Structural Dynamics

(Multi storey shear building)

Multistorey shear building-General

- In Part I we analyzed and obtained the dynamic response for structures modeled as a single degree-of-freedom system.
- Only if the structure can assume a unique shape during its motion will the single degree model provide the exact dynamic response.
- Otherwise, when the structure takes more than one possible shape during motion, the solution obtained from a single degree model will be an approximation to the true dynamic behavior.
- Structures cannot always be described by a single degree model, and, in general, have to be represented by multiple degree models.
- In fact, structures are continuous systems and as such possess an intlnite number ot' degrees of freedom.

Multistorey shear building-General

- There are analytical methods to describe the dynamic behavior of continuous structures which have uniform material properties and regular geometry.
- These methods of analysis, though interesting in revealing information for the discrete modeling of structures, are rather complex and are applicable only to relatively simple actual structures.
- They require considerable mathematical analysis, including the solution of partial differential equations which will be presented in next section.
- For the present, we shall consider one of the most instructive and practical types of structure which involve many degrees of freedom, the multistory shear building.

Multistorey shear building-Stiffness Equation

- A shear building may be defined as a structure in which there is no rotation of a horizontal section at the level of the floors.
- In this respect, the deflected building will have many of the features of a cantilever beam that is deflected by shear forces only; hence, the name shear building.
- To accomplish such deflection in a building, we must assume that:
- (1) the total mass of the structure is concentrated at the levels of the floors.
- (2) the girders on the floors are infinitely rigid as compared to the columns.
- (3) the deformation of the structure is independent of the axial forces present in the columns.

Multistorey shear building-Stiffness Equation

- The first assumption transforms the problem from a structure with an infinite number of' degrees of freedom (due to the distributed mass) to a structure which has only as many degrees as it has lumped masses at the floor levels.

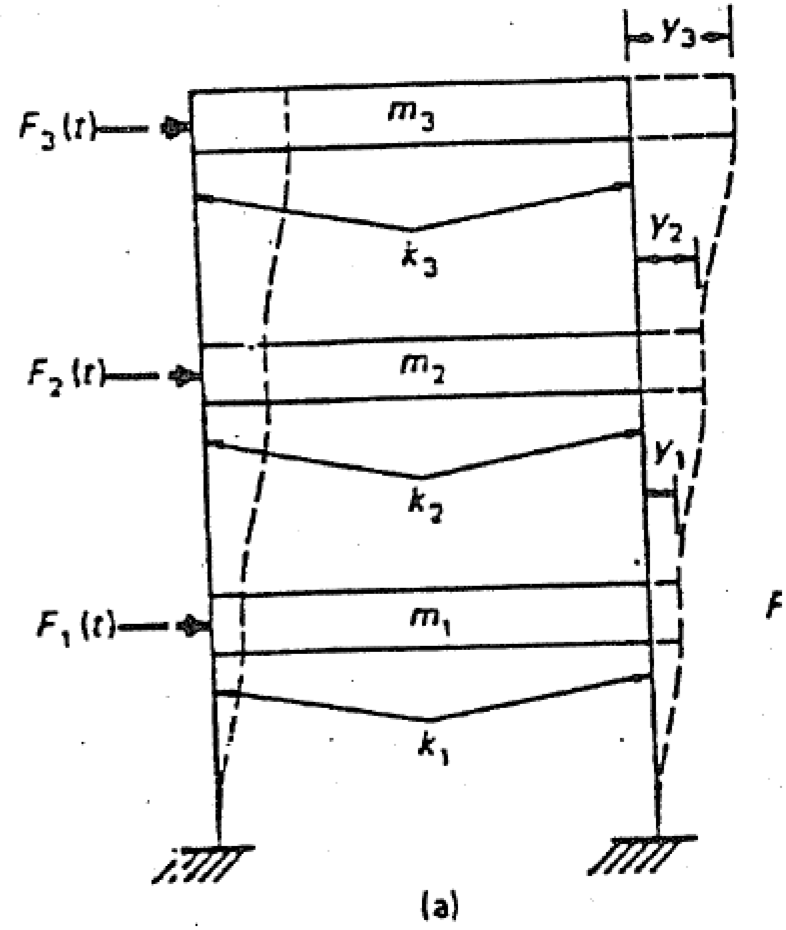
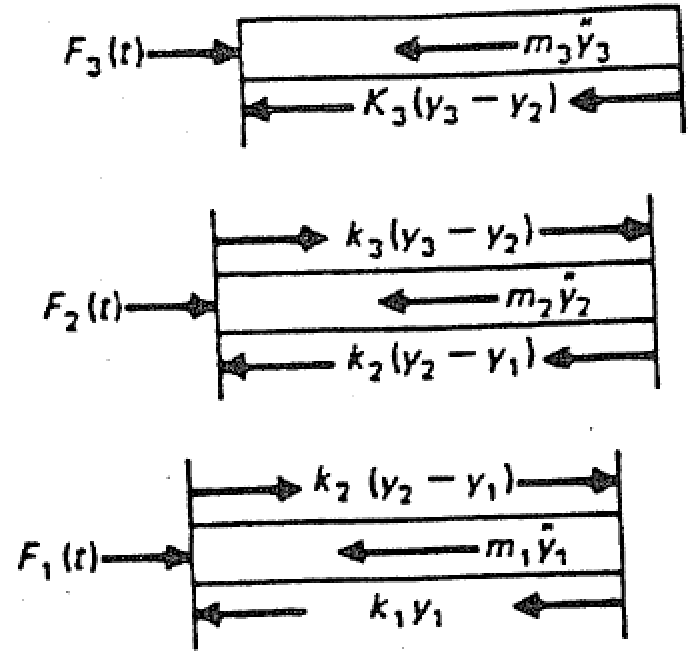


