Centrifugal pumps

A centrifugal pump consists essentially of a runner or impeller which carries a number of backward curved vanes and rotates in a casing, Fig. 14.1(*a*). Liquid enters the pump at the centre and work is done on it as it passes centrifugally outwards so that it leaves the impeller with high velocity and increased pressure. In the casing, part of the kinetic energy of the fluid is converted into pressure energy as the flow passes to the delivery pipe. Fig. 14.1(*a*) shows a volute casing which increases in area towards the delivery thus reducing the velocity of the liquid and increasing the pressure to overcome the delivery head. This type of casing has a low efficiency as there is a large loss of energy in eddies.

Figure 14.1(*b*) shows a pump with a vortex or whirlpool chamber which is a combination of a circular chamber and a spiral volute. This type of chamber has a higher efficiency of conversion of kinetic energy to pressure energy than the volute. A higher efficiency still can be obtained by using a diffuser consisting of a ring of stationary guide vanes, Fig. 14.1(*c*), an arrangement known as a *turbine pump* since it resembles a turbine operating in reverse.

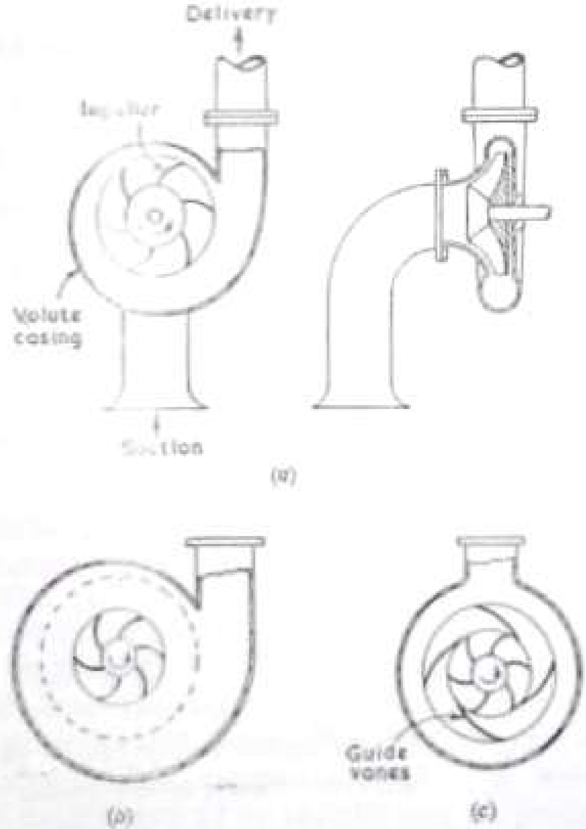

Figure 14.1

Teacher: Prof. Dr. Habib-ur-Rehman

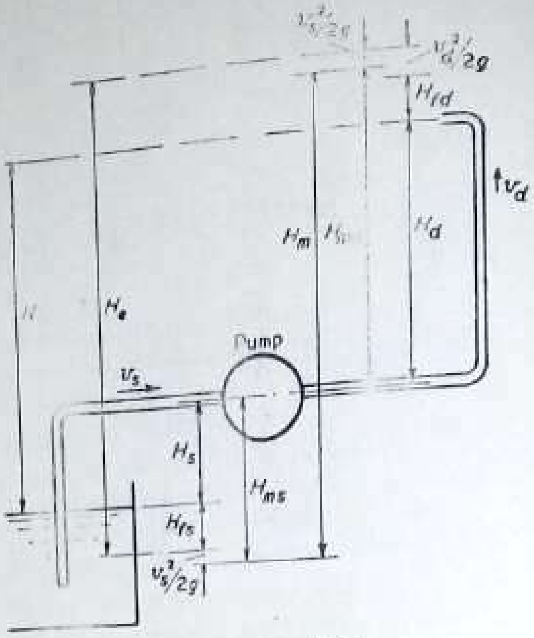

Figure 14.2

Figure 14.2 shows diagrammatically a pump with its suction and delivery pipes.

H_s = suction lift, H_d = delivery lift

H = total static head = $H_s + H_d$

There will be losses of head in the pipelines due to friction and shock losses at fittings.

