Plain & Reinforced Concrete-II

CE-413

Footings

By Dr. Usman Akmal

ALLOWABLE DISTORTION

The value of distortion normally allowed in design is 1/3300 for $l/h \leq 3.0$ in sandy soils, 1/2500 for $l/h \leq 3.0$ in clay.

1/2000 for $l/h \geq 5.0$ in sandy soils, 1/1400 for $l/h \geq 5.0$ in clay.

where

$$l = width\ of\ the\ building$$
$$h = height\ of\ the\ building$$

Flow Chart for Design of Footings

Load

↓

q_a

↓

q_{na}

↓

$A_f)_{req}$

↓

q_n, q_u

↓

Mu, Vu → $\varphi_v V_c$ → d, h → $As)x, As)y$

↑

Joint Design

↑

Sketch

SHEAR

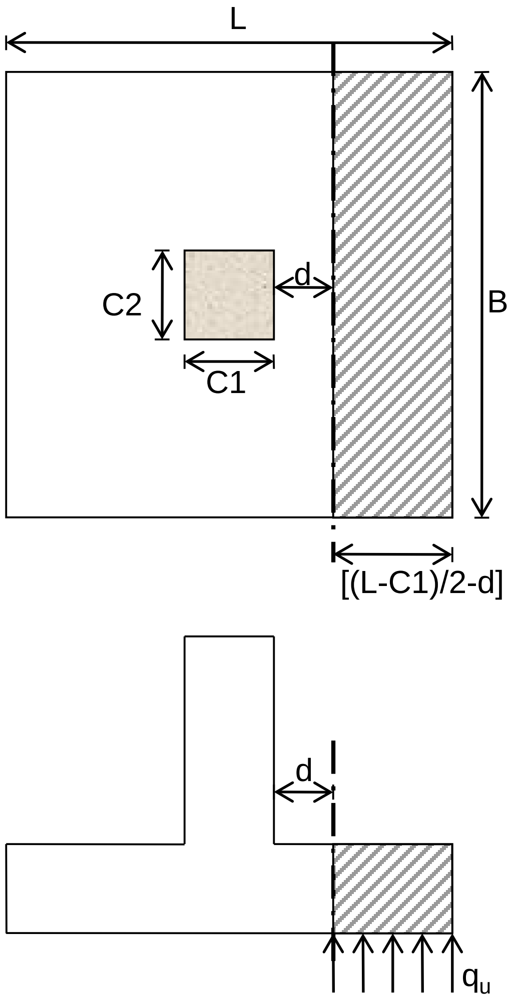
Footing may fail in shear as a wide beam or due to punching. These are referred as one-way shear and two-way shear respectively.

One-Way Shear or Beam Shear

Critical section for one-way shear is taken at distance “d” from the face of the support.

If shear is not satisfied no extra shear reinforcement is provided but the depth is increased.

ONE-WAY SHEAR OR BEAM SHEAR (CONT.)



One-way shear is only calculated in longer direction because it will be smaller along the width. Applied one-way shear, *Vu* is given as

$$Vu = q_{nu} B \left[\left(\frac{L - C1}{2} \right) - d \right]$$

One-way shear strength, Vc is given as

$$Vc = \frac{1}{6} \sqrt{f_c^{'}} Bd$$

Where

B = width of the footing

ONE-WAY SHEAR OR BEAM SHEAR (CONT.)

For a footing to be safe against one-way shear following condition should be satisfied

$$Vu \leq \phi_v V_c$$

Where $\Phi_v = 0.75$

If the above condition is not satisfied calculate revised d from following

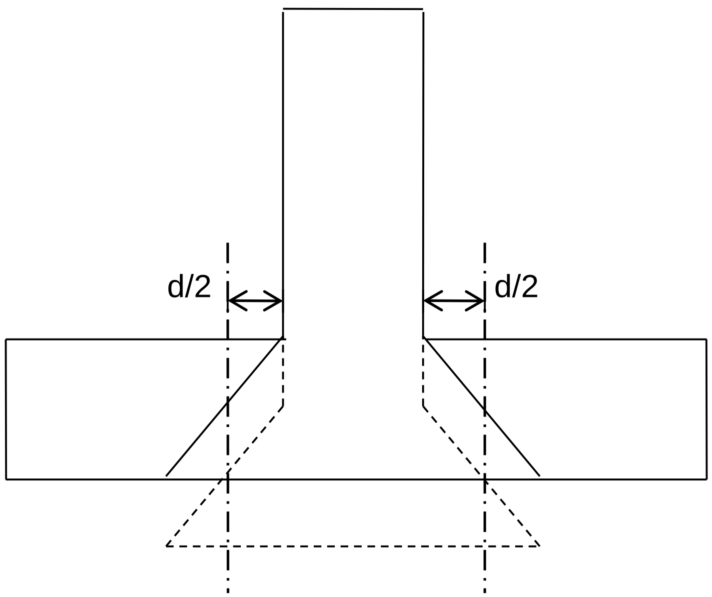
$$d = \frac{Vu}{\phi_v \frac{1}{6}\sqrt{f_c'}B}$$

TWO-WAY SHEAR OR PUNCHING SHEAR

It is a shear which acts all around the perimeter such that column punches down through foundation slab and starts settling.

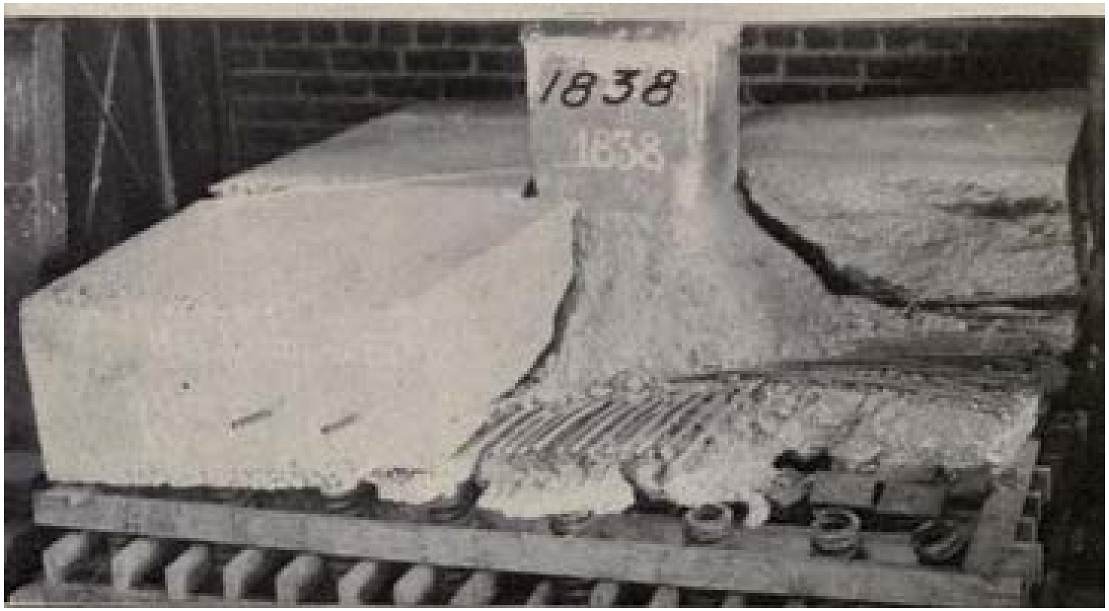
It is very high in magnitude and most critical aspect in design of column footing.

Punching shear normally controls the depth of the column footing.



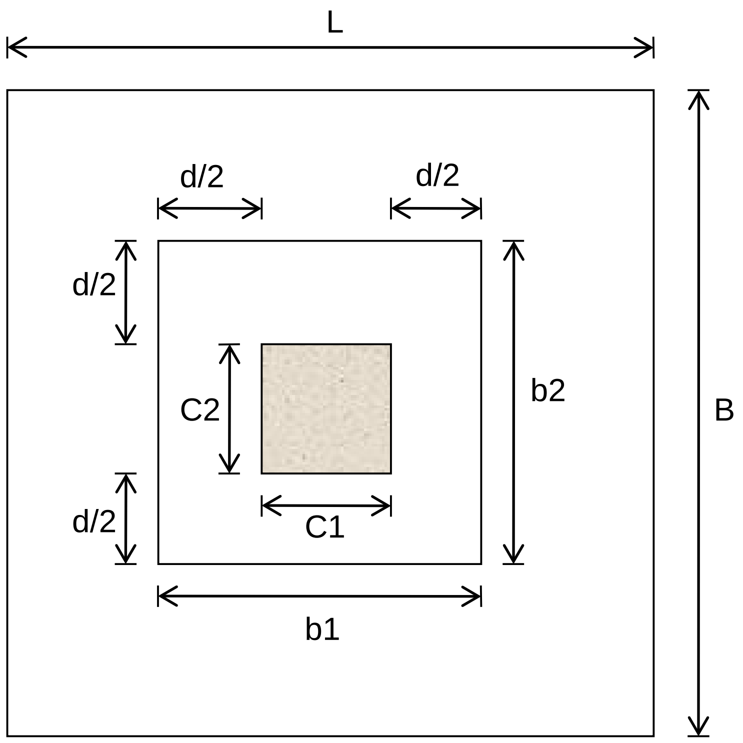
TWO-WAY SHEAR OR PUNCHING SHEAR (cont.)

