Civil Engineering Department

Flexural Design of Prestressed Concrete Members – 1

(Introduction and Selection of Geometric Dimensions)

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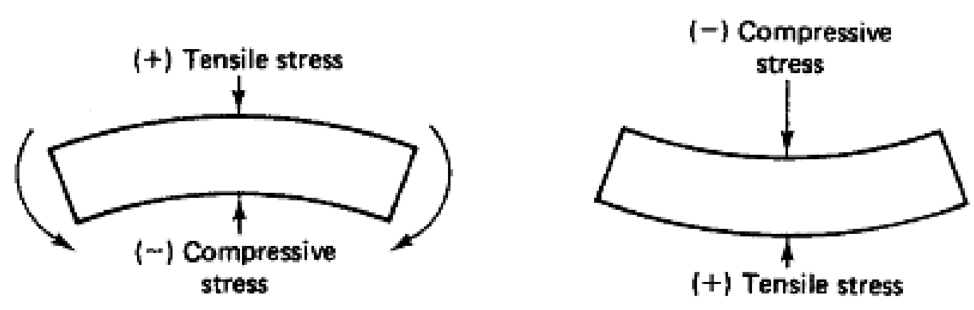
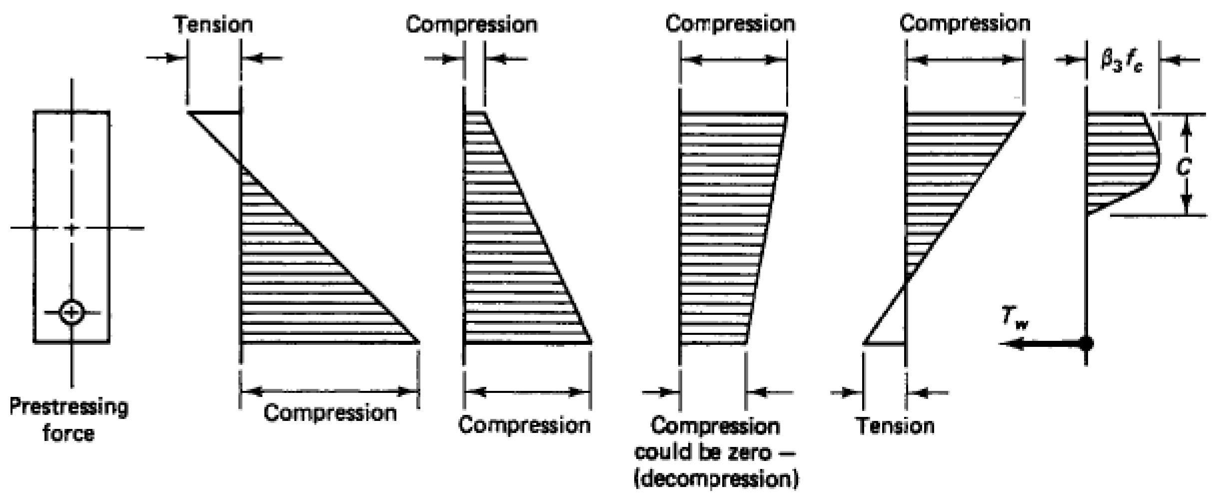
Introduction

- What Are We Designing For?
  - Reinforced Concrete – **<u>Strength</u>** - Should Resist Factored Load
    <u>Serviceability</u> -Crack Control, Deflection check etc.
  - Prestressed Concrete – **<u>Stress Limits</u>** at Different Loading Stages
    <u>Serviceability</u> – Crack Control, Deflection Check etc.
    <u>Strength</u> – Should Resist Factored Load
  - Sign Convention – **Stresses:** Tension +ve, Compression –ve.
    Bending:



Loading Stages and Stress Checks

- ***Transfer of Prestress*** - Application of Initial Prestress Force, P_i.
- ***Self Weight, W_D*** Acts Instantaneously at Transfer.
- ***Superimposed Dead Load, W_{sD}*** is Applied – Short Term Losses Reduce P_i to P_{eo}.
- ***Service Load is Applied*** – Long Term Losses Reduce P_{eo} to P_e.
- ***Overloading of Member Occurs*** – Member has to Resist Applied Factored Loading



Selection of Geometric Dimensions – Minimum Section Modulus

- ***Problem Statement:*** Determine Minimum Section Modulus that can Withstand All Loads at Different Loading Stages
- ***Stress Limits***

ACI MAXIMUM PERMISSIBLE STRESSES IN CONCRETE AND REINFORCEMENT

Following are definitions of some important mathematical terms used in this section:

f_{py} = specified yield strength of prestressing tendons, in psi
f_y = specified yield strength of nonprestressed reinforcement, in psi
f_{pu} = specified tensile strength of prestressing tendons, in psi
f'_c = specified compressive strength of concrete, in psi
f'_{ci} = compressive strength of concrete at time of initial prestress

2.8.1 Concrete Stresses in Flexure

Stresses in concrete immediately after prestress transfer (before time-dependent prestress losses) shall not exceed the following:

(a) Extreme fiber stress in compression . . . . $0.60 f'_{ci}$
(b) Extreme fiber stress in tension except as permitted in (c) . . . . $3\sqrt{f'_{ci}}$
(c) Extreme fiber stress in tension at ends of simply supported members . . . . $6\sqrt{f'_{ci}}$

Selection of Geometric Dimensions – ACI Maximum Stress Limits

Where computed tensile stresses exceed these values, bonded auxiliary reinforcement (nonprestressed or prestressed) shall be provided in the tensile zone to resist the total tensile force in concrete computed under the assumption of an uncracked section.