H_{fs} = loss of head in suction pipe

H_{fd} = loss of head in delivery pipe

If v_s and v_d are the velocities in the suction and delivery pipe

$\frac{v_s^2}{2g}$ = velocity head in suction pipe

$\frac{v_d^2}{2g}$ = velocity head in delivery pipe

The effective head H_e which the pump must provide must be equal to the total lift plus the friction loss plus the kinetic energy of the fluid at discharge:

$$H_e = H_s + H_d + H_{fs} + H_{fd} + \frac{v_d^2}{2g}$$

If manometers or pressure gauges are placed at the same level on the inlet and outlet at the pump

$$\text{Manometric suction head} = H_{ms} = H_s + H_{fs} + \frac{v_s^2}{2g}$$

$$\text{Manometric delivery head} = H_{md} = H_d + H_{fd} + \frac{v_d^2}{2g} - \frac{v_s^2}{2g}$$

$$\text{Manometric head} = H_m = H_{ms} + H_{md}$$

$$= H_s + H_d + H_{fs} + H_{fd} + \frac{v_d^2}{2g}$$

$$= \text{head rise through pump}$$

Efficiencies

Considering a pump together with suction and delivery pipes, if W is the weight discharged per sec and H is the head of fluid, then

$$\text{Overall efficiency} = \frac{\text{useful work done}}{\text{energy supplied to pump shaft}}$$

$$= \frac{WH}{\text{shaft input power}}$$

$$\text{Manometric efficiency} = \frac{\text{head rise through pump}}{\text{energy/unit weight given to fluid by impeller}}$$

$$= \frac{H_m}{u_2 w_2/g} \quad \text{(see example 14.1)}$$

$$\text{Mechanical efficiency} = \frac{\text{energy/unit wt given to fluid by impeller}}{\text{mechanical energy/unit wt supplied to shaft}}$$

14.1 Work done per unit weight and turning moment

> A centrifugal pump has an impeller of outer radius r_2 and inner radius r_1 and the corresponding peripheral velocities are u_2 and u_1. If the flow enters the impeller radially obtain an expression for the work done/unit wt on the fluid by the impeller in terms of u_2 and the velocity of whirl at outlet w_2.
>
> The diameter of the impeller of a pump is 1·2 m and its peripheral speed is 9 m/s. Water enters radially and is discharged from the impeller with a velocity whose radial component is 1·5 m/s. The vanes are curved backwards at exit and make an angle of 30 deg with the periphery. If the pump discharges 3·4 m³/min, what will be the turning moment on the shaft?

Solution Fig. 14.3 shows the triangles of velocities of the inlet and outlet of an impeller blade. To avoid shock the relative velocity at inlet should be tangential to the blade, but this will not be the case at all speeds and discharges. The relative velocity at outlet will also be tangential to the blade. The absolute velocity at outlet v_2 is found by compounding v_{r2} and u_2, but it is often convenient to consider the components of v_2 radially and tangentially which are f_2 and w_2.

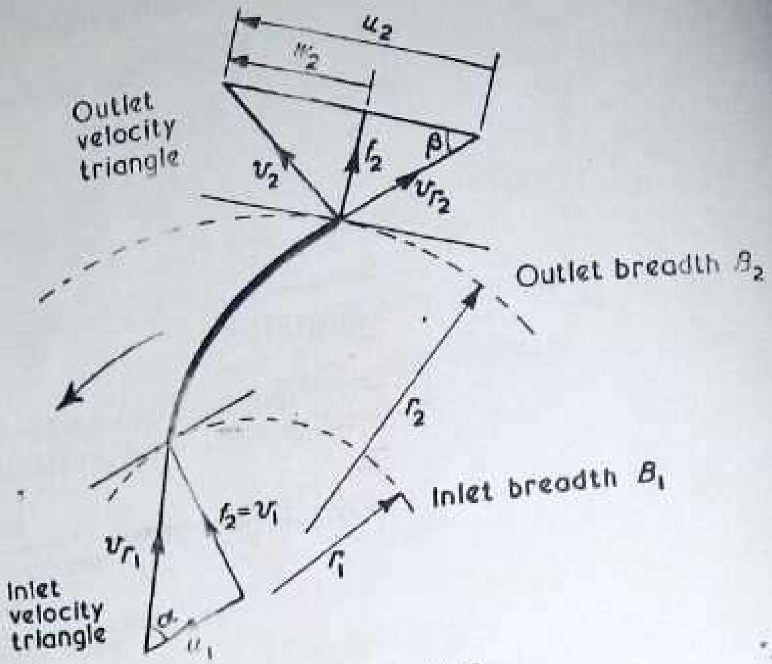

Figure 14.3

Note that the values given for blade angles in problems are often $(180° - \alpha)$ and $(180° - \beta)$.

$$\text{Discharge} = \text{peripheral area} \times \text{radial velocity}$$
$$Q = 2\pi r_1 B_1 f_1 = 2\pi r_2 B_2 f_2$$

If ω = angular velocity of impeller

$$u_1 = \omega r_1, \quad u_2 = \omega r_2$$

In passing through the impeller the tangential component of the absolute velocity of the fluid is changed and there is a change of moment of momentum:

Torque on impeller = T = rate of change of moment of momentum.

