Load Contour Method

If the interaction diagram is horizontally sliced at a particular load level, as done in Fig. 14.27, the resulting slice is actually a graph between M_{nx} and M_{ny} at a constant load and is called a ***load contour***.

The values of the moments, M_{nx} and M_{ny}, are usually normalized with M_{nxo} (when $M_{ny} = 0$) and M_{nyo} (when $M_{nx} = 0$), respectively.

In this method, a curve of the following form is passed through the failure line of a load contour:

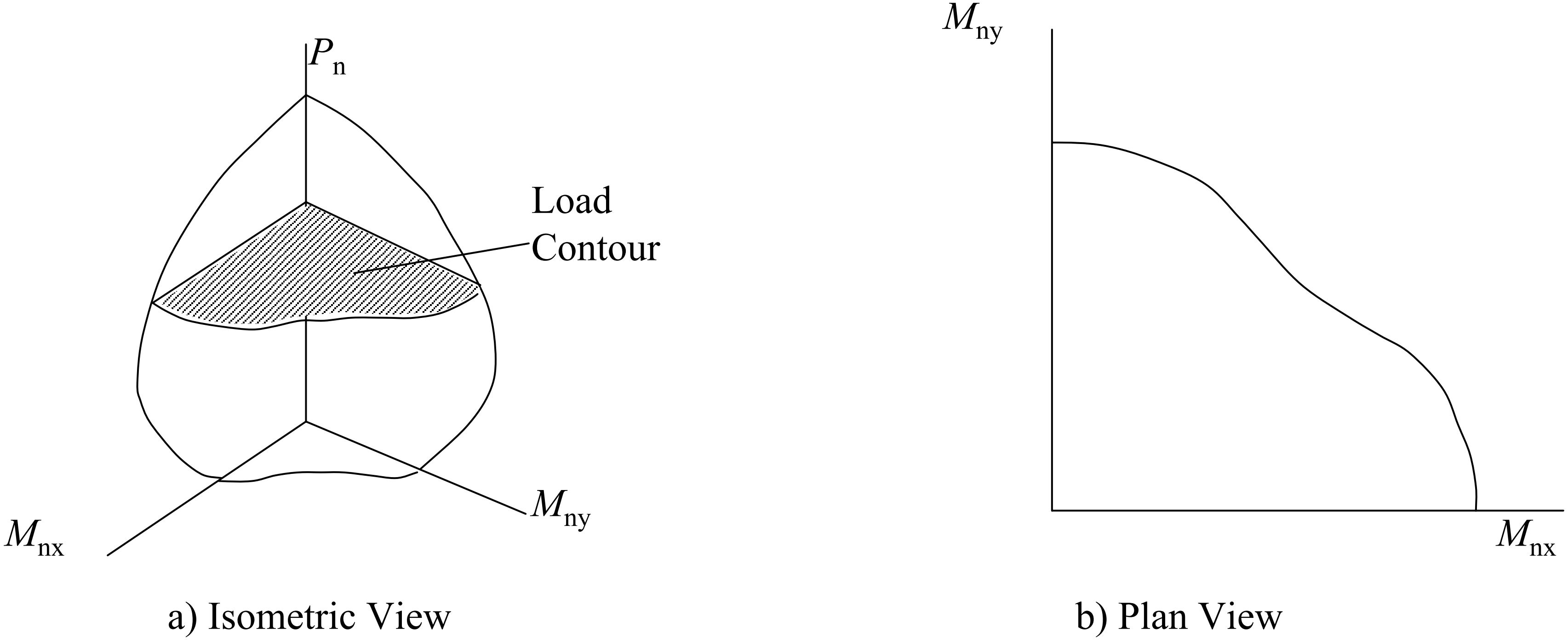


Fig. 14.27. A Typical Load Contour.

$$\left(\frac{M_{nx}}{M_{nxo}}\right)^{\alpha 1} + \left(\frac{M_{ny}}{M_{nyo}}\right)^{\alpha 2} = 1.0$$

The constants α_1 and α_2 are the exponents depending on

- column dimensions,
- amount and distribution of steel reinforcement,
- stress-strain characteristics of steel and concrete,
- amount of concrete cover and
- size of lateral ties or spirals.

Generally, the values of α_1 and α_2 equal to a constant value α give satisfactory results reducing the equation to:

$$\left(\frac{M_{nx}}{M_{nxo}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{nyo}}\right)^{\alpha} = 1.0$$

The range of values of α for square and rectangular columns is between 1.15 and 1.55 and the lower values of α are more conservative.

A value of $\alpha = 1.5$ is reasonably accurate for the most square and rectangular sections having uniformly distributed reinforcement.

According to some researchers, the value of α may be more accurately obtained in the following form:

$$\alpha \quad = \quad \log 0.5 \,/\, \log \beta$$

Values of β are given in the form of charts on pages 7-17 to 7-19 of PCA Notes on ACI-02 against the values of P_u / P_o and the reinforcement index.

$$\text{Reinforcement index} = \rho_t \frac{f_y}{f_c'}$$

Separate curves are available for various bar arrangements.

The column must be rectangular with larger to shorter sides ratio of less than *4.0*, f_c' must be between 12 and 41 *MPa,* and γ must be between 0.6 and 1.0.

Example 14.5: Select a square tied column cross-section to resist $P_u = 1600$ kN, $M_{ux} = 95$ kN-m and $M_{uy} = 110$ kN-m. $f_c' = 20$ MPa, $f_y = 300$ MPa, and clear cover = 40 mm. Use the following two methods:

a) Equivalent uniaxial eccentricity method
b) Reciprocal load method.

<u>Solution:</u>

<u>a) Equivalent Uniaxial Eccentricity Method</u>

$$A_g\ (\text{trial}) = \frac{P_u + 2M_{ux} + 2M_{uy}}{0.43 f_c' + 0.008 f_y} \quad = \quad \frac{(1600 + 2 \times 95 + 2 \times 110)(1000)}{0.43 \times 20 + 0.008 \times 300}$$

$$= 182{,}727 \text{ mm}^2$$

Try 450 × 450 mm column.

$$\gamma = \frac{450 - 2\times 10 - 25 - 2\times 40}{450} = 0.72$$

$$e_{\mathrm{x}} = \frac{M_{uy}}{P_u} = \frac{110\times 1000}{1600} = 68.8 \text{ mm}$$

$$e_{\mathrm{y}} = \frac{M_{ux}}{P_u} = \frac{95\times 1000}{1600} = 59.4 \text{ mm}$$

$$e_{\mathrm{x}} / b = 68.8 / 450 \quad \geq \quad e_{\mathrm{y}} / h = 59.4 / 450$$

$$\frac{P_u}{f_c' A_g} = \frac{1600\times 1000}{20\times 450^2} = 0.395 \quad \leq \quad 0.4$$

$$\alpha = \left(0.5 + \frac{P_u}{f_c' A_g}\right) \frac{f_y + 300}{720} \geq 0.6$$

$$= (0.5 + 0.395) \frac{300 + 300}{720} = 0.746$$

$$e_{ox} = e_x + \alpha \frac{e_y}{h} b$$

$$= 68.8 + 0.746 \times 59.4 \times 450 / 450 = 113.1 \text{ mm}$$

Equivalent uniaxial moment is:

$$M_{oy} = P_u e_{ox} = 1600 \times 113.1 / 1000 = 180.98 \text{ kN-m}$$

Use uniaxial interaction diagrams with bars in all the four faces to determine the total steel ratio, ρ_t.