Fig. 9.1 Single bay model representation of a shear building.

Multistorey shear building-Stiffness Equation

- A three-story structure modeled as a shear building [Fig 9.1(a)] will have three degrees of freedom, that is, the three horizontal displacements at the floor levels.
- The second assumption introduces the requirement that the joints between girders and columns are fixed against rotation.
- The third assumption leads to the condition that the rigid girders will remain horizontal during motion.



(b)

Fig. 9.1 Single bay model representation of a shear building.

Multistorey shear building-Stiffness Equation

- It should be noted that the building may have any number of bays and that it is only as a matter of convenience that we represent the shear building solely in terms of a single bay.
- Actually, we can further idealize the shear building as a single column [Fig 9.2(a)] having concentrated masses at the floor levels with the understanding that only horizontal displacements of these masses are possible.

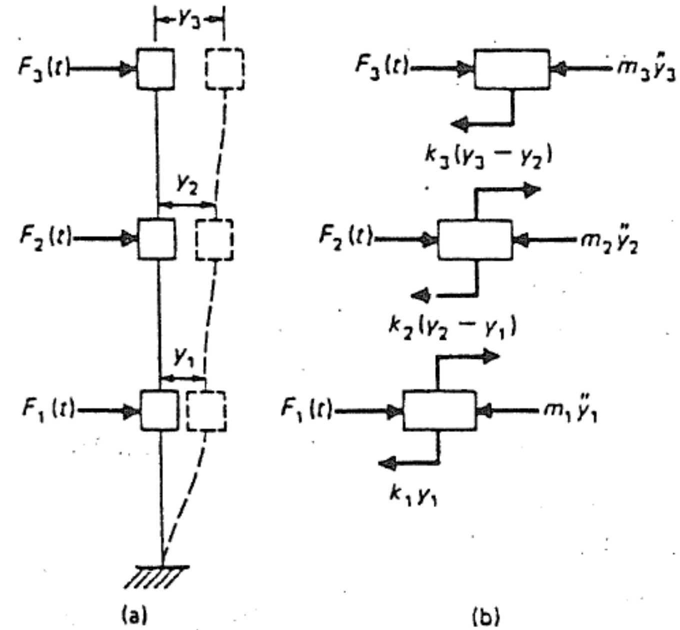


Fig. 9.2 Single column model representation of shear building.

Multistorey shear building-Stiffness Equation

- Another alternative is to adopt a multi-mass spring system shown in Fig. 9.3(a) to represent the shear building.
- In any of the three representations depicted in these figures, the stiffness coefficient or spring constant k_i shown between any two consecutive masses is the force required to produce relative unit displacement of the two adjacent floor levels.