Stresses in concrete at service loads (after allowance for all prestress losses) shall not exceed the following:

(a) Extreme fiber stress in compression due to prestress plus sustained load, where sustained dead load and live load are a large part of the total service load . . . . $0.45 f'_c$

(b) Extreme fiber stress in compression due to prestress plus total load, if the live load is transient . . . . $0.60 f'_c$

(c) Extreme fiber stress in tension in precompressed tensile zone. . . . . $6\sqrt{f'_c}$

(d) Extreme fiber stress in tension in precompressed tensile zone of members (except two-way slab systems), where analysis based on transformed cracked sections and on bilinear moment-deflection relationships shows that immediate and long-time deflections comply with the ACI definition requirements and minimum concrete cover requirements . . . . $12\sqrt{f'_c}$

2.8.2 Prestressing Steel Stresses

Tensile stress in prestressing tendons shall not exceed the following:

(a) Due to tendon jacking force . . . . $0.94 f_{py}$
but not greater than the lesser of $0.80 f_{pu}$ and the maximum value recommended by the manufacturer of prestressing tendons or anchorages.

Selection of Geometric Dimensions – AASHTO Maximum Stress Limits

(b) Immediately after prestress transfer. . . . . $0.82\ f_{py}$
but not greater than $0.74\ f_{pu}$.

(c) Post-tensioning tendons, at anchorages and couplers, immediately after tendon anchorage . . . . $0.70\ f_{pu}$

AASHTO Maximum Permissible Limits

2.9.1 Concrete Stresses before Creep and Shrinkage Losses

Compression

Pretensioned members. . . . . $0.60\ f'_{ci}$

Post-tensioned members . . . . $0.55\ f'_{ci}$

Tension

Precompressed tensile zone . . . . No temporary allowable stresses are specified.

Other Areas

In tension areas with no bonded reinforcement. . . . . 200 psi or $3\sqrt{f'_{ci}}$

Where the calculated tensile stress exceeds this value, bonded reinforcement shall be provided to resist the total tension force in the concrete computed on the assumption of an uncracked section. The maximum tensile stress shall not exceed. . . . . $7.5\sqrt{f'_{ci}}$

Selection of Geometric Dimensions – AASHTO Maximum Stress Limits

2.9.2 Concrete Stresses at Service Load after Losses

Compression	$0.40\, f'_c$
Tension in the precompressed tensile zone	
(a) For members with bonded reinforcement	$6\sqrt{f'_c}$
For severe corrosive exposure conditions, such as coastal areas	$3\sqrt{f'_c}$
(b) For members without bonded reinforcement	0

Tension in other areas is limited by the allowable temporary stresses specified in Section 2.8.1.

2.9.2.1 Cracking Stresses. Modulus of rupture from tests or if not available.

For normal-weight concrete	$7.5\sqrt{f'_c}$
For sand-lightweight concrete	$6.3\sqrt{f'_c}$
For all other lightweight concrete	$5.5\sqrt{f'_c}$

2.9.2.2 Anchorage-Bearing Stresses

Post-tensioned anchorage at service load .... 3,000 psi
(but not to exceed $0.9\, f'_{ci}$)

2.9.3 Prestressing Steel Stresses

(a) Due to tendon jacking for	$0.94\, f_{py} \leq 0.80\, f_{pu}$
(b) Immediately after prestress transfer	$0.82\, f_{py} \leq 0.74\, f_{pu}$
(c) Post-tensioning tendons at anchorage, immediately after tendon anchorage	$0.70\, f_{pu}$

$f_{py} \approx 0.85\, f_{pu}$ (for low-relaxation, $f_{py} = 0.90\, f_{pu}$)

Selection of Geometric Dimensions – Minimum Section Modulus

- **Variable Eccentricity Tendons – Critical Section: Mid-span**
  - If Residual Prestress Ratio, $\gamma = P_e/P_i$

    $\Rightarrow$ **Loss of Prestress** = $P_i - P_e = \mathbf{(1-\gamma)\,P_i}$
  - Change in Stress From Allowable Limits

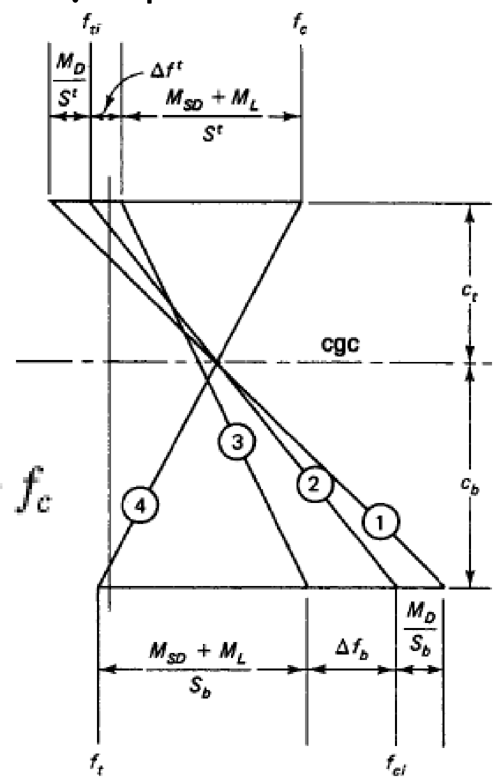
$$\Delta f^t = (1-\gamma)\left(f_{ti} + \frac{M_D}{S^t}\right)$$

$$\Delta f_b = (1-\gamma)\left(-f_{ci} + \frac{M_D}{S_b}\right)$$

  - Net Stress (Stress Capacity) at Top and Bottom

$$f_n^t = f_{ti} - \Delta f^t - f_c \quad \Rightarrow \quad f_n^t = \gamma f_{ti} - (1-\gamma)\frac{M_D}{S^t} - f_c$$

$$f_{bn} = f_t - f_{ci} - \Delta f_b \quad \Rightarrow \quad f_{bn} = f_t - \gamma f_{ci} - (1-\gamma)\frac{M_D}{S_b}$$



