# **Equivalent Static Load Method**

- $\triangleright$  There are two commonly used methods for specifying seismic design forces:
	- 1. Equivalent Static Force Method
	- 2. Dynamic Analysis
- $\triangleright$  In the equivalent static force method, the inertial forces are specified as static forces using empirical formulas. The empirical formulas do not explicitly account for the "dynamic characteristics" of the particular structure being designed or analyzed.
- $\triangleright$  The formulas were, however, developed to adequately represent the dynamic behavior of what are called "regular" structures, which have a reasonably uniform distribution of mass and stiffness. For such structures, the equivalent static force method is most often adequate.

The concept employed in *equivalent static lateral method* is to place static loads on a structure with magnitudes and direction that closely approximate the effects of dynamic loading caused by earthquakes.

Concentrated lateral forces due to dynamic loading tend to occur at floor and ceiling/roof levels in buildings, where concentration of mass is the highest.

Furthermore, concentrated lateral forces tend to be larger at higher elevations in a structure.

Thus, the greatest lateral displacements and the largest lateral forces often occur at the top level of a structure (particularly for tall buildings). These effects are modeled in *equivalent static lateral force* procedures of the IBC and UBC by placing a force at each story level in a structure

In general, the distribution of lateral story forces is associated with the first (fundamental) mode of vibration of a cantilevered structure. (In this case, a typical structure is idealized as a vertical cantilever rigidly attached to the ground.)

The effects of higher modes of vibration are approximated in the UBC by considering an additional lateral force, **F<sup>t</sup>** , applied to the top level of a structure.

The summation of the lateral story forces (plus the additional lateral force at the top, Ft, in the UBC) must be equivalent to the base shear (V) force applied to the structure due to seismic ground motion.



where

- **V** = base shear force associated with ground motion at the base of the structure
- **F<sup>x</sup>** = lateral story force applied at each story level of the structure
- **F<sup>t</sup>** = additional lateral force applied at the top level of the structure (per the UBC)

#### **Base Shear**

*Base shear* is an estimate of the **maximum expected lateral force** that will occur due to seismic ground motion at the base of a structure. Calculations of base shear (**V**) depend on:

- $\triangleright$  soil conditions at the site
- $\triangleright$  proximity to potential sources of seismic activity (such as geological faults)
- $\triangleright$  probability of significant seismic ground motion
- $\triangleright$  the level of ductility and overstrength associated with various structural configurations and the total weight of the structure
- $\triangleright$  the fundamental (natural) period of vibration of the structure when subjected to dynamic loading

## Equivalent Static Force Method: Seismic Zones

The UBC addresses the probability of significant seismic activity in various locations by categorizing geographic regions of the into Seismic Zones 1 through 4. Seismic Zone 1 indicates a geographic location where least seismic activity is expected to occur. Seismic Zone 4 indicates a geographic location with a high probability of significant seismic activity.



• The total design base shear in a given direction shall be determined from the following formula:

$$
V = \frac{C_v I}{RT} W \qquad UBC97 \quad 30-4
$$

- The total design base shear need not exceed the following:  $V = \frac{2.5C_aI}{R}W$  $UBC97 30-5$
- The total design base shear shall not be less than the following:

$$
V = 0.11C_a I W \qquad \qquad UBC97 \quad 30-6
$$

• In addition, for seismic zone 4, the total base shear shall also be not less than the following:

$$
V = \frac{0.8ZN_vI}{R}W \qquad UBC97 \quad 30-7
$$

Where:

- Z= seismic zone factor, Table 16-I.
- I= importance factor, Table 16K. ٠
- R= numerical coefficient representative of the inherent over strength and global ductility capacity of lateral- force- resisting systems, Table 16-N.
- $C_a$ =acceleration seismic coefficient, Table 16-Q.
- $C_v$  velocity seismic coefficient, Table 16-R.
- $N_v$ = near source factor, Table 16-T.

