.

Check compression:

$f_c = C/A = 242{,}619/(12 \times 16) = 1264$ psi O.K.

Design of reinforcing steel for tensile forces:

$T = A_s \times 0.9 \times f_y$

Capacity of #9 bars at 8 in. E.F.

$T = 12/8 \times 2.0 \times 0.9 \times 60{,}000 = 162{,}000$ lbs

Capacity of #8 bars at 8 in. E.F.

$T = 12/8 \times 1.58 \times 0.9 \times 60{,}000 = 127{,}980$ lbs

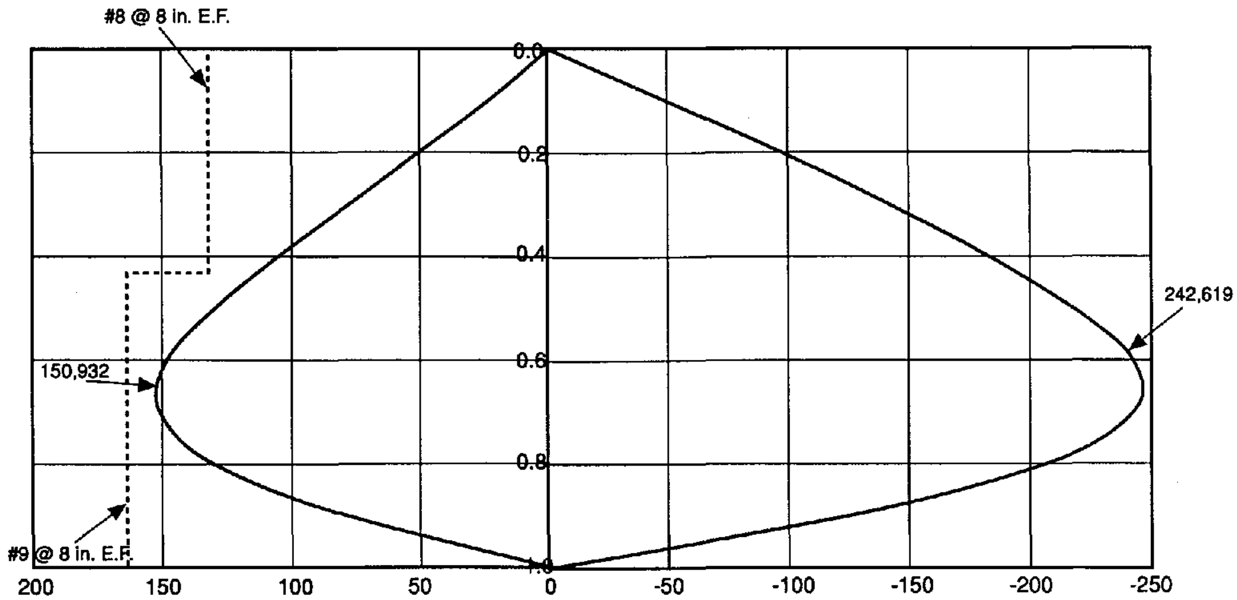


Figure E-3—Ring tension envelope for tank wall

∴use #9 @ 8 in. E.F. (162,000 > 150,932 lbs) for the bottom 15 ft of the wall and #8 @ 8 in. E.F. (127,980 > 120,523 lbs) for the top 13 ft of the wall.

Class B splices will be used for the ring steel since in most cases, the area of reinforcement is not greater than twice that required by analysis. The basic development length, ℓ_{db}, for #8 and #9 bars is calculated as follows:

$$\ell_{db} = 0.04\ A_b f_y/\sqrt{f_c'}$$ 12.15.1

#8 bars: $\ell_{dh} = 0.04 \times 0.79 \times 60{,}000/\sqrt{4000} = 30.0$ in.

#9 bars: $\ell_{dh} = 0.04 \times 1.00 \times 60{,}000/\sqrt{4000} = 37.9$ in.

The development length is determined by multiplying the basic development length by the modifiers provided in ACI 318-89. For the present case, the following modifiers are applicable:

- Horizontal bars so placed that more than 12 in. of fresh concrete is cast in the member below the development length or splice. Use modifier of 1.3.
- For #11 bars or smaller, with clear spacing not less than $5d_b$ and with edge face cover not less than $2.5d_b$. Use modifier of 0.8.
- Bars in inner layer of wall with clear spacing of not less than $3d_b$. Use modifier of 1.0.

Using these modifiers, the development lengths for #8 and #9 is:

#8 bars: $\ell_d = 1.3 \times 0.8 \times 1.0 \times 30.0 = 31.2$ in.

but not less than $0.03 d_b f_y/\sqrt{f_c'}$ 12.2.3.6

$\ell_{min} = 0.03 \times 1.0 \times 60{,}000/\sqrt{4000} = 28.4 < 31.2$ O.K.

#9 bars: $\ell_d = 1.3 \times 0.8 \times 1.0 \times 37.9 = 39.4$ in.

but not less than $0.03 d_b f_y/\sqrt{f_c'}$

$\ell_{min} = 0.03 \times 9/8 \times 60{,}000/\sqrt{4000} = 32$ in. < 39.4 in. O.K.

The length of a Class B splice is equal to $1.3 \times \ell_d$.

Splice length (#8 bars) = $1.3 \times 31.2 = 40.6$ in. (use 42 in.)

Splice length (#9 bars) = $1.3 \times 39.4 = 51.2$ in. (use 52 in.)

Adjacent reinforcing splices should be staggered horizontally (center-to-center of lap) by not less than one lap length nor 3 ft and should not coincide in vertical arrays more frequently than every third bar.

Minimum temperature and shrinkage reinforcement requirements are shown in Fig. 3 of Section 4. The minimum reinforcement ratio provided in the horizontal direction is:

$$\rho = (2 \times 0.79)/(8 \times 16)$$
$$= 0.0123$$

The minimum required reinforcement ratio when the length between shrinkage-dissipating joints is 50 ft is $0.00475 < 0.0123$. Therefore, the minimum reinforcement requirements are satisfied.

VII. Design of Moment Reinforcing

Figure E-4 shows the distribution of bending moments along the height of the wall.

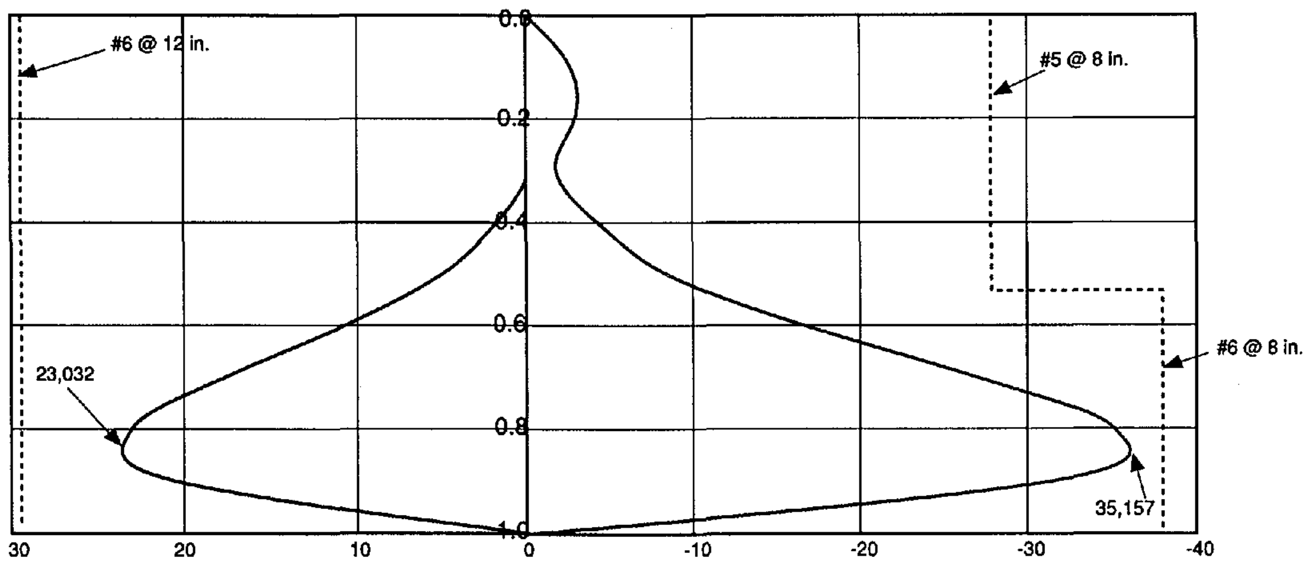


Figure E-4—Bending moments envelope for tank wall

The required vertical reinforcement for the exterior face of the wall with a moment of 23,032 ft-lbs is:

$$M_u/(\phi f'_c bd^2) = 23{,}032 \times 12/(0.9 \times 4000 \times 12 \times (13.51)^2)$$
$$= 0.0351$$