$$\frac{P_u}{A_g} = 1600{,}000 / 450^2 \qquad = 7.9 \text{ MPa}$$

$$\frac{M_{oy}}{A_g h} = 180.98\times10^6 / 450^3 \qquad = 2.0 \text{ MPa}$$

For $\gamma = 0.6 \; \rho_t = 0.023$

For $\gamma = 0.75 \qquad \rho_t = 0.018$

For $\gamma = 0.72$

$$\rho_t = 0.018 + 0.005 / 0.15 \times (0.75 - 0.72) = 0.019$$

$$A_{st} = \rho_t \times A_g = 0.019 \times 450^2 = 3848 \; mm^2$$

(Use 8 – #25 bars)

Reciprocal Load Method

Trial Size: 450 × 450 mm column

Use 8 – #25 bars as the first try.

$$\rho_t \quad = \; 8 \times 510 \,/\, 450^2 \; = \; 0.02$$

$$\frac{M_{uy}}{A_g h} = \frac{\phi M_{ny}}{A_g h} = \; 110\times10^6 \,/\, 450^3 \; = \; 1.21 \text{ MPa}$$

$$\frac{\phi P_n}{A_g} = \; 1600{,}000 \,/\, 450^2 \; = 7.90 \text{ MPa}$$

Join this point with the origin and extend to the 2% steel curve to get the value of the required capacity.

For $\gamma = 0.60$ $\qquad \dfrac{\phi P_{nx}}{A_g} = 9.8$ MPa

For $\gamma = 0.75$ $\qquad \dfrac{\phi P_{nx}}{A_g} = 10.2$ MPa

For $\gamma = 0.72$ $\qquad \dfrac{\phi P_{nx}}{A_g} = 10.12$ MPa

$$\Rightarrow \qquad \phi P_{\mathrm{nx}} = 2049\ kN$$

$$\frac{M_{ux}}{A_g h} = \frac{\phi M_{nx}}{A_g h} = 95\times10^6 / 450^3 = 1.04 \text{ MPa}$$

$$\frac{\phi P_n}{A_g} = 1600{,}000 / 450^2 = 7.90 \text{ MPa}$$

For $\gamma = 0.60$ $$\frac{\phi P_{ny}}{A_g} = 10.6 \text{ MPa}$$

For $\gamma = 0.75$ $$\frac{\phi P_{ny}}{A_g} = 10.75 \text{ MPa}$$

$$\frac{\phi P_{ny}}{A_g} = 10.72 \text{ MPa for } \gamma = 0.72 \Rightarrow \phi P_{\text{ny}} = 2171\ kN$$

The point to calculate ϕP_{no} is located on the diagram where the interaction curve for $\rho = 0.02$ intersects the vertical load line.

$$\frac{\phi P_{no}}{A_g} = 14.70 \text{ MPa} \quad \Rightarrow \quad \phi P_{\text{no}} = 2977\ kN$$

As the loads ϕP_{nx} and ϕP_{ny} are quite closer to ϕP_{no}, ϕ-factor of 0.65 seems reasonable for all the loads.

$$\frac{1}{\phi P_{ni}} = \frac{1}{\phi P_{nx}} + \frac{1}{\phi P_{ny}} - \frac{1}{\phi P_o}$$

$$= \frac{1}{2049} + \frac{1}{2171} - \frac{1}{2977}$$

$$\phi P_{\mathrm{ni}} = 1632\ kN > P_{\mathrm{u}} = 1600\ kN$$

Design is OK according to the Reciprocal Load Method.

Example 14.6: Check the adequacy of a rectangular tied column X-section of size 300 × 450 mm to resist P_{u} = 1000 kN acting at e_{x} = 125 mm and e_{y} = 50 mm, as shown in Fig. 14.28. f_{c}' = 25 MPa, f_{y} = 420 MPa, and cover to centroid of bars = 60 mm. The reinforcement is arranged around the perimeter of the column consisting of 8 – #25 bars.

Use the following two methods:

a) Reciprocal load method
b) Load contour method.

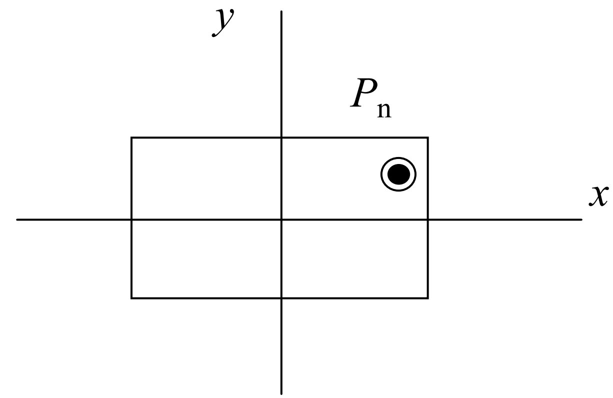


Fig. 14.28.Column For Example 14.28.

Solution:

$$\rho_t = 4080 / (300 \times 450) = 0.03$$

a) Reciprocal Load Method

i) Considering bending about y-axis:

$$\gamma = 330 / 450 = 0.73 \cong 0.75$$

$$\frac{P_u}{A_g} = 1000 \times 1000 / (300 \times 450) = 7.41 \text{ MPa}$$

$$\frac{M_u}{A_g h} = \frac{P_u \times e_x}{A_g h}$$

$$= 1000 \times 1000 \times 125 \,/\, (300 \times 450^2) \;=\; 2.06 \text{ MPa}$$

Join the point $[P_u/A_g,\, M_u/(A_g h)]$ with the origin and extend to $\rho = 0.03$ to get the following:

$$\frac{\phi P_{nx}}{A_g} = 10.8 \text{ MPa} \qquad \Rightarrow \qquad \phi P_{nx} = 1458\ kN$$

ii) Considering bending about x-axis:

