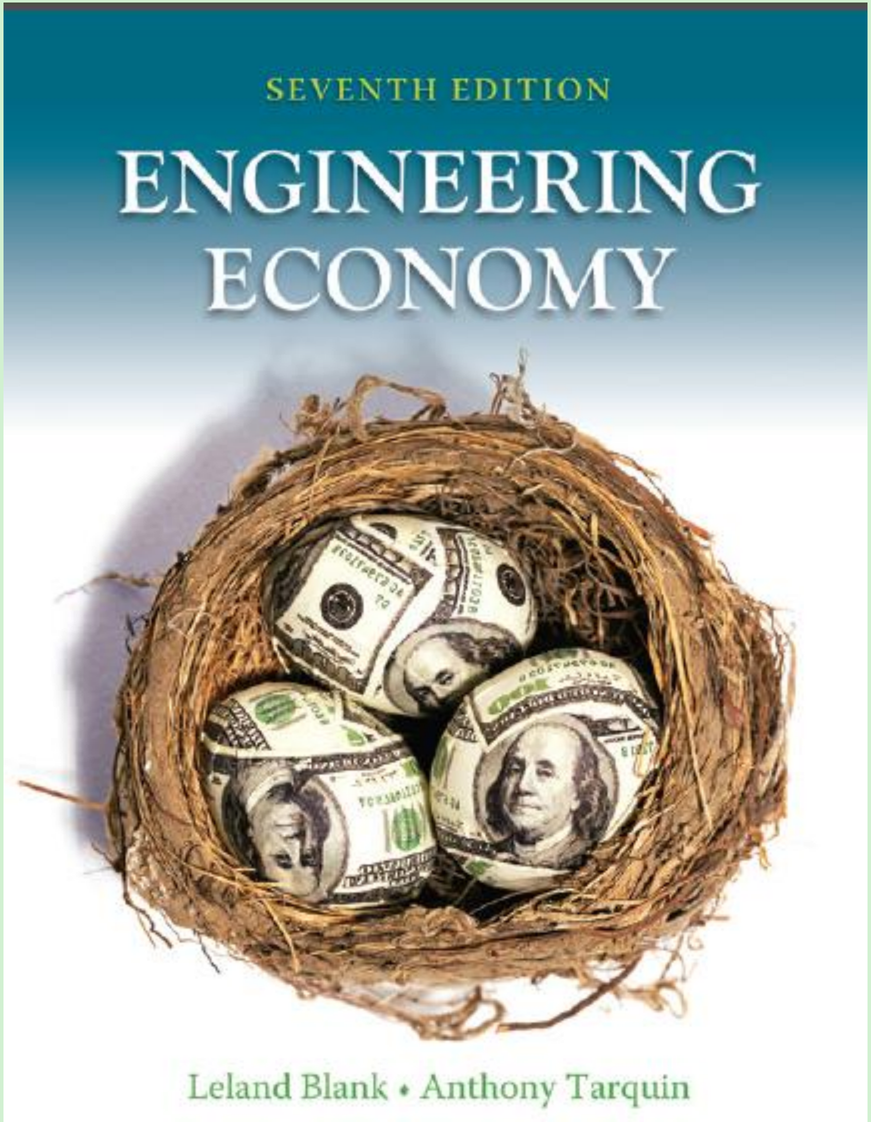

Chapter 5
Present Worth Analysis

Lecture slides to accompany

Engineering Economy

7th edition

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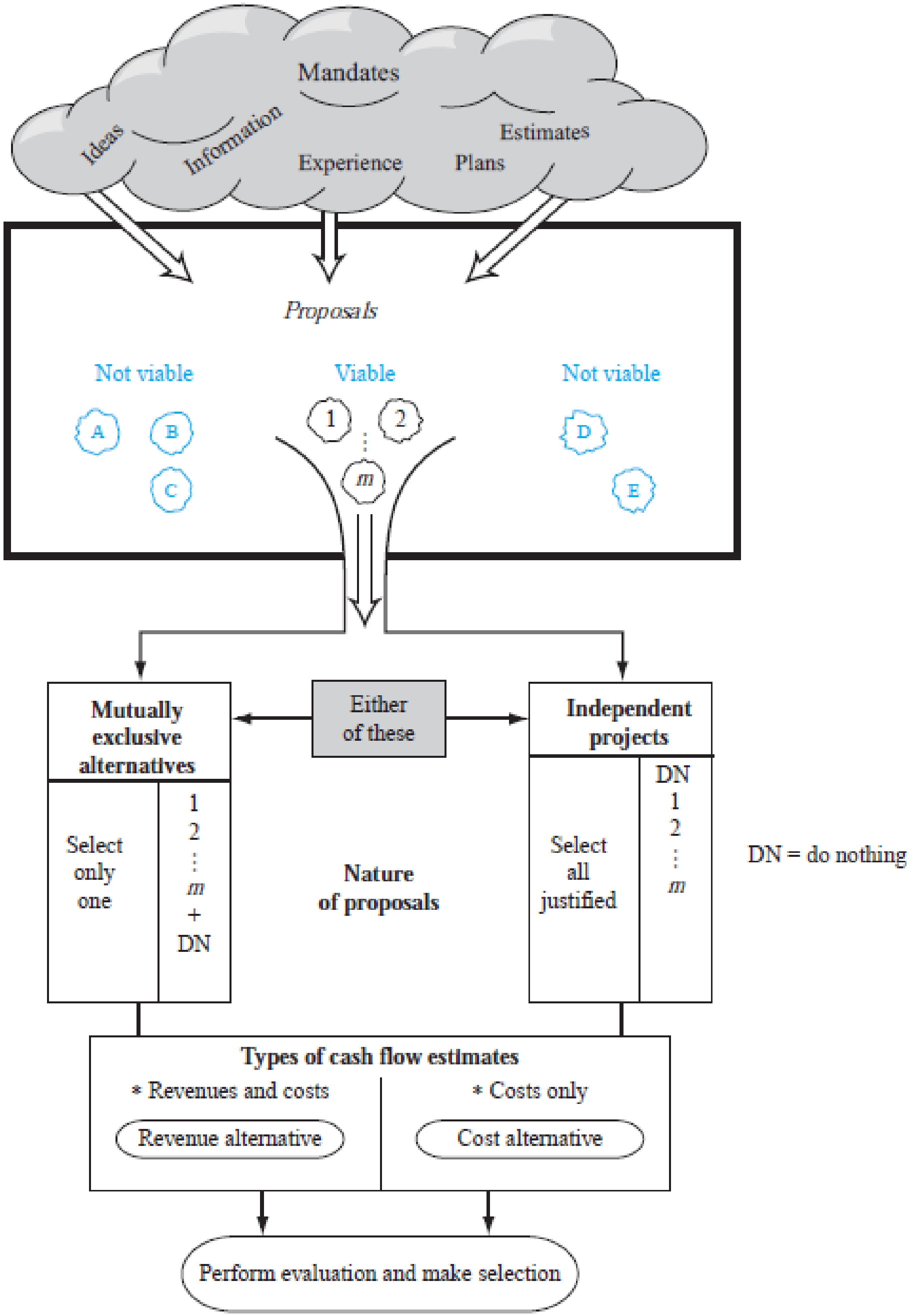
LEARNING OUTCOMES

1. Formulate Alternatives
2. PW of equal-life alternatives
3. PW of different-life alternatives
4. Future Worth analysis
5. Capitalized Cost analysis



Progression from proposals to economic evaluation to selection.

Formulating Alternatives

Two types of economic proposals

- **Mutually Exclusive (ME) Alternatives:** ***Only one*** **can be selected; Compete against each other**

- **Independent Projects:** ***More than one*** **can be selected; Compete only against DN**

Do Nothing (DN) – An ME alternative or independent project to maintain the current approach; no new costs, revenues or savings

Formulating Alternatives

A **mutually exclusive** selection takes place, for example, when an engineer must select the best diesel-powered engine from several available models. Only one is chosen, and the rest are rejected. If none of the alternatives are economically justified, then all can be rejected and, by default, the DN alternative is selected. For **independent projects** one, two or more, in fact, all of the projects that are economically justified can be accepted, provided capital funds are available. This leads to the two following fundamentally different evaluation bases:

Mutually exclusive alternatives ***compete with one another and are compared pair wise.***

Independent projects are evaluated one at a time and ***compete only with the DN project.***

Formulating Alternatives

Independent projects

Assume there are m independent projects. Zero, one, two, or more may be selected. Since each project may be in or out of the selected group of projects, there are a total of **2^m mutually exclusive alternatives.** This number includes the DN alternative. For example, if the engineer has three diesel engine models (A, B, and C) and may select any number of them, there are $2^3 = 8$ alternatives: DN, A, B, C, AB, AC, BC, ABC. Commonly, in real-world applications, there are restrictions, such as an upper budgetary limit, that eliminate many of the 2^m alternatives.

