Dynamics of Earthquake Analysis

Earthquake or seismic analysis

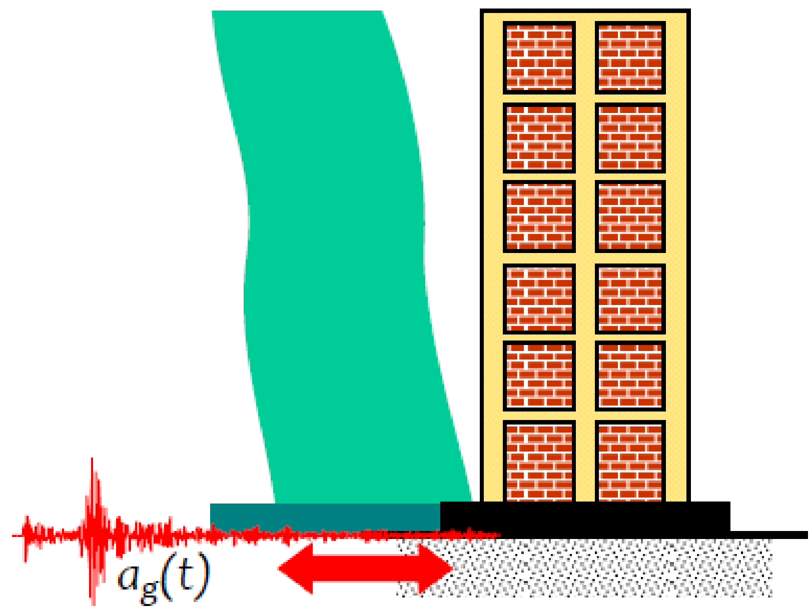
- Earthquake or seismic analysis is a subset of structural analysis which involves the calculation of the response of a structure subjected to earthquake excitation.

- The analysis is based on an appropriate ground motion representation and is performed using accepted principles of dynamics.

- The main methods of dynamic analysis are time- history analysis and response spectrum analysis.

1- Time History Method of Analysis:

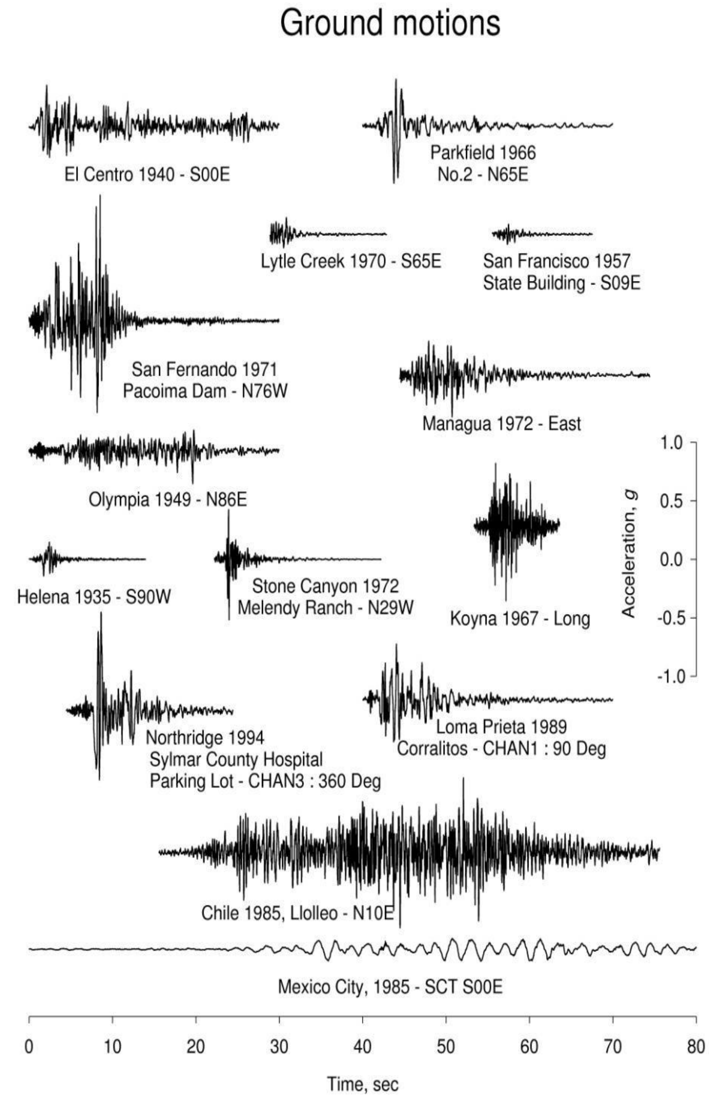
- Time-History analysis is an analysis of the dynamic response of the structure at each increment of time, when its base is subjected to a specific ground motion time history.
- It is a step-by-step procedure where the loading and the response history are evaluated at successive time increments.
- The time-history response method provides more detailed information regarding the seismic behaviour of a structure and is therefore used for more specific earthquake analyses.