Assuming that v_1 is radial, tangential velocity at inlet is zero for unit mass:

$$\text{Moment of momentum at inlet} = 0$$

Tangential component of absolute velocity at exit will be w_2.

$$\text{Moment of momentum at outlet} = w_2 r_2$$
$$\text{Change of moment of momentum} = w_2 r_2$$
$$\text{Mass per sec passing} = \rho Q \quad \text{where} \quad \rho = \text{mass density}$$
$$T = \rho Q w_2 r_2 \tag{1}$$
$$\text{Work done per sec} = \text{torque} \times \text{angular velocity}$$
$$= \rho Q w_2 r_2 \omega = \rho Q w_2 u_2$$
$$\text{Weight flowing per sec} = \rho g Q$$
$$\text{Work done per unit wt} = \frac{w_2 u_2}{g}$$

From equation (1)

$$\text{Turning moment on shaft} = T = \rho Q w_2 r_2$$

From outlet velocity triangle

$$w_2 = u_2 - f_2 \cot \beta$$

Putting $u_2 = 9\,\text{m/s}$, $f_2 = 1{\cdot}5\,\text{m/s}$, $\beta = 30°$

$$w_2 = 9 - 1{\cdot}5\sqrt{3} = 6{\cdot}4\,\text{m/s}$$

Also $\rho = 1000\,\text{kg/m}^3$, $Q = \dfrac{3{\cdot}4}{60}\,\text{m}^3/\text{s}$, $r_2 = 0{\cdot}6\,\text{m}$

$$\text{Torque on shaft} = 1000 \times \frac{3{\cdot}4}{60} \times 6{\cdot}4 \times 0{\cdot}6$$

$$= \mathbf{217\,N\text{-}m}$$

14.2 Speed to commence pumping

If the static lift for a centrifugal pump is h metres, the speed of rotation N rev/min and the external diameter of the impeller d metres, deduce the expression

$$N = 83{\cdot}5\,\frac{\sqrt{h}}{D}$$

for the speed at which pumping commences, assuming only rotation of water in the impeller at the "no-flow" condition.

Such a pump delivers $1{\cdot}27\,\text{m}^3$ of water per minute at 1200 rev/min. The impeller diameter is 350 mm and breadth at outlet 12·7 mm. The pressure difference between inlet and outlet flanges is 272 kN/m^2. Taking the manometric efficiency at 63 per cent, calculate the impeller exit blade angle.

Solution. Under no-flow conditions a forced vortex is formed by the impeller. Pumping can commence when the pressure difference from centre to outside of vortex is equal to the static lift or

$$\frac{u_2^2}{2g} = h \quad \text{where} \quad u_2 = \text{peripheral velocity}$$

so that $$u_2 = \sqrt{(2gh)}$$

Now $$u_2 = \frac{\pi DN}{60}$$

therefore $$\frac{\pi DN}{60} = \sqrt{(2gh)}$$

$$N = \frac{60\sqrt{(2g)}}{\pi D}\sqrt{h} = 83{\cdot}5\,\frac{\sqrt{h}}{D}$$

The outlet velocity diagram will be as Fig. 14.3. The exit blade angle is β and can be found from

$$\tan \beta = \frac{f_2}{u_2 - w_2}$$

$$\text{Peripheral speed} = u_2 = \frac{\pi d_2 N}{60} = \pi \times 0{\cdot}35 \times \frac{1\,200}{60} = 22\,\text{m/s}$$

$$\text{Velocity of flow} = f_2 = \frac{Q}{\pi d_2 B_2} = \frac{1{\cdot}27}{60\pi \times 0{\cdot}35 \times 0{\cdot}0127} = 1{\cdot}52\,\text{m/s}$$

Since the manometric efficiency and the pressure rise through the pump are known the work done per unit wt can be found and used to calculate w_2 (*see* example **14.1**)

$$\text{Work done/unit wt} = \frac{u_2 w_2}{g} = \frac{\text{head rise through pump}}{\text{manometric efficiency}} = \frac{p/\rho g}{\eta_m}$$

$$\therefore \qquad w_2 = \frac{p}{\rho \eta_m u_2}$$

Putting $p = 272 \times 10^3\,\text{N/m}^2$, $\rho = 10^3\,\text{kg/m}^3$, $\eta_m = 0{\cdot}63$, $u_2 = 22\,\text{m/s}$

$$w_2 = \frac{272 \times 10^3}{10^3 \times 0{\cdot}63 \times 22} = 19{\cdot}6\,\text{m/s}$$

Thus $$\tan \beta = \frac{f_2}{u_2 - w_2} = \frac{1{\cdot}52}{22 - 19{\cdot}6} = \frac{1{\cdot}52}{2{\cdot}4} = 0{\cdot}633$$

$$\text{Exit blade angle} = \beta = 32°20'$$

14.3 Efficiency and losses

A centrifugal blower has an impeller of outer diameter 500 mm and width 75 mm with vanes set back at 70 deg to the tangent at the outer periphery. When the blower is delivering air weighing $1{\cdot}25\,\text{kg/m}^3$ at a rate of $3{\cdot}1\,\text{m}^3/\text{s}$ the speed is 900 rev/min and the pressure difference across the blower measured by a manometer is 33 mm of water. The power supplied to the blower shaft is 1·65 kW and the mechanical efficiency is 93 per cent.