Formulating Alternatives

Two types of cash flow estimates

- **Revenue:** Alternatives include estimates of costs (cash outflows) *and* revenues (cash inflows)
- **Cost:** Alternatives include *only* costs; revenues and savings assumed equal for all alternatives; also called *service alternatives*

PW Analysis of Alternatives

- Convert all cash flows to PW using MARR
- Precede *costs* by *minus* sign; receipts by plus sign

EVALUATION

- For one project, if PW > 0, it is justified
- For mutually exclusive alternatives, select one with *numerically largest PW*
- For independent projects, select all with *PW > 0*

Selection of Alternatives by PW

For the alternatives shown below, which should be selected if they are (a) mutually exclusive; (b) independent?

Project ID	Present Worth
A	$30,000
B	$12,500
C	$-4,000
D	$ 2,000

Solution: (a) **Select numerically largest PW; alternative A**

(b) **Select all with PW > 0; projects A, B & D**

PW Evaluation of Equal-Life Mutually Exclusive Alternatives.

If the alternatives have the same capacities for the same time period (life), the **equal-service requirement is met. Calculate the PW value at the stated MARR for each alternative.**

*For **mutually exclusive (ME) alternatives, whether they are revenue or cost alternatives, the following** guidelines are applied to justify a single project or to select one from several alternatives.*

- **One alternative: If PW ≥ 0, the requested MARR is met or exceeded and the alternative is** economically justified.

- **Two or more alternatives: Select the alternative with the PW that is numerically largest,** that is, less negative or more positive. This indicates a lower PW of cost for cost alternatives or a larger PW of net cash flows for revenue alternatives.

Example: PW Evaluation of Equal-Life ME Alts.

Alternative X has a first cost of $20,000, an operating cost of $9,000 per year, and a $5,000 salvage value after 5 years. Alternative Y will cost $35,000 with an operating cost of $4,000 per year and a salvage value of $7,000 after 5 years. At an MARR of 12% per year, which should be selected?

Solution: Find PW at MARR and select numerically larger PW value

PW_X = -20,000 - 9000(P/A,12%,5) + 5000(P/F,12%,5)
= -$49,606

PW_Y = -35,000 - 4000(P/A,12%,5) + 7000(P/F,12%,5)
= -$45,447

Select alternative Y

PW of Different-Life Alternatives

The PW of the alternatives must be compared over the **same number of years and must end** at the same time to satisfy the equal-service requirement.

For cost alternatives, failure to compare equal service will always favor the shorter-lived mutually exclusive alternative, even if it is not the more economical choice, because fewer periods of costs are involved.

PW of Different-Life Alternatives

The equal-service requirement is satisfied by using either of two approaches:

LCM: **Compare the PW of alternatives over a period of time equal to the least common multiple (LCM) of their estimated lives.**

Study period: **Compare the PW of alternatives using a specified study period of *n years.***
This approach does not necessarily consider the useful life of an alternative. The study period is also called the *planning horizon.*

Assumptions of LCM approach

The assumptions when using the LCM approach are that

1. **The service provided will be needed over the entire LCM years or more.**

2. **The selected alternative can be repeated over each life cycle of the LCM in exactly the same manner.**

3. **Cash flow estimates are the same for each life cycle.**

Example: Different-Life Alternatives

Compare the machines below using present worth analysis at i = 10% per year

	Machine A	Machine B
First cost, $	20,000	30,000
Annual cost, $/year	9000	7000
Salvage value, $	4000	6000
Life, years	3	6

Solution: **LCM = 6 years; repurchase A after 3 years**

$$PW_A = -20{,}000 - 9000(P/A,10\%,6) - 16{,}000(P/F,10\%,3) + 4000(P/F,10\%,6)$$
$$= \$-68{,}961$$

20,000 – 4,000 in year 3

$$PW_B = -30{,}000 - 7000(P/A,10\%,6) + 6000(P/F,10\%,6)$$
$$= \$-57{,}100$$

Select alternative B

PW Evaluation Using a Study Period

- A **study period analysis** is necessary if the first assumption about the length of time the alternatives are needed cannot be made.

- The time horizon chosen might be relatively short, especially when short-term business goals are very important.

- For the study period approach, a time horizon is chosen over which the economic analysis is conducted, and only those cash flows which occur during that time period are considered relevant to the analysis.