Ref.:
[ARTHUR N. TALBOT, 1913]



TWO-WAY SHEAR OR PUNCHING SHEAR (cont.)

Critical section for two-way shear is taken at a distance ***"d/2"*** form the face of support in both directions.



Critical perimeter for two-way shear is denoted by ***"b_o"***.

Critical perimeter ***"b_o"*** is given as

b_o = 2(b1+b2)

b_o = 2[(C1+d)+(C2+d)]

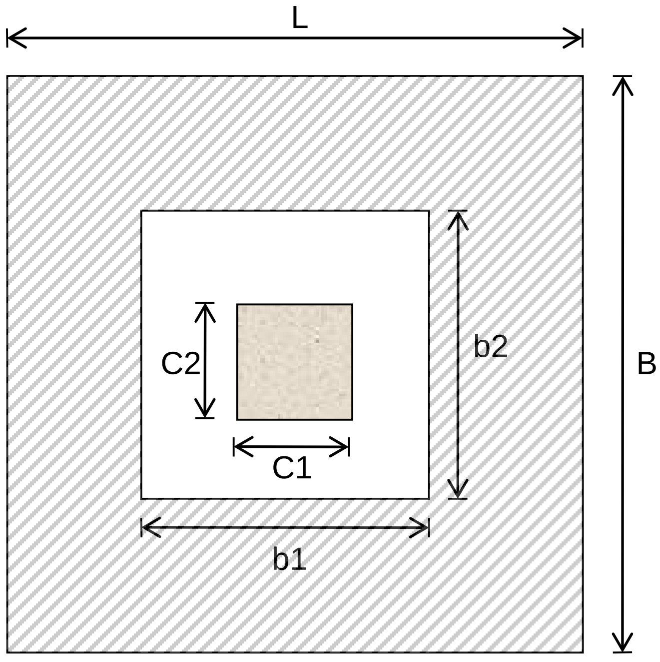
b_o = 2(C1+C2+2d)

TWO-WAY SHEAR OR PUNCHING SHEAR (cont.)

Applied two-way shear ***"Vu"*** is given as

$$Vu = q_{nu}(Area \text{ outside the critical perimeter})$$

$$Vu = q_{nu}(L \times B - b1 \times b2)$$



For a footing to be safe against two-way shear following condition should be satisfied

$$Vu \le \phi_v V_c$$

Where $\Phi_v = 0.75$

TWO-WAY SHEAR OR PUNCHING SHEAR (cont.)

Punching shear strength ***"Vc"*** is taken as

Lesser of following

1) $$Vc = \frac{1}{3}\sqrt{f_c^{'}}\,b_o d$$

Where b_o is critical perimeter

2) $$Vc = \frac{1}{12}\left[2 + \frac{4}{\beta_c}\right]\sqrt{f_c^{'}}\,b_o d$$

$$where\ \beta_c = \frac{\textit{longer}\ \text{col. side}}{\text{shorter col. side}}$$

3) $$Vc = \frac{1}{12}\left[\frac{\alpha_s d}{b_o} + 2\right]\sqrt{f_c^{'}}\,b_o d$$

Where

α_s = 40 for interior column

α_s = 30 for exterior column

α_s = 20 for corner column

Critical Section for Moment

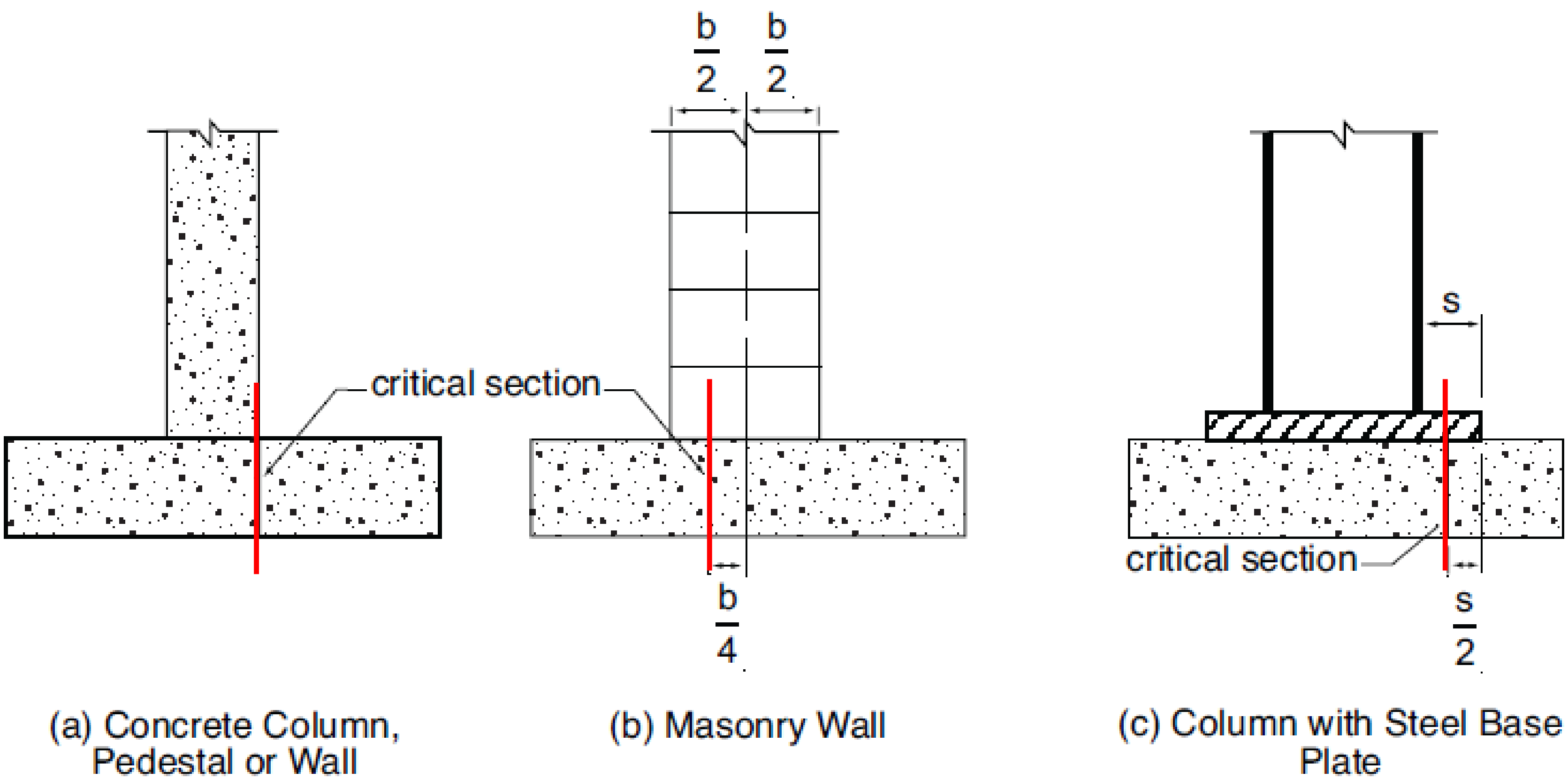
The critical sections for moment are taken as (ACI 15.3 and 15.4.2)

- For footings supporting square or rectangular concrete columns or walls, at the face of the column or wall.

- For footings supporting masonry walls, halfway between the middle and edge of the wall.

- For footing supporting a column with steel base plate, halfway between face of the column and edge of the base plate.

Critical Section for Moment (cont.)



[Ref.: PCA NOTES on ACI 318-08]

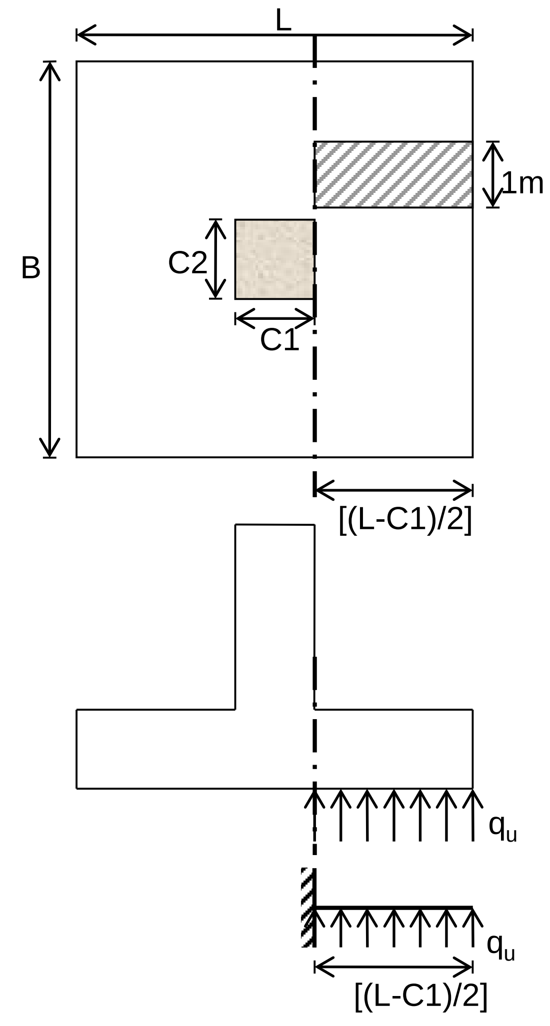
Design Moment Calculation

Design moment is calculated considering a unit strip of a cantilever fixed at column and free at edges in both directions.