TIME HISTORY DATA

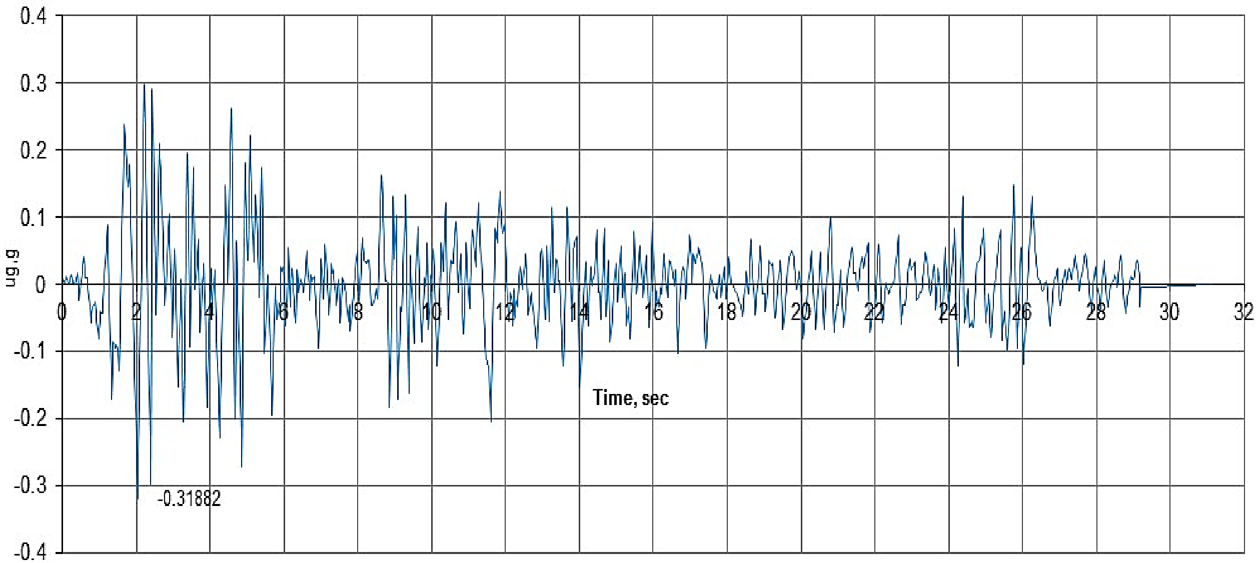
- The most direct description of an earthquake motion in time domain is provided by accelerograms that are recorded by instruments called *strong motion accelerographs*.

- The peak ground acceleration, duration, and frequency content of earthquake can be obtained from an accelerograms. And accelerogram can be integrated to obtain the time variations of the ground velocity and ground displacement.



TIME HISTORY DATA

El Centro ground motion (N-S Component)
May 18, 1940

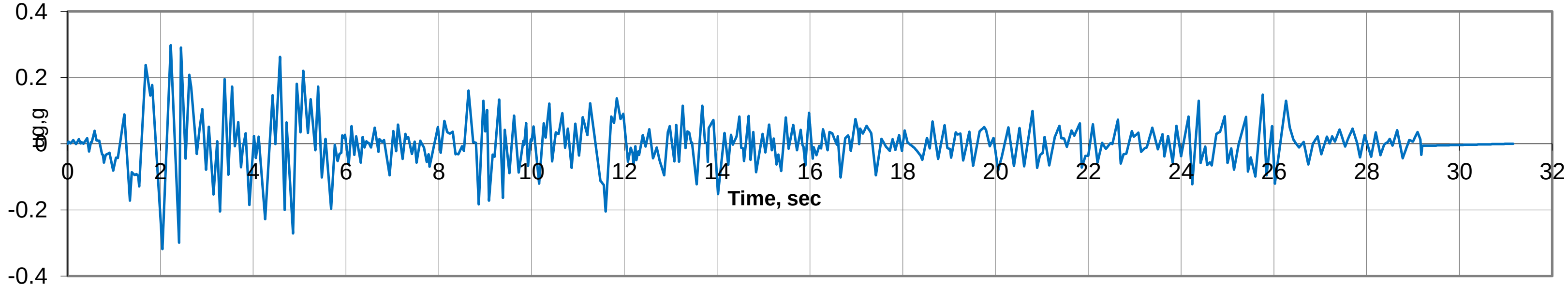


Time, sec	Acceleration, g
0.00	0.00630
0.02	0.00364
0.04	0.00099
0.06	0.00428
0.08	0.00758
0.10	0.01087
0.12	0.00682
0.14	0.00277
0.16	-0.00128
0.18	0.00368
0.20	0.00864
0.22	0.01360
0.24	0.00727
0.26	0.00094
0.28	0.00420
0.30	0.00221