Assuming radial inlet to the impeller and neglecting the thickness of the vanes, find the manometric and the overall efficiencies. Also determine the power lost in (*a*) bearing friction and windage, (*b*) the diffuser and (*c*) the impeller.

Solution. $$\text{Manometric efficiency} = \frac{\text{manometric head}}{\text{work done/unit wt in impeller}}$$

Since the fluid is air, express the manometric pressure difference as a head of air.

$$\text{Manometric head} = 0{\cdot}033 \times \frac{10^3}{1{\cdot}25} = 26{\cdot}4\,\text{m of air}$$

From example 14.1

$$\text{Work done per unit wt in impeller} = \frac{u_2 w_2}{g}$$

The outlet velocity is similar to Fig. 14.3.

$$\text{Peripheral velocity at outlet} = u_2 = \frac{\pi d_2 N}{60}$$

$$= \pi \times 0{\cdot}5 \times \frac{900}{60} = 23{\cdot}55\,\text{m/s}$$

$$w_2 = u_2 - f_2 \cot \beta$$

$$f_2 = \frac{\text{discharge}}{\text{outlet area}} = \frac{Q}{\pi d_2 B_2} = \frac{3{\cdot}1}{\pi \times 0{\cdot}5 \times 0{\cdot}075} = 26{\cdot}35\,\text{m/s}$$

Thus $w_2 = 23{\cdot}55 - 26{\cdot}35 \cot 70°$
$= 23{\cdot}55 - 9{\cdot}6 = 14{\cdot}05\,\text{m/s}$

Work done/unit wt in impeller

$$= \frac{u_2 w_2}{g} = \frac{23{\cdot}55 \times 14{\cdot}05}{9{\cdot}81} = 33{\cdot}8\,\text{J/N}$$

$$\text{Manometric efficiency} = \frac{H_m}{u_2 w_2/g} = \frac{26{\cdot}4}{33{\cdot}8} = \textbf{78·3 per cent}$$

Weight of air delivered per sec $= W = \rho g Q = 1{\cdot}25 \times 9{\cdot}81 \times 3{\cdot}1\,\text{N/s}$
Output power of blower $= WH_m = 38{\cdot}1 \times 26{\cdot}4\,\text{N-m/s} = 1{\cdot}005\,\text{kW}$
Mechanical input to shaft $= 1{\cdot}65\,\text{kW}$

$$\text{Overall efficiency} = \frac{\text{output}}{\text{input}} = \frac{1{\cdot}005}{1{\cdot}65} = \textbf{60·9 per cent}$$

The losses are:

(*a*) The mechanical efficiency is 93 per cent, therefore

Bearing and windage loss $= \frac{7}{100} \times$ power supplied
$= 0{\cdot}07 \times 1{\cdot}65 =$ **0·115kW**

(*b*) The loss in the diffuser is the difference between the power put into the fluid leaving the impeller and the output power.

$$\text{Work done in impeller per sec} = W \times \frac{u_2 w_2}{g}$$

$$= 38{\cdot}1 \times 33{\cdot}8 = 1290\,\text{W} = 1{\cdot}29\,\text{kW}$$

Output power $= 1{\cdot}005\,\text{kW}$
Power lost in diffuser $= 1{\cdot}29 - 1{\cdot}005 =$ **0·285kW**

(*c*) Loss in impeller = power supplied − bearing loss − diffuser loss − output power

$= 1{\cdot}65 - 0{\cdot}115 - 0{\cdot}285 - 1{\cdot}005$
$= 1{\cdot}65 - 1{\cdot}405 =$ **0·245kW**

14.4 Diffuser efficiency

A centrifugal pump running at 700 rev/min is supplying 9 m³/min against a head of 19·8 m. The blade angle at exit is 135° from the direction of motion of the blade tip. Assume that the relative velocity of the water at exit is along the blade and that the absolute velocity at inlet is radial. The velocity of flow is constant at 1·8 m/s. Calculate the necessary impeller diameter (*a*) if none of the energy corresponding to the velocity at the exit from the impeller is recovered; (*b*) if 40 per cent of this energy is recovered.

In case (*b*) find also the width of impeller at exit, allowing 8 per cent for vane thickness.

Solution The velocity diagram at outlet is similar to Fig. 14.3, with $\beta = 180° - 135° = 45°$.