$\gamma \quad = 180 / 300 = 0.60$

$$\frac{P_u}{A_g} = 1000 \times 1000 \,/\, (300 \times 450) = 7.41 \text{ MPa}$$

$$\frac{M_u}{A_g h} = \frac{P_u \times e_y}{A_g h}$$

$$= 1000 \times 1000 \times 50 / (450 \times 300^2) = 1.23 \text{ MPa}$$

Join the point $[P_u/A_g, M_u/(A_g h)]$ with the origin and extend to $\rho = 0.03$ to get the following:

$$\frac{\phi P_{ny}}{A_g} = 13.6 \text{ MPa} \qquad \Rightarrow \qquad \phi P_{ny} = 1836\ kN$$

iii) No eccentricity case:

$$\phi P_{no} = 21.4 \times 300 \times 450 = 2889\ kN$$

Assuming same ϕ-factors for all the loads, we have,

$$\frac{1}{\phi P_{ni}} = \frac{1}{\phi P_{nx}} + \frac{1}{\phi P_{ny}} - \frac{1}{\phi P_o}$$

$$= \frac{1}{1458} + \frac{1}{1836} - \frac{1}{2889}$$

$$\phi P_{\mathrm{ni}} = \mathit{1131\ kN} > P_{\mathrm{u}} = \mathit{1000\ kN} \qquad \textbf{(\textit{OK})}$$

a) Load Contour Method

i) Considering bending about x-axis:

$$\gamma = 180 / 300 = 0.60$$

$$\frac{P_u}{A_g} = 1000 \times 1000 / (300 \times 450) = 7.41 \text{ MPa}$$

$$\rho_t = 0.03$$

$$\frac{\phi M_{nx}}{A_g h} = 2.92 \text{ MPa} \qquad \Rightarrow$$

$$\phi M_{\text{nxo}} = 2.92 \times 450 \times 300^2 / 10^6 = 118.26 \; kN\text{-}m$$

ii) Considering bending about y-axis:

$$\gamma = 320 / 450 = 0.71 \cong 0.75$$

$$\frac{P_u}{A_g} = 1000 \times 1000 / (300 \times 450) = 7.41 \; MPa$$

$$\rho_t = 0.03$$

$$\frac{\phi M_{ny}}{A_g h} = 3.47 \text{ MPa} \qquad \Rightarrow$$

$$\phi M_{\text{nyo}} = 3.47 \times 300 \times 450^2 / 10^6 = 210.80 \; kN\text{-}m$$

$$M_{ux} = P_u \times e_y = 1000 \times 50 / 1000 = 50 \text{ kN-m}$$

$$M_{uy} = P_u \times e_x = 1000 \times 125 / 1000 = 125 \text{ kN-m}$$

Selecting the conservative value of $\alpha = 1.15$, we have,

$$\left(\frac{M_{nx}}{M_{nxo}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{nyo}}\right)^{\alpha} = \left(\frac{M_{ux}}{\phi M_{nxo}}\right)^{\alpha} + \left(\frac{M_{uy}}{\phi M_{nyo}}\right)^{\alpha}$$

$$= \left(\frac{50.00}{118.26}\right)^{1.15} + \left(\frac{125.00}{210.80}\right)^{1.15}$$

$$= 0.92 < 1.0 \qquad (\boldsymbol{OK})$$

Hence the column is safe according to the load contour method. This method is relatively more exact for portion of curve below the balanced condition.

SLENDER COLUMNS

There are three methods of design of slender columns described below:

1. Perform exact $P - \Delta$ analysis to find P_u and $M_{u,max}$ and then use the standard interaction diagram for the short columns. This method is called the second order analysis of frames.

2. Find P_u and approximately magnified moment (M_u) and then use the standard interaction diagram for short columns. This method is called ***Moment Magnification Method***. In this method, the moments obtained from the first order analysis are multiplied with an empirical moment magnifier.

3. Modify the interaction diagram to account for the slenderness effects, having the coordinates P_u and unmagnified moment M_u just like a regular diagram.

Elastic Buckling Load For Concentrically Loaded Columns

$$P_c = \frac{\pi^2 E_t I}{(k\ell_u)^2} = \frac{\pi^2 E_t A}{(k\ell_u / r)^2}$$

Concrete is an inelastic material and hence the modulus of elasticity varies all along the stress-strain curve, as shown in Fig. 14.29.

By replacing the E-value with the tangent modulus of elasticity (E_t), Euler's formula may be used for materials like concrete.

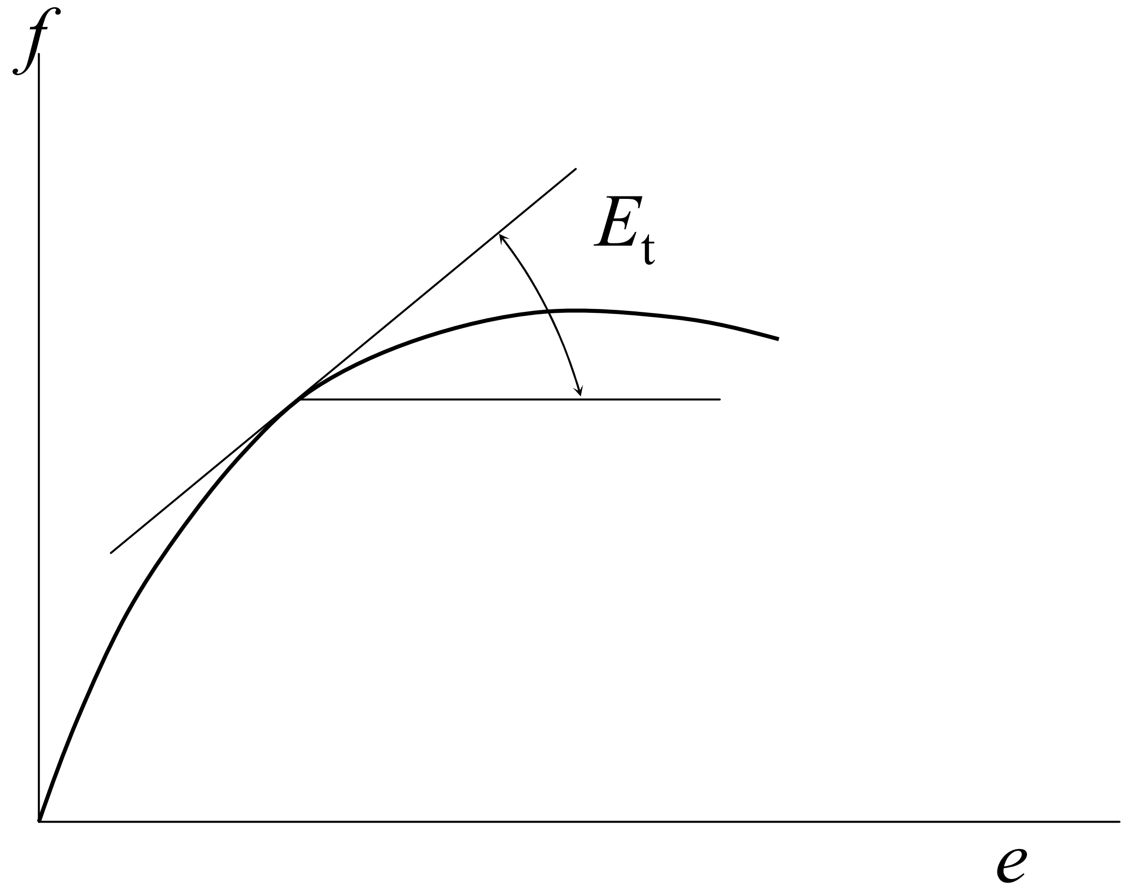


Fig. 14.29.Modulus of Elasticity For Concrete.

The tangent modulus of elasticity is different at all points of the stress-strain curve and is difficult to estimate precisely.

Hence approximate methods are used to calculate the effective *E*-value along with the reduced moment of inertia due to cracking and long-term effects.

A typical load – slenderness ratio curve is shown in Fig.14.30 to represent the buckling behavior.