#### • **Seismic Dead Load (***W***)**

The seismic dead load (*W*) consists of the following:

- Dead load of the structure.
- **25 percent** of the floor live load for storage and warehouses.
- A minimum allowance of **50 kg/m<sup>2</sup>** for movable partitions.
- The total weight of permanent equipment and fittings.



#### **TABLE 16-I-SEISMIC ZONE FACTOR Z**

NOTE: The zone shall be determined from the seismic zone map in Figure 16-2.

#### I= importance factor, Table 16K.  $\bullet$

#### **TABLE 16-K-OCCUPANCY CATEGORY**



The limitation of  $I_p$  for panel connections in Section 1633.2.4 shall be 1.0 for the entire connector.<br><sup>2</sup>Structural observation requirements are given in Section 1702.

<sup>3</sup>For anchorage of machinery and equipment required for life-safety systems, the value of  $I_p$  shall be taken as 1.5.

#### **Ductility**

• **Ductility** of an element shows its capacity to deform in the inelastic range without collapse.



#### **Response Modification Factor (R)**

- The value of the response modification factor (R) is determined from consideration of a structure's over-strength capacity beyond the point at which the elastic response of the structure is exceeded.
- This reduced force level is made possible by the energy absorption and dissipation capacity of the structure at displacements in excess of initial yield.

#### R= Response modification

factor, numerical coefficient representative of the inherent over strength and global ductility capacity of lateralforce- resisting systems, **Table 16-N.** 



TABLE 16-N-STRUCTURAL SYSTEMS<sup>1</sup>

#### Ca=acceleration seismic coefficient, **Table 16-Q.**



#### TABLE 16-Q-SEISMIC COEFFICIENT C

<sup>1</sup>Site-specific geotechnical investigation and dynamic site response analysis shall be performed to determine seismic coefficients for Soil Profile Type  $S_E$ 

**TABLE 16-J-SOIL PROFILE TYPES** 

		AVERAGE SOIL PROPERTIES FOR TOP 100 FEET (30 480 mm) OF SOIL PROFILE		
<b>SOIL PROFILE</b> <b>TYPE</b>	SOIL PROFILE NAME/GENERIC <b>DESCRIPTION</b>	Shear Wave Velocity, $v_s$ feet/second (m/s)	Standard Penetration Test, $\overline{N}$ [or $\overline{N}_{CH}$ for cohesionless soil layers] (blows/foot)	Undrained Shear Strength, $\overline{s}_U$ psf (kPa)
$S_A$	Hard Rock	> 5,000 (1,500)		
$S_{B}$	Rock	2,500 to 5,000 $(760 \text{ to } 1,500)$		
$S_C$	Very Dense Soil and Soft Rock	1,200 to 2,500 $(360 \text{ to } 760)$	> 50	> 2,000 (100)
Sр	Stiff Soil Profile	600 to 1,200 (180 to 360)	15 to 50	1,000 to 2,000 $(50 \text{ to } 100)$
$S_{E}^{-1}$	Soft Soil Profile	< 600 (180)	$\leq 15$	$\leq 1,000$ (50)
SF	Soil Requiring Site-specific Evaluation. See Section 1629.3.1.			

<sup>1</sup>Soil Profile Type S<sub>E</sub> also includes any soil profile with more than 10 feet (3048 mm) of soft clay defined as a soil with a plasticity index,  $PI > 20$ ,  $w_{mc} \ge 40$  percent and  $s_u < 500$  psf (24 kPa). The Plasticity Ind

#### $C_v$ = velocity seismic coefficient, **Table 16-R.**



#### TABLE 16-R-SEISMIC COEFFICIENT Cv

<sup>1</sup>Site-specific geotechnical investigation and dynamic site response analysis shall be performed to determine seismic coefficients for Soil Profile Type S<sub>F</sub>.