PEER Ground Motion Database University of California, Berkeley

http://peer.berkeley.edu/smcat/

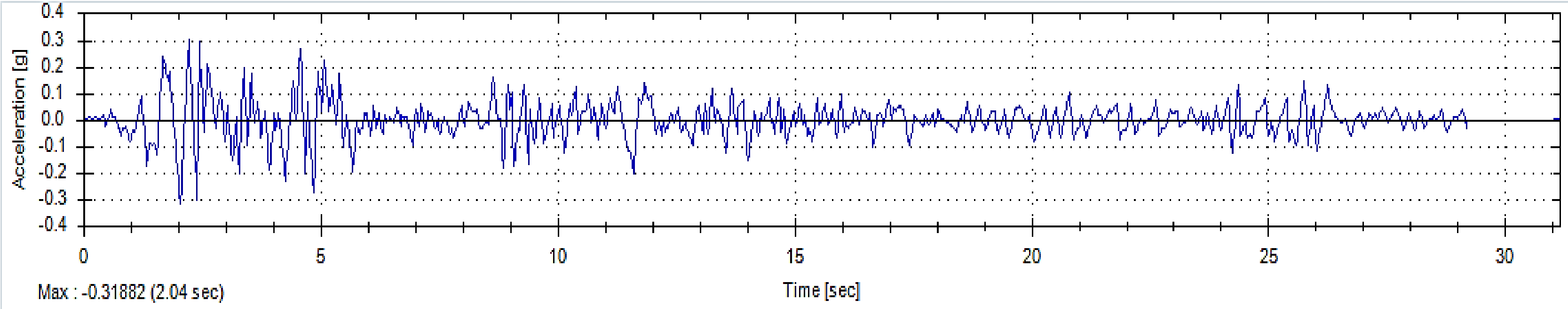
TIME HISTORY DATA

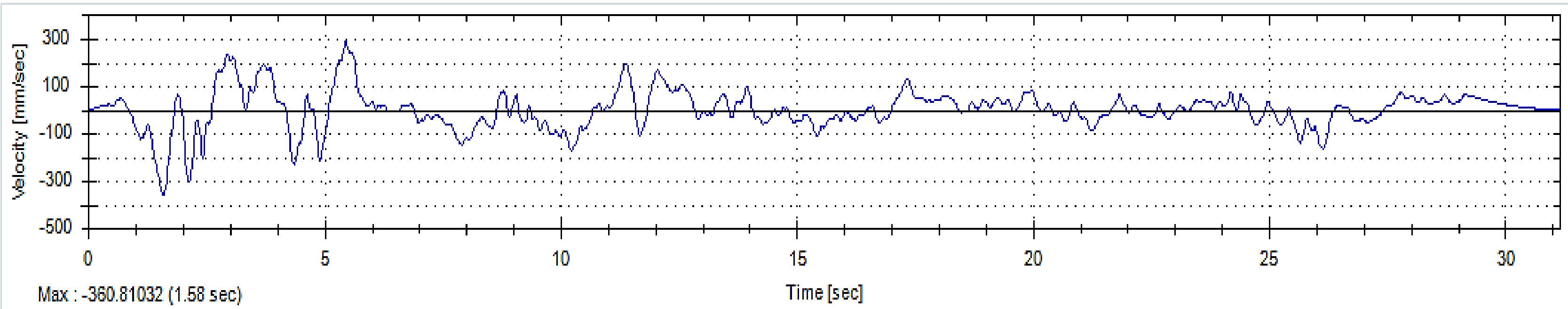


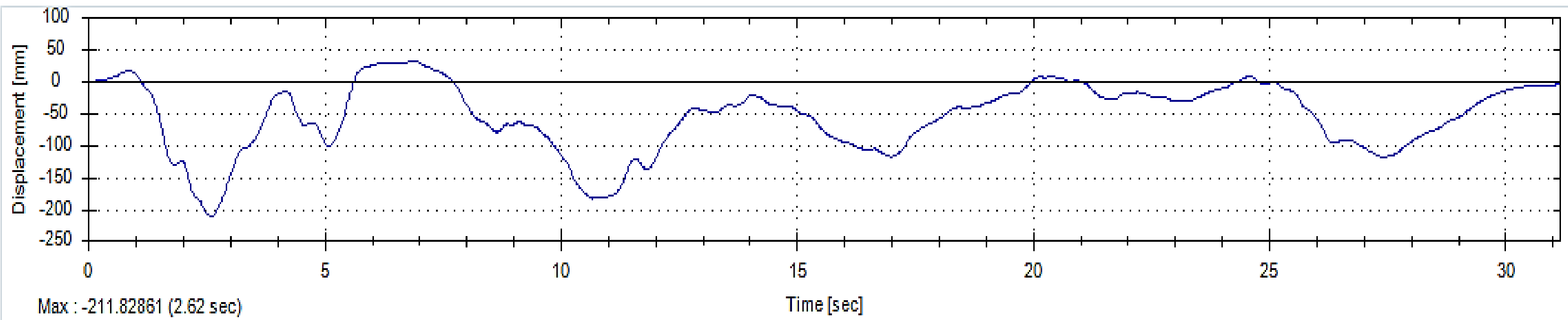
For earthquake excitation –

- Analytical solution is not possible;
- Numerical Methods are employed for dynamic analysis. Different numerical methods are:
  1. Central difference method
  2. Average acceleration method
  3. Newmark's method etc.