(*a*) If no kinetic energy is recovered

Work done/unit wt in impeller = static head + velocity head

$$\frac{u_2 w_2}{g} = H + \frac{v_2^2}{2g} \qquad (1)$$

From the outlet triangle

$$w_2 = u_2 - f_2 \cot 45°, \quad \text{put} \quad f_2 = 1{\cdot}8\,\text{m/s}$$

$$w_2 = u_2 - 1{\cdot}8$$

Also $\quad v_2^2 = f_2^2 + w_2^2 = 1{\cdot}8^2 + (u_2^2 - 3{\cdot}6u_2 + 1{\cdot}8^2)$

$$= u_2^2 - 3{\cdot}6u_2 + 6{\cdot}48$$

Substituting in equation (1) and multiplying both sides by $2g$,

$$2u_2(u_2 - 1{\cdot}8) = 2gH + (u_2^2 - 3{\cdot}6u_2 + 6{\cdot}48)$$

Putting $H = 19{\cdot}8$ m and solving for u_2

$$2u_2^2 - 3{\cdot}6u_2 - (u_2^2 - 3{\cdot}6u_2 + 6{\cdot}48) = 2 \times 9{\cdot}81 \times 19{\cdot}8$$

$$u_2^2 = 389 + 6{\cdot}48 = 395{\cdot}48$$

$$u_2 = 19{\cdot}9\,\text{m/s}$$

$$\text{Impeller diameter} = \frac{u_2 \times 60}{\pi N}$$

$$= \frac{19{\cdot}9 \times 60}{\pi \times 700} = \mathbf{0{\cdot}542\,m}$$

(*b*) If 40 per cent of the kinetic energy is recovered, 60 per cent is lost.

Work done/unit wt in impeller = static head + 0·6 velocity head

$$\frac{u_2 w_2}{g} - 0{\cdot}6\frac{v_2^2}{2g} = H$$

$$2u_2(u_2 - 1{\cdot}8) - 0{\cdot}6(u_2^2 - 3{\cdot}6u_2 + 6{\cdot}48) = 19{\cdot}8 \times 2g$$

$$1\cdot 1u_2^2 - 1\cdot 44u_2 - 399\cdot 37 = 0$$

$$u_2 = 17\cdot 35\,\text{m/s}$$

$$\text{Impeller diameter} = \frac{u_2 \times 60}{\pi N}$$

$$= \frac{17\cdot 35 \times 60}{\pi \times 700} = \mathbf{0\cdot 473\,m}$$

Effective area at outlet is reduced by 8 per cent owing to blade thickness.

$$\text{Discharge} = \text{effective peripheral area} \times \text{radial velocity}$$

$$Q = 0\cdot 92\pi D_2 B_2 f_2$$

$$B_2 = \frac{Q}{0\cdot 92\pi D_2 f_2}$$

Putting $Q = 9\,\text{m}^3/\text{min} = 9/60\ \text{m}^3/\text{s}$

$$D_2 = 0\cdot 473\,\text{m},\quad f_2 = 1\cdot 8\,\text{m/s}$$

$$\text{Width at exit} = B_2 = \frac{9}{60 \times 0\cdot 92 \times \pi \times 0\cdot 473 \times 1\cdot 8}\,\text{m}$$

$$= 0\cdot 061\,\text{m} = \mathbf{61\,mm}$$

14.5 Specific speed

Explain what is meant by the specific speed of a centrifugal pump and show that its value is $NQ^{1/2}/H^{3/4}$ where N is the rotational speed of the impeller, Q the discharge and H the operating head.

A centrifugal pump, having four stages in parallel, delivers 11 m³/min of liquid against a head of 24·7 m, the diameter of the impellers being 225 mm and the speed 1700 rev/min.

A pump is to be made up with a number of identical stages in series, of similar construction to those in the first pump, to run at 1250 rev/min and to deliver 14·5 m³/min against a head of 248 m. Find the diameter of the impellers and the number of stages required.

Solution. The *specific speed* is used as a basis of comparison of the performance of different pumps and is defined as the theoretical speed at which the given pump would deliver unit quantity against unit head. For example the speed in rev/min at which the pump would discharge 1 m³/min under 1 metre head. The specific speed of a given pump depends on the system of units chosen.

To find this theoretical speed for unit discharge under unit head it is necessary to scale down the operating values for the pump. This is done by assuming that in scaling-down proportions are kept geometrically similar and all linear dimensions are proportional to the impeller diameter. It is also assumed that the velocity diagrams are similar and all velocities are proportional to the square root of the head H.

$$\text{Breadth of impeller } B \propto \text{diameter } D$$
$$\text{Impeller velocity } u \propto H^{1/2}$$

Also if the speed of the impeller is N rev/min

$$u \propto ND \quad \text{or} \quad D \propto \frac{u}{N}$$

or

$$D \propto \frac{H^{1/2}}{N}$$

$$\text{Discharge } Q \propto \text{area of flow} \times \text{velocity of flow}$$
$$\propto \pi DBf$$