For concrete columns, the chances of elastic buckling are usually very less and only moment magnification and at the most inelastic buckling are the important parameters.

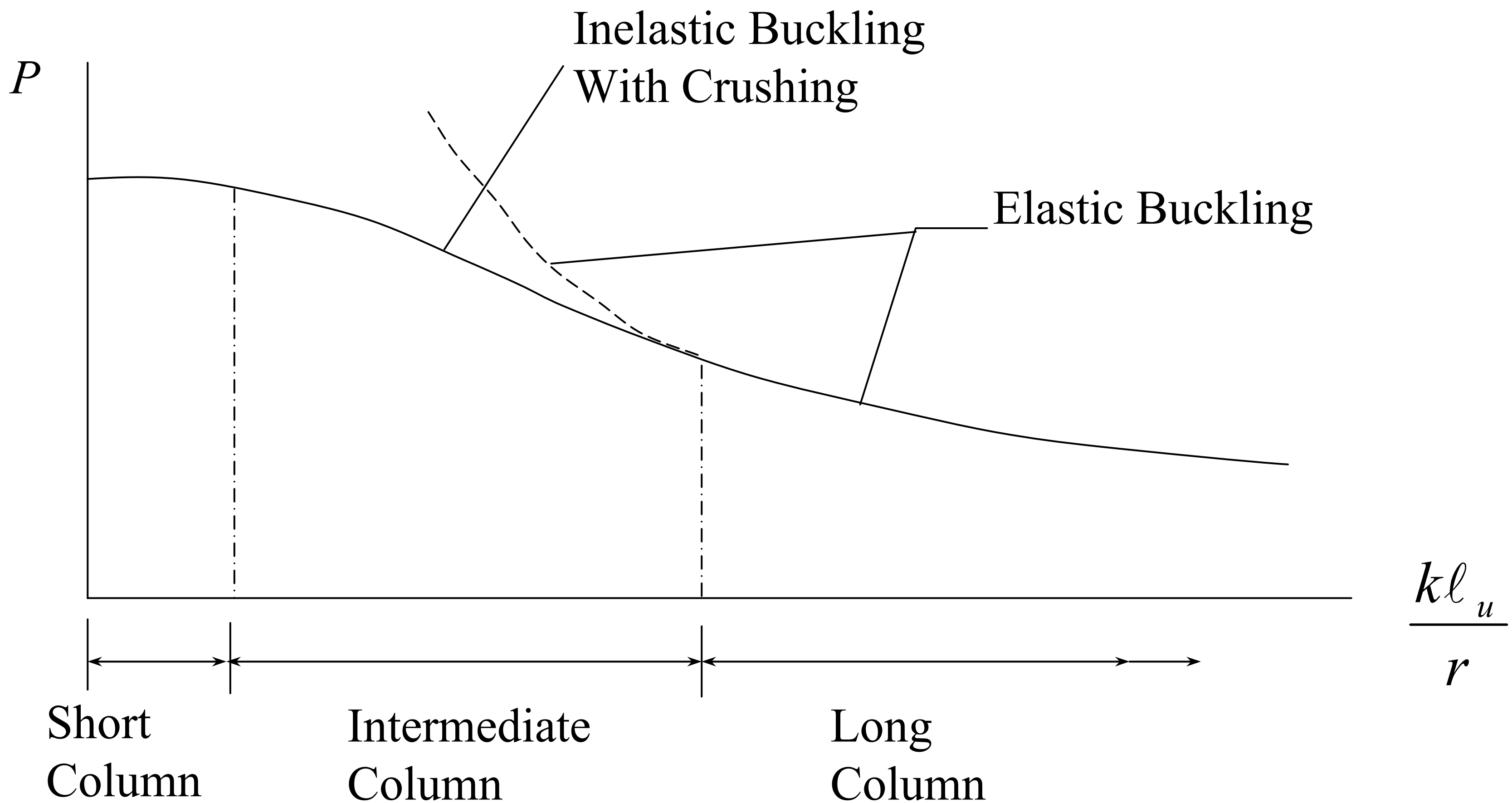


Fig. 14.30. Buckling Load Versus Slenderness Ratio For Columns.

Effective Length Factor

This factor gives the ratio of length of half sine wave portion of defected shape after buckling (distance between two points of contra-flexure) to full-unsupported length of column.

This depends upon the end conditions of the column and the fact that whether side-sway is permitted or not. Greater the *k*-value, greater is the effective length and slenderness ratio and hence smaller is the buckling load.

The value of *k*-factor in case of no side-sway is between 0.5 and 1.0 whereas, in case of appreciable side-sway, it is always greater than or equal to 1.0.

Any appreciable lateral or sideward movement of top of a vertical column relative to its bottom is called side-sway, sway or lateral drift.

If side-sway is possible, *k*-value increases by a greater degree and column buckles at a lesser load.

Side-sway in a frame takes place due to the following factors shown in Fig. 24.31:-

1. Lengths of different columns are unequal.
2. Sections of columns have different cross-sectional properties.
3. Loads are un-symmetrical.
4. Lateral loads are acting.