- All cash flows occurring beyond the study period are ignored. An estimated market value at the end of the study period must be made.

- The study period approach is often used in replacement analysis (Chapter 11).

- It is also useful when the LCM of alternatives yields an unrealistic evaluation period, for example, 5 and 9 years.

Example: Study Period PW Evaluation

Compare the alternatives below using present worth analysis at i = 10% per year and a 3-year study period

	Machine A	Machine B
First cost, $	-20,000	-30,000
Annual cost, $/year	-9,000	-7,000
Salvage/market value, $	4,000	6,000 (after 6 years) 10,000 (after 3 years)
Life, years	3	6

Solution: **Study period = 3 years; disregard all estimates after 3 years**

$$PW_A = -20{,}000 - 9000(P/A,10\%,3) + 4000(P/F,10\%,3) = \$-39{,}376$$

$$PW_B = -30{,}000 - 7000(P/A,10\%,3) + 10{,}000(P/F,10\%,3) = \$-39{,}895$$

Marginally, select A; different selection than for LCM = 6 years

Future Worth Analysis

- Analysis of alternatives using FW values is especially applicable to large capital investment decisions when a prime goal is to maximize the *future wealth of a corporation's stockholders.*

- Future worth analysis over a specified study period is often utilized if the asset (equipment, a building, etc.) might be sold or traded at some time before the expected life is reached.

- Suppose an entrepreneur is planning to buy a company and expects to trade it within 3 years. FW analysis is the best method to help with the decision to sell or keep it 3 years hence.

Future Worth Analysis

FW exactly like PW analysis, except calculate FW

Must compare alternatives for ***equal service***
(i.e. alternatives must ***end*** at the same time)

Two ways to compare equal service:

- **Least common multiple (LCM) of lives**
- **Specified study period**

(The LCM procedure is used unless otherwise specified)

FW of Different-Life Alternatives

Compare the machines below using future worth analysis at i = 10% per year

	Machine A	Machine B
First cost, $	-20,000	-30,000
Annual cost, $/year	-9000	-7000
Salvage value, $	4000	6000
Life, years	3	6

Solution: **LCM = 6 years; repurchase A after 3 years**

$$FW_A = -20{,}000(F/P,10\%,6) - 9000(F/A,10\%,6) - 16{,}000(F/P,10\%,3) + 4000$$
$$= \$-122{,}168$$

$$FW_B = -30{,}000(F/P,10\%.6) - 7000(F/A,10\%,6) + 6000$$
$$= \$-101{,}157$$

Select B (Note: PW and FW methods will *always* result in *same selection*)

Capitalized Cost (CC) Analysis

Many public sector projects such as bridges, dams, highways and toll roads, railroads, and hydroelectric and other power generation facilities have very long expected useful lives. A **perpetual or infinite life is the effective planning horizon.**

Capitalized Cost (CC) is the present worth of a project that has a very long life (more than, say, 35 or 40 years) or when the planning horizon is considered very long or infinite.

Capitalized Cost (CC) Analysis

The formula to calculate CC is derived from the PW relation $P = A(P/A,i\%,n)$, where $n = \infty$ time periods. Take the equation for P using the P/A factor and divide the numerator and denominator by $(1 + i)^n$ to obtain

$$P = A\left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right]$$

As n approaches ∞, the bracketed term becomes $1/i$. We replace the symbols P and PW with CC as a reminder that this is a capitalized cost equivalence. Since the A value can also be termed AW for annual worth, the capitalized cost formula is simply

$$\text{CC} = \frac{A}{i} \quad \text{or} \quad \text{CC} = \frac{\text{AW}}{i}$$

Solving for A or AW, the amount of new money that is generated each year by a capitalization of an amount CC is

$$\text{AW} = \text{CC}(i)$$

Capitalized Cost (CC) Analysis

CC refers to the present worth of a project with a very long life, that is, PW as n becomes *infinite*

Basic equation is: $CC = P = \frac{A}{i}$

If $20,000 is invested now (this is the capitalization) at 10% per year, the maximum amount of money that can be withdrawn at the end of every year for eternity is $2000, which is the interest accumulated each year. This leaves the original $20,000 to earn interest so that another $2000 will be accumulated the next year.