Now

$$f \propto H^{1/2} \quad \text{and} \quad B \propto D$$

so that

$$Q \propto D^2 H^{1/2}$$

Substituting,

$$D \propto \frac{H^{1/2}}{N}$$

$$Q \propto \frac{H}{N^2} H^{1/2}$$

$$N = N_s \frac{H^{3/4}}{Q^{1/2}}$$

$$\text{Specific speed } N_s = \frac{NQ^{1/2}}{H^{3/4}}$$

Considering pump with 4 stages in parallel

$$\text{Discharge for one stage} = \frac{11}{4} \text{m}^3/\text{min}$$

$$Q_1 = 2{\cdot}75\,\text{m}^3/\text{min}$$
$$\text{Operating head } H_1 = 24{\cdot}7\,\text{m}$$
$$\text{Operating speed } N_1 = 1\,700\,\text{rev/min}$$
$$\text{Specific speed} = \frac{N_1 Q_1^{1/2}}{H_1^{3/4}} = \frac{1\,700 \times \sqrt{2{\cdot}75}}{24{\cdot}7^{3/4}} = 254$$

For the multi-stage pump

If each stage is similar to those of the first pump

Specific speed of each stage $= N_s = 254$

The whole discharge passes through each stage, so that

$$Q_2 = 14{\cdot}5\,\text{m}^3/\text{min}$$
$$N_2 = 1\,250\,\text{rev/min}$$
$$N_s = \frac{N_2 \sqrt{Q_2}}{H_2^{3/4}}$$

where H_2 = head rise per stage.

$$254 = \frac{1250\sqrt{14{\cdot}5}}{H_2^{3/4}}$$

$$H_2^{3/4} = 18{\cdot}7; \quad H_2 = 49{\cdot}64\,\text{m}$$

$$\text{Total head required} = 248\,\text{m}$$

$$\text{Number of stages required} = \frac{248}{H_2} = 5$$

Since the head H is proportional to the square of the impeller velocity u and $u \propto ND$,

$$H = kN^2D^2$$

Comparing the original pump and one stage of the second pump, for similarity

$$\frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2$$

Thus

$$D_2 = D_1\frac{N_1}{N_2}\sqrt{\frac{H_2}{H_1}}$$

$D_1 = 0{\cdot}225\,\text{m}$, $N_1 = 1\,700\,\text{rev/min}$, $N_2 = 1\,250\,\text{rev/min}$.
$H_1 = 24{\cdot}7\,\text{m}$, $H_2 = 49{\cdot}64\,\text{m}$.

$$\text{Diameter of impeller } D_2 = 0{\cdot}225 \times \frac{1\,700}{1\,250}\sqrt{\frac{49{\cdot}64}{24{\cdot}7}} = 0{\cdot}433\,\text{m}$$

$$= \mathbf{433\,mm}$$

14.6 Type number or dimensionless specific speed

(*a*) By dimensional analysis derive expressions for the head coefficient K_H, flow coefficient K_Q, and power coefficient K_p of a centrifugal pump or fan and explain how these can be combined to give the type number or dimensionless specific speed.

(*b*) A centrifugal pump running at 2950 rev/min under test at peak efficiency gave the following results: Effective head H = 75 m of water, Rate of flow $Q = 0{\cdot}05$ m³/s, overall efficiency η = 76 per cent. Calculate the dimensionless specific speed of this pump based on rotational speed in rev/s.

(*c*) A dynamically similar pump is to operate at a corresponding point of its characteristic when delivering $0{\cdot}5$ m³/s through a pipe 800 m long and 1 m diameter for which the friction coefficient $f = 0{\cdot}05$. The pipe discharges 90 m above reservoir level. Determine the rotational speed at which the pump should run to meet the duty and the ratio of its impeller diameter to that of the pump in (*b*), stating any assumptions made. What will be the power consumed by the pump.

Solution (*a*) For the general dimensional analysis of any rotodynamic machine the variables to be considered are:

Q = Volumetric flow rate $[L^3T^{-1}]$
P = Power transferred from impeller to fluid $[ML^2T^{-3}]$
N = Rotational speed of the impeller $[T^{-1}]$
H = Difference of head across machine $[L]$

D = Diameter of impeller $[L]$
ρ = Density of fluid $[ML^{-3}]$
μ = Dynamic viscosity of fluid $[ML^{-1}T^{-1}]$
K = Bulk modulus of elasticity of fluid $[ML^{-1}T^{-2}]$
ε = Roughness of internal passages represented by a typical dimension $[L]$

The head H is the energy per unit weight of the fluid and it is convenient to substitute gH which is the energy per unit mass.

Using Buckingham's theorem (see example 1.7) there are nine variables and three fundamental dimensions therefore there will be six dimensionless groups in the relationship. These can be

$$\Pi_1 = \frac{gH}{N^2D^2}$$ known as the head coefficient K_H

$$\Pi_2 = \frac{Q}{ND^3}$$ known as the flow coefficient K_Q

$$\Pi_3 = \frac{P}{N^3D^5\rho}$$ known as the power coefficient K_P

$$\Pi_4 = \frac{\mu}{ND^2\rho}$$ which, since ND is the peripheral velocity, is proportional to $1/Re$ where Re is Reynolds number based on impeller diameter.

$$\Pi_5 = \frac{K}{N^2D^2\rho}$$ which, since $\sqrt{K/\rho}$ is the velocity of sound a, is proportional to $1/Ma$ where Ma is the Mach number.

$$\Pi_6 = \frac{\varepsilon}{D}$$ which is the relative roughness of the internal passages of the machine.