From standard design aid:

$\omega = 0.036$

$A_s = \omega bd f'_c/f_y = 0.036 \times 12 \times 13.5 \times 4/60 = 0.389$

Use #6 bars at 10 in. on exterior face ($A_s = 0.53$ in.^{2})

The required vertical reinforcement for the interior face of the wall with a moment of 35,157 ft-lbs is:

$$M_u/(\phi f'_c bd^2) = 35{,}157 \times 12/(0.9 \times 4000 \times 12 \times (13.51)^2)$$
$$= 0.0535$$

From standard design aid:

$\omega = 0.0555$

$A_s = \varphi b d f'_c / f_y = 0.0555 \times 12 \times 13.5 \times 4/60 = 0.60$

Use #6 bars at 8 in. on interior face ($A_s < 0.66$ in.^{2}) for the bottom half of the wall. Use #5 bars at 8 in. on the interior face for the top half of the wall ($M_u = 27{,}600$).

This gives a minimum vertical reinforcement ratio of:

$\rho = 0.53/(12 \times 13.5) = 0.0033$

The bars will be spliced at midheight immediately above the pour line. Since all the vertical bars will be spliced at this level, a Class B splice will be required. The basic development length, ℓ_{db}, for #6 bars is calculated as follows:

$$\begin{aligned} \ell_{db} &= 0.04\, A_b f_y / \sqrt{f'_c} \\ &= 0.04 \times 0.44 \times 60{,}000/\sqrt{4000} \\ &= 16.7 \text{ in.} \end{aligned}$$

The development length is determined by multiplying the basic development length by the applicable modifiers in ACI 318-89. For the present case, the following multipliers apply:

- Since bars are vertical, there will not be 12 in. of fresh concrete below the bars. Use modifier of 1.0.
- Cover is greater than $2d_b$, clear bar spacing is greater than $5d_b$, and since the tank is circular, side cover requirements are satisfied. Use modifier of 1.0.

The development length for #7 bars using these modifiers is:

$\ell_d = 1.0 \times 1.0 \times 16.7 = 16.7$ in.

but not less than $0.03 d_b f_y / \sqrt{f'_c}$

$$\begin{aligned} \ell_{min} &= 0.03 \times 0.75 \times 60{,}000/\sqrt{4000} \\ &= 21.3 \text{ in.} > 16.7 \text{ in.} \end{aligned}$$

The length of a Class B splice is equal to $1.3\ell_d$.

Splice length = $1.3 \times 21.3 = 27.7$ in. (use 30 in.)

The same size lap can be used for the splicing of the bars on the exterior face.

The development length of the standard hook that will be embedded in the base slab is equal to $1200 d_b / \sqrt{f'_c}$ multiplied by the appropriate modifiers of Section 12.5 of ACI 318-89. Since the cover is 2 in., the yield of reinforcement is 60,000 and no ties or stirrups are used, all applicable modifiers are 1.0 and the development length is:

$$\begin{aligned} \ell_{dh} &= 1200\, d_b / \sqrt{f'_c} \\ &= 1200 \times (6/8)/\sqrt{4000} \\ &= 14.2 \text{ in.} \end{aligned}$$

but not less than $8d_b$ (6 in.) nor 6 in. Therefore, use 15 in.

As discussed in Section 7, the maximum bar spacing must be limited to control flexural cracking. The maximum flexural moment for the exterior of the wall occurs at 0.8H. The maximum unfactored moment is equal to 0.0073 multiplied by wH^3. Note that this is the unfactored load. Therefore, the maximum unfactored moment for the exterior face of the wall is:

$M_{max} = 0.0073 \times 65.0 \times (28)^3 = 10{,}416$ ft-lbs per ft

The increase in the bending moments from the effect of restraint of the top of the tank from the roof slab is omitted since that increase is very small at this location. The stress in the reinforcing is calculated using the working stress method as follows:

$$f_s = \frac{M}{A_s jd}$$

where:

$A_s = 0.528$ in.^{2}/ft

$d = 13.5$

$n = 8$

$\rho = 0.528/(12 \times 13.5) = 0.0033$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$
$$= 0.204$$

$j = 1 - k/3 = 0.932$

Therefore:

$$f_s = (10{,}416 \times 12)/(0.528 \times 0.932 \times 13.5)$$
$$= 18{,}815 \text{ psi}$$

The maximum spacing to control cracking is:

$$s_{max} = z^3/(2 \times d_c^2 \times f_s^3)$$

where:

d_c = Cover + bar radius
$= 2 + 0.375 = 2.375$

$z = 115$ kips/in.

$f_s = 18.8$ kips

$$s_{max} = (115)^3/(2 \times (2.375)^2 \times (18.8)^3)$$
$$= 20 \text{ in.} > 8 \text{ in. O.K.}$$

s_{max} for the interior #6 bars is 15 in.

VIII. Shear Strength

The shear capacity of the wall is given by:

$$V_c = 2\sqrt{f'_c} \times b_w d$$
$$= 2\sqrt{4000} \times 12 \times 13.5$$
$$= 20{,}492 \text{ lbs}$$

$$\phi V_c = 0.85 \times 20{,}492 = 17{,}418 \text{ lbs}$$

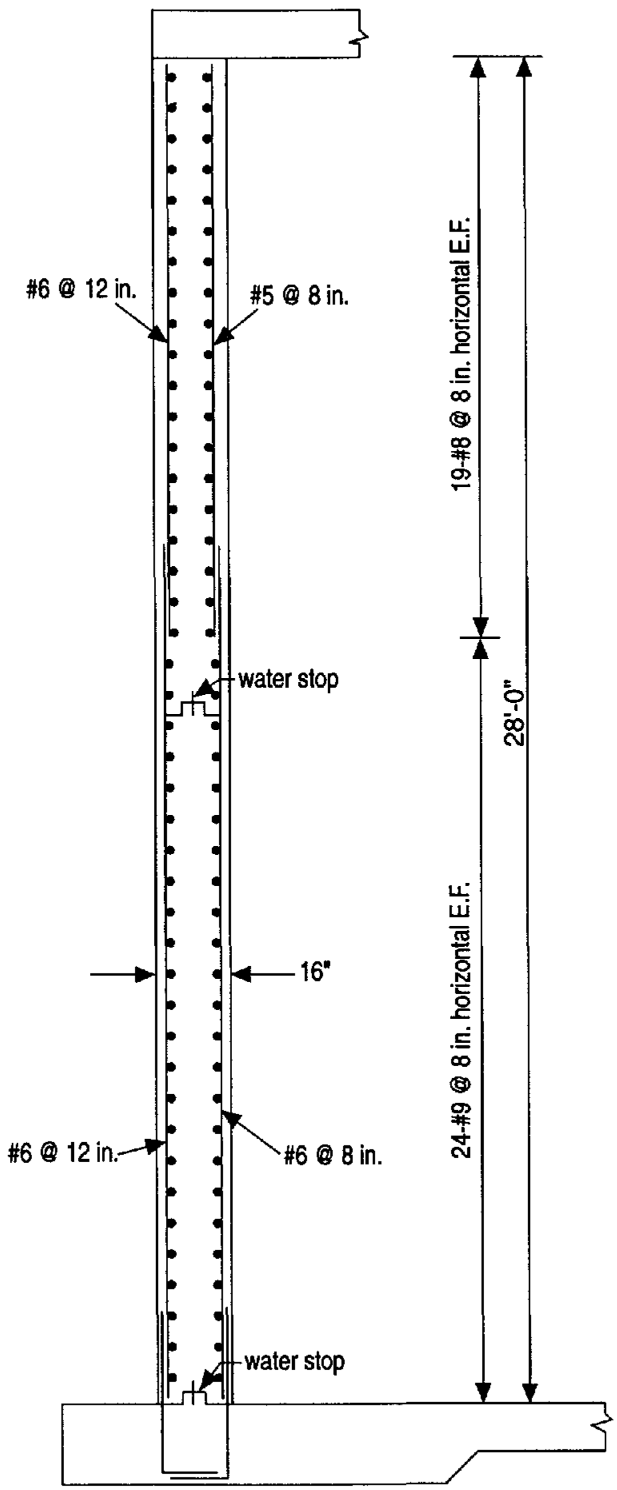
The maximum shear force is given by:

V_{max} = Coefficient × w_u × H^2

where Coefficient = 0.1065 (see Table A-12)

w_u = 1.7 × 90.0 = 153.0 lbs/ft^3

V_{max} = 0.1065 × 153.0 × $(28)^2$ = 12,775 lbs < 17,418 lbs O.K.