Capitalized Cost (CC) Analysis

The cash flows (costs, revenues, and savings) in a capitalized cost calculation are usually of two types: recurring, also called periodic, and nonrecurring. An annual operating cost of $50,000 and a rework cost estimated at $40,000 every 12 years are examples of recurring cash flows. Examples of nonrecurring cash flows are the initial investment amount in year 0 and one-time cash flow estimates at future times, for example, $500,000 in fees 2 years hence.

Capitalized Cost (CC) Analysis

The procedure to determine the CC for an infinite sequence of cash flows is as follows:

1. Draw a cash flow diagram showing all nonrecurring (one time) cash flows and at least two cycles of all recurring (periodic) cash flows.
2. Find the present worth of all nonrecurring amounts. This is their CC value.
3. Find the A value through one life cycle of all recurring amounts. (This is the same value in all succeeding life cycles, as explained in Chapter 6.) Add this to all other uniform amounts (A) occurring in years 1 through infinity. The result is the total equivalent uniform annual worth (AW).
4. Divide the AW obtained in step 3 by the interest rate i to obtain a CC value.
5. Add the CC values obtained in steps 2 and 4.

Example: Capitalized Cost

Compare the machines shown below on the basis of their capitalized cost. Use i = 10% per year

	Machine 1	Machine 2
First cost,$	-20,000	-100,000
Annual cost,$/year	-9000	-7000
Salvage value, $	4000	-----
Life, years	3	∞

Solution: **Convert machine 1 cash flows into A and then divide by i**

$$A_1 = -20{,}000(A/P,10\%,3) - 9000 + 4000(A/F,10\%,3) = \$-15{,}834$$

$$CC_1 = -15{,}834 / 0.10 = \$-158{,}340$$

$$CC_2 = -100{,}000 - 7000/ 0.10 = \$-170{,}000$$

Select machine 1

Example Capitalized Cost

Two sites are currently under consideration for a bridge to cross a river :

The north site, which connects a major state highway with an interstate loop around the city, would alleviate much of the local through traffic. The disadvantages of this site are that the bridge would do little to ease local traffic congestion during rush hours, and the bridge would have to stretch from one hill to another to span the widest part of the river, railroad tracks, and local highways below. This bridge would therefore be a suspension bridge.

The south site would require a much shorter span, allowing for construction of a truss bridge, but it would require new road construction.

Problem Parameters

The suspension bridge will cost $50 million with annual inspection and maintenance - costs of $35,000. In addition, the concrete deck would have to be resurfaced every 10 years at a cost of $100,000. The cost of purchasing right-of-way is expected' to be $2 million.

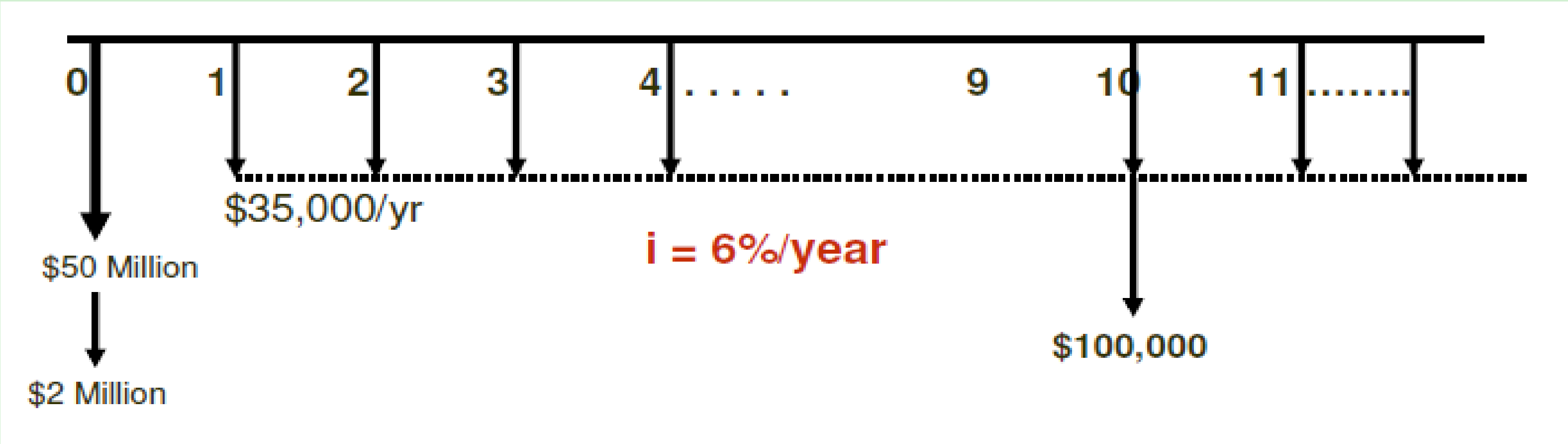
Problem Parameters

The truss bridge and, approach roads are expected to cost $25 million and have annual maintenance costs of $20,000. The bridge would have to be painted every 3 years at a cost of $40,000. In addition, the bridge would have to be sandblasted every 10 years at a cost of $190,000. The cost' of purchasing right-of-way is expected' to be $15 million.