Selection of Geometric Dimensions – Minimum Section Modulus

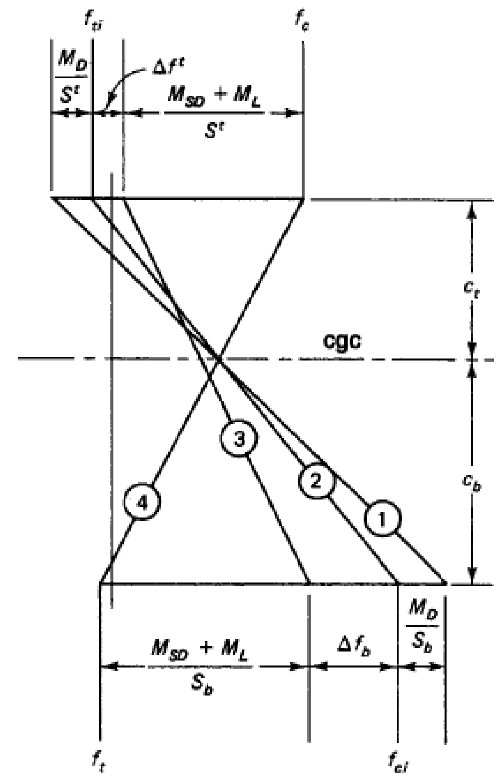
$$S^t \geq \frac{(1-\gamma)M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c} \qquad S_b \geq \frac{(1-\gamma)M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

– Calculate Initial Prestress Force, P_i

$$P_i = \bar{f}_{ci} A_c \qquad \bar{f}_{ci} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci})$$

– The Required Eccentricity, e

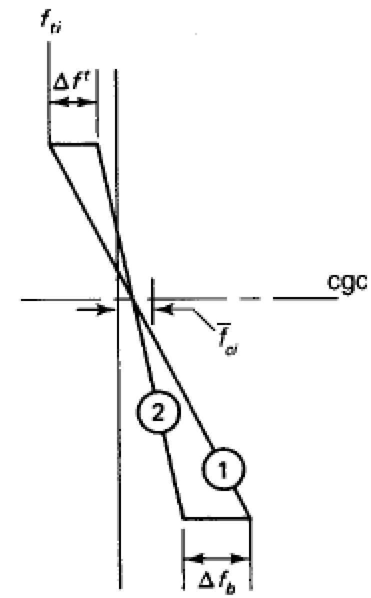
$$e_c = (f_{ti} - \bar{f}_{ci})\frac{S^t}{P_i} + \frac{M_D}{P_i}$$



Selection of Geometric Dimensions – Minimum Section Modulus

- **Constant Eccentricity Tendons – Critical Section: Supports**
  - The Change in Stress Due to Prestress Losses

$$\Delta f^t = (1 - \gamma)(f_{ti}) \qquad \Delta f_b = (1 - \gamma)(-f_{ci})$$



  - Net Stress (Stress Capacity) at Top and Bottom

$$f_n^t = f_{ti} - \Delta f^t - f_c \Rightarrow f_n^t = \gamma f_{ti} - f_{cs}$$

$$f_{bn} = f_t - f_{ci} - \Delta f_b \Rightarrow f_{bn} = f_t - \gamma f_{ci}$$

  - The Minimum Section Modulus

$$S^t \geq \frac{M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c} \qquad S_b \geq \frac{M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

  - The Require Eccentricity at Support: $e_e = (f_{ti} - \bar{f}_{ci})\dfrac{S^t}{P_i}$

Selection of Geometric Dimensions – General Guidelines

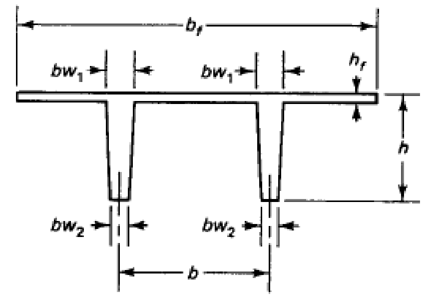
- $e \propto 1/P_i$
- Large e => Large Concrete Area at Top --- Hence a T or wide I
- Ends are Usually Solid to: i. Provide Anchorage of Tendons
  ii. Increase in Shear Capacity
- If e > Section Depth (for Allowable Stresses) => Uneconomical Section => Revise Section to Increase Depth

- **Gross Area, Transformed Section and Presence of Ducts**
  - Use of Gross Area is Adequate for use of Service Load Design.
  - The Accuracy Gained by Using Transformed Section Properties is Insignificant.
  - For Long Span Bridges and Industrial Beams: Taking out Area of Ducts and using Transformed Sectional Properties.

Selection of Geometric Dimensions – Some Standardized Sections

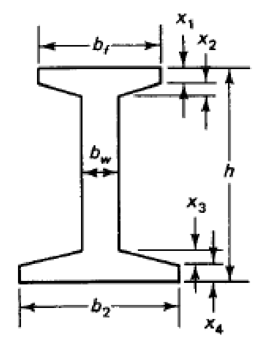
Table 4.4(a) Geometrical Details of As-Built PCI and AASHTO Sections

Designation	b_f (in.)	h_f (in.)	b_{w1} (in.)	$bw2$ (in.)	h (in.)	b (in.)
8DT12	96	2	5.75	3.75	12	48
8DT14	96	2	5.75	3.75	14	48
8DT16	96	2	5.75	3.75	16	48
8DT18	96	2	5.75	3.75	18	48
8DT20	96	2	5.75	3.75	20	48
8DT24	96	2	5.75	3.75	24	48
8DT32	96	2	7.75	4.75	32	48
10DT32	120	2	7.75	4.75	32	60
12DT34	144	4	7.75	4.75	34	60
15DT34	180	4	7.75	4.75	34	90