#### N<sub>v</sub>= near source factor, **Table 16-T.**

#### TABLE 16-T-NEAR-SOURCE FACTOR N<sub>v</sub>1



# Equivalent Static Force Method: Fundamental Time Period of a Structure

- The time period of a structure may exactly be calculated by performing free vibration analysis of the structure, which involves lengthy calculations
- Following empirical methods are also available to reasonably guess the fundamental time period of a structure:

## • **Approximate method**

• Fundamental time period,

$$
T = \frac{\text{number of stories}}{10} \text{ sec}
$$

# Equivalent Static Force Method: Fundamental Time Period of a Structure

• **Method A of UBC**

$$
T_A = C_t (h_n)^{\frac{3}{4}}
$$

- where
- *h*<sub>n</sub> = height of the roof above the base in meters, not including the height of parapets.
- *C<sup>t</sup>* = 0.085 for steel moment resisting frames

= 0.073 for reinforced concrete moment resisting frames and eccentric braced steel frames

= 0.050 for all other buildings

# Equivalent Static Force Method: Fundamental Time Period of a Structure

• **Method B of UBC**

$$
T_B = 2\pi \sqrt{\frac{\sum W_i \delta_i^2}{g \sum f_i \delta_i}}
$$

 $\leq 1.4T_A$  for Zones 1,2 and 3

 $\leq 1.3$   $T_A$  for Zones 4

- where
	- $\delta_i$  = static elastic deflection at level "*i*" due to the forces applied at all levels, increasing in a linear way with height. The value of deflection must be with respect to the base in mm.

$$
= \frac{\text{total lateral force at } i - \text{th floor}}{k_i} + \delta_{i-1}
$$

- $k_i$  = shear stiffness of columns under floor "*i*"
- *f <sup>i</sup>* = lateral force at level "*i*", N
- *wi*= dead load located at level "*i*", N
- *g* = acceleration due to gravity
	- = 9810 mm/sec

Calculate the empirical time period for the steel moment resisting frame, shown in Fig. , located on a site in zone-3



#### Solution:

1. Approximate Method:

$$
T = number of stories / 10 = 0.3 sec
$$

2. Method A:

$$
h_n
$$
 = 3 × 3.7 = 11.1 m  
\n $T_A$  =  $C_t (h_n)^{3/4}$  = 0.085(11.1)<sup>3/4</sup> = 0.517 sec

#### 3. Method B:

$$
k_1 = 105000 \text{ N/mm}, k_2 = 70000 \text{ N/mm} \text{ and } k_3 = 52500 \text{ N/mm}
$$
  
\n
$$
\delta_1 = (f_3 + f_2 + f_1) / k_1
$$
  
\n
$$
= (525 + 350 + 175) (1000) / 105000
$$
  
\n
$$
= 10 \text{ mm}
$$
  
\n
$$
\delta_2 = (f_3 + f_2) / k_2 + \delta_1
$$
  
\n
$$
= (525 + 350) (1000) / 70000 + 10
$$
  
\n
$$
= 22.5 \text{ mm}
$$
  
\n
$$
\delta_3 = f_3 / k_3 + \delta_2
$$
  
\n
$$
= 525000 / 52500 + 22.5
$$
  
\n
$$
= 32.5 \text{ mm}
$$

$$
\sum W_i \delta_i^2 = (455000 \times 10^2 + 455000 \times 22.5^2 + 455000 \times 32.5^2) \times 9.81
$$

- $= 7.421 \times 10^9$  N-mm<sup>2</sup>
- $= 175000 \times 10 + 350000 \times 22.5 + 525000 \times 32.5$  $\sum f_i \delta_i$ 
	- $= 2.669 \times 10^7$  N-mm<sup>2</sup>

$$
T_B = 2\pi \sqrt{\frac{\sum W_i \delta_i^2}{g \sum f_i \delta_i}} \le 1.4T_A \text{ for Zones } 1,2 \text{ and } 3
$$
  