Design moment along length for unit strip is given as

$$Mu = q_u\left(\frac{L-C1}{2}\right)\times\frac{1}{2}\left(\frac{L-C1}{2}\right)$$

$$Mu = q_u \times \frac{1}{8}(L-C1)^2$$

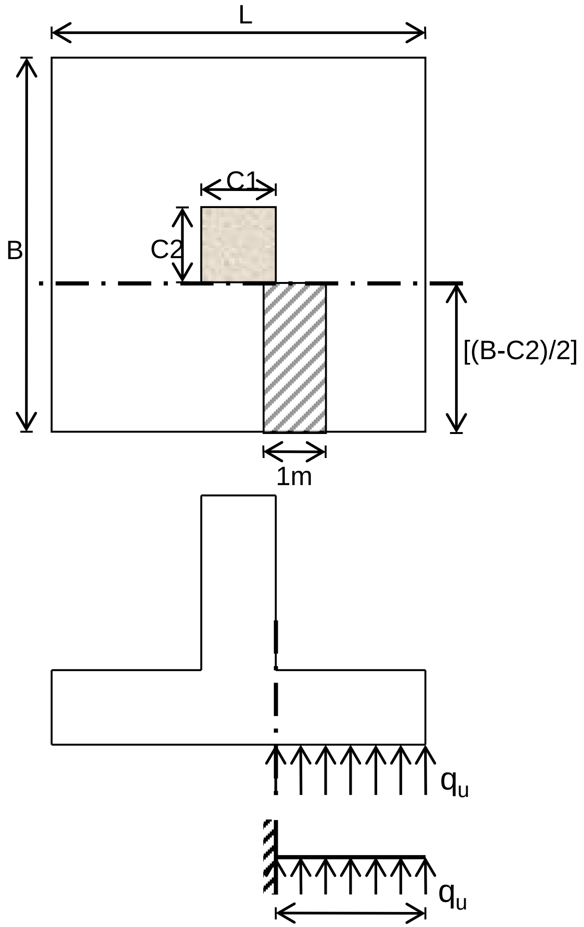


Design Moment Calculation (cont.)

Design moment along width for unit strip is given as

$$Mu = q_u\left(\frac{B-C2}{2}\right)\times\frac{1}{2}\left(\frac{B-C2}{2}\right)$$

$$Mu = q_u\times\frac{1}{8}\left(B-C2\right)^2$$



BENDING MOMENT AND STEEL REINFORCEMENT

Calculate area of steel corresponding to respective moments.

Minimum flexural reinforcement is given as

$As)_{min}$ = 0.002bh for f_y=300MPa

$As)_{min}$ = 0.0018bh for f_y=420MPa

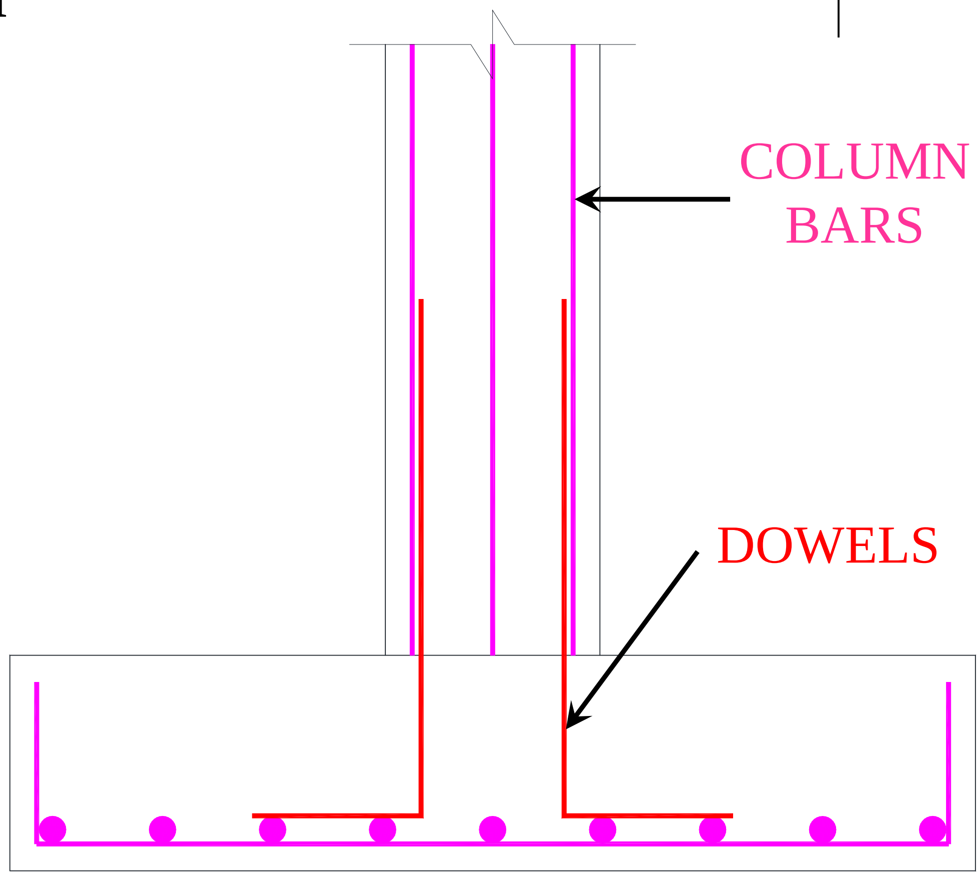
NOTE: Maximum spacing must not exceed 450mm.

FORCE AND MOMENT TRANSFER AT COLUMN BASE

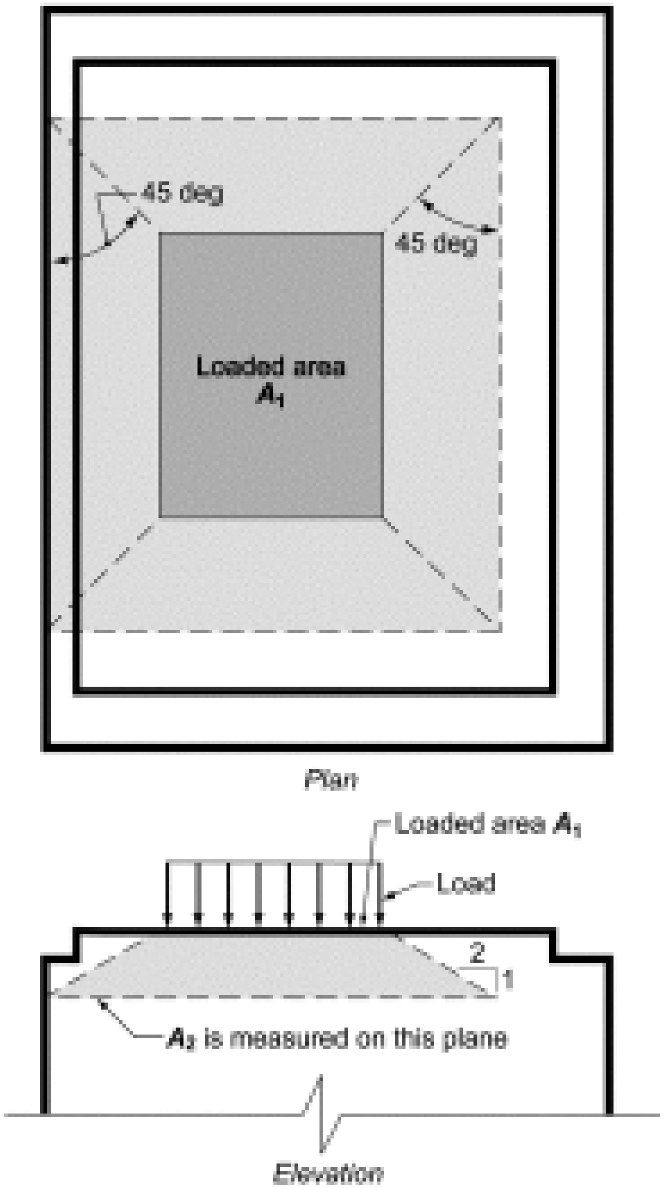
The concentrated load in the columns is transferred to the footing by