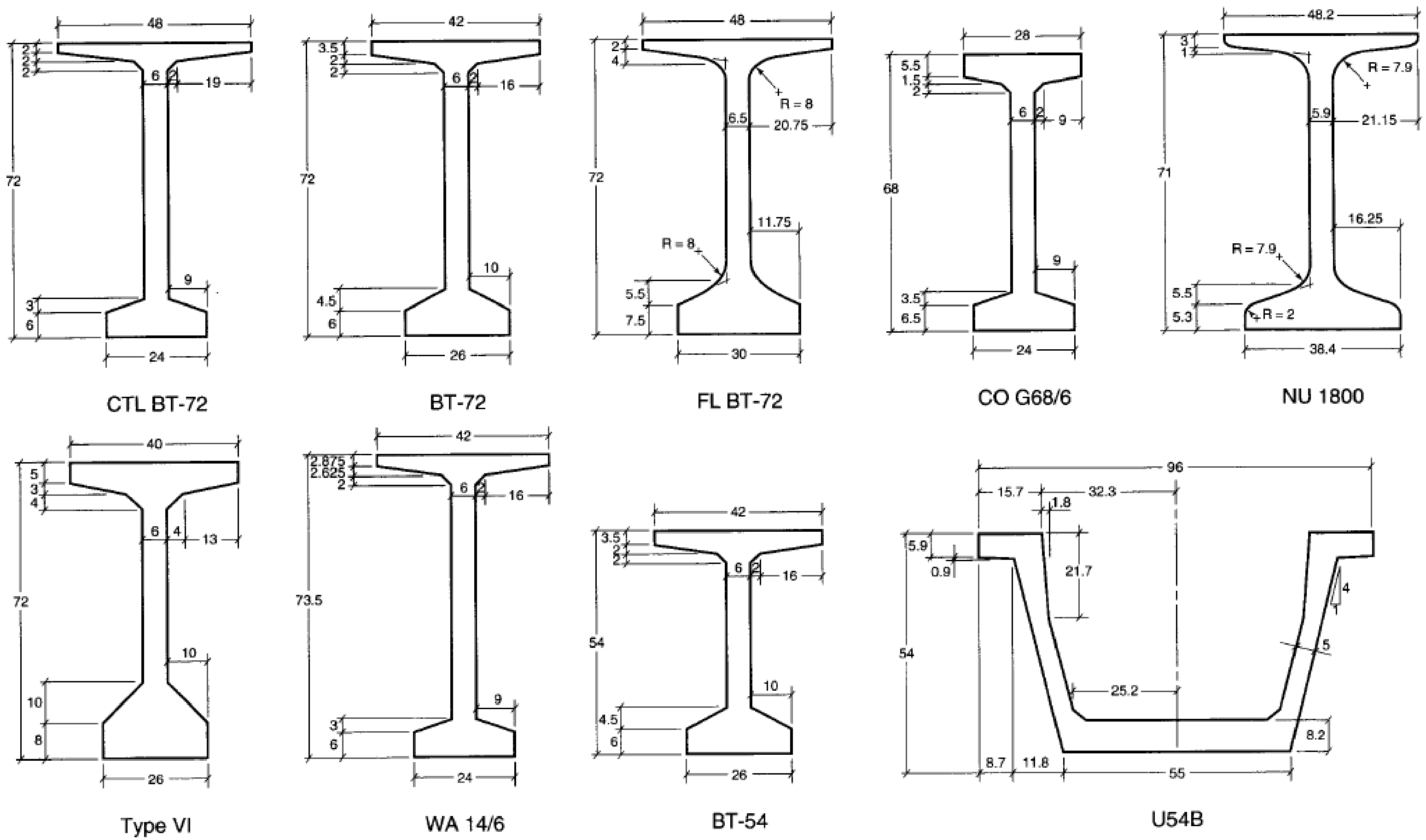
Actual double-T sections

Designation	b_f (in.)	x_1 (in.)	x_2 (in.)	b_2 (in.)	x_3 (in.)	x_4 (in.)	b_w (in.)	h (in.)
AASHTO 1	12	4	3	16	5	5	6	28
AASHTO 2	12	6	3	18	6	6	6	36
AASHTO 3	16	7	4.5	22	7.5	7	7	45
AASHTO 4	20	8	6	26	9	8	8	54
AASHTO 5	42	5	7	28	10	8	8	63
AASHTO 6	42	5	7	28	10	8	8	72



Actual I sections

Selection of Geometric Dimensions – Optimized Bridge Girder Sections



Selection of Geometric Dimensions – Optimized Bridge Girder Sections

Agency	Girder Type	Depth (in.)	Web Width (in.)	area (in.^{2})	Inertia (in.^{4})	y_t (in.)	y_b (in.)	S_t (in.^{3})	S_b (in.^{3})	ρ	α
CTL	BT-48	48	6	557	177,736	23.53	24.47	7,553	7,264	0.554	0.940
	BT-60	60	6	629	308,722	29.59	30.41	10,432	10,154	0.545	0.931
	BT-72	72	6	701	484,993	35.64	36.36	13,606	13,340	0.534	0.914
PCI	BT-54	54	6	659	268,077	26.37	27.63	10,166	9,702	0.558	0.943
	BT-63	63	6	713	392,638	30.82	32.12	12,715	12,224	0.556	0.942
	BT-72	72	6	767	545,894	35.40	36.60	15,421	14,915	0.549	0.934
AASHTO	Type VI	72	8	1,085	733,320	35.62	36.38	20,587	20,157	0.522	0.893
	Mod. Type VI	72	6	941	671,088	35.56	36.44	18,871	18,417	0.550	0.941
Washington	80/6	50	6	513	159,191	27.24	22.76	5,844	6,994	0.501	0.943
	100/6	58	6	591	256,560	30.01	27.99	8,549	9,166	0.517	0.925
	120/6	73.5	6	688	475,502	37.68	35.82	12,619	13,275	0.512	0.908
	14/6	73.5	6	736	534,037	35.30	38.20	15,122	13,985	0.538	0.894
Colorado	G54/6	54	6	631	242,592	27.33	26.67	8,877	9,095	0.527	0.924
	G68/6	68	6	701	426,575	33.99	34.01	12,548	12,544	0.526	0.911
Nebraska	1600	63	5.9	852	494,829	32.64	30.36	15,159	16,300	0.586	1.051
	1800	70.9	5.9	898	659,505	36.72	34.18	17,959	19,297	0.585	1.049
	2000	78.7	5.9	944	849,565	40.74	37.96	20,854	22,380	0.582	1.042
	2400	94.5	5.9	1,038	1,323,985	48.84	45.66	27,106	28,999	0.572	1.023
Florida	BT-54	54	6.5	785	311,765	28.11	25.89	11,091	12,042	0.546	0.983
	BT-63	63	6.5	843	458,521	32.88	30.12	13,945	15,223	0.549	0.992
	BT-72	72	6.5	901	638,672	37.64	34.36	16,968	18,588	0.548	0.991
Texas	U54A	54	10.2	1,022	379,857	30.12	23.90	12,612	15,895	0.516	0.996
	U54B	54	10.2	1,118	403,878	31.54	22.48	12,807	17,966	0.509	1.029