TIME HISTORY DATA ANALYSIS



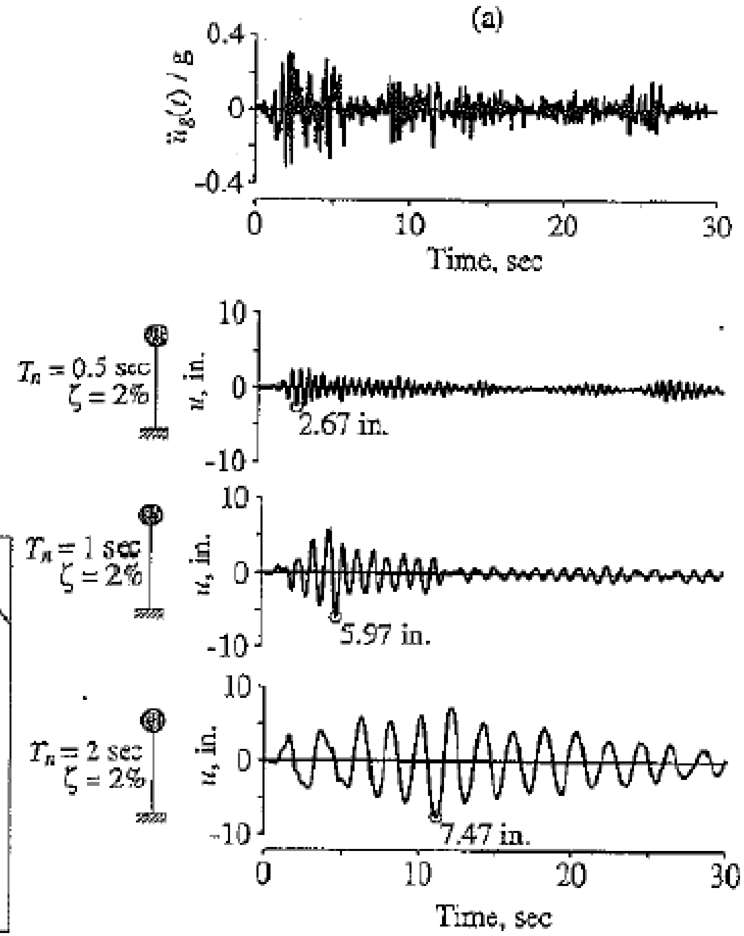




2-Response Spectrum Method

It is a plot of the peak value of response quantity of interest (e.g. relative displacement u,) as a function of the natural time period, T or any related parameter (e.g. circular frequency ω_n or cyclic frequency, f).

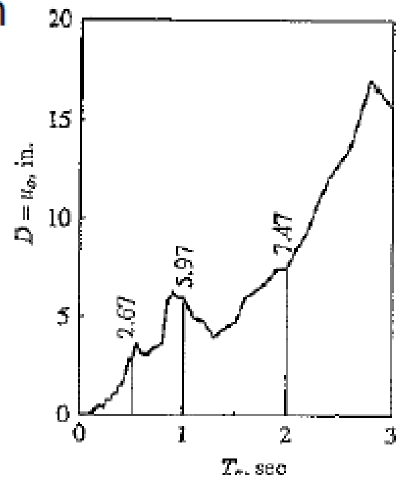
- The ground acceleration history is given.
- Select a Certain Value of Damping Ratio, ξ
- Calculate the response of the SDOF for each value of Time Period, T



For Example:

For $\xi = 2\%$, when

- $T = 0.5$ sec, u_{max} =2.67 in
- $T = 1$ sec, $u_{max} = 5.97$ in
- $T = 2$ sec, $u_{max} = 7.47$ in
- Similarly calculate and plot u_{max} for other values of T.



Response Spectrum Method

- In order to perform the seismic analysis and design of a structure to be built at a particular location, the actual time history record is required.

- However, it is not possible to have such records at each and every location. Further, the seismic analysis of structures cannot be carried out simply based on the peak value of the ground acceleration as the response of the structure depend upon the frequency content of ground motion and its own dynamic properties.

- To overcome the above difficulties, earthquake response spectrum is the most popular tool in the seismic analysis of structures.

- There are computational advantages in using the response spectrum method of seismic analysis for prediction of displacements and member forces in structural systems.

Response Spectrum

- The method involves the calculation of only the maximum values of the displacements and member forces in each mode of vibration using smooth design spectra that are the average of several earthquake motions.