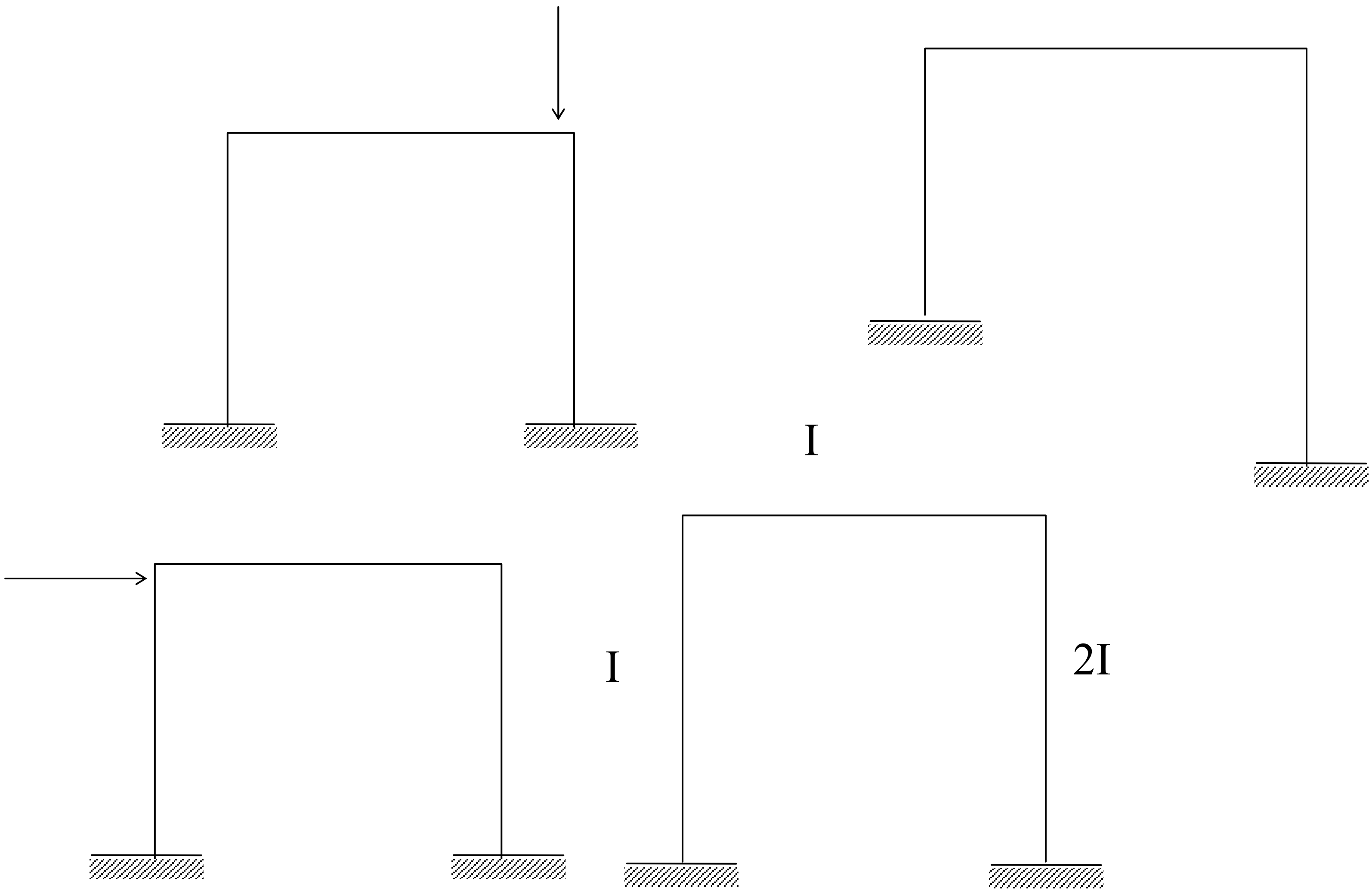
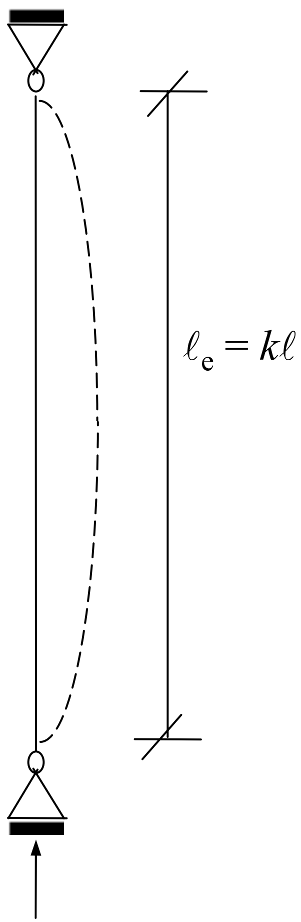


Fig. 14.31. Chances of Side-sway.

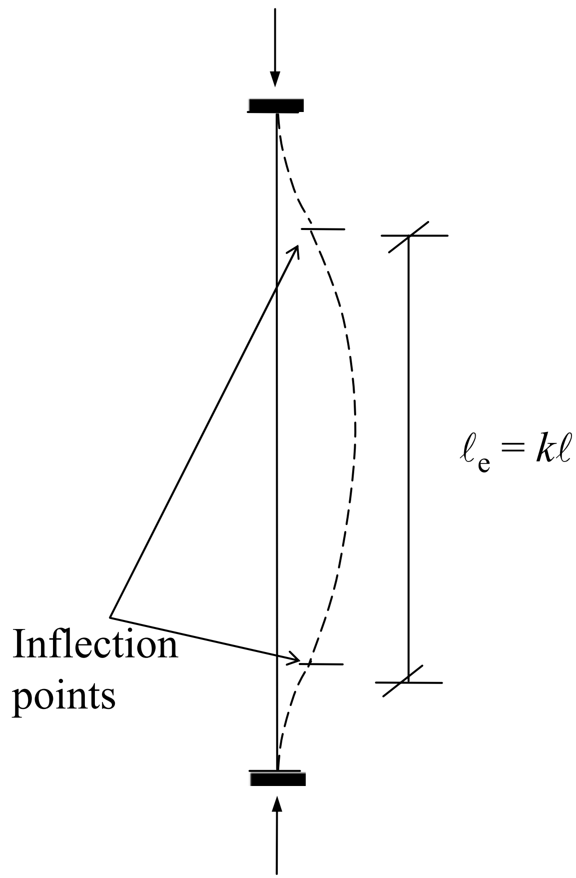
Side-sway can be prevented in a frame by:-

1. Providing shear or partition walls.
2. Fixing the top of frame with adjoining rigid structures.
3. Provision of properly designed lift well in a building, which may act like backbone of the structure reducing the lateral deflections.
4. Provision of lateral bracing, which may be of following two types:
    A. Diagonal bracing.
    B. Longitudinal bracing.

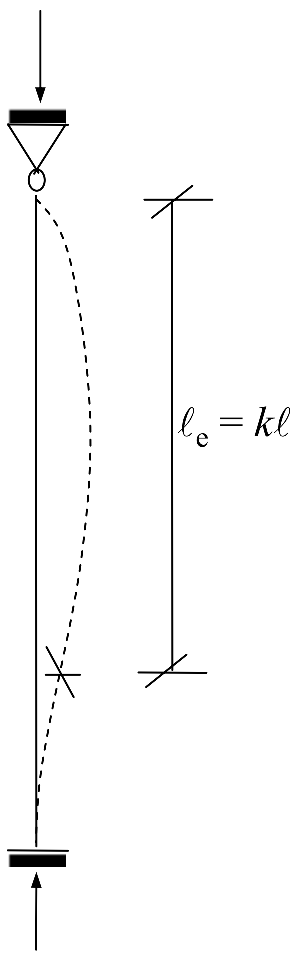
Effective length factor and the buckled shape of columns having well-defined end conditions are given in Fig. 14.32.



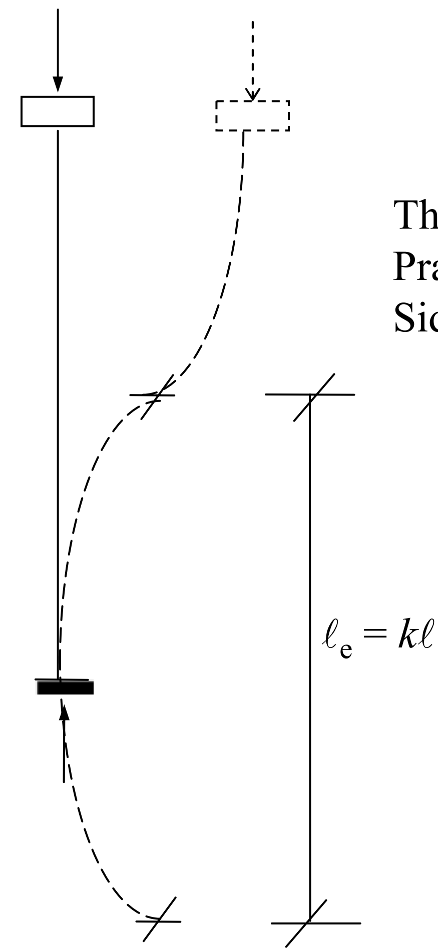
Theoretical k = 1.0
Practical k = 1.0
No side-sway