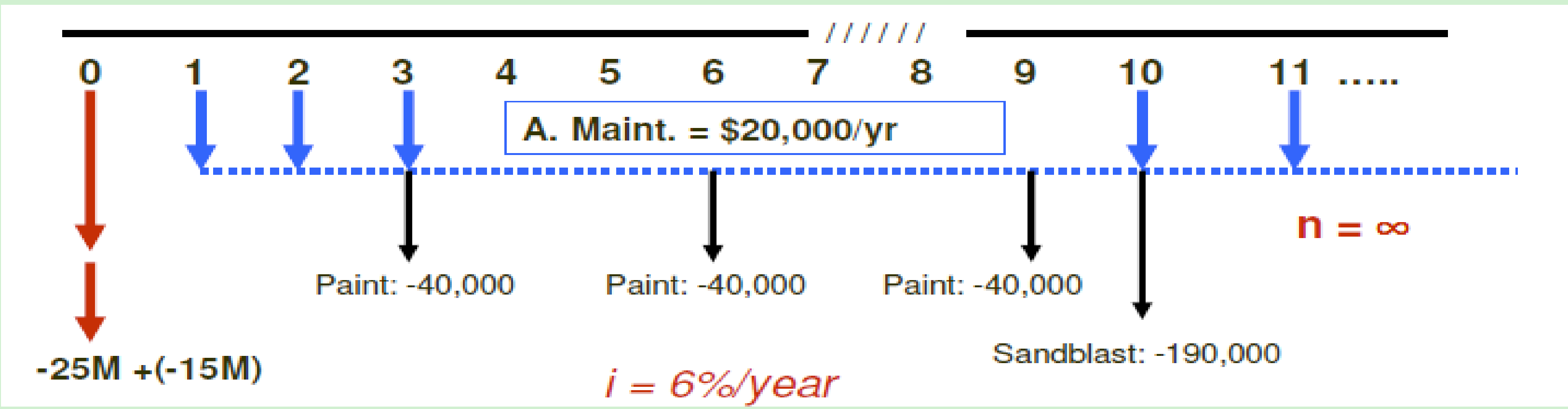
Compare the alternatives on the basis of their capitalized cost if the interest rate is 6% per, year.

Cash Flow Diagrams

Suspension Bridge



Truss Bridge



Alternative 1 Suspension Bridge

- CC_1= -52 million at t = 0.

$$A_1 = -\$35,000$$

$$A_2 = -100,000(A/F,6\%,10) = -\$7,587$$

$$CC_2 = \frac{A_1 + A_2}{i} = \frac{-35,000 + (-7,587)}{0.06} = -\$709,783.$$

Total CC – suspension bridge is:

-52 million + (709,783) = -$52.71 million

Alternative 2 Truss Bridge

CC_1 Initial Cost:

-\$25M + (-15M) = -\$40M

Annual Maintenance is already an "A" amount so: A_1 = -\$20,000/year

For any given cycle of painting compute:

A_2 = -\$40,000(A/F,6%,3) = -\$12,564/year

For any given cycle of Sandblasting Compute

A_3 = -\$190,000(A/F,6%,10) =-\$14,421

- $CC_2 = (A_1+A_2+A_3)/i$
- CC_2 = -(20,000+12,564+14,421)/0.06
- CC_2 – \$783,083/year
- $CC_{Total} = CC_1 + CC_2$ =-40.783 million

- $CC_{Suspension}$ = -$52.71 million
- CCT_{russ} - -40.783 million

Select the Truss Design!

Summary of Important Points

- PW method converts all cash flows to *present value at MARR*
- Alternatives can be *mutually exclusive* or *independent*
- *Cash flow estimates can be for* *revenue* *or* *cost* *alternatives*
- PW comparison must always be made for *equal service*
- Equal service is achieved by using *LCM* or *study period*
- Capitalized cost is PW of project with *infinite life*; CC = P = A/i