A

Appendix

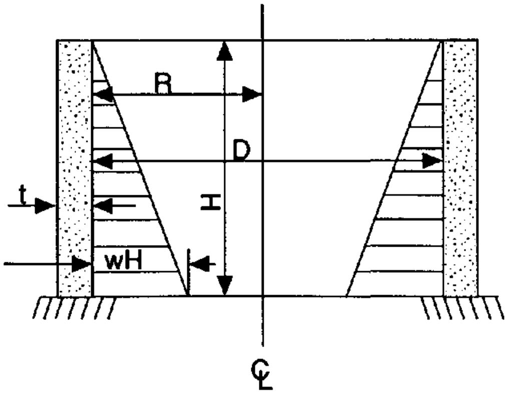


Table A-1 — Tension in circular rings

T = coef. × wHR lb per ft
Positive sign indicates tension

	Coefficients at point									
$\frac{H^2}{Dt}$	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+0.149	+0.134	+0.120	+0.101	+0.082	+0.066	+0.049	+0.029	+0.014	+0.004
0.8	+0.263	+0.239	+0.215	+0.190	+0.160	+0.130	+0.096	+0.063	+0.034	+0.010
1.2	+0.283	+0.271	+0.254	+0.234	+0.209	+0.180	+0.142	+0.099	+0.054	+0.016
1.6	+0.265	+0.268	+0.268	+0.266	+0.250	+0.226	+0.185	+0.134	+0.075	+0.023
2.0	+0.234	+0.251	+0.273	+0.285	+0.285	+0.274	+0.232	+0.172	+0.104	+0.031
3.0	+0.134	+0.203	+0.267	+0.322	+0.357	+0.362	+0.330	+0.262	+0.157	+0.052
4.0	+0.067	+0.164	+0.256	+0.339	+0.403	+0.429	+0.409	+0.334	+0.210	+0.073
5.0	+0.025	+0.137	+0.245	+0.346	+0.428	+0.477	+0.469	+0.398	+0.259	+0.092
6.0	+0.018	+0.119	+0.234	+0.344	+0.441	+0.504	+0.514	+0.447	+0.301	+0.112
8.0	-0.011	+0.104	+0.218	+0.335	+0.443	+0.534	+0.575	+0.530	+0.381	+0.151
10.0	-0.011	+0.098	+0.208	+0.323	+0.437	+0.542	+0.608	+0.589	+0.440	+0.179
12.0	-0.005	+0.097	+0.202	+0.312	+0.429	+0.543	+0.628	+0.633	+0.494	+0.211
14.0	-0.002	+0.098	+0.200	+0.306	+0.420	+0.539	+0.639	+0.666	+0.541	+0.241
16.0	0.000	+0.099	+0.199	+0.304	+0.412	+0.531	+0.641	+0.687	+0.582	+0.265

Supplemental Coefficients

	Coefficients at point				
$\frac{H^2}{Dt}$	.75H	.80H	.85H	.90H	.95H
20	+0.716	+0.654	+0.520	+0.325	+0.115
24	+0.746	+0.702	+0.577	+0.372	+0.137
32	+0.782	+0.768	+0.663	+0.459	+0.182
40	+0.800	+0.805	+0.731	+0.530	+0.217
48	+0.791	+0.828	+0.785	+0.593	+0.254
56	+0.763	+0.838	+0.824	+0.636	+0.285

Table A-2 — Moments in cylindrical wall

Mom. = coef. × wH^3 ft-lb per ft
Positive sign indicates tension in the outside

	Coefficients at point									
$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	+.0005	+.0014	+.0021	+.0007	-.0042	-.0150	-.0302	-.0529	-.0816	-.1205
0.8	+.0011	+.0037	+.0063	+.0080	+.0070	+.0023	-.0068	-.0224	-.0465	-.0795
1.2	+.0012	+.0042	+.0077	+.0103	+.0112	+.0090	+.0022	-.0108	-.0311	-.0602
1.6	+.0011	+.0041	+.0075	+.0107	+.0121	+.0111	+.0058	-.0051	-.0232	-.0505
2.0	+.0010	+.0035	+.0068	+.0099	+.0120	+.0115	+.0075	-.0021	-.0185	-.0436
3.0	+.0006	+.0024	+.0047	+.0071	+.0090	+.0097	+.0077	+.0012	-.0119	-.0333
4.0	+.0003	+.0015	+.0028	+.0047	+.0066	+.0077	+.0069	+.0023	-.0080	-.0268
5.0	+.0002	+.0008	+.0016	+.0029	+.0046	+.0059	+.0059	+.0028	-.0058	-.0222
6.0	+.0001	+.0003	+.0008	+.0019	+.0032	+.0046	+.0051	+.0029	-.0041	-.0187
8.0	.0000	+.0001	+.0002	+.0008	+.0016	+.0028	+.0038	+.0029	-.0022	-.0146
10.0	.0000	.0000	+.0001	+.0004	+.0007	+.0019	+.0029	+.0028	-.0012	-.0122
12.0	.0000	-.0000	+.0001	+.0002	+.0003	+.0013	+.0023	+.0026	-.0005	-.0104
14.0	.0000	.0000	.0000	.0000	+.0001	+.0008	+.0019	+.0023	-.0001	-.0090
16.0	.0000	.0000	-.0001	-.0002	-.0001	+.0004	+.0013	+.0019	+.0001	-.0079

Supplemental Coefficients

	Coefficient at point				
$\frac{H^2}{Dt}$	.80H	.85H	.90H	95H	1.00H
20	+.0015	+.0014	+.0005	-.0018	-.0063
24	+.0012	+.0012	+.0007	-.0013	-.0053
32	+.0007	+.0009	+.0007	-.0008	-.0040
40	+.0002	+.0005	+.0006	-.0005	-.0032
48	.0000	+.0001	+.0006	-.0003	-.0026
56	.0000	.0000	+.0004	-.0001	-.0023

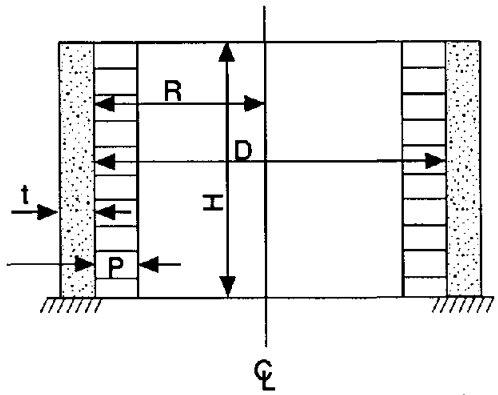


Table A-3— Tension in circular rings

T = coef. × pR lb per ft
Positive sign indicates tension

	Coefficients at point									
$\frac{H^2}{Dt}$	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+0.582	+0.505	+0.431	+0.353	+0.277	+0.206	+0.145	+0.092	+0.046	+0.013
0.8	+1.052	+0.921	+0.796	+0.669	+0.542	+0.415	+0.289	+0.179	+0.089	+0.024
1.2	+1.218	+1.078	+0.946	+0.808	+0.665	+0.519	+0.378	+0.246	+0.127	+0.034
1.6	+1.257	+1.141	+1.009	+0.881	+0.742	+0.600	+0.449	+0.294	+0.153	+0.045
2.0	+1.253	+1.144	+1.041	+0.929	+0.806	+0.667	+0.514	+0.345	+0.186	+0.055
3.0	+1.160	+1.112	+1.061	+0.998	+0.912	+0.796	+0.646	+0.459	+0.258	+0.081
4.0	+1.085	+1.073	+1.057	+1.029	+0.977	+0.887	+0.746	+0.553	+0.322	+0.105
5.0	+1.037	+1.044	+1.047	+1.042	+1.015	+0.949	+0.825	+0.629	+0.379	+0.128
6.0	+1.010	+1.024	+1.038	+1.045	+1.034	+0.986	+0.879	+0.694	+0.430	+0.149
8.0	+0.989	+1.005	+1.022	+1.036	+1.044	+1.026	+0.953	+0.788	+0.519	+0.189
10.0	+0.989	+0.998	+1.010	+1.023	+1.039	+1.040	+0.996	+0.859	+0.591	+0.226
12.0	+0.994	+0.997	+1.003	+1.014	+1.031	+1.043	+1.022	+0.911	+0.652	+0.262
14.0	+0.997	0.998	+1.000	+1.007	+1.022	+1.040	+1.035	+0.949	+0.705	+0.294
16.0	+1.000	0.999	+0.999	+1.003	+1.015	+1.032	+1.040	+0.975	+0.750	+0.321