1. Direct bearing of the column over the footing (Producing bearing stresses at the interface)

2. By forces in the dowels or column main steel bars that cross the joint.



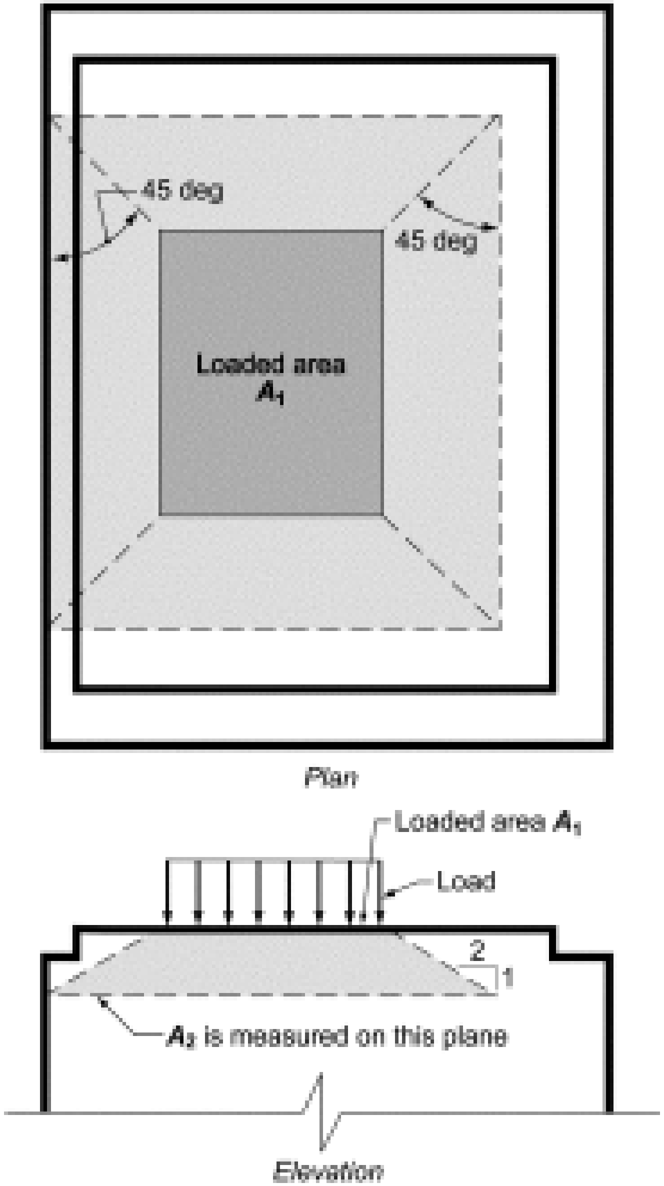
FORCE AND MOMENT TRANSFER AT COLUMN BASE



The maximum bearing load on the concrete is defined by ACI 318-11 Sec. 10.14.1 "Design bearing strength on concrete shall not exceed ϕ(0.85 fc'A1), except when the supporting surface is wider on all sides than the loaded area, then the design bearing strength on the loaded area shall be permitted to be multiplied by $\sqrt{\frac{A_2}{A_1}}$ but not more than 2."

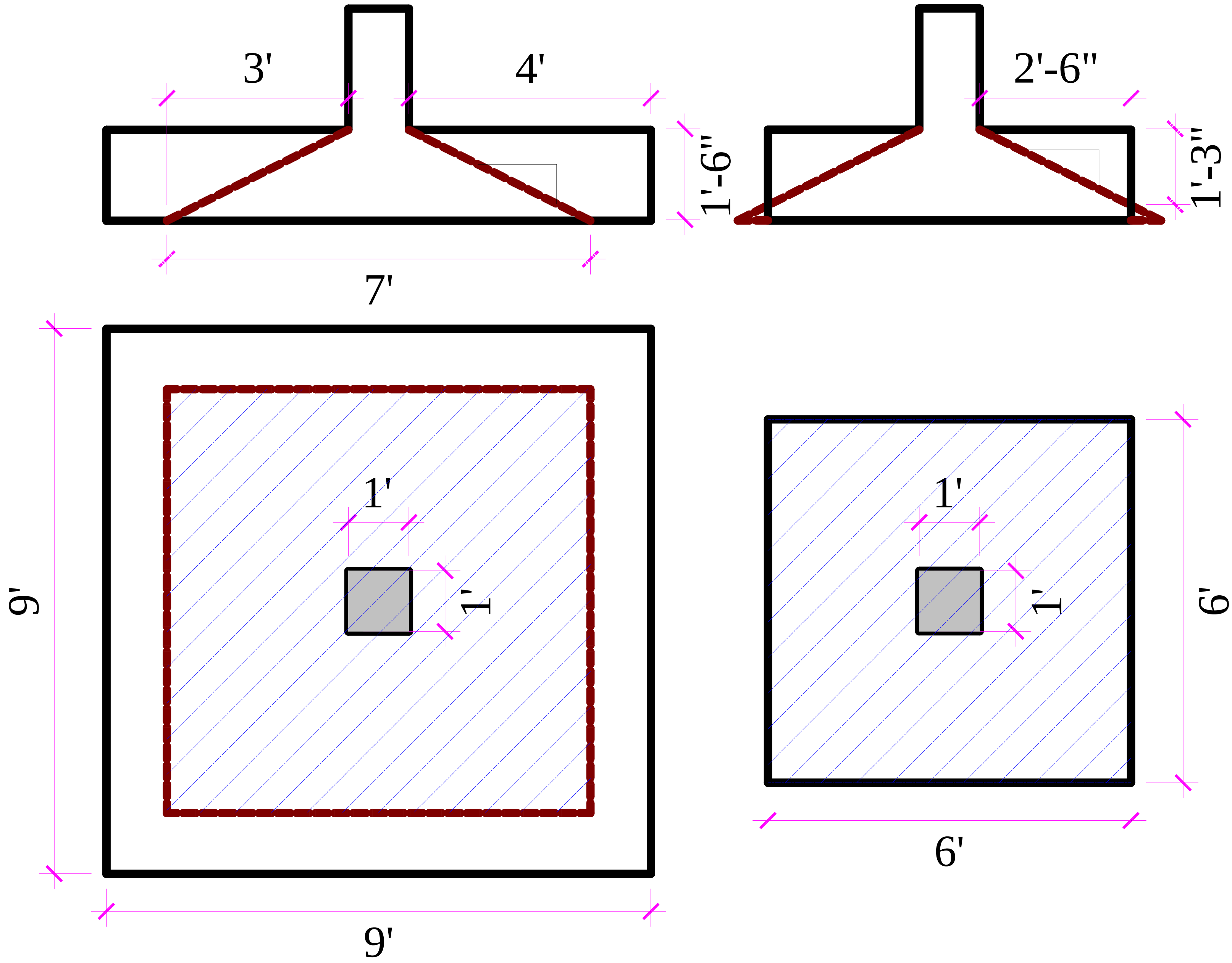
The loaded area or the area of cross-section of column is denoted by A_1, whereas, a larger area of footing may be considered effective in resisting the bearing stresses and is denoted by A_2.

FORCE AND MOMENT TRANSFER AT COLUMN BASE

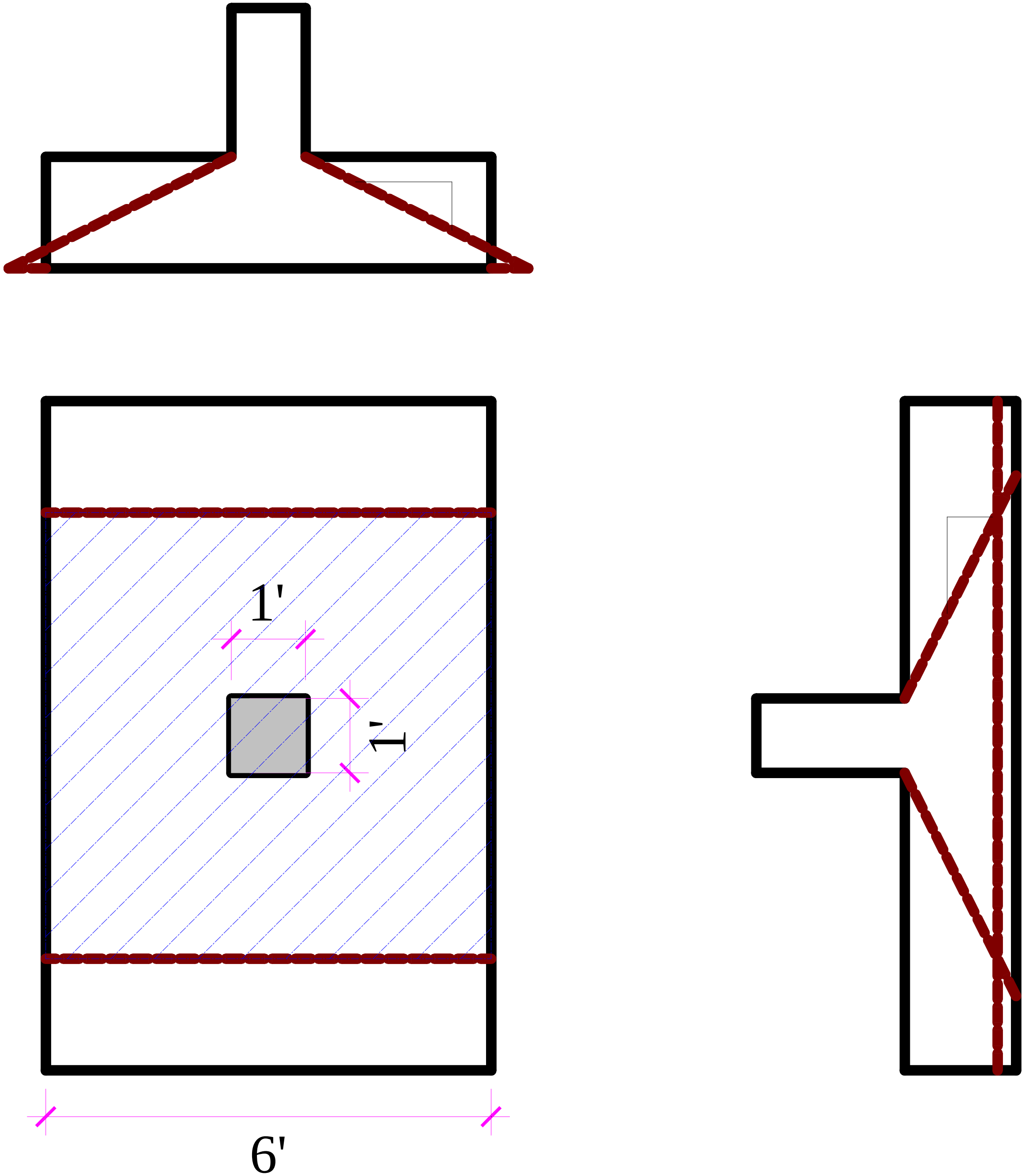


- The area A_2 at a certain depth inside the footing is found by spreading the area A_1 at a slope of lesser of 2 Hz. to 1 Ver. for solid footings and actually available steeper slope on each side.