- Response spectra are curves plotted between maximum response of SDOF system subjected to specified earthquake ground motion and its time period (or frequency).

- Response spectrum can be interpreted as the locus of maximum response of a SDOF system for given damping ratio.

- Response spectra thus helps in obtaining the peak structural responses under linear range, which can be used for obtaining lateral forces developed in structure due to earthquake thus facilitates in earthquake-resistant design of structures.

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- Response spectra thus helps in obtaining the peak structural responses under linear range, which can be used for obtaining lateral forces developed in structure due to earthquake thus facilitates in earthquake-resistant design of structures.

- Usually response of a SDOF system is determined by time domain or frequency domain analysis, and for a given time period of system, maximum response is picked. This process is continued for all range of possible time periods of SDOF system.

- Final plot with system time period on x-axis and response quantity (Displacement, velocity or acceleration) on y-axis is the required response spectra pertaining to specified damping ratio and input ground motion. Same process is carried out with different damping ratios to obtain overall response spectra.

Origin of the Response Spectrum Method

- Response Spectrum Method began in 1932, in the doctoral dissertation of M.A. Biot (1905–1985)

- The RSM remained in the academic sphere of research for many years and did not gain widespread engineering acceptance until the early 1970s because there were only a few well-recorded accelerograms that could be used for that purpose.

- Then, in 1971, with the occurrence of the San Fernando, California, earthquake, the modern era of RSM was launched.

- This earthquake was recorded by 241 accelerographs, and by combining these data with all previous strong-motion records it became possible to perform the first comprehensive empirical scaling analyses of response spectral amplitudes (Lee 2002).

Procedure for Calculation of Response Spectrum

The response spectrum for a given ground motion component $\ddot{u}_g(t)$ can be developed by implementation of the following steps:

1. Numerically define the ground acceleration $\ddot{u}_g(t)$; typically, the ground motion ordinates are defined every 0.02 sec.
2. Select the natural vibration period T_n and damping ratio ζ of an SDF system.
3. Compute the deformation response $u(t)$ of this SDF system due to the ground motion $\ddot{u}_g(t)$ by any of the numerical methods .
4. Determine u_o, the peak value of $u(t)$.
5. The spectral ordinates are $D = u_o$, $V = (2\pi/T_n)D$, and $A = (2\pi/T_n)^2 D$.
6. Repeat steps 2 to 5 for a range of T_n and ζ values covering all possible systems of engineering interest.
7. Present the results of steps 2 to 6 graphically to produce three separate spectra

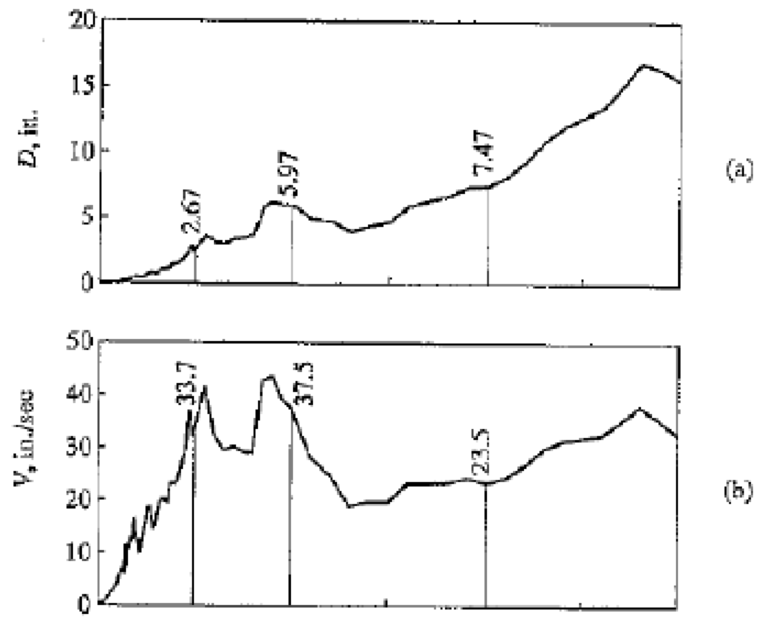
Pseudo Velocity Spectrum

The strain energy stored in a linear system is given by

$$E_{so} = \frac{ku_o^2}{2}$$

If u_o is taken from displacement response spectrum so that $D = (u_o)_{max}$

$$E_{so} = \frac{kD^2}{2} \quad Equation\ 1$$



To express the above equation in terms m,

$$V \equiv \omega D = \frac{2\pi}{T} D$$

Plugging in the value of D in Equation 1

$$E_{so} = \frac{k\left(\frac{T}{2\pi} \times V\right)^2}{2} = \frac{m\left(\frac{2\pi}{T}\right)^2\left(\frac{T}{2\pi} \times V\right)^2}{2} = \frac{mV^2}{2}$$

The R.H.S of the above equation is K.E. of mass m, moving with pseudo-velocity 'V'.