Now

$$\Pi_1 = \phi\{\Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6\}$$

or

$$\frac{gH}{N^2D^2} = \phi\left\{\frac{Q}{ND^3}, \frac{P}{N^3D^5\rho}, \frac{\mu}{ND^2\rho}, \frac{K}{N^2D^2\rho}, \frac{\varepsilon}{D}\right\}$$

or

$$K_H = \phi(K_Q, K_P, Re, Ma, \varepsilon/D)$$

Comparison of rotodynamic machines can be made on the basis of the values of K_Q, K_H and K_P. For pumps K_Q and K_H are the most important factors and their ratio K_Q/K_H indicates whether a particular pump is suitable for large or small flows for a given head. For geometrical similar machines the impeller diameter can be eliminated by using the ratio of $K_Q^{1/2}$ to $K_H^{3/4}$ which is known as the *type number* or *dimensionless specific speed* n_s

$$n_s = \frac{(K_Q)^{1/2}}{(K_H)^{3/4}} = \left(\frac{Q}{ND^3}\right)^{1/2}\left(\frac{N^2D^2}{gH}\right)^{3/4}$$

$$n_s = N\frac{Q^{1/2}}{(gH)^{3/4}}$$

The value of n_s which is the type number is calculated from the values of N, Q and H corresponding to the *design point*, that is to say the particular duty for which the machine is designed.

(*b*) Putting $N = 2950$ rev/min $= 49\cdot17$ rev/s

$$Q = 0\cdot05 \text{ m}^3/\text{s} \quad \text{and} \quad H = 75 \text{ m of water}$$

$$n_s = 49\cdot17 \times \frac{0\cdot05^{1/2}}{(9\cdot81 \times 75)^{3/4}} = 7\cdot79 \times 10^{-2}$$

(*c*) Lift of pump $= 90$ m

$$\text{Friction loss in pipe} = h_f = \frac{fLQ^2}{3d^5}$$

$$= \frac{0\cdot05 \times 800 \times (0\cdot45)^2}{3 \times (1)^5} = 27 \text{ m}$$

$$\text{Effective head required} = H_2 = h + h_f = 90 + 27 = 117 \text{ m}$$

For operation at the same point of the characteristic curve n_s will be the same. Therefore $n_s = 7\cdot79 \times 10^{-2}$. Substituting in the expression for specific speed

$$7\cdot79 \times 10^{-2} = N\frac{(0\cdot45)^{1/2}}{(9\cdot81 \times 117)^{3/4}} = 3\cdot4 \times 10^{-3}N$$

$$N = \frac{7\cdot79}{0\cdot34} \text{ rev/s} = 1375 \text{ rev/min}$$

Since the head coefficient must be the same for both pumps

$$\frac{gH_1}{(N_1D_1)^2} = \frac{gH_2}{(N_2D_2)^2}$$

$$\frac{D_2}{D_1} = \left(\frac{H_2}{H_1}\right)^{1/2}\left(\frac{N_1}{N_2}\right) = \left(\frac{117}{75}\right)^{1/2}\left(\frac{2950}{1375}\right) = 2\cdot68$$

Assuming no scale effects and no variation in efficiency

$$\text{Power consumed by pump} = \frac{\text{power transferred to fluid}}{\text{efficiency}}$$

$$= \frac{\rho gQH}{\eta}$$

$$= \frac{1000 \times 9\cdot81 \times 0\cdot45 \times 117}{0\cdot7} \text{ W}$$

$$= \mathbf{736 \text{ kW}}$$

14.7 Performance of pump and pipeline

A centrifugal pump running at 1000 rev/min gave the following relation between head and discharge:

Discharge (m^3/min)	0	4·5	9·0	13·5	18·0	22·5
Head (m)	22·5	22·2	21·6	19·5	14·1	0

The pump is connected to a 300 mm suction and delivery pipe the total length of which is 69 m and the discharge to atmosphere is 15 m above sump level. The entrance loss is equivalent to an additional 6 m of pipe and f is assumed at 0·006. Calculate the discharge in m^3 per minute.

If it is required to adjust the flow by regulating the pump speed, estimate the speed to reduce the flow to one-half.

Solution.