- The concrete present in larger area A_2 around the loaded area A_1 provides lateral confinement to the concrete and causes increase in the bearing strength.



3'
4'
1'-6"
7'
2'-6"
1'-3"
1'
1'
9'
9'
1'
1'
6'
6'


1'
1'
6'

FORCE AND MOMENT TRANSFER AT COLUMN BASE

Bearing strength at the base of the column is given as

$$\phi P_n = \phi(0.85 f_c^{'} \mathrm{A1}) \qquad Where\ \varphi = 0.65$$

Were, f_c' is the cylinder strength of the column concrete.

Bearing strength at the bottom of the footing is given as

$$\phi P_n = \phi(0.85 f_c^{'} \mathrm{A1})\sqrt{\frac{\mathrm{A}_2}{\mathrm{A}_1}} \leq \phi 2(0.85 f_c^{'} \mathrm{A1})$$

Were, f_c' is the cylinder strength of the footing concrete.

The above expression is used to check the concrete in the footing just below the column, without dowels having development length beyond this region.

Dowel Bars

Excess force to be resisted by dowels, having development length beyond the bottom of the footing is

$$Dowel\ Force = P_u - \phi P_n$$

Area of steel required for dowels within the footing is

$$A_s = \frac{Pu - \varphi Pn}{\varphi f_y} \qquad \text{where } \varphi = 0.65 \quad (ACI\ 15.8.1.2)$$

A ***minimum area*** of reinforcement of ***0.005Ag*** (but not less than four bars) has to be provided across the interface of the column and footing. *Ag* is the gross area of column cross section. *(ACI 15.8.2.1)*

Dowel Bars (cont).

However, if some of the column steel is discontinued at the top of footing, the portion of the column just above the footing will be lacking fully developed bars.

The bearing strength of this portion of the column is to be checked. The continuing steel must provide a dowel steel area to develop a force equal to $P_u - \varphi 0.85 f_c' A_g$.

Area of steel required for dowels within the column is

$$A_s = \frac{Pu - \varphi 0.85 f_c' A_g}{\varphi f_y} \qquad \text{where } \varphi = 0.65$$

Dowel Bars (cont).

Dowels must have diameter less than or equal to No. 35 and must extend into the supported member by a distance equal to larger of

1. Development length in compression of the column main steel bars.
2. Compression splice length of the dowels.

Similarly, these dowels must extend into the footing by a minimum distance equal to the development length of dowels in compression. (ACI 318-11 15.8.2.3)

1. Development Length in Tension

$$l_d = 0.63 \times \frac{f_y}{\sqrt{f_c^{'}}} d_b \geq 300\text{mm} \qquad \text{for top bars and } d_b \leq \text{No. } 20$$

$$l_d = 0.485 \times \frac{f_y}{\sqrt{f_c^{'}}} d_b \geq 300\text{mm} \qquad \text{for bottom bars and } d_b \leq \text{No. } 20$$

$$l_d = 0.788 \times \frac{f_y}{\sqrt{f_c^{'}}} d_b \geq 300\text{mm} \qquad \text{for top bars and } d_b > \text{No. } 20$$

$$l_d = 0.606 \times \frac{f_y}{\sqrt{f_c^{'}}} d_b \geq 300\text{mm} \qquad \text{for bottom bars and } d_b > \text{No. } 20$$

2. Development Length in Compression

$$l_{dc} = 0.24\frac{f_y}{\sqrt{f_c'}}d_b \geq 200\text{mm} \qquad \text{for } f_c' \leq 31\text{MPa}$$

$$l_{dc} = 0.043 f_y d_b \geq 200\text{mm} \qquad \text{for } f_c' > 31\text{MPa}$$

Where

d_b = Dia of bar

3. Tension Splice Length

$$Class\ B\ Splice = 1.3 l_d \geq 300mm$$

Where l_d = Development length in tension

4. Compression Splice Length

$$Compression\,splice\,length = 0.071 f_y d_b F \geq 300mm \qquad for\ f_y \leq 420MPa$$

$$= (0.13 f_y - 24) d_b F \geq 300mm \qquad for\ f_y > 420MPa$$

$$where, if\ \ f_c' > 20MPa, F = 1.0, otherwise, F = 1.3$$

When bars of different diameters are lap spliced in compression, the required splice length is to be taken larger of the development length in compression for the larger diameter bar and splice length for the smaller diameter bar.

PROCEDURE FOR DESIGN OF ISOLATED FOOTING

Step1:

Collect all the required information i.e. allowable bearing capacity, depth of footing, type of load coming and decide type of footing.

Step2:

For service DL, LL and net allowable bearing capacity find the size of the footing.

Step3:

Calculate the net factored contact pressure **"q_u"** at the interface of soil and concrete surface.

Step4:

Select or assume a suitable thickness of footing satisfying two-way punching shear.

Step5:

Calculate one-way shear in longer direction and check for its capacity.

Step6:

Calculate the moment in shorter and longer direction.

PROCEDURE FOR DESIGN OF ISOLATED FOOTING

Step 7:

Calculate the total amount of steel in shorter and longer direction and find the spacing.

Step 8:

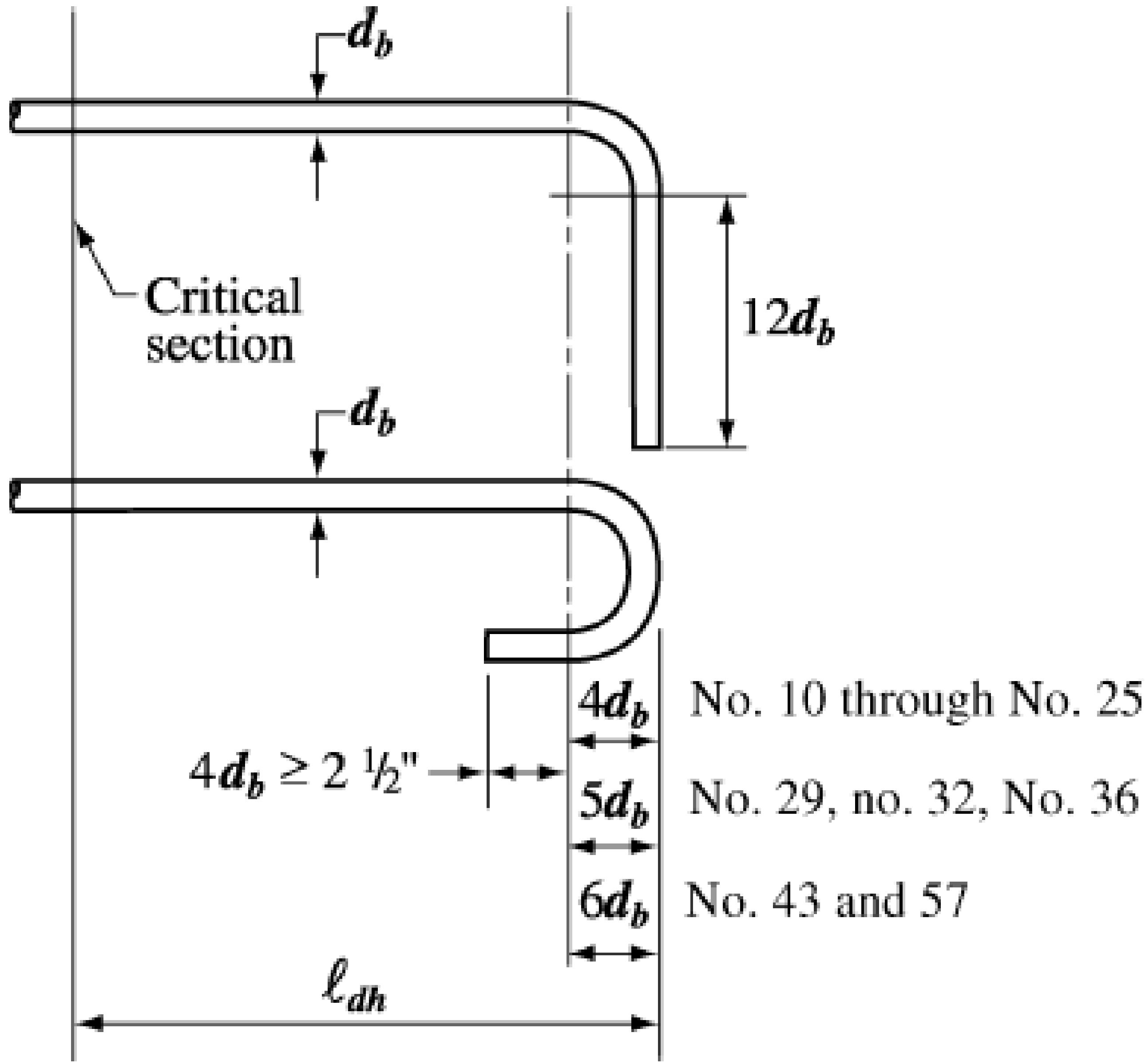
Check the bearing pressure at the bottom of column.