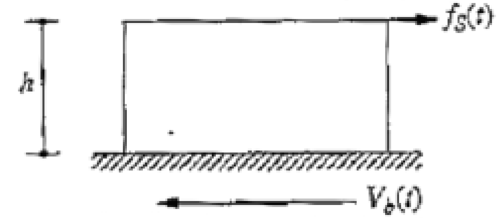
Pseudo Acceleration Spectrum

The equivalent static force f_S is given by

$$f_S(t) = ku(t)$$

Substituting the value of k

$$f_S(t) = m\,\omega_n^2\,u(t) = mA(t)$$



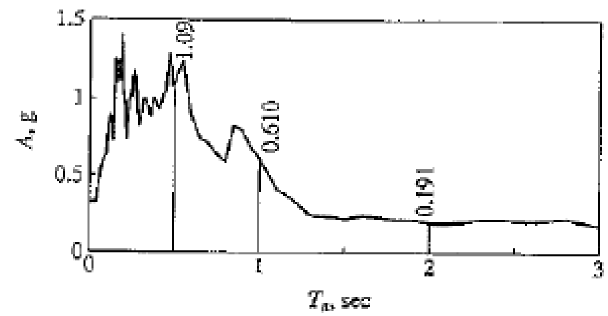
If u(t) is taken from the displacement response spectrum

i.e. $u(t) = D$

$$A = \omega_n^2 D = \left(\frac{2\pi}{T_n}\right)^2 D$$

For a portal frame, base shear in all columns is given by

$$V_b = f_s = mA = A/g \times W$$



Note:

- $\ddot{u} \neq A$
- The units of A are that of acceleration.
- A/g may be interpreted as base shear coefficient

The Combined D-V-A Spectrum

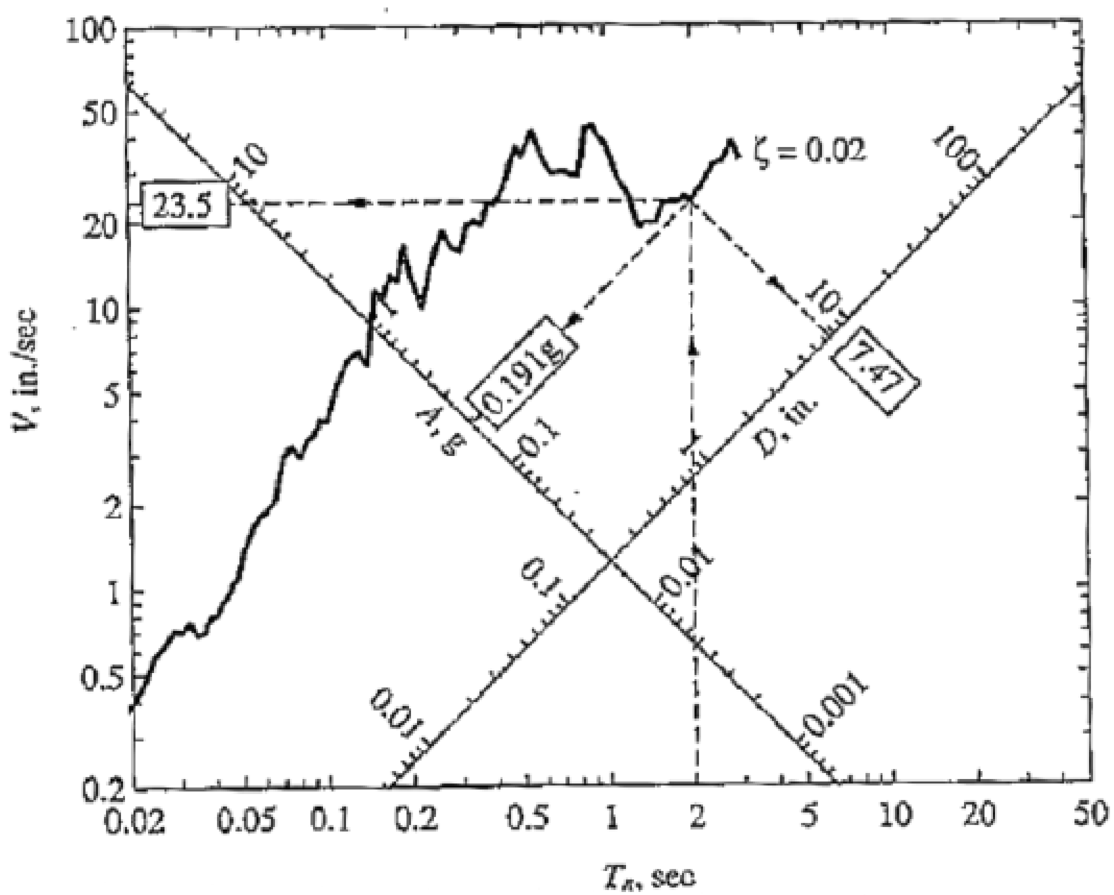
- This type of plot was first developed by A/S/ Veletsos and N.M. Newmark in 1960

- It is beneficial to plot Pseudo values of velocity and acceleration, along with displacement response spectrum on one figure.