Selection of Geometric Dimensions – Variable Tendon Eccentricity Example 1

- **Variable Tendon Eccentricity Example**

Design a simply supported pretensioned double-T-beam for a parking garage with harped tendon and with a span of 60 ft (18.3 m) using the ACI 318 Building Code allowable stresses. The beam has to carry a superimposed sustained service live load of 1,100 plf (16.1 kN/m) and superimposed dead load of 100 plf (1.5 kN/m), and has no concrete topping. Assume the beam is made of normal-weight concrete with $f'_c = 5{,}000$ psi (34.5 MPa) and that the concrete strength f'_{ci} at transfer is 75 percent of the cylinder strength. Assume also that the time-dependent losses of the initial prestress are 18 percent of the initial prestress, and that $f_{pu} = 270{,}000$ psi (1,862 MPa) for stress-relieved tendons, $f_t = 12\sqrt{f'_c}$.

Solution:

$$\gamma = 100 - 18 = 82\%$$

$$f'_{ci} = 0.75 \times 5{,}000 = -3{,}750 \text{ psi (25.9 MPa)}$$

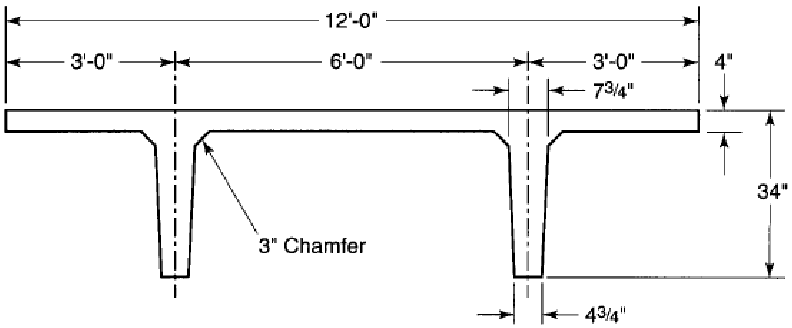
Use $f_t = 12\sqrt{5{,}000} = 849$ psi (5.9 MPa) as the maximum stress in tension, and assume a self-weight of approximately 1,000 plf (14.6 kN/m). Then the self-weight moment is given by

$$M_D = \frac{wl^2}{8} = \frac{1{,}000(60)^2}{8} \times 12 = 5{,}400{,}000 \text{ in.-lb (610 kN-m)}$$

and the superimposed load moment is

$$M_{SD} + M_L = \frac{(1{,}100 + 100)\,(60)^2}{8} \times 12 = 6{,}480{,}000 \text{ in.-lb (732 kN-m)}$$

Selection of Geometric Dimensions – Variable Tendon Eccentricity Example 1



$$S^t \geq \frac{(1-\gamma)M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$$

$$\geq \frac{(1-0.82)5{,}400{,}000 + 6{,}480{,}000}{0.82 \times 184 - (-2{,}250)} = 3{,}104 \text{ in}^3 \ (50{,}860 \text{ cm}^3)$$

$$S_b \geq \frac{(1-\gamma)M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

$$\geq \frac{(1-0.82)\ 5{,}400{,}000 + 6{,}480{,}000}{849 - 0.82(-2{,}250)} = 2{,}766 \text{ in.}^3 \ (45{,}330 \text{ cm})^3$$

From the PCI design handbook, select a nontopped normal weight concrete double-T 12 DT 34 168-D1, since it has the bottom-section modulus value S_b closest to the required value.

The section properties of the concrete are as follows:

$A_c = 978 \text{ in.}^2$ $\qquad c_t = 8.23 \text{ in.}$

$I_c = 86{,}072 \text{ in.}^4$ $\qquad c_b = 25.77 \text{ in.}$

$r^2 = \dfrac{I_c}{A_c} = 88.0 \text{ in.}^2$ $\qquad e_c = 22.02 \text{ in.}$

$S^t = 10{,}458 \text{ in.}^3$ $\qquad e_e = 12.77 \text{ in.}$

$S_b = 3{,}340 \text{ in.}^3$ $\qquad W_D = 1{,}019 \text{ plf}$

$\dfrac{V}{S} = 2.39 \text{ in.}$

Selection of Geometric Dimensions – Variable Tendon Eccentricity Example 1

Design of Strands and Check of Stresses. The assumed self-weight is close to the actual self-weight of Fig. 4-7. Hence, use

$$M_D = \frac{1{,}019}{1{,}000} \times 5{,}400{,}000 = 5{,}502{,}600 \text{ in.-lb}$$

$$f_{pi} = 0.70 \times 270{,}000 = 189{,}000 \text{ psi}$$

$$f_{pe} = 0.82 f_{pi} = 0.82 \times 189{,}000 = 154{,}980 \text{ psi}$$

(a) *Analysis of Stresses at Transfer.* From Equation 4.1a,

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} \le f_{ti} = 184 \text{ psi}$$

Then

$$184 = -\frac{P_i}{978}\left(1 - \frac{22.02 \times 8.23}{88.0}\right) - \frac{5{,}502{,}600}{10{,}458}$$

$$P_i = (184 + 526.16)\frac{978}{1.06} = 655{,}223 \text{ lb.}$$

Required number of tendons $= \dfrac{655{,}223}{189{,}000 \times 0.153} = 22.66$ $\frac{1}{2}$-in. dia. tendons.