Step 9:

Check the development length for the steel provided.

Step 10:

Show reinforcement using neat sketches.



Development Length ℓ_{dh} of Standard Hooks

FOR RECTANGULAR FOOTINGS

According to ACI 15.4.4 — In two-way rectangular footings, reinforcement shall be distributed in accordance with ACI 15.4.4.1 and 15.4.4.2.

15.4.4.1 — Reinforcement *in long direction* shall be distributed uniformly across entire width of footing.

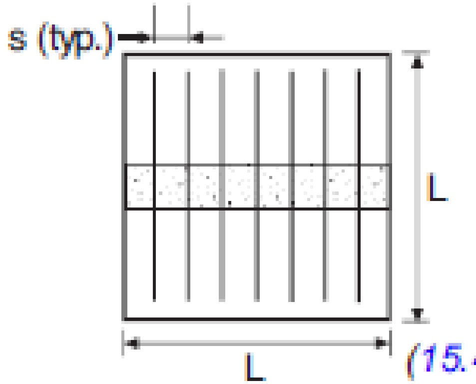
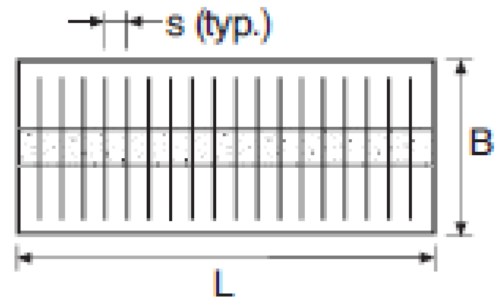
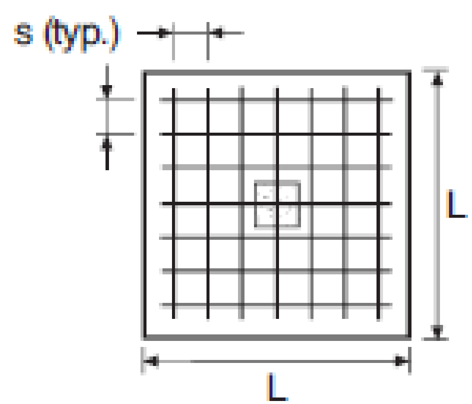
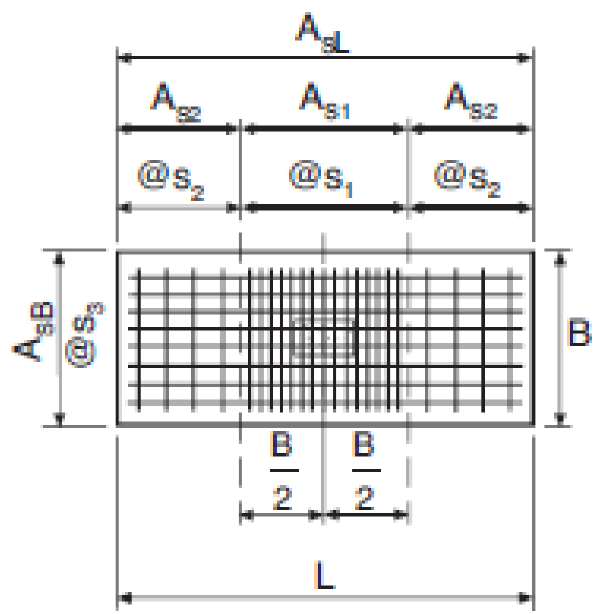
15.4.4.2 — For reinforcement *in short direction*, a portion of the total reinforcement, $\gamma_s A_s$, shall be distributed uniformly over a band width (centered on centerline of column or pedestal) equal to the length of short side of footing. Remainder of reinforcement required in short direction, $(1 - \gamma_s)A_s$, shall be distributed uniformly outside center band width of footing.

FOR RECTANGULAR FOOTINGS

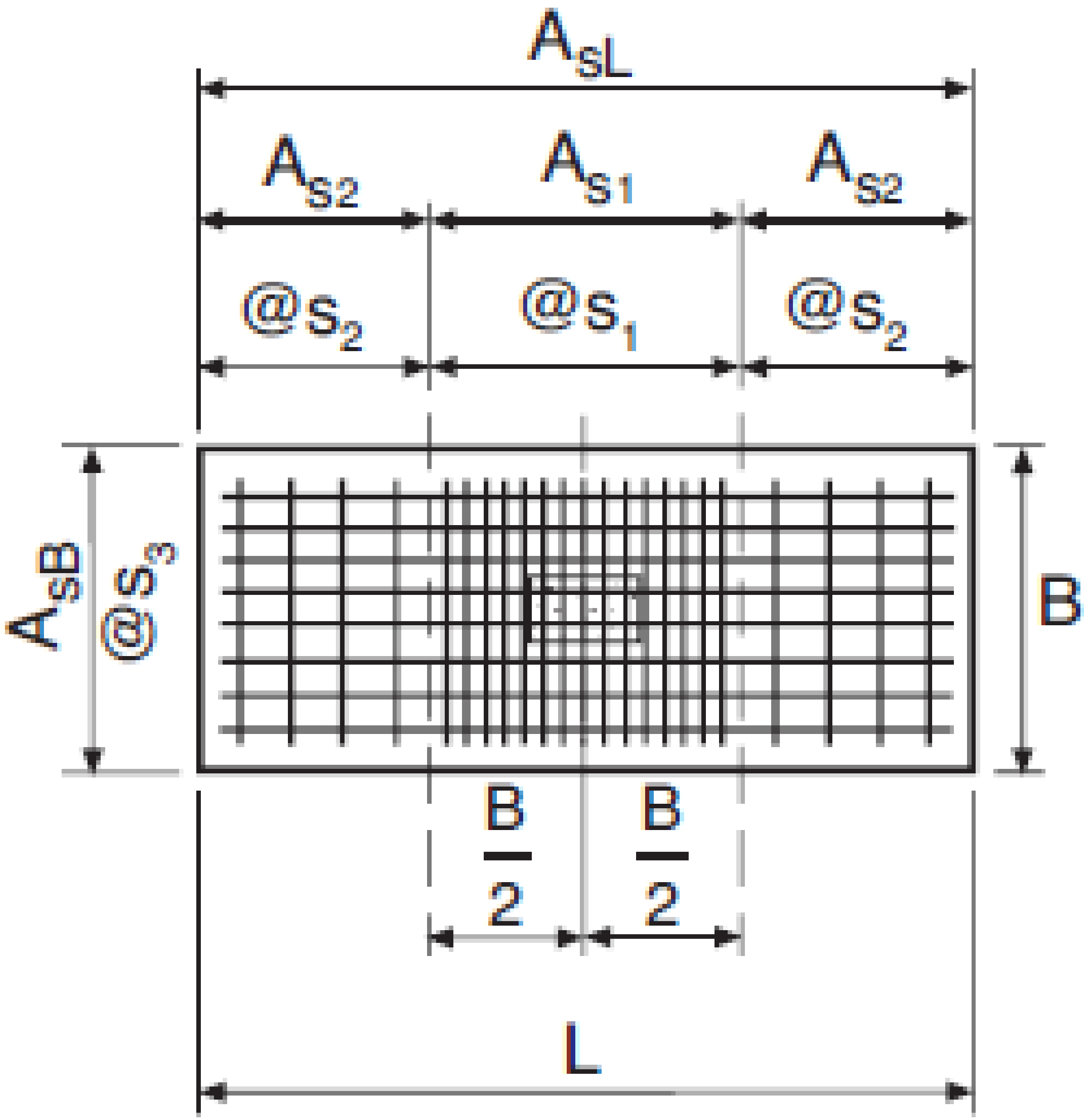
$$\gamma_s = \frac{2}{\beta + 1}$$

Where

$$\beta = \frac{long\ side\ of\ footing}{short\ side\ of\ footing}$$

Footing Type	Square Footing	Rectangular Footing
One-way	 *(15.4.3)*	 *(15.4.3)*
Two-way	 *(15.4.3)*	 $A_{s1} = \gamma_s A_{sL}$ $A_{s2} = \frac{(1-\gamma_s)A_{sL}}{2}$ $\beta = \frac{L}{B}$ $\gamma_s = \left(\frac{2}{\beta+1}\right)$ *(15.4.4)*

[Ref.: PCA NOTES on ACI 318-08]