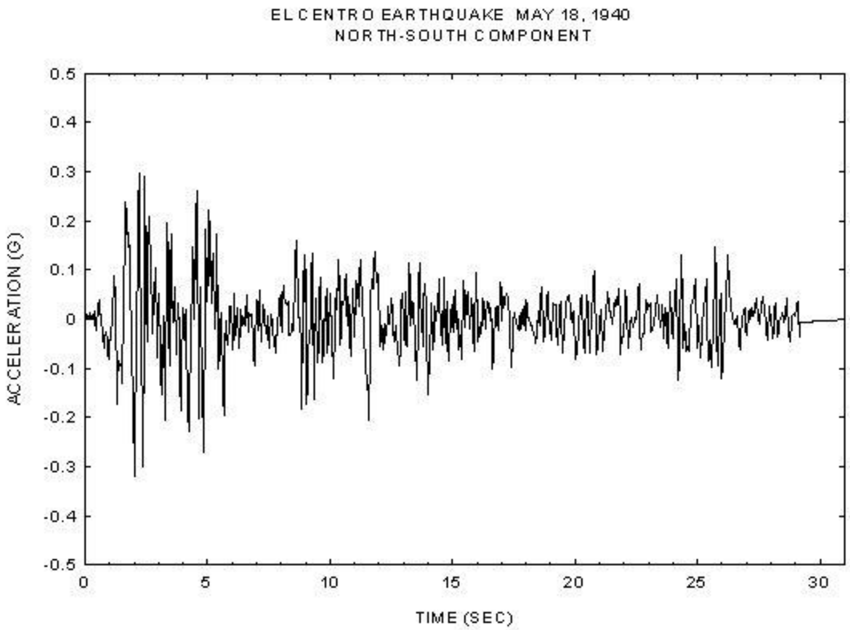
- Plotting of all three quantities on one figure is possible because the three are related i.e.