Supplemental Coefficients

	Coefficient at point				
$\frac{H^2}{Dt}$	.75H	.80H	.85H	.90H	.95H
20	+0.949	+0.825	+0.629	+0.379	+0.128
24	+0.986	+0.879	+0.694	+0.430	+0.149
32	+1.026	+0.953	+0.788	+0.519	+0.189
40	+1.040	+0.996	+0.859	+0.591	+0.226
48	+1.043	+1.022	+0.911	+0.652	+0.262
56	+1.040	+1.035	+0.949	+0.705	+0.294

Table A-4— Moments in cylindrical wall

Mom. = coef. × pH^2 ft-lb per ft
Positive sign indicates tension in the outside

	Coefficients at point									
$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	-.0023	-.0093	-.0227	-.0439	-.0710	-.1018	-.1455	-.2000	-.2593	-.3310
0.8	.0000	-.0006	-.0025	-.0083	-.0185	-.0362	-.0594	-.0917	-.1325	-.1835
1.2	+.0008	+.0026	+.0037	+.0029	-.0009	-.0089	-.0227	-.0468	-.0815	-.1178
1.6	+.0011	+.0036	+.0062	+.0077	+.0068	+.0011	-.0093	-.0267	-.0529	-.0876
2.0	+.0010	+.0036	+.0066	+.0088	+.0089	+.0059	-.0019	-.0167	-.0389	-.0719
3.0	+.0007	+.0026	+.0051	+.0074	+.0091	+.0083	+.0042	-0053	-.0223	-.0483
4.0	+.0004	+.0015	+.0033	+.0052	+.0068	+.0075	+.0053	-.0013	-.0145	-.0365
5.0	+.0002	+.0008	+.0019	+.0035	+.0051	+.0061	+.0052	+.0007	-.0101	-.0293
6.0	+.0001	+.0004	+.0011	+.0022	+.0036	+.0049	+.0048	+.0017	-.0073	-.0242
8.0	.0000	+.0001	+.0003	+.0008	+.0018	+.0031	+.0038	+.0024	-.0040	-.0184
10.0	.0000	-.0001	.0000	+.0002	+.0009	+.0021	+.0030	+.0026	-.0022	-.0147
12.0	.0000	.0000	-.0001	.0000	+.0004	+.0014	+.0024	+.0022	-.0012	-.0123
14.0	.0000	.0000	.0000	.0000	+.0002	+.0010	+.0018	+.0021	-.0007	-.0105
16.0	.0000	.0000	.0000	-.0001	+.0001	+.0006	+.0012	+.0020	-.0005	-.0091

Supplemental Coefficients

	Coefficient at point				
$\frac{H^2}{Dt}$	.80H	.85H	.90H	95H	1.00H
20	+.0015	+.0013	+.0002	-.0024	-.0073
24	+.0012	+.0012	+.0004	-.0018	-.0061
32	+.0008	+.0009	+.0006	-.0010	-.0046
40	+.0005	+.0007	+.0007	-.0005	-.0037
48	+.0004	+.0006	+.0006	-.0003	-.0031
56	+.0002	+.0004	+.0005	-.0001	-.0026

Hinged Base-Free Top (Triangular Load)

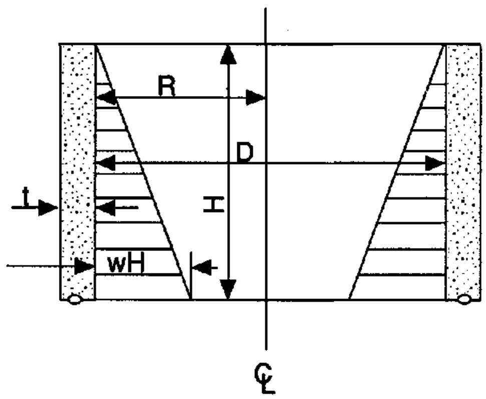


Table A-5— Tension in circular rings

T = coef. × wHR lb per ft
Positive sign indicates tension

	Coefficients at point									
$\frac{H^2}{Dt}$	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+0.474	+0.440	+0.395	+0.352	+0.308	+0.264	+0.215	+0.165	+0.111	+0.057
0.8	+0.423	+0.402	+0.381	+0.358	+0.330	+0.297	+0.249	+0.202	+0.145	+0.076
1.2	+0.350	+0.355	+0.361	+0.362	+0.358	+0.343	+0.309	+0.256	+0.186	+0.098
1.6	+0.271	+0.303	+0.341	+0.369	+0.385	+0.385	+0.362	+0.314	+0.233	+0.124
2.0	+0.205	+0.260	+0.321	+0.373	+0.411	+0.434	+0.419	+0.369	+0.280	+0.151
3.0	+0.074	+0.179	+0.281	+0.375	+0.449	+0.506	+0.519	+0.479	+0.375	+0.210
4.0	+0.017	+0.137	+0.253	+0.367	+0.469	+0.545	+0.579	+0.553	+0.447	+0.256
5.0	-0.008	+0.114	+0.235	+0.356	+0.469	+0.562	+0.617	+0.606	+0.503	+0.294
6.0	-0.011	+0.103	+0.223	+0.343	+0.463	+0.566	+0.639	+0.643	+0.547	+0.327
8.0	-0.015	+0.096	+0.208	+0.324	+0.443	+0.564	+0.661	+0.697	+0.621	+0.386
10.0	-0.008	+0.095	+0.200	+0.311	+0.428	+0.552	+0.666	+0.730	+0.678	+0.433
12.0	-0.002	+0.097	+0.197	+0.302	+0.417	+0.541	+0.664	+0.750	+0.720	+0.477
14.0	0.000	+0.098	+0.197	+0.299	+0.408	+0.531	+0.659	+0.761	+0.752	+0.513
16.0	+0.002	+0.100	+0.198	+0.299	+0.403	+0.521	+0.650	+0.764	+0.776	+0.536

Supplemental Coefficients

	Coefficient at point				
$\frac{H^2}{Dt}$	.75H	.80H	.85H	.90H	.95H
20	+0.812	+0.817	+0.756	+0.603	+0.344
24	+0.816	+0.839	+0.793	+0.647	+0.377
32	+0.814	+0.861	+0.847	+0.721	+0.436
40	+0.802	+0.866	+0.880	+0.778	+0.483
48	+0.791	+0.864	+0.900	+0.820	+0.527
56	+0.781	+0.859	+0.911	+0.852	+0.563