=  $2\pi \sqrt{\frac{7.421 \times 10^9}{9810 \times 2.669 \times 10^7}} \le 1.4 \times 0.517$   
= 1.058  $\le 0.724$ 

$$
\sum W_i \delta_i^2 = (455000 \times 10^2 + 455000 \times 22.5^2 + 455000 \times 32.5^2) \times 9.81
$$

- $= 7.421 \times 10^9$  N-mm<sup>2</sup>
- $\sum f_i \delta_i$  $=$  175000  $\times$  10 + 350000  $\times$  22.5 + 525000  $\times$  32.5

 $= 2.669 \times 10^7 \text{ N-mm}^2$ 

$$
T_B = 2\pi \sqrt{\frac{\sum W_i \delta_i^2}{g \sum f_i \delta_i}} \le 1.4T_A \text{ for Zones } 1, 2 \text{ and } 3
$$
  
=  $2\pi \sqrt{\frac{7.421 \times 10^9}{9810 \times 2.669 \times 10^7}} \le 1.4 \times 0.517$   
= 1.058  
= 0.724 sec

## **Vertical Distribution of Forces**

A concentrated force  $F_t$  that shall be applied at the top of the structure as part of the base shear shall be determined from the following formula:  $F_t = 0.07TV \le 0.25V$  for  $T > 0.7$  seconds And

$$
F_t = 0.0 \, \text{for} \, T \leq 0.7 \, \text{seconds}
$$

#### So,

The total base shear, V will be:

$$
V = F_t + \sum_{i=1}^{n} F_i
$$

$$
F_x = \frac{(V - F_t)w_x h_x}{\sum_{i=1}^{n} w_i h_i}
$$

# **Vertical Distribution of Forces**

$$
F_x = \frac{(V - F_t)w_x h_x}{\sum_{i=1}^n w_i h_i}
$$

Where:

- $F_r$  = design seismic force applied to level x,
- $W<sub>x</sub>$  = that portion of weight, W located at or assigned to level x
- $W_i$ = that portion of weight, w located to or assigned to level i
- $h_{y}$ ,  $h_i$  = height in meters above the base to level x or i, respectively

# **Horizontal Distribution of Forces**

$$
V_{yi} = \frac{K_{yi}}{\sum K_{yi}} V_y \mp \frac{K_{yi} X_i}{K_t} V_y e
$$

$$
K_t = \sum K_{yi} X_i^2 + \sum K_{xi} Y_i^2
$$

Where:

- $V_{vi}$  = shear force to frame i
- $V_v =$  shear force
- $K_{vi}$ = stiffness of frame i
- $X_i$  = distance from frame i to center of stiffness of the floor in xdirection
- $Y_i$  = distance from frame i to center of stiffness of the floor in ydirection
- $\bullet$  e= eccentricity
- $K_t$  = torsional stiffness of resisting frames



- $\triangleright$  Number of stories = five
- $\triangleright$  Story height= 3.5m
- $\triangleright$  Concrete cylinder compressive strength at 28 days, f'c= 28MPa
- $\triangleright$  Steel yield strength, fy= 420MPa
- $\triangleright$  All columns are square with side length equals to 500mm
- All beams are 300mm section width and 600mm total thickness
- $\triangleright$  The slab is one way solid slab of 200mm thickness.

- $\triangleright$  The live load is 3kN/m<sup>2</sup>
- $\triangleright$  The superimposed dead load is 4kN/m<sup>2</sup>
- $\triangleright$  The perimeter wall weight is 3kN/m
- $\triangleright$  Importance factor =1
- $\triangleright$  Soil: soft rock, Sc type in accordance with UBC 97 provisions.
- $\triangleright$  Zone 2B, Zone Factor factor: Z=0.2 (Table 16-I)
- $\triangleright$  Ca= 0.24 (Table 16-Q)
- $\triangleright$  Cv= 0.32 (Table 16-R)