Try sixteen $\frac{1}{2}$in. dia. strands for the standard section:

$$A_{ps} = 16 \times 0.153 = 2.448 \text{ in}^2 \ (15.3 \text{ cm}^2)$$

$$P_i = 2.448 \times 189{,}000 = 462{,}672 \text{ lb } (2{,}058 \text{ kN})$$

$$P_e = 2.448 \times 154{,}980 = 379{,}391 \text{ lb } (1{,}688 \text{ kN})$$

Selection of Geometric Dimensions – Variable Tendon Eccentricity Example 1

(b) *Analysis of Stresses at Service Load at Midspan*

$$P_e = 379{,}391 \text{ lb}$$

$$M_{SD} = \frac{100(60)^2 12}{8} = 540{,}000 \text{ in.-lb (61 kN-m)}$$

$$M_L = \frac{1{,}100(60)^2 12}{8} = 5{,}940{,}000 \text{ in.-lb (788 kN-m)}$$

$$\text{Total moment } M_T = M_D + M_{SD} + M_L = 5{,}502{,}600 + 6{,}480{,}000$$
$$= 11{,}982{,}600 \text{ in.-lb (1,354 kN-m)}$$

From Equation 4.3a,

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t}$$

$$= -\frac{379{,}391}{978}\left(1 - \frac{22.02 \times 8.23}{88.0}\right) - \frac{11{,}982{,}600}{10{,}458}$$

$$= +411 - 1146 = -735 \text{ psi} < f_c = -2{,}250 \text{ psi, O.K.}$$

From Equation 4.3b,

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b}$$

$$= -\frac{379{,}391}{978}\left(1 + \frac{22.02 \times 25.77}{88.0}\right) + \frac{11{,}982{,}600}{3{,}340}$$

$$= -2{,}889 + 3{,}587 = +698 \text{ psi (T)} < f_t = +849 \text{ psi, O.K.}$$

Selection of Geometric Dimensions – Variable Tendon Eccentricity Example 1

(c) *Analysis of Stresses at Support Section*

$$e_e = 12.77 \text{ in. } (324 \text{ mm})$$

$$f_{ti} = 6\sqrt{f'_{ci}} = 6\sqrt{3{,}750} \cong 367 \text{ psi}$$

$$f_t = 12\sqrt{f'_c} = 12\sqrt{5{,}000} = 849 \text{ psi}$$

(i) *At Transfer*

$$f^t = -\frac{462{,}672}{978}\left(1 - \frac{12.77 \times 8.23}{88.0}\right) - 0 = +92 \text{ psi } (T)$$

$$f_b = -\frac{462{,}672}{978}\left(1 + \frac{12.77 \times 25.77}{88.0}\right) + 0 = -2{,}240 \text{ psi } (C)$$

$$< f_{ci} = -2{,}250 \text{ psi, O.K.}$$

If $f_b > f_{ci}$, the support eccentricity has to be changed.

(ii) *At Service Load*

$$f^t = -\frac{379{,}391}{978}\left(1 - \frac{12.77 \times 8.23}{88.0}\right) - 0 = +75 \text{ psi } (T) \quad < f_t = 849 \text{ psi, O.K.}$$

$$f_b = -\frac{379{,}391}{978}\left(1 + \frac{12.77 \times 25.77}{88.0}\right) + 0 = -1{,}840 \text{ psi } (C)$$

$$< f_c = -2{,}250 \text{ psi, O.K.}$$

Adopt the section for service-load conditions using sixteen $\frac{1}{2}$-in. (1.7 mm) strands with midspan eccentricity $e_c = 22.02$ in. (560 mm) and end eccentricity $e_e = 12.77$ in. (324 mm).

Selection of Geometric Dimensions – Variable Tendon Eccentricity Example 2

- **Variable Tendon Eccentricity with No Height Limit**

Design an I-section for a beam having a 65-ft (19.8 m) span to satisfy the following section modulus values: Use the same allowable stresses and superimposed loads as in Example 4.1.

$$\text{Required } S^t = 3{,}570 \text{ in}^3 \ (58{,}535 \text{ cm}^3)$$

$$\text{Required } S_b = 3{,}780 \text{ in}^3 \ (61{,}940 \text{ cm}^3)$$

Solution

Since the section moduli at the top and bottom fibers are almost equal, a symmetrical section is adequate. Next, analyze the section in Figure 4.8 chosen by trial and adjustment.

Analysis of Stresses at Transfer. From Equation 4.4d,

$$\bar{f}_{ci} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci})$$

$$= +184 - \frac{21.16}{40}(+184 + 2{,}250) \cong -1{,}104 \text{ psi } (C) \ (7.6 \text{ MPa})$$

$$P_i = A_c \bar{f}_{ci} = 377 \times 1{,}104 = 416{,}208 \text{ lb } (1{,}851 \text{ kN})$$

$$M_D = \frac{393(65)^2}{8} \times 12 = 2{,}490{,}638 \text{ in.-lb } (281 \text{ kN-m})$$

From Equation 4.4c, the eccentricity required at the section of maximum moment at midspan is

Selection of Geometric Dimensions – Variable Tendon Eccentricity Example 2

$$e_c = (f_{ti} - \bar{f}_{ci})\frac{S^t}{P_i} + \frac{M_D}{P_i}$$

$$= (184 + 1{,}104)\frac{3{,}340}{416{,}208} + \frac{2{,}490{,}638}{416{,}208}$$

$$= 10.34 + 5.98 = 16.32 \text{ in. (415 mm)}$$

Since $c_b = 18.84$ in., and assuming a cover of 3.75 in., try $e_c = 18.84 - 3.75 \cong 15.0$ in. (381 mm).

$$\text{Required area of strands } A_p = \frac{P_i}{f_{pi}} = \frac{416{,}208}{189{,}000} = 2.2 \text{ in}^2 \text{ (14.2 cm}^2\text{)}$$

$$\text{Number of strands} = \frac{2.2}{0.153} = 14.38$$

Try thirteen $\frac{1}{2}$-in. strands, $A_p = 1.99$ in.^{2} (12.8 cm^2), and an actual $P_i = 189{,}000 \times 1.99 =$ 376,110 lb (1,673 kN), and check the concrete extreme fiber stresses. From Equation 4.1a

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t}$$

$$= -\frac{376{,}110}{377}\left(1 - \frac{15.0 \times 21.16}{187.5}\right) - \frac{2{,}490{,}638}{3{,}340}$$

$$= +691.2 - 745.7 = -55 \text{ psi } (C)\text{, no tension at transfer, O.K.}$$

From Equation 4.1b

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b}$$

$$= -\frac{376{,}110}{377}\left(1 + \frac{15 \times 18.84}{187.5}\right) + \frac{2{,}490{,}638}{3{,}750}$$

$$= -2{,}501.3 + 664.2 = -1{,}837 \text{ psi } (C) < f_{ci} = 2{,}250 \text{ psi, O.K.}$$