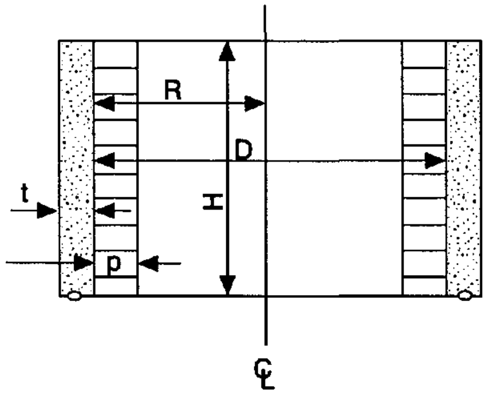


Table A-6— Tension in circular rings

T = coef. × pR lb per ft

Positive sign indicates tension

	Coefficients at point									
$\frac{H^2}{Dt}$	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+1.474	+1.340	+1.195	+1.052	+0.908	+0.764	+0.615	+0.465	+0.311	+0.154
0.8	+1.423	+1.302	+1.181	+1.058	+0.930	+0.797	+0.649	+0.502	+0.345	+0.166
1.2	+1.350	+1.255	+1.161	+1.062	+0.958	+0.843	+0.709	+0.556	+0.386	+0.198
1.6	+1.271	+1.203	+1.141	+1.069	+0.985	+0.885	+0.756	+0.614	+0.433	+0.224
2.0	+1.205	+1.160	+1.121	+1.073	+1.011	+0.934	+0.819	+0.669	+0.480	+0.251
3.0	+1.074	+1.079	+1.081	+1.075	+1.049	+1.006	+0.919	+0.779	+0.575	+0.310
4.0	+1.017	+1.037	+1.053	+1.067	+1.069	+1.045	+0.979	+0.853	+0.647	+0.356
5.0	+0.992	+1.014	+1.035	+1.056	+1.069	+1.062	+1.017	+0.906	+0.703	+0.394
6.0	+0.989	+1.003	+1.023	+1.043	+1.063	+1.066	+1.039	+0.943	+0.747	+0.427
8.0	+0.985	+0.996	+1.008	+1.024	+1.043	+1.064	+1.061	+0.997	+0.821	+0.486
10.0	+0.992	+0.995	+1.000	+1.011	+1.028	+1.052	+1.066	+1.030	+0.878	+0.533
12.0	+0.998	+0.997	+0.997	+1.002	+1.017	+1.041	+1.064	+1.050	+0.920	+0.577
14.0	+1.000	+0.998	+0.997	+0.999	+1.008	+1.031	+1.059	+1.060	+0.952	+0.613
16.0	+1.002	+1.000	+0.998	+0.999	+1.003	+1.021	+1.050	+1.064	+0.976	+0.636

Supplemental Coefficients

	Coefficient at point				
$\frac{H^2}{Dt}$	.75H	.80H	.85H	.90H	.95H
20	+1.062	+1.017	+0.906	+0.703	+0.394
24	+1.066	+1.039	+0.943	+0.747	+0.427
32	+1.064	+1.061	+0.997	+0.821	+0.486
40	+1.052	+1.066	+1.030	+0.878	+0.533
48	+1.041	+1.064	+1.050	+0.920	+0.577
56	+1.021	+1.059	+1.061	+0.952	+0.613

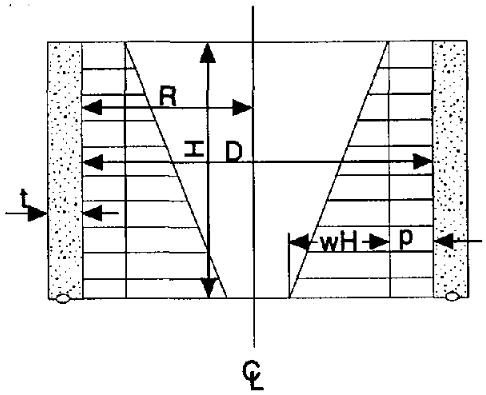


Table A-7— Moments in cylindrical wall

Mom. = coef. × ($wH^3 + pH^2$) ft-lb per ft
Positive sign indicates tension in the outside

	Coefficients at point									
$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	+.0020	+.0072	+.0151	+.0230	+.0301	+.0348	+.0357	+.0312	+.0197	0
0.8	+.0019	+.0064	+.0133	+.0207	+.0271	+.0319	+.0329	+.0292	+.0187	0
1.2	+.0016	+.0058	+.0111	+.0177	+.0237	+.0280	+.0296	+.0263	+.0171	0
1.6	+.0012	+.0044	+.0091	+.0145	+.0195	+.0236	+.0255	+.0232	+.0155	0
2.0	+.0009	+.0033	+.0073	+.0114	+.0158	+.0199	+.0219	+.0205	+.0145	0
3.0	+.0004	+.0018	+.0040	+.0063	+.0092	+.0127	+.0152	+.0153	+.0111	0
4.0	+.0001	+.0007	+.0016	+.0033	+.0057	+.0083	+.0109	+.0118	+.0092	0
5.0	.0000	+.0001	+.0006	+.0016	+.0034	+.0057	+.0080	+.0094	+.0078	0
6.0	.0000	.0000	+.0002	+.0008	+.0019	+.0039	+.0062	+.0078	+.0068	0
8.0	.0000	.0000	-.0002	.0000	+.0007	+.0020	+.0038	+.0057	+.0054	0
10.0	.0000	.0000	-.0002	-.0001	+.0002	+.0011	+.0025	+.0043	+.0045	0
12.0	.0000	.0000	-.0001	-.0002	.0000	+.0005	+.0017	+.0032	+.0039	0
14.0	.0000	.0000	-.0001	-.0001	-.0001	.0000	+.0012	+.0026	+.0033	0
16.0	.0000	.0000	.0000	-.0001	.0002	-.0004	+.0008	+.0022	+.0029	0

Supplemental Coefficients

	Coefficient at point				
$\frac{H^2}{Dt}$	.75H	.80H	.85H	.90H	.95H
20	+.0008	+.0014	+.0020	+.0024	+.0020
24	+.0005	+.0010	+.0015	+.0020	+.0017
32	.0000	+.0005	+.0009	+.0014	+.0013
40	.0000	+.0003	+.0006	+.0011	+.0011
48	.0000	+.0001	+.0004	+.0008	+.0010
56	.0000	.0000	+.0003	+.0007	+.0008

Fixed Base-Free Top (Shear Applied at Top)

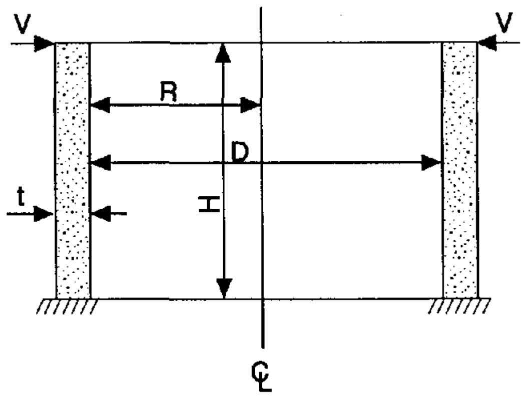


Table A-8— Tension in circular rings

T = coef. × VR/H lb per ft
Positive sign indicates tension

$\frac{H^2}{Dt}$	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	-1.57	-1.32	-1.08	-0.86	-0.65	-0.47	-0.31	-0.18	-0.08	-0.02
0.8	-3.09	-2.55	-2.04	-1.57	-1.15	-0.80	-0.51	-0.28	-0.13	-0.03
1.2	-3.95	-3.17	-2.44	-1.79	-1.25	-0.81	-0.48	-0.25	-0.10	-0.02
1.6	-4.57	-3.54	-2.60	-1.80	-1.17	-0.69	-0.36	-0.16	-0.05	-0.01
2.0	-5.12	-3.83	-2.68	-1.74	-1.02	-0.52	-0.21	-0.05	+0.01	+0.01
3.0	-6.32	-4.37	-2.70	-1.43	-0.58	-0.02	+0.15	+0.19	+0.13	+0.04
4.0	-7.34	-4.73	-2.60	-1.10	-0.19	+0.26	+0.38	+0.33	+0.19	+0.06
5.0	-8.22	-4.99	-2.45	-0.79	+0.11	+0.47	+0.50	+0.37	+0.20	+0.06
6.0	-9.02	-5.17	-2.27	-0.50	+0.34	+0.59	+0.53	+0.35	+0.17	+0.01
8.0	-10.42	-5.36	-1.85	-0.02	+0.63	+0.66	+0.46	+0.24	+0.09	+0.01
10.0	-11.67	-5.43	-1.43	+0.36	+0.78	+0.62	+0.33	+0.12	+0.02	0.00
12.0	-12.76	-5.41	-1.03	+0.63	+0.83	+0.52	+0.21	+0.04	-0.02	0.00
14.0	-13.77	-5.34	-0.68	+0.80	+0.81	+0.42	+0.13	0.00	-0.03	-0.01
16.0	-14.74	-5.22	-0.33	+0.96	+0.76	+0.32	+0.05	-0.04	-0.05	-0.02

Supplemental Coefficients

$\frac{H^2}{Dt}$	.00H	.05H	.10H	.15H	.20H
20	-16.44	-9.98	-4.90	-1.59	+0.22
24	-18.04	-10.34	-4.54	-1.00	+0.68
32	-20.84	-10.72	-3.70	-0.04	+1.26
40	-23.34	-10.86	-2.86	+0.72	+1.56
48	-25.52	-10.82	-2.06	+1.26	+1.66
56	-27.54	-10.68	-1.36	+1.60	+1.62

When this table is used for shear applied at the base while the top is fixed, 0.0H is the bottom of the wall and 1.0H is the top. Shear acting inward is positive, outward is negative.