Theoretical k = 0.5
Practical k = 0.65
No side-sway



Theoretical k = 0.7
Practical k = 0.8
No side-sway



Theoretical k =1.0
Practical k = 1.2
Side-sway present

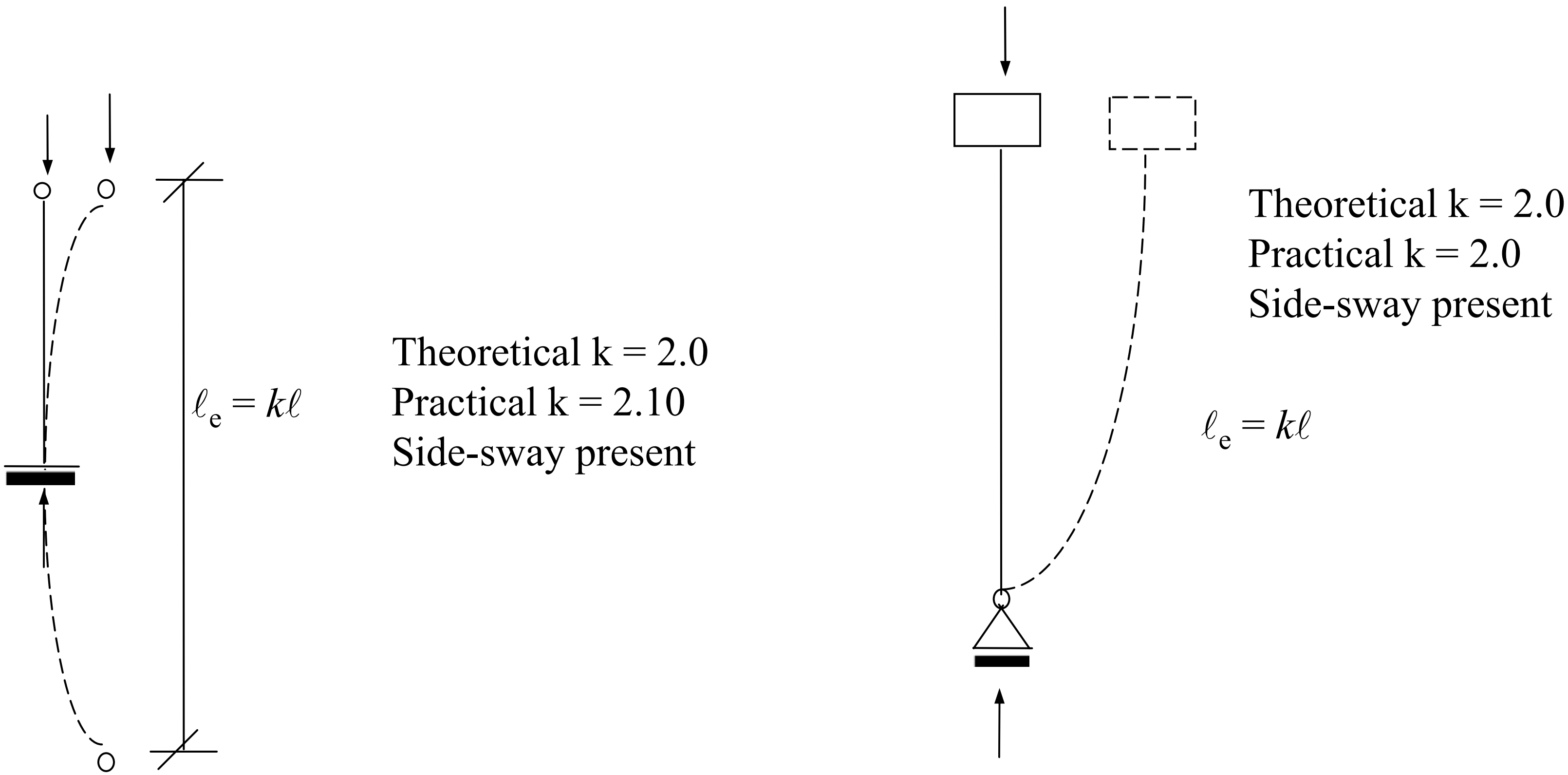


Fig. 14.32. Values of k-factor For Various End Conditions.

Consider the example of column AB shown in Fig. 14.33.

The ends are not free to rotate.

However, these ends are also not perfectly fixed.

Instead the ends are partially fixed with the fixity determined by the ratio of relative flexural stiffness of columns meeting at a joint to the flexural stiffness of beams meeting at the joint.