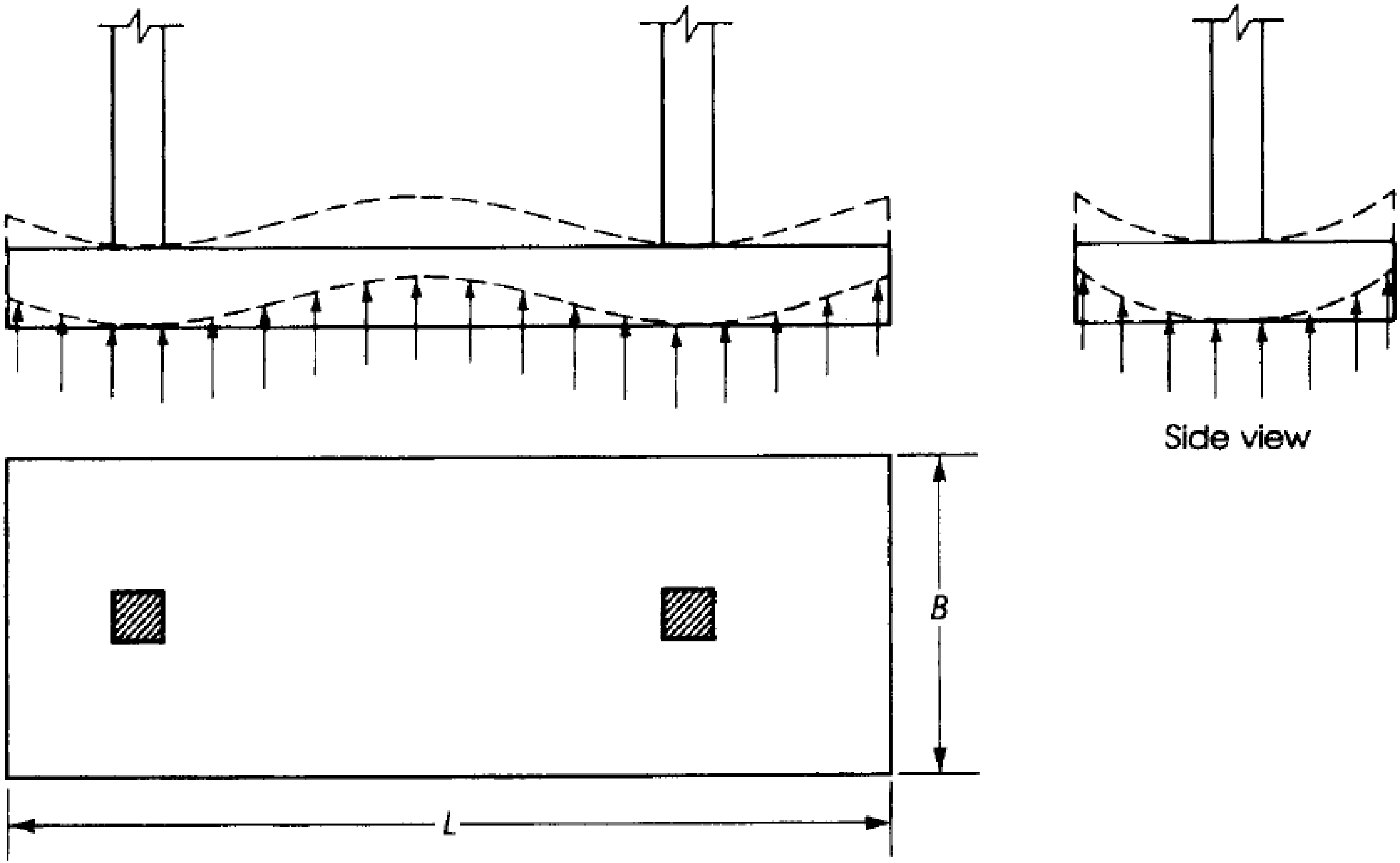
$$A_{s1} = \gamma_s A_{sL}$$

$$A_{s2} = \frac{(1-\gamma_s)A_{sL}}{2}$$

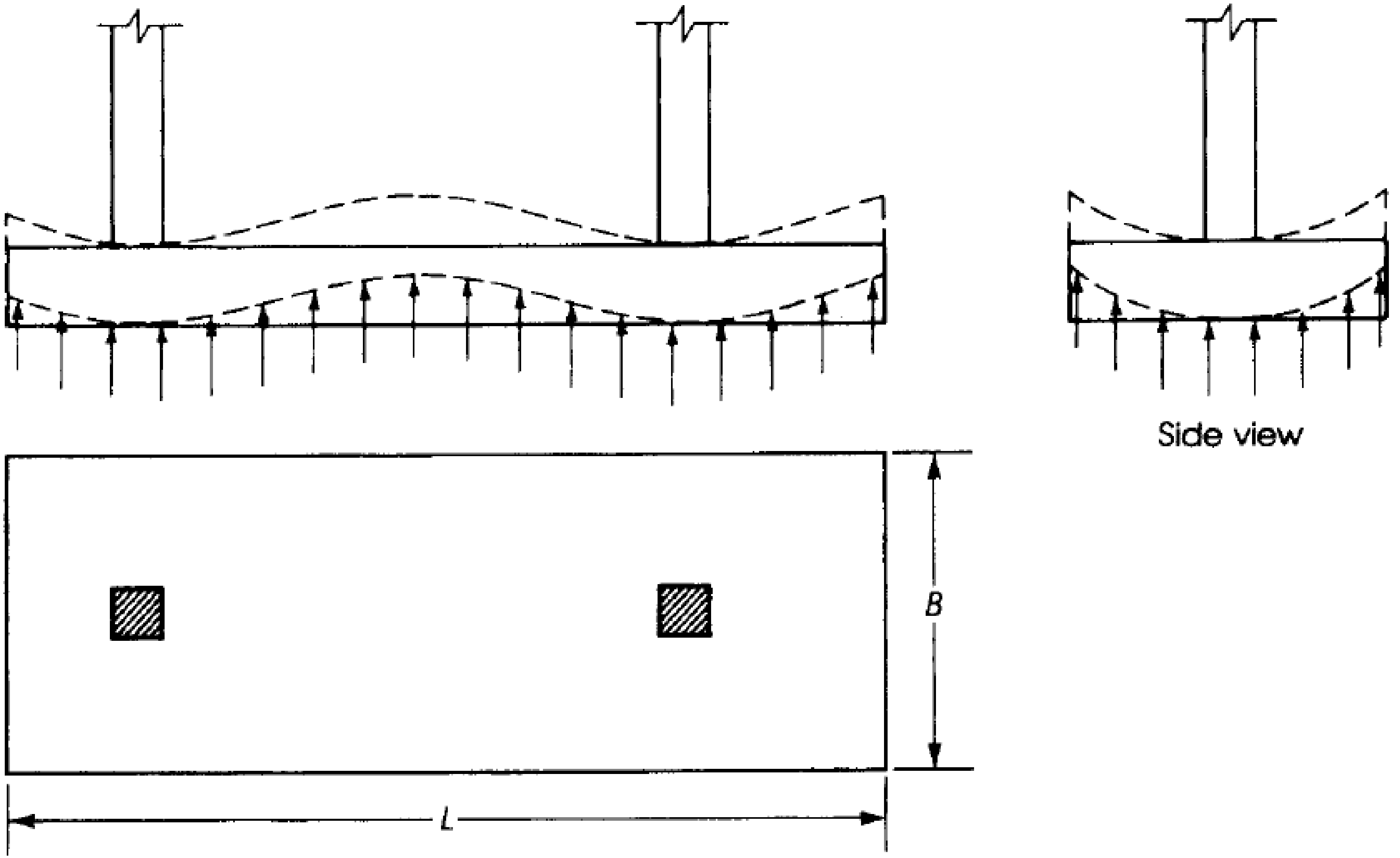
$$\beta = \frac{L}{B}$$

$$\gamma_s = \left(\frac{2}{\beta+1}\right)$$

[Ref.: PCA NOTES on ACI 318-08]



[Ref.: Structural Concrete, Theory and Design, 4th Ed.
by Nadim Hassoun]



[Ref.: Structural Concrete, Theory and Design, 4th Ed. by Nadim Hassoun]