Table A-9— Moments in cylindrical wall

Mom. = coef. × VH ft-lb per ft
Positive sign indicates tension in the outside

$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	+0.093	+0.172	+0.240	+0.300	+0.354	+0.402	+0.448	+0.492	+0.535	+0.578
0.8	+0.085	+0.145	+0.185	+0.208	+0.220	+0.224	+0.223	+0.219	+0.214	+0.208
1.2	+0.082	+0.132	+0.157	+0.164	+0.159	+0.145	+0.127	+0.106	+0.084	+0.062
1.6	+0.079	+0.122	+0.139	+0.138	+0.125	+0.105	+0.081	+0.056	+0.030	+0.004
2.0	+0.077	+0.115	+0.126	+0.119	+0.103	+0.080	+0.056	+0.031	+0.006	-0.019
3.0	+0.072	+0.100	+0.100	+0.086	+0.066	+0.044	+0.025	+0.006	-0.010	-0.024
4.0	+0.068	+0.088	+0.081	+0.063	+0.043	+0.025	+0.010	-0.001	-0.010	-0.019
5.0	+0.064	+0.078	+0.067	+0.047	+0.028	+0.013	+0.003	-0.003	-0.007	-0.011
6.0	+0.062	+0.070	+0.056	+0.036	+0.018	+0.006	0.000	-0.003	-0.005	-0.006
8.0	+0.057	+0.058	+0.041	+0.021	+0.007	0.000	-0.002	-0.003	-0.002	-0.001
10.0	+0.053	+0.049	+0.029	+0.012	+0.002	-0.002	-0.002	-0.002	-0.001	0.000
12.0	+0.049	+0.042	+0.022	+0.007	+0.000	-0.002	-0.002	-0.001	0.000	0.000
14.0	+0.046	+0.036	+0.017	+0.004	-0.001	-0.002	-0.001	-0.001	0.000	0.000
16.0	+0.044	+0.031	+0.012	+0.001	-0.002	-0.002	-0.001	0.000	0.000	0.000

Supplemental Coefficients

$\frac{H^2}{Dt}$	.05H	.10H	.15H	.20H	.25H
20	+0.032	+0.039	+0.033	+0.023	+0.014
24	+0.031	+0.035	+0.028	+0.018	+0.009
32	+0.028	+0.029	+0.020	+0.011	+0.004
40	+0.026	+0.025	+0.015	+0.006	+0.001
48	+0.024	+0.021	+0.011	+0.003	0.000
56	+0.023	+0.018	+0.008	+0.002	0.000

When this table is used for shear applied at the base while the top is fixed, 0.0H is the bottom of the wall and 1.0H is the top. Shear acting inward is positive, outward is negative.

Hinged Base-Free Top (Moment Applied at Base)

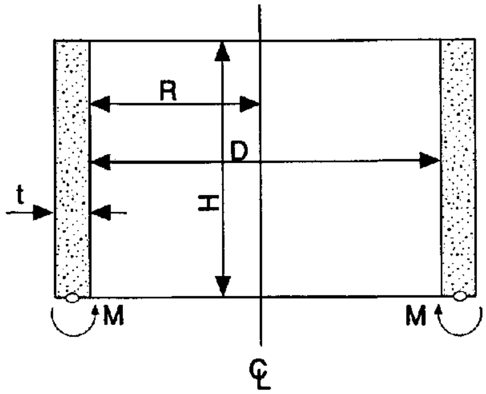


Table A-10— Tension in circular rings

T = coef. × MR/H^2 lb per ft
Positive sign indicates tension

H^2/Dt	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+2.70	+2.50	+2.30	+2.12	+1.91	+1.69	+1.41	+1.13	+0.80	+0.44
0.8	+2.02	+2.06	+2.10	+2.14	+2.10	+2.02	+1.95	+1.75	+1.39	+0.80
1.2	+1.06	+1.42	+1.79	+2.03	+2.46	+2.65	+2.80	+2.60	+2.22	+1.37
1.6	+0.12	+0.79	+1.43	+2.04	+2.72	+3.25	+3.56	+3.59	+3.13	+2.01
2.0	-0.68	+0.22	+1.10	+2.02	+2.90	+3.69	+4.30	+4.54	+4.08	+2.75
3.0	-1.78	-0.71	+0.43	+1.60	+2.95	+4.29	+5.66	+6.58	+6.55	+4.73
4.0	-1.87	-1.00	-0.08	+1.04	+2.47	+4.31	+6.34	+8.19	+8.82	+6.81
5.0	-1.54	-1.03	-0.42	+0.45	+1.86	+3.93	+6.60	+9.41	+11.03	+9.02
6.0	-1.04	-0.86	-0.59	-0.05	+1.21	+3.34	+6.54	+10.28	+13.08	+11.41
8.0	-0.24	-0.53	-0.73	-0.67	-0.02	+2.05	+5.87	+11.32	+16.52	+16.06
10.0	+0.21	-0.23	-0.64	-0.94	-0.73	+0.82	+4.79	+11.63	+19.48	+20.87
12.0	+0.32	-0.05	-0.46	-0.96	-1.15	-0.18	+3.52	+11.27	+21.80	+25.73
14.0	+0.26	+0.04	-0.28	-0.76	-1.29	-0.87	+2.29	+10.55	+23.50	+30.34
16.0	+0.22	+0.07	-0.08	-0.64	-1.28	-1.30	+1.12	+9.67	+24.53	+34.65

Supplemental Coefficients

H^2/Dt	.75H	.80H	.85H	.90H	.95H
20	+15.30	+25.9	+36.9	+43.3	+35.3
24	+13.20	+25.9	+40.7	+51.8	+45.3
32	+8.10	+23.2	+45.9	+65.4	+63.6
40	+3.28	+19.2	+46.5	+77.9	+83.5
48	-0.70	+14.1	+45.1	+87.2	+103.0
56	-3.40	+9.2	+42.2	+94.0	+121.0

When this table is used for moment applied at the top, while the top is hinged, 0.0H is the bottom of the wall and 1.0H is the top. Moment applied at an edge is positive when it causes outward rotation at that edge.

Table A-11— Moments in cylindrical wall

Mom. = coef. × M ft-lb per ft
Positive sign indicates tension in the outside

H^2/Dt	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	+0.013	+0.051	+0.109	+0.196	+0.296	+0.414	+0.547	+0.692	+0.843	+1.000
0.8	+0.009	+0.040	+0.090	+0.164	+0.253	+0.375	+0.503	+0.659	+0.824	+1.000
1.2	+0.006	+0.027	+0.063	+0.125	+0.206	+0.316	+0.454	+0.616	+0.802	+1.000
1.6	+0.003	+0.011	+0.035	+0.078	+0.152	+0.253	+0.393	+0.570	+0.775	+1.000
2.0	-0.002	-0.002	+0.012	+0.034	+0.096	+0.193	+0.340	+0.519	+0.748	+1.000
3.0	-0.007	-0.022	-0.030	-0.029	+0.010	+0.087	+0.227	+0.426	+0.692	+1.000
4.0	-0.008	-0.026	-0.044	-0.051	-0.034	+0.023	+0.150	+0.354	+0.645	+1.000
5.0	-0.007	-0.024	-0.045	-0.061	-0.057	-0.015	+0.095	+0.296	+0.606	+1.000
6.0	-0.005	-0.018	-0.040	-0.058	-0.065	-0.037	+0.057	+0.252	+0.572	+1.000
8.0	-0.001	-0.009	-0.022	-0.044	-0.068	-0.062	+0.002	+0.178	+0515	+1.000
10.0	0.000	-0.002	-0.009	-0.028	-0.053	-0.067	-0.031	+0.123	+0.467	+1.000
12.0	0.000	0.000	-0.003	-0.016	-0.040	-0.064	-0.049	+0.081	+0.424	+1.000
14.0	0.000	0.000	0.000	-0.008	-0.029	-0.059	-0.060	+0.048	+0.387	+1.000
16.0	0.000	0.000	+0.002	-0.003	-0.021	-0.051	-0.066	+0.025	+0.354	+1.000

Supplemental Coefficients

H^2/Dt	.80H	.85H	.90H	.95H	1.00H
20	-0.015	+0.095	+0.296	+0.606	+1.000
24	-0.037	+0.057	+0.250	+0.572	+1.000
32	-0.062	+0.002	+0.178	+0.515	+1.000
40	-0.067	-0.031	+0.123	+0.467	+1.000
48	-0.064	-0.049	+0.081	+0.424	+1.000
56	-0.059	-0.060	+0.048	+0.387	+1.000

When this table is used for moment applied at the top, while the top is hinged, 0.0H is the bottom of the wall and 1.0H is the top. Moment applied at an edge is positive when it causes outward rotation at that edge.