Selection of Geometric Dimensions – Variable Tendon Eccentricity Example 2

Analysis of Stresses at Service Load. From Equation 4.3a

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t}$$

$$P_e = 13 \times 0.153 \times 154{,}980 = 308{,}255 \text{ lb } (1{,}371 \text{ kN})$$

$$M_{SD} + M_L = \frac{(100 + 1100)(65)^2}{8} \times 12 = 7{,}605{,}000 \text{ m} - \text{lb}$$

$$\text{Total moment } M_T = M_D + M_{SD} + M_L = 2{,}490{,}638 + 7{,}605{,}000$$

$$= 10{,}095{,}638 \text{ in.-lb } (1{,}141 \text{ kN-m})$$

$$f^t = -\frac{308{,}225}{377}\left(1 - \frac{15.0 \times 21.16}{187.5}\right) - \frac{10{,}095{,}638}{3{,}340}$$

$$= +566.5 - 3{,}022.6 = -2{,}456 \text{ psi } (C) \; > f_c = -2{,}250 \text{ psi}$$

Hence, either enlarge the depth of the section or use higher strength concrete. Using $f'_c = 6{,}000$ psi,

$$f_c = 0.45 \times 6{,}000 = -2{,}700 \text{ psi, O.K.}$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b} = -\frac{308{,}255}{377}\left(1 + \frac{15.0 \times 18.84}{187.5}\right) + \frac{10{,}095{,}638}{3{,}750}$$

$$= -2{,}050 + 2{,}692.2 = 642 \text{ psi } (T)\text{, O.K.}$$

Selection of Geometric Dimensions – Variable Tendon Eccentricity Example 2

Check Support Section Stresses

Allowable $f'_{ci} = 0.75 \times 6{,}000 = 4{,}500$ psi

$f_{ci} = 0.60 \times 4{,}500 = 2{,}700$ psi

$f_{ti} = 3\sqrt{f'_{ci}} = 201$ psi for midspan

$f_{ti} = 6\sqrt{f'_{ci}} = 402$ psi for support

$f_c = 0.45 f'_c = 2{,}700$ psi

$f_{t1} = 6\sqrt{f'_c} = 465$ psi

$f_{t2} = 12\sqrt{f'_c} = 930$ psi

(a) *At Transfer*. Support section compressive fiber stress.

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + 0$$

$$P_i = 376{,}110 \text{ lb}$$

or

$$-2{,}700 = -\frac{376{,}110}{377}\left(1 + \frac{e \times 18.84}{187.5}\right)$$

so that

$$e_e = 16.98 \text{ in.}$$

Selection of Geometric Dimensions – Variable Tendon Eccentricity Example 2

To ensure a tensile stress at the top fibers within the allowable limits, try $e_e = 12.49$ in.:

$$f^t = -\frac{376{,}110}{377}\left(1 - \frac{12.49 \times 21.16}{187.5}\right) - 0$$

$$= 409 \text{ psi } (T) > f_{ti} = 402 \text{ psi}$$

$$f_b = -2250 \text{ psi}$$

Thus, use mild steel at the top fibers at the support section to take all tensile stresses in the concrete, or use a higher strength concrete for the section, or reduce the eccentricity.

(b) *At Service Load*

$$f^t = -\frac{308{,}255}{377}\left(1 - \frac{12.49 \times 21.16}{187.5}\right) - 0 = 335 \text{ psi } (T) < 930 \text{ psi, O.K.}$$

$$f_b = -\frac{308{,}255}{377}\left(1 + \frac{12.49 \times 18.84}{187.5}\right) + 0 = -1{,}844 \text{ psi } (C) < -2{,}700 \text{ psi, O.K.}$$

Hence, adopt the 40-in. (102-cm)-deep I-section prestressed beam of f'_c equal to 6,000 psi (41.4 MPa) normal-weight concrete with thirteen $\frac{1}{2}$-in. tendons having midspan eccentricity $e_c = 15.0$ in. (381 mm) and end section eccentricity $e_e = 12.5$ in. (318 m).

An alternative to this solution is to continue using $f'_c = 5{,}000$ psi, but change the number of strands and eccentricities.

Selection of Geometric Dimensions – Constant Tendon Eccentricity Example

- **Constant Tendon Eccentricity Example**

Solve Example 4.2 assuming that the prestressing tendon has constant eccentricity. Use $f'_c = 5{,}000$ psi (34.5 MPa) normal-weight concrete, permitting a maximum concrete tensile stress $f_t = 12\sqrt{f'_c} = 849$ psi.

Solution: Since the tendon has constant eccentricity, the dead-load and superimposed dead- and live-load moments at the support section of the simply supported beam are zero. Hence, the support section controls the design. The required section modulus at the support, from Equation 4.5a, is

$$S^t \geq \frac{M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$$

$$S_b \geq \frac{M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

Assume $W_D = 425$ plf. Then

$$M_D = \frac{425 \times (65)^2}{8} \times 12 = 2{,}693{,}438 \text{ in.-lb (304 kN-m)}$$

$$M_{SD} + M_L = 7{,}605{,}000 \text{ in.-lb (859 kN-m)}$$

Thus, the total moment $M_T = 10{,}298{,}438$ in.-lb (1,164 kN-m), and we also have

$$\text{Allowable } f_{ci} = -2{,}250 \text{ psi}$$

$$f'_{ci} = -3{,}750 \text{ psi}$$

$$f_{ti} = 6\sqrt{f'_{ci}} \text{ for support section} = 367 \text{ psi}$$

Selection of Geometric Dimensions – Constant Tendon Eccentricity Example

$$f_c = -2{,}250 \text{ psi } (15.5 \text{ MPa})$$

$$f_t = +849 \text{ psi}$$

$$\gamma = 0.82$$

$$\text{Required } S^t = \frac{10{,}298{,}438}{0.82 \times 184 + 2{,}250} = 4{,}289 \text{ in}^3 \ (72{,}210 \text{ cm}^3)$$