This ratio is denoted by ψ or G using the expression given below:-

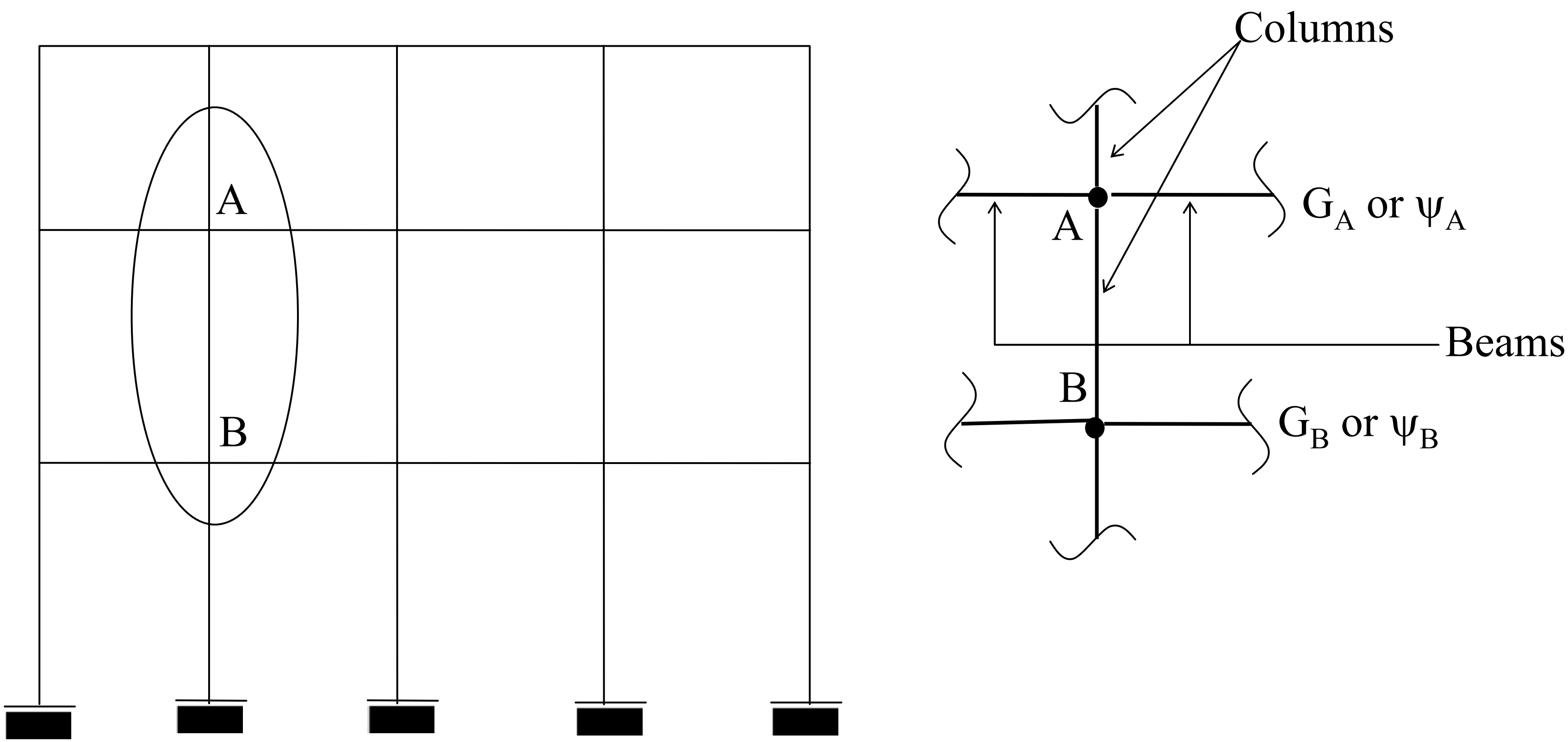


Fig. 14.33. Partially Fixed Column Ends.

$$\psi \text{ or } G \text{ at each end} = \frac{\sum\left(\mathrm{EI}/\ell\right) \text{ of columns}}{\sum\left(\mathrm{EI}/\ell\right) \text{ of beams}}$$

This value is calculated at both ends of the columns, denoted by points A and B, and summation is taken for all members meeting at a particular end.

The lower columns of Fig. 14.33 have their top ends partially fixed but the bottom ends may have well-defined end conditions.

The ψ or G value at these ends are decided as follows:

Hinged Support

G or Ψ = 10.0 for braced columns

= 20.0 for sway columns

Fixed Support

G or Ψ = 0.5 for braced columns

= 1.0 for sway columns

To find out the k-value, the alignment charts given in Figs. 14.34 and 14.35 are used. The alignment chart of Fig. 14.34 is for columns without any side-sway and the alignment chart of Fig. 14.35 is for columns having appreciable side-sway.

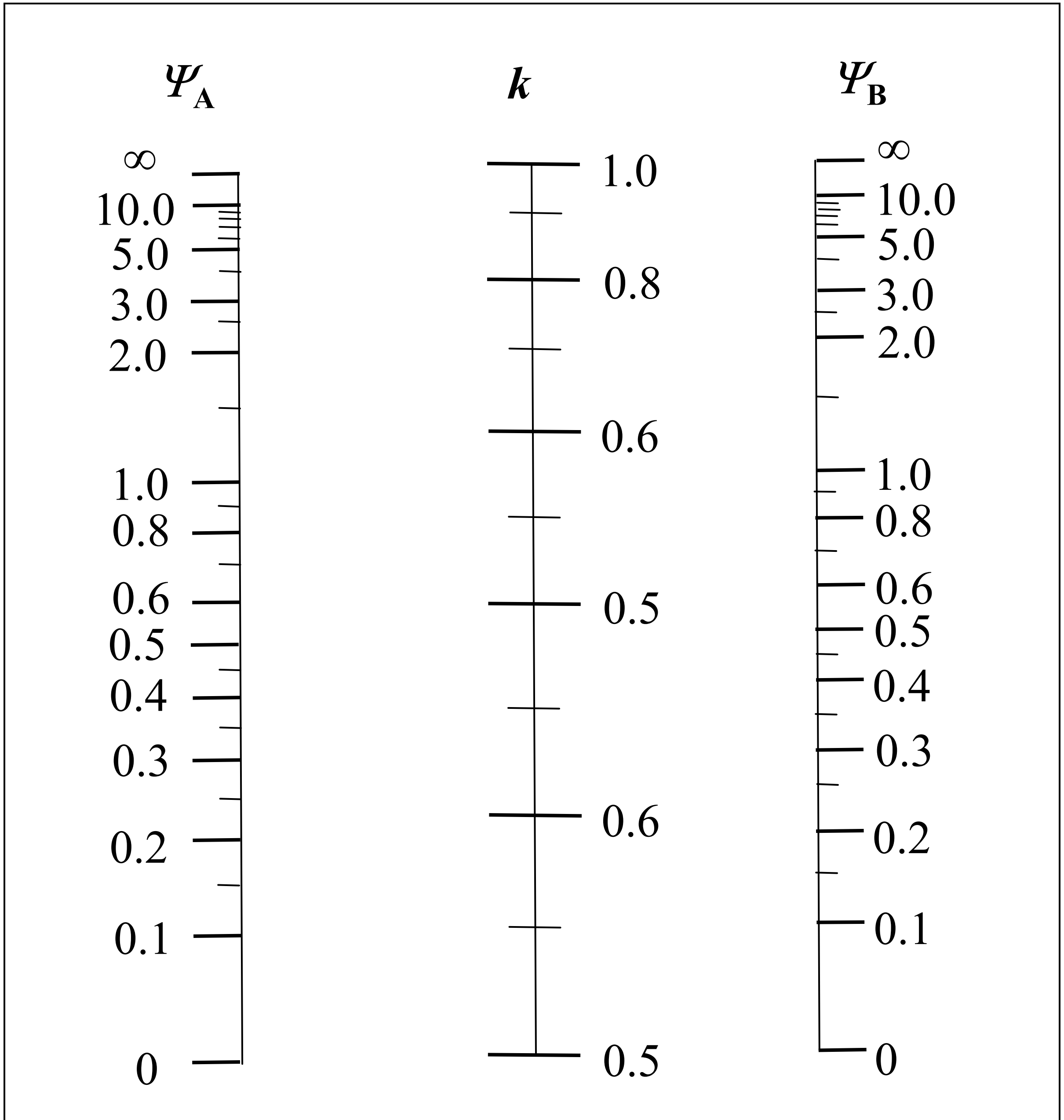


Fig. 14.34. Effective Length Factor For Braced Columns.