Shear at Base of Cylindrical Wall

Table A-12—Shear at base of cylindrical wall

$$V = \text{coef.} \times \begin{cases} wH^2 \text{ lb. (triangular)} \\ pH \text{ lb. (rectangular)} \\ M/H \text{ lb. (moment at base)} \end{cases}$$

Positive sign indicates shear acting inward

$\frac{H^2}{Dt}$	Triangular load, fixed base	Rectangular load, fixed base	Triangular or rectangular load, hinged base	Moment at edge
0.4	+0.436	+0.755	+0.245	-1.58
0.8	+0.374	+0.552	+0.234	-1.75
1.2	+0.339	+0.460	+0.220	-2.00
1.6	+0.317	+0.407	+0.204	-2.26
2.0	+0.299	+0.370	+0.189	-2.57
3.0	+0.262	+0.310	+0.158	-3.18
4.0	+0.236	+0.271	+0.137	-3.68
5.0	+0.213	+0.243	+0.121	-4.10
6.0	+0.197	+0.222	+0.110	-4.49
8.0	+0.174	+0.193	+0.096	-5.18
10.0	+0.158	+0.172	+0.087	-5.81
12.0	+0.145	+0.158	+0.079	-6.38
14.0	+0.135	+0.147	+0.073	-6.88
16.0	+0.127	+0.137	+0.068	-7.36
20.0	+0.114	+0.122	+0.062	-8.20
24.0	+0.102	+0.111	+0.055	-8.94
32.0	+0.089	+0.096	+0.048	-10.36
40.0	+0.080	+0.086	+0.043	-10.62
48.0	+0.072	+0.079	+0.039	-12.76
56.0	+0.067	+0.074	+0.036	-13.76

Load on Center Support for Circular Slab

Table A-13—Load on center support for circular slab

$$\text{Load} = \text{coef.} \times \begin{cases} pR^2 \text{ (hinged and fixed)} \\ M \text{ (moment at edge)} \end{cases}$$

c/D	0.05	0.10	0.15	0.20	0.25
Hinged	1.320	1.387	1.463	1.542	1.625
Fixed	0.839	0.919	1.007	1.101	1.200
M at edge	8.16	8.66	9.29	9.99	10.81

Moments in Circular Slab Without Center Support

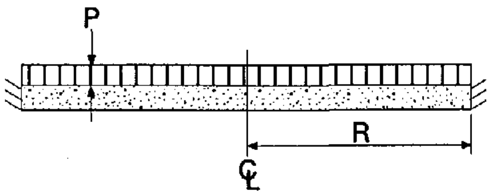


Table A-14— Moments in circular slab without center support

Mom. = coef. × pR^2 ft-lb per ft

Positive sign indicates compression in surface loaded

Coefficients at point										
0.00R	0.10R	0.20R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
Radial Moments, M_r										
+.075	+.073	+.067	+.057	+.043	+.025	+.003	-.023	-.053	-.087	-.125
Tangential Moments, M_t										
+.075	+.074	+.071	+.066	+.059	+.050	+.039	+.026	+.011	-.006	-.025

Stiffness of Cylindrical Wall

Table A-15— Stiffness of cylindrical wall, near edge hinged, far edge free

k = coef. × Et^3/H

$\frac{H^2}{Dt}$	Coefficient	$\frac{H^2}{Dt}$	Coefficient
0.4	0.139	10	1.010
0.8	0.270	12	1.108
1.2	0.345	14	1.198
1.6	0.399	16	1.281
2.0	0.445	20	1.430
3.0	0.548	24	1.566
4.0	0.635	32	1.810
5.0	0.713	40	2.025
6.0	0.783	48	2.220
8.0	0.903	56	2.400

Stiffness of Circular Plates

Table A-16— Stiffness of circular plates with center support

k = coef. × Et^3/R

c/D	0.05	0.10	0.15	0.20	0.25
Coef.	0.290	0.309	0.332	0.358	0.387

Without center support
Coef. = 0.104

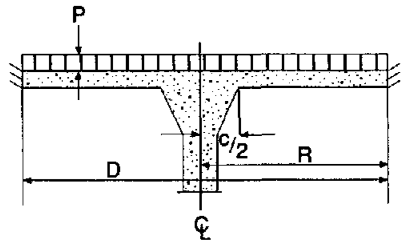


Table A-17—Uniform load, fixed edge

Mom. = coef. × pR^2 ft-lb per ft
Positive sign indicates compression in surface loaded

	Coefficients at point												
c/D	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
	Radial Moments, M_r												
0.05	-0.2100	-0.0729	-0.0275	-0.0026	+0.0133	+0.0238	+0.0342	+0.0347	+0.0277	+0.0142	-0.0049	-0.0294	-0.0589
0.10		-0.1433	-0.0624	-0.0239	-0.0011	+0.0136	+0.0290	+0.0326	+0.0276	+0.0158	-0.0021	-0.0255	-0.0541
0.15			-0.1089	-0.0521	-0.0200	+0.0002	+0.0220	+0.0293	+0.0269	+0.0169	+0.0006	-0.0216	-0.0490
0.20				-0.0862	-0.0429	-0.0161	+0.0133	+0.0249	+0.0254	+0.0176	+0.0029	-0.0178	-0.0441
0.25					-0.0698	-0.0351	+0.0029	+0.0194	+0.0231	+0.0177	+0.0049	-0.0143	-0.0393
	Tangential Moments, M_t												
0.05	-0.0417	-0.0700	-0.0541	-0.0381	-0.0251	-0.0145	+0.0002	+0.0085	+0.0118	+0.0109	+0.0065	-0.0003	-0.0118
0.10		-0.0287	-0.0421	-0.0354	-0.0258	-0.0168	-0.0027	+0.0059	+0.0099	+0.0098	+0.0061	-0.0009	-0.0108
0.15			-0.0218	-0.0284	-0.0243	-0.0177	-0.0051	+0.0031	+0.0080	+0.0086	+0.0057	-0.0006	-0.0098
0.20				-0.0172	-0.0203	-0.0171	-0.0070	+0.0013	+0.0063	+0.0075	+0.0052	-0.0003	-0.0088
0.25					-0.0140	-0.0150	-0.0083	-0.0005	+0.0046	+0.0064	+0.0048	0.0000	-0.0078

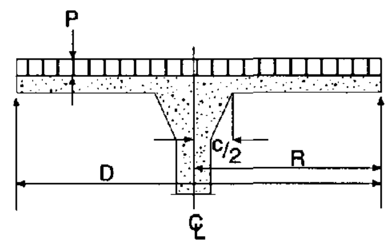


Table A-18—Uniform load, hinged edge

Mom. = coef. × pR^2 ft-lb per ft
Positive sign indicates compression in surface loaded