$$\text{Required } S_b = \frac{M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}} = \frac{10{,}298{,}438}{849 + 0.82 \times 2{,}250}$$

$$= 3{,}823 \text{ in}^3 \ (62{,}713 \text{ cm}^3)$$

First Trial. Since the required $S^t = 4{,}289$ in^3, which is greater than the available S^t in Example 4.2, choose the next larger I-section with $h = 44$ in. as shown in Figure 4.9. The section properties are:

$$I_c = 92{,}700 \text{ in}^4$$

$$r^2 = 228.9 \text{ in}^2$$

$$A_c = 405 \text{ in}^2$$

$$c_t = 23.03 \text{ in.}$$

$$S^t = 4{,}030 \text{ in}^3$$

$$c_b = 20.97 \text{ in}$$

$$S_b = 4{,}420 \text{ in}^3$$

$$W_D = 422 \text{ plf}$$

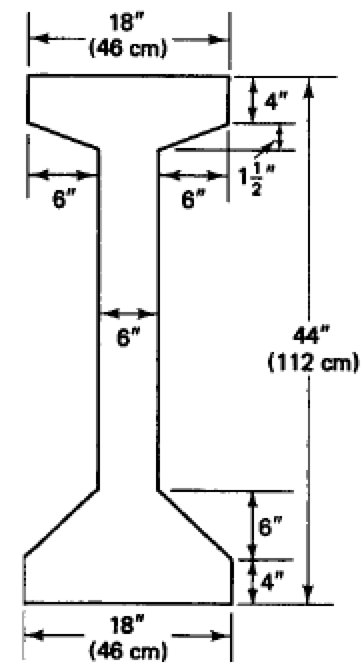


Figure 4.9 I-beam section

Selection of Geometric Dimensions – Constant Tendon Eccentricity Example

From Equation 4.5c, the required eccentricity at the critical section at the support is

$$e_e = (f_{ti} - \bar{f}_{ci})\frac{S^t}{P_i}$$

where

$$\bar{f}_{ci} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci})$$

$$= 367 - \frac{23.03}{44}(367 + 2{,}250) = -1{,}002 \text{ psi (6.9 MPa)}$$

and

$$P_i = A_c\bar{f}_{ci} = 405 \times 1{,}002 = 405{,}810 \text{ lb (1,805 kN)}$$

Hence,

$$e = (367 + 1{,}002)\frac{4{,}030}{405{,}810} = 13.60 \text{ in. (346 mm)}$$

The required prestressed steel area is

$$A_p = \frac{P_i}{f_{pi}} = \frac{405{,}810}{189{,}000} = 2.15 \text{ in}^2 \text{ (14.4 cm}^2\text{)}$$

So we try $\frac{1}{2}$in. strands tendon. The required number of strands is 2.15/0.153 = 14.05. Accordingly, use fourteen $\frac{1}{2}$in. (12.7 mm) tendons. As a result,

$$P_i = 14 \times 0.153 \times 189{,}000 = 404{,}838 \text{ lb (1,801 kN)}$$

Selection of Geometric Dimensions – Constant Tendon Eccentricity Example

(a) *Analysis of Stresses at Transfer at End Section.* From Equation 4.1a,

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} = -\frac{404{,}838}{405}\left(1 - \frac{13.60 \times 23.03}{228.9}\right) - 0$$

$$= +368.2 \text{ psi } (T) \cong f_{ti} = 367, \text{ O.K.}$$

From Equation 4.1b,

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b} = -\frac{404{,}838}{405}\left(1 + \frac{13.6 \times 20.97}{228.9}\right) + 0$$

$$= -2{,}245 \text{ psi } (C) \cong f_{ci} = -2{,}250, \text{ O.K.}$$

(b) *Analysis of Final Service-Load Stresses at Support*

$$P_e = 14 \times 0.153 \times 154{,}980 = 331{,}967 \text{ lb } (1{,}477 \text{ kN})$$

$$\text{Total moment } M_T = M_D + M_{SD} + M_L = 0$$

From Equation 4.3a,

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t}$$

$$= -\frac{331{,}967}{405}\left(1 - \frac{13.60 \times 23.03}{228.9}\right) - 0 = 302 \text{ psi } (T) < f_t = 849 \text{ psi, O.K.}$$

Selection of Geometric Dimensions – Constant Tendon Eccentricity Example

This is also applicable to midspan since eccentricity e is constant. From Equation 4.3b

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b}$$

$$= -\frac{331,967}{405}\left(1 + \frac{13.60 \times 20.97}{228.9}\right) + 0$$

$$= -1,841 \text{ psi } (12.2 \text{ MPa})\ (C) < f_c = -2,250 \text{ psi, O.K.}$$

(c) *Analysis of Final Service-Load Stresses at Midspan.* From before, the total moment $M_T = M_D + M_{SD} + M_L = 10,298,438$ in.-lb. Revised $w_D = 422$ plf = assumed $w_D = 425$ plf; hence, $M_T \simeq 10,298,438$ in.-lb is sufficiently accurate. So the extreme concrete fiber stress due to M_T is

$$f_1^t = \frac{M_T}{S^t} = -\frac{10,298,438}{4,030} = -2,555 \text{ psi } (C)\ (17.6 \text{ MPa})$$

$$f_{1b} = \frac{M_T}{S_b} = \frac{10,298,438}{4,420} = +2,330 \text{ psi } (T)\ (16.1 \text{ MPa})$$

Hence, the final midspan fiber stresses are

$$f^t = +302 - 2,555 = -2,253 \text{ psi } (C) \cong f_c = -2,250 \text{ psi, accept}$$

$$f_b = -1,841 + 2,330 = +489 \text{ psi } (T) < f_t = 849 \text{ psi, O.K.}$$

Consequently, accept the trial section with a constant eccentricity $e = 13.60$ in. (345 mm) for the fourteen $\frac{1}{2}''$ (12.7 mm dia.) tendons.