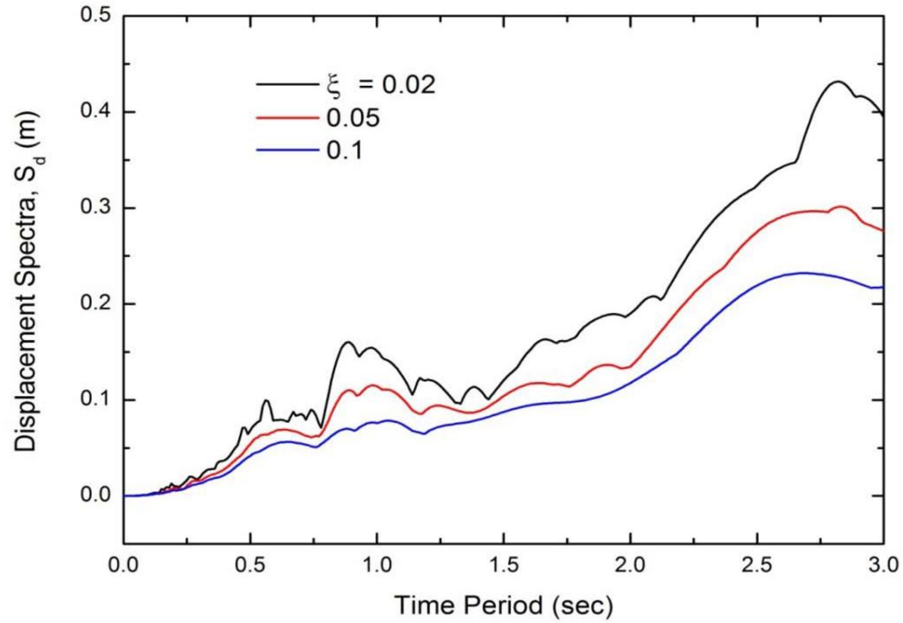
$$\frac{A}{\omega_n} = V = \omega_n D$$

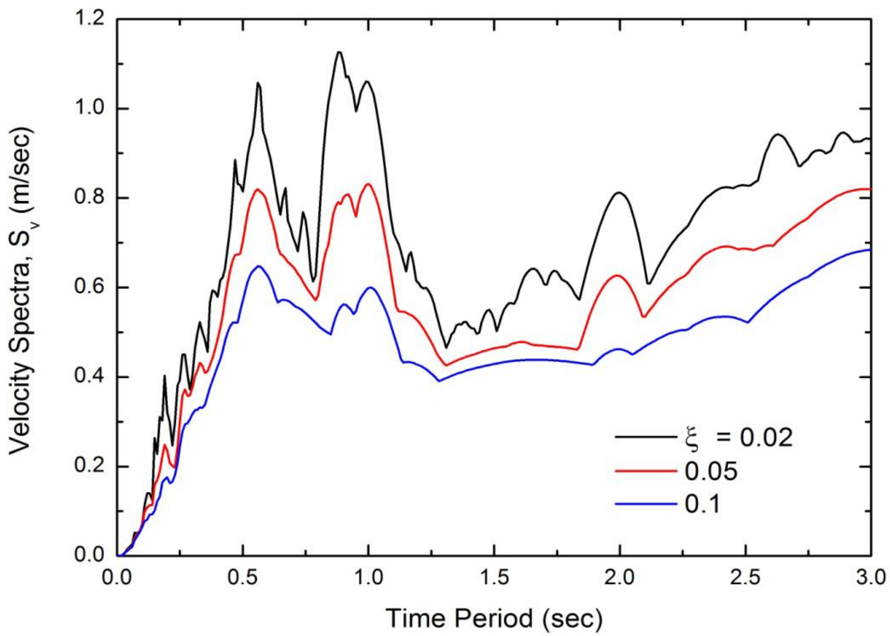
$$\frac{T_n}{2\pi} A = V = \frac{2\pi}{T_n} D$$

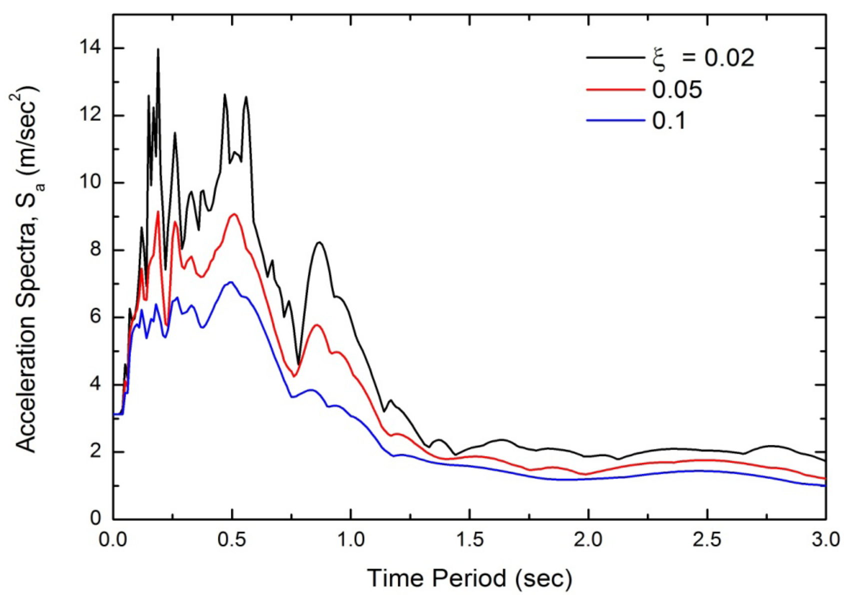


Response spectra of El-Centro, 1940 earthquake ground motion.









Example 1

Consider a SDOF system with mass, $m = 2 \times 10^3$ kg, stiffness, $k = 60$ kN/m and damping, $c = 0.44$ kN.sec/m. Using the response spectra of El-Centro, 1940 earthquake, compute (a) Maximum relative displacement, (b) Maximum base shear and (c) Maximum strain energy.

Solution: The natural frequency, time period and damping ratio of the SDOF system are

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{60 \times 10^3}{2 \times 10^3}} = 5.48 \; rad/\sec$$

$$T_0 = \frac{2\pi}{\omega_0} = 1.15 \text{ sec}$$

$$\xi = \frac{c}{2m\omega_0} = \frac{0.44 \times 10^3}{2 \times 2 \times 10^3 \times 5.48} = 0.02$$

From the response spectrum curve of El-Centro, 1940 earthquake ground motion for the time period of 1.15 sec and damping ratio of 0.02

$S_d = 0.11147$m and $S_a = 3.321$ m/sec^2

Example 1

(a) The maximum displacement

$$x_{\max} = S_d = 111.47 \text{ mm}$$

Alternatively, $x_{\max} \simeq \dfrac{S_a}{\omega_0^{\,2}} = \dfrac{3.321}{5.48^2} = 0.11055 \text{ m} = 110.55 \text{ mm}$

(b) The maximum base shear

$$V_{\max} = m S_a = 2\times10^3 \times 3.321 = 6.64 \text{ kN}$$

Alternatively, $V_{\max} = k\, x_{\max} = 60\times10^3 \times 0.11147 = 6.688 \text{ kN}$

(c) The maximum strain energy

$$E_{\max} = \frac{1}{2} k\, x_{\max}^{\;2} = \frac{1}{2} \times 60\times10^3 \times (0.11147)^2 = 372.76 \text{ N.m}$$