Problem 1

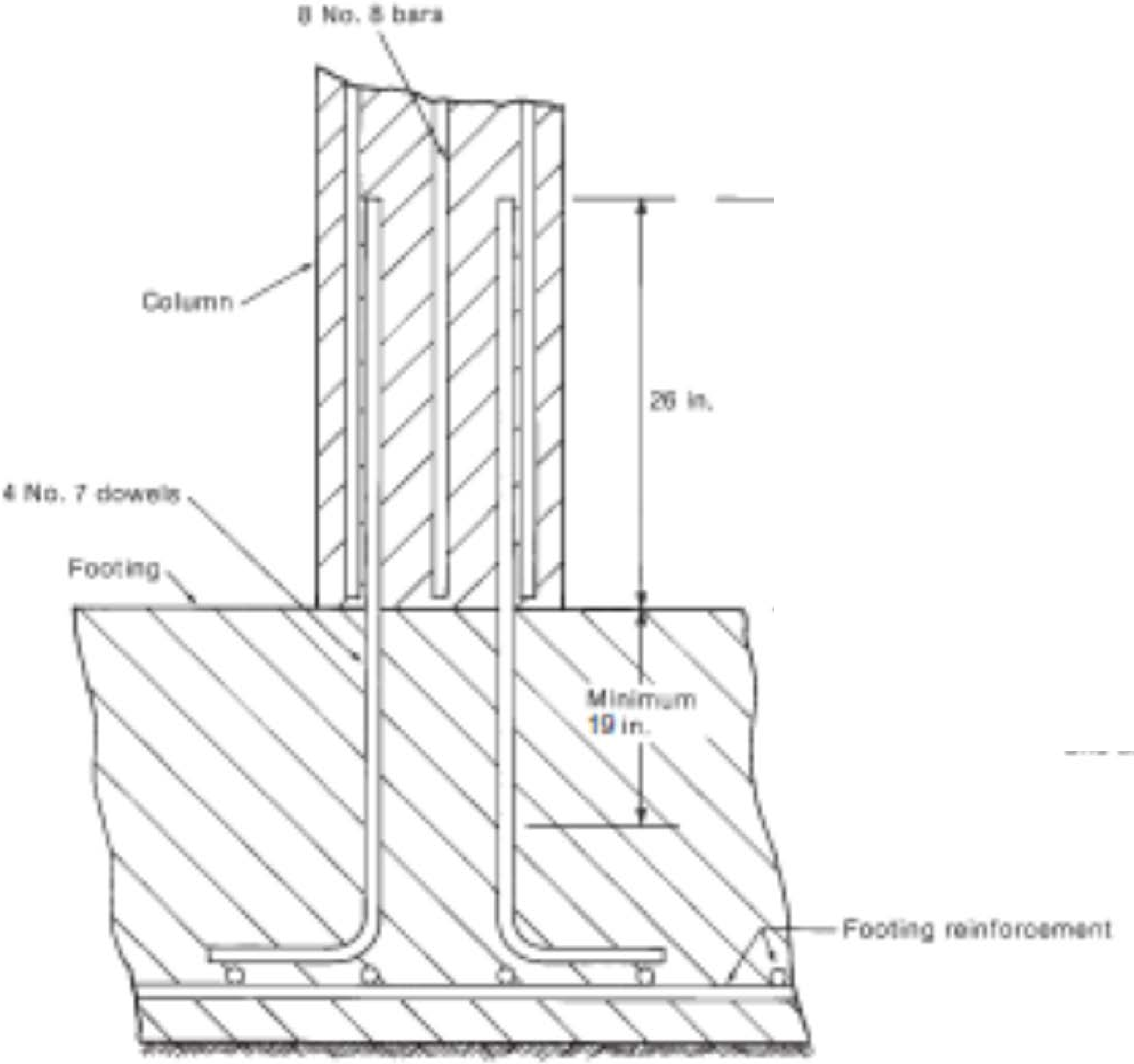
A 3.9 m × 3.9 m square footing with 825 mm thickness supports a tied rectangular column having dimensions 750 mm × 300 mm reinforced with 6-No. 25 longitudinal bars. For the following data, check force transfer at interface of column and footing. Footing flexural steel is #19@200 c/c (B.W.)

fc′ (column) = 35 MPa, normalweight concrete
fc′ (footing) = 20 MPa, normalweight concrete
fy = 420 MPa
Pu = 3825 kN

Problem 2

For the design conditions given below, check for transfer of force between the column and footing. 300 mm × 300 mm tied reinforced column with 4-No. 43 longitudinal bars. Footing Size 2800 × 2800 × 450 mm

fc′ = 28 MPa (column and footing),
Concrete = Normal weight concrete
fy = 420 MPa
PD = 890 kN
PL = 445 kN


8 No. 8 bars
Column
26 in.
4 No. 7 dowels
Footing
Minimum
19 in.
Footing reinforcement

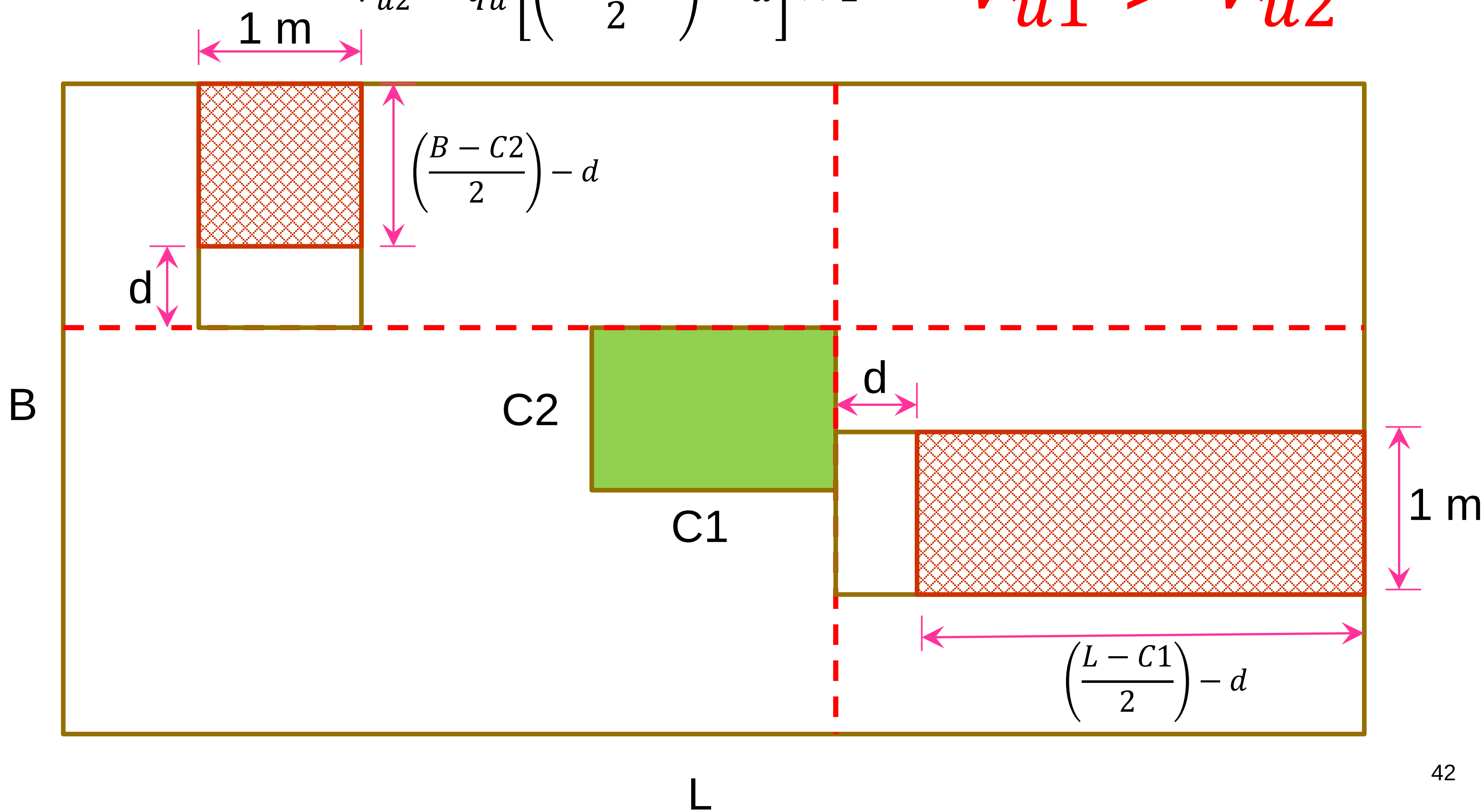
Shear in longer direction strip

$$V_{u1} = q_u \left[\left(\frac{L - C1}{2} \right) - d \right] \times 1$$

Shear in shorter direction strip

$$V_{u2} = q_u \left[\left(\frac{B - C2}{2} \right) - d \right] \times 1$$

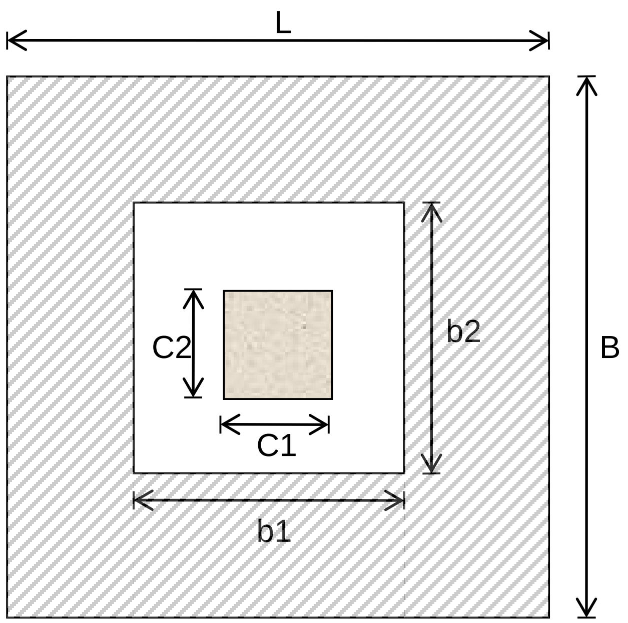
$$V_{u1} > V_{u2}$$



$$V_u = q_u(Area \text{ outside the critical perimeter})$$

$$V_u = q_u(L \times B - b1 \times b2)$$

$$V_u = q_u(L \times B) - q_{nu}(b1 \times b2)$$



$$P_u = q_u(L \times B)$$

$$V_u = P_u - q_u(b1 \times b2)$$

END OF LECTURE