	Coefficients at point												
c/D	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
	Radial Moments, M_r												
0.05	-0.3658	-0.1388	-0.0640	-0.0221	+0.0058	+0.0255	+0.0501	+0.0614	+0.0629	+0.0566	+0.0437	+0.0247	0
0.10		-0.2487	-0.1180	-0.0557	-0.0176	+0.0081	+0.0391	+0.0539	+0.0578	+0.0532	+0.0416	+0.0237	0
0.15			-0.1869	-0.0977	-0.0467	-0.0135	+0.0258	+0.0451	+0.0518	+0.0494	+0.0393	+0.0226	0
0.20				-0.1465	-0.0800	-0.0381	+0.0109	+0.0352	+0.0452	+0.0451	+0.0368	+0.0215	0
0.25					-0.1172	-0.0645	-0.0055	+0.0245	+0.0381	+0.0404	+0.0340	+0.0200	0
	Tangential Moments, M_t												
0.05	-0.0731	-0.1277	-0.1040	-0.0786	-0.0569	-0.0391	-0.0121	+0.0061	+0.0175	+0.0234	+0.0251	+0.0228	+0.0168
0.10		-0.0498	-0.0768	-0.0684	-0.0539	-0.0394	-0.0153	+0.0020	+0.0134	+0.0197	+0.0218	+0.0199	+0.0145
0.15			-0.0374	-0.0516	-0.0470	-0.0375	-0.0175	-0.0014	+0.0097	+0.0163	+0.0186	+0.0172	+0.0123
0.20				-0.0293	-0.0367	-0.0333	-0.0184	-0.0042	+0.0065	+0.0132	+0.0158	+0.0148	+0.0103
0.25					-0.0234	-0.0263	-0.0184	-0.0062	+0.0038	+0.0103	+0.0132	+0.0122	+0.0085

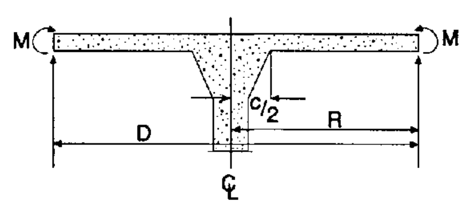


Table A-19—Moment per ft, M, applied at edge, hinged edge

Mom. = coef. × M ft-lb per ft
Positive sign indicates compression in top surface

	Coefficients at point												
c/D	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
	Radial Moments, M_r												
0.05	-2.650	-1.121	-0.622	-0.333	-0.129	+0.029	+0.268	+0.450	+0.596	+0.718	+0.824	+0.917	+1.000
0.10		-1.950	-1.026	-0.584	-0.305	-0.103	+0.187	+0.394	+0.558	+0.692	+0.808	+0.909	+1.000
0.15			-1.594	-0.930	-0.545	-0.280	+0.078	+0.323	+0.510	+0.663	+0.790	+0.900	+1.000
0.20				-1.366	-0.842	-0.499	-0.057	+0.236	+0.451	+0.624	+0.768	+0.891	+1.000
0.25					-1.204	-0.765	-0.216	+0.130	+0.392	+0.577	+0.740	+0.880	+1.000
	Tangential Moments, M_t												
0.05	-0.530	-0.980	-0.847	-0.688	-0.544	-0.418	-0.211	-0.042	+0.095	+0.212	+0.314	+0.405	+0.486
0.10		-0.388	-0.641	-0.608	-0.518	-0.419	-0.233	-0.072	+0.066	+0.185	+0.290	+0.384	+0.469
0.15			-0.319	-0.472	-0.463	-0.404	-0.251	-0.100	+0.035	+0.157	+0.263	+0.363	+0.451
0.20				-0.272	-0.372	-0.368	-0.261	-0.123	+0.007	+0.129	+0.240	+0.340	+0.433
0.25					-0.239	-0.305	-0.259	-0.145	-0.020	+0.099	+0.214	+0.320	+0.414

Design Aid for Bending Moment Reinforcing

Table A-20—Design Aid for Bending Moment Reinforcing

ω	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
0.0	0	.0010	.0020	.0030	.0040	.0050	.0060	.0070	.0080	.0090
0.01	.0099	.0109	.0119	.0129	.0139	.0149	.0159	.0168	.0178	.0188
0.02	.0197	.0207	.0217	.0226	.0236	.0246	.0256	.0266	.0275	.0285
0.03	.0295	.0304	.0314	.0324	.0333	.0343	.0352	.0362	.0372	.0381
0.04	.0391	.0400	.0410	.0420	.0429	.0438	.0448	.0457	.0467	.0476
0.05	.0485	.0495	.0504	.0513	.0523	.0532	.0541	.0551	.0560	.0569
0.06	.0579	.0588	.0597	.0607	.0616	.0625	.0634	.0643	.0653	.0662
0.07	.0671	.0680	.0689	.0699	.0708	.0717	.0726	.0735	.0744	.0753
0.08	.0762	.0771	.0780	.0789	.0798	.0807	.0816	.0825	.0834	.0843
0.09	.0852	.0861	.0870	.0879	.0888	.0897	.0906	.0915	.0923	.0932
0.10	.0941	.0950	.0959	.0967	.0976	.0985	.0994	.1002	.1011	.1020
0.11	.1029	.1037	.1046	.1055	.1063	.1072	.1081	.1089	.1098	.1106
0.12	.1115	.1124	.1133	.1141	.1149	.1158	.1166	.1175	.1183	.1192
0.13	.1200	.1209	.1217	.1226	.1234	.1243	.1251	.1259	.1268	.1276
0.14	.1284	.1293	.1301	.1309	.1318	.1326	.1334	.1342	.1351	.1359
0.15	.1367	.1375	.1384	.1392	.1400	.1408	.1416	.1425	.1433	.1441
0.16	.1449	.1457	.1465	.1473	.1481	.1489	.1497	.1506	.1514	.1522
0.17	.1529	.1537	.1545	.1553	.1561	.1569	.1577	.1585	.1593	.1601
0.18	.1609	.1617	.1624	.1632	.1640	.1648	.1656	.1664	.1671	.1679
0.19	.1687	.1695	.1703	.1710	.1718	.1726	.1733	.1741	.1749	.1756
0.20	.1764	.1772	.1779	.1787	.1794	.1802	.1810	.1817	.1825	.1832
0.21	.1840	.1847	.1855	.1862	.1870	.1877	.1885	.1892	.1900	.1907
0.22	.1914	.1922	.1929	.1937	.1944	.1951	.1959	.1966	.1973	.1981
0.23	.1988	.1985	.2002	.2010	.2017	.2024	.2031	.2039	.2046	.2053
0.24	.2060	.2067	.2075	.2082	.2089	.2096	.2103	.2110	.2117	.2124
0.25	.2131	.2138	.2145	.2152	.2159	.2166	.2173	.2180	.2187	.2194
0.26	.2201	.2208	.2215	.2222	.2229	.2236	.2243	.2249	.2256	.2263
0.27	.2270	.2277	.2284	.2290	.2297	.2304	.2311	.2317	.2324	.2331
0.28	.2337	.2344	.2351	.2357	.2364	.2371	.2377	.2384	.2391	.2397
0.29	.2404	.2410	.2417	.2423	.2430	.2437	.2443	.2450	.2456	.2463
0.30	.2469	.2475	.2482	.2488	.2495	.2501	.2508	.2514	.2520	.2527
0.31	.2533	.2539	.2546	.2552	.2558	.2565	.2571	.2577	.2583	.2590
0.32	.2596	.2602	.2608	.2614	.2621	.2627	.2633	.2639	.2645	.2651
0.33	.2657	.2664	.2670	.2676	.2682	.2688	.2694	.2670	.2706	.2712
0.34	.2718	.2724	.2730	.2736	.2742	.2748	.2754	.2760	.2766	.2771
0.35	.2777	.2783	.2789	.2795	.2801	.2807	.2812	.2818	.2824	.2830
0.36	.2835	.2841	.2847	.2853	.2858	.2864	.2870	.2875	.2881	.2887
0.37	.2892	.2898	.2904	.2909	.2915	.2920	.2926	.2931	.2937	.2943
0.38	.2948	.2954	.2959	.2965	.2970	.2975	.2981	.2986	.2992	.2997
0.39	.3003	.3003	.3013	.3019	.3024	.3029	.3035	.3040	.3045	.3051

Design: Using factored moment M_u, enter table with $M_u/\phi f'_c bd^2$; find ω and compute steel percentage ρ from $\rho = \omega f'_c/f_y$

Investigation: Enter table with ω from $\omega = \rho f_y/f'_c$; find value of $M_n/f'_c bd^2$ and solve for nominal strength M_n.

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