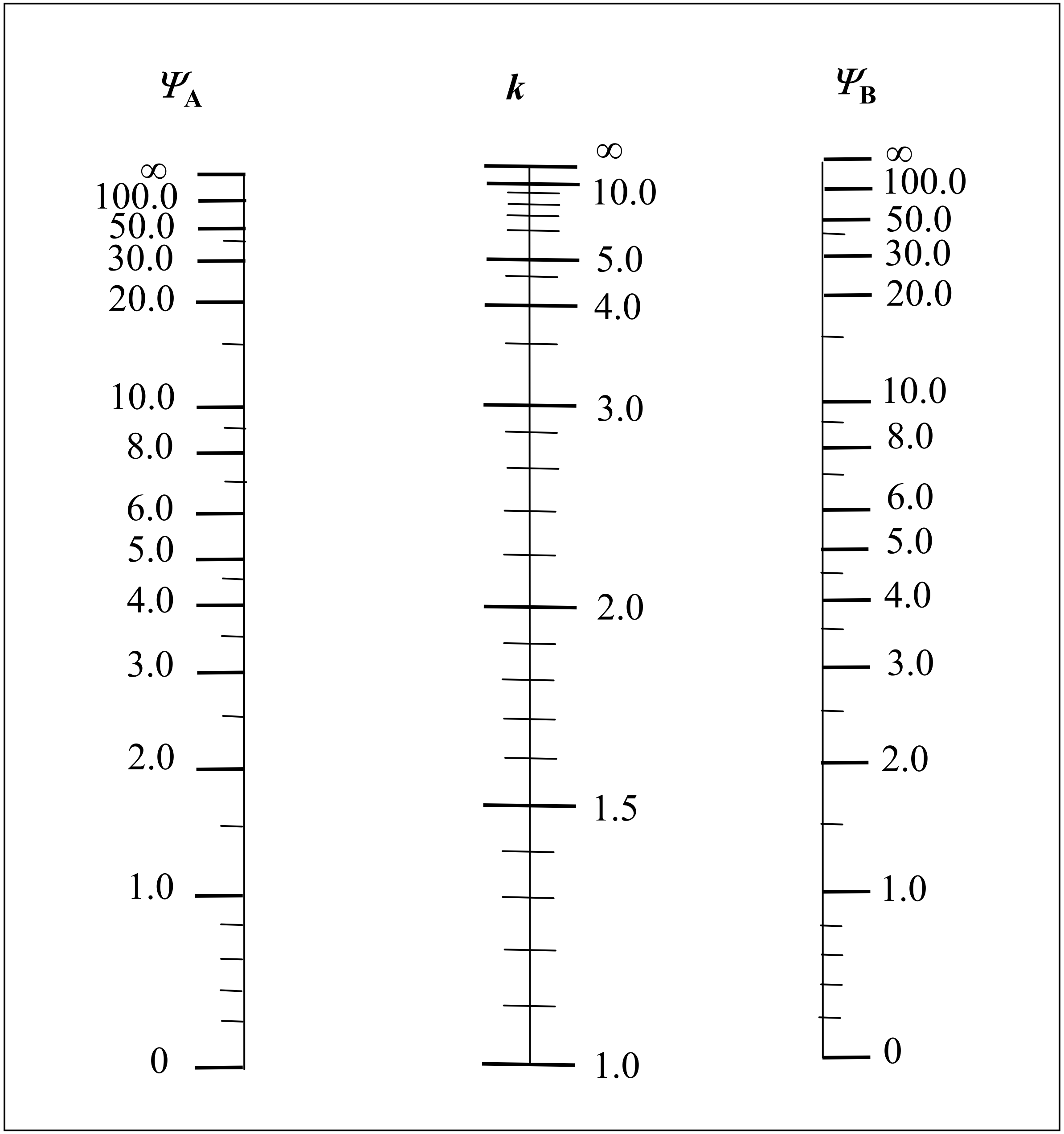


Fig. 14.35. Effective Length Factor For Unbraced Columns.

Restraint Provided By Footings For Calculation Of Ψ Factor

In case of footing resting on soil, the Ψ value is calculated by using the foundation stiffness (K_f) in place of the beam stiffness in the usual expression as under:

$$\psi \quad = \quad \frac{\sum(EI/\ell)\,\text{of columns}}{\sum(EI/\ell)\,\text{of footing}} \quad = \quad \frac{K_c}{K_f}$$

To calculate the stiffness of the footing, a moment M is applied to the footing of Fig. 14.36 and the corresponding rotation (θ_f), settlement at extreme compression end due to moment alone leaving the uniform downward settlement (Δ) and the contact stress under the footing due to the rotation (σ) are observed.

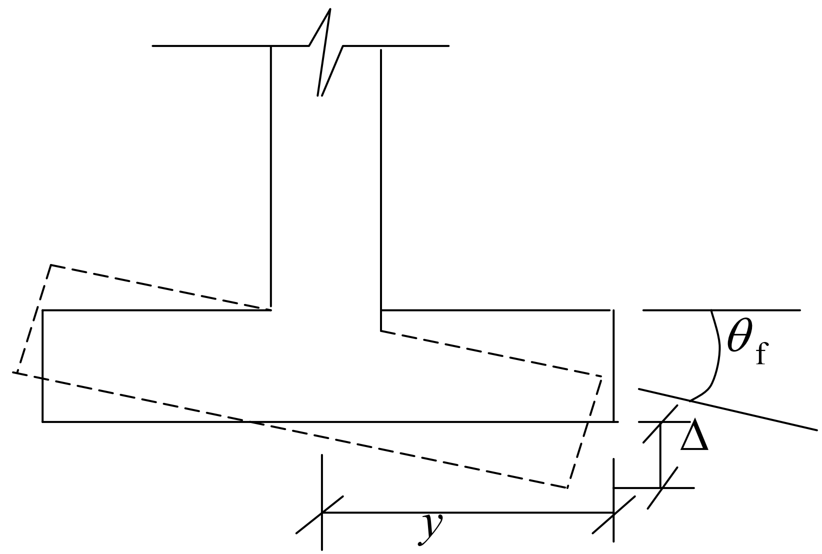


Fig. 14.36. Rotation and Stiffness of a Foundation.

The extreme edge contact stress may be calculated as follows:

$$\sigma \quad = M / S_f = M y / I_f$$

where S_f = Section modulus of the contact surface

and I_f = Moment of inertia of the contact surface.

$$\theta_f = \frac{\Delta}{y} = \frac{\sigma / K_s}{y} = \frac{M}{I_f K_s}$$

where K_s is the modulus of sub-grade reaction (pressure corresponding to 1mm settlement), which may approximately be found from Table 14.1.

Table 14.1. Soil Sub-grade Reaction.

Allowable Soil Bearing Capacity	Sub-grade Modulus
kN/m^2 or kPa	$N/mm^2/mm$
60	0.0136
100	0.0272
180	0.0450
240	0.0504
360	0.0654
480	0.0736

$$\therefore \quad K_f = \frac{M}{\theta_f} = I_f K_s$$

The value of Ψ at a footing to column joint is estimated as follows:

$$\psi = \frac{K_c}{K_f} = \frac{4E_c I_c / \ell_c}{I_f K_s}$$

After calculation of Ψ factors at both ends of the column, the value of k may also be calculated from the following ACI equations:

For braced members, k should be the lesser of the following two values:

$$k = 0.7 + 0.05\,(\Psi_A + \Psi_B) \le 1.0$$

$$k = 0.85 + 0.05\,\Psi_{min} \le 1.0$$

where Ψ_{min} = smaller of Ψ_A and Ψ_B

For unbraced compression members, k may be evaluated as follows:

For $\Psi_m = \dfrac{\psi_A + \psi_B}{2} < 2$, $k = \dfrac{20 + \psi_m}{20}\sqrt{1 + \psi_m}$

For unbraced compression members hinged at one end and partially fixed at the other end, *k*-value is obtained as follows:

$$k = 2.0 + 0.3\,\Psi$$