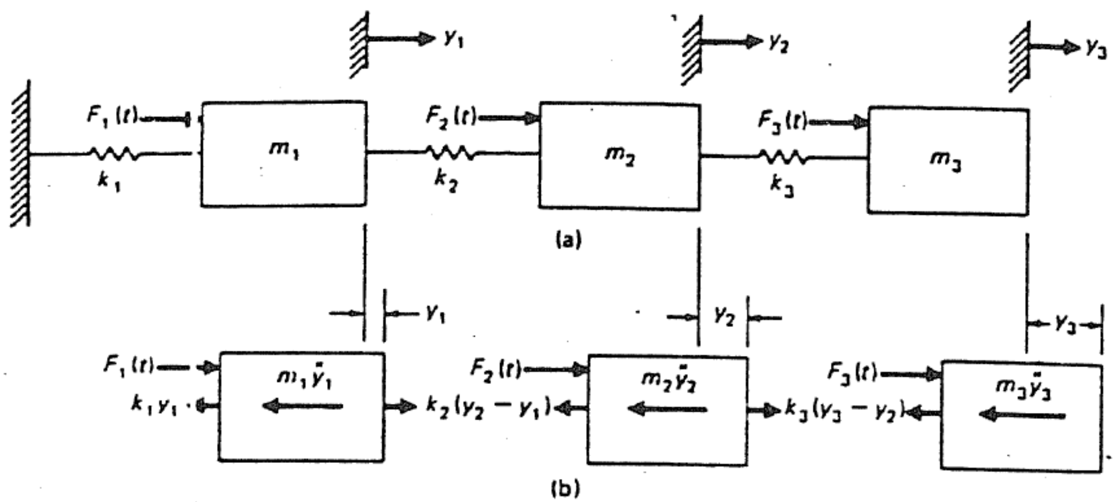


Fig. 9.3 Multimass spring model representation of a shear building.

Multistorey shear building-Stiffness Equation

- For a uniform column with the two ends fixed against rotation, the spring constant is given by $k = \frac{12EI}{L^3}$
- and for a column with one end fixed and the other pinned by $k = \frac{3EI}{L^3}$
- where E is the material modulus of elasticity, I the cross-sectional moment of inertia, and L the height of the story.
- It should_ be clear that all three representations shown in Figs. 9.1 to 9.3 for the shear building are equivalent.
- Consequently, the following equations of motion for the three-story shear building are obtained from any of the corresponding free body diagrams shown in these figures by equating to zero the sum of the forces acting on each mass.

Multistorey shear building-Stiffness Equation

- Hence,

$$m_1\ddot{y}_1 + k_1 y_1 - k_2(y_2 - y_1) - F_1(t) = 0,$$
$$m_2\ddot{y}_2 + k_2(y_2 - y_1) - k_3(y_3 - y_2) - F_2(t) = 0,$$
$$m_3\ddot{y}_3 + k_3(y_3 - y_2) - F_3(t) = 0. \qquad (9.2)$$

- This system of equations constitutes the stiffness formulation of the equations of motion for a three story shear building.
- It may conveniently be written in matrix notation as

$$[M]\{\ddot{y}\} + [K]\{y\} = [F], \qquad (9.3)$$

- where [M] and [K] are, respectively, the mass and stiffness matrices given, respectively, by

Multistorey shear building-Stiffness Equation

$$[M] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \tag{9.4}$$

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \tag{9.5}$$

and $\{y\}$, $\{\ddot{y}\}$, and $\{F\}$ are, respectively, the displacement, acceleration, and force vectors given by

$$\{y\} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \qquad \{\ddot{y}\} = \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{bmatrix}, \qquad \{F\} = \begin{bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{bmatrix}. \tag{9.6}$$

Multi storey shear building-Stiffness Equation

- It should be noted that the mass matrix, eq; (9.4), corresponding to the shear building is a diagonal matrix (the nonzero elements are only in the main diagonal).
- The elements of the stiffness matrix, eq. (9.5); are designated stiffness coefficients.
- In general, the stiffness coefficient kij is defined as the force at coordinate i when a. unit displacement is given at j, all other coordinates being fixed.
- For example the coefficient in the second row and second column of eq. (9.5) $k_{22} = k_2 + k_3$ is the force required at the second floor when a unit displacement is given to the floor.