## • Step 1: weight of the building

Solid slab own weight= slab thickness x unit weight of concrete

 $W_D = 0.20 \times 25 = 5kN/m^2$  $W_{slab} = 25 \times 21 \times 5 = 2625 kN$ 

Beams weight= length of beams x cross section area x unit weight of concrete

 $W_{beams} = (21 \times 6 + 25 \times 2)(0.30 \times 0.40 \times 25) = 528kN$ It shall be noted that the beam depth below slab is used which is 0.60-0.20= 0.40m

## • Step 1: weight of the building

Columns weight= length of columns x cross section area x unit weight of concrete

 $W_{columns} = 2.9 \times 24 \times 0.50 \times 0.50 \times 25 = 435 kN$ Perimeter wall weight= length of wall x weight of wall/m

 $W_{walls} = (25 \times 2 + 21 \times 2)(3) = 276kN$ 

Superimposed dead load on slab= area of slab x superimposed dead load/ $m<sup>2</sup>$ 

```
W_{superimposed} = 25 × 21 × 4 = 2100kN
```
Total weight of one story= 5964kN Total weight of building= 5964 x 5= 29820kN

• Step 2: base shear

**Building period:** 

$$
T = C_t(h_n)^{\frac{3}{4}}
$$

$$
h_n = 5 \, \text{floors} \times 3.5 = 17.5m
$$
\n
$$
C_t = 0.0731
$$
\n
$$
T = 0.0731(17.5)^{\frac{3}{4}} = 0.625 \, \text{seconds}
$$

# • Step 2: base shear

Notice that the stiffness, k, which is the force divided by the lateral deformation, varies in each direction. But, in this example use this period, T, in both directions.

The base shear, V, is computed as follows:

 $R = 5.5$  from Table16-N (intermediate moment resisting frame)

$$
V = \frac{C_v I}{RT} W \qquad \qquad V = \frac{0.32 \times 1}{5.5 \times 0.625} W = 0.093W
$$

#### • Step 2: base shear

The base shear, V, need not exceed

$$
V = \frac{2.5C_aI}{R}W
$$
  
\n
$$
V = \frac{2.5 \times 0.24 \times 1}{5.5}W = 0.109W > 0.093W
$$
  
\nThe base shear shall not be less than  
\n
$$
V = 0.11C_aIW
$$
  
\n
$$
V = 0.11 \times 0.24 \times 1 \times W = 0.026W < 0.093W
$$
  
\nSo, total base shear will be  
\n
$$
V = 0.093 \times 29820 = 2773kN
$$
  
\nSince, T < 0.7 seconds, F<sub>r</sub>=0.0 kN

• Step 3: vertical distribution of base shear

$$
F_x = \frac{(V - F_t)w_x h_x}{\sum_{i=1}^n w_i h_i}
$$
  

$$
F_x = \frac{w_x x h}{w_x (1h + 2h + 3h + 4h + 5h)} = \frac{x}{15}V
$$

Where x is the number of the story. Table 1 shows a summary of distribution of forces for the stories.

#### Table 1: Distribution of forces to stories



• Step 4: horizontal distribution of story shear to frames

The force in each frame of y- direction is

$$
\frac{V}{6} = \frac{2773}{6} = 462kN
$$

The force in each frame of x- direction is

$$
\frac{V}{2} = \frac{2773}{2} = 1387kN
$$



If 5% eccentricity is assumed to base shear in ydirection; parallel to frames FY1 to FY6, then, for ydirection

Let the stiffness of each frame be K, so

$$
\sum K_{yi} = 6K
$$

$$
e = 0.05 \times 25 = 1.25m
$$
  

$$
K_t = 2(K(2.5)^2 + K(7.5)^2 + K(12.5)^2) = 437.5K
$$

So, for frame FY1 or FY6

$$
V_{y1,6} = \frac{K}{6K}V_y + \frac{K(12.5)}{437.5K}V_y(1.25) = 0.167V_y + 0.036V_y
$$
  
= 0.203V\_y  
0.203V\_y = 0.203(2773) = 563kN

It should be noted that the force is increased on the frame by

$$
\frac{563}{462} = 1.22
$$
 22% increase