Head required from pump = static + friction + velocity head

$$H = 15 + H_f + H_v$$

Both H_f and H_v depend on the discharge Q so that both the head required and the head available are functions of the discharge and if these are plotted on a base of quantity the intersection of the two curves will give the discharge required.

$$\text{Head lost in friction} = H_f = \frac{fLQ^2}{3d^5}$$

$$= \frac{0{\cdot}006 \times (69 + 6)Q^2}{3 \times (0{\cdot}3)^5}$$

where Q is in m^3/s

If q = discharge in m^3/min, then $q = 60Q$.

$$\therefore \qquad Q^2 = \frac{q^2}{3600}$$

$$\text{Head lost in friction} = \frac{0{\cdot}006 \times 75 \times q^2}{3 \times (0{\cdot}3)^5 \times 3600} = 17{\cdot}15 \times 10^{-3}q^2$$

$$\text{Velocity head} = H_v = \frac{v^2}{2g} = \frac{Q^2}{2g\left(\frac{\pi}{4}d^2\right)^2}$$

$$= \frac{q^2 \times 16}{3600 \times 2g \times \pi^2 \times (0{\cdot}3)^4}$$

$$= 2{\cdot}83 \times 10^{-3}q^2$$

Head required $= H = 15 + 19{\cdot}98 \times 10^{-3}q^2$ where q is in m^3/min

From this expression and the figures given in the problem the following table is compiled:

Table 14.1

Discharge q (m³/min)	0	4·5	9·0	13·5	18·0	22·5
Head available (m)	22·5	22·2	21·6	19·5	14·1	0
Head required (m)	15	15·4	16·6	18·6	21·5	25·1

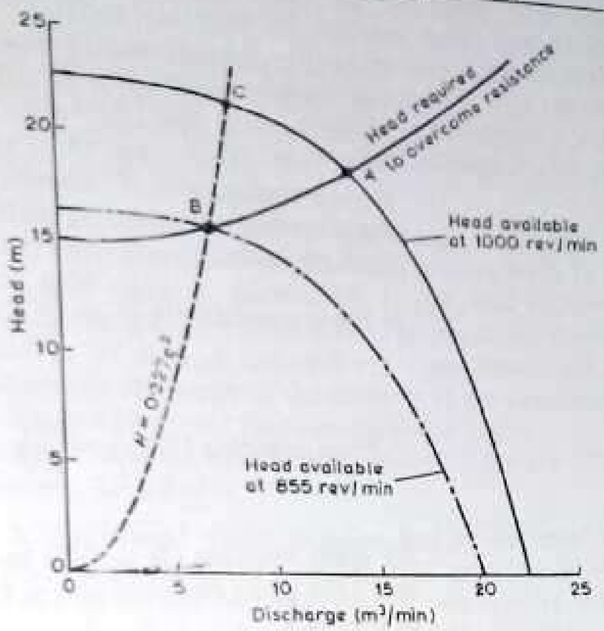

Figure 14.4

These are plotted in Fig. 14.4 from which the operating point of the system will be at the point A at which

$$\text{Head available} = \text{head required}$$

$$\text{Operating head } H_A = 19\,\text{m}, \quad \text{Discharge } q_A = \mathbf{14\,m^3/min}$$

At reduced speed. For half-flow there will be a new operating point B at which

$$\text{Discharge} = q_B = 7\,\text{m}^3/\text{min}$$

From Fig. 14.4

$$\text{Head required to overcome resistance} = H_B = 16{\cdot}0\,\text{m}$$

For every speed N curves of H against q could be drawn for the pump similar to that already drawn for $N = 1000$ rev/min. The problem therefore is to find the speed N_2 of the pump for which the corresponding curve of H against q passes through B.

Since for a given pump $q \propto N$ and $H \propto N^2$ we have

$$\frac{q}{N} = \frac{q_B}{N_2} \qquad (1)$$

and

$$\frac{H}{N^2} = \frac{H_B}{N_2^{\,2}} \qquad (2)$$

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Eliminating N $$H = H_B\left(\frac{q}{q_B}\right)^2.$$

Putting $H_B = 16{\cdot}0\,\text{m}$ when $q_B = 7\,\text{m}^3/\text{min}$

$$H = \frac{16{\cdot}0}{7^2}q^2 = 0{\cdot}327q^2$$

This is a parabola passing through the origin and B which intersects the original curve of H against q for $N_1 = 1000\,\text{rev/min}$ at C, the corresponding operating point, for which $H_C = 21{\cdot}9\,\text{m}$ and $q_C = 8{\cdot}2\,\text{m}^3/\text{min}$. This curve links the corresponding values of H and q as they change with N according to equations (1) and (2) and we can therefore find the speed N_2 corresponding to H_B and q_B from either of these equations.

From equation (1) $$\frac{q_C}{N_1} = \frac{q_B}{N_2} \therefore N_2 = N_1\frac{q_B}{q_C} = 1000 \times \frac{7}{8{\cdot}2}$$
$$= 855\,\text{rev/min}$$

From equation (2) $$\frac{H_C}{N_1^2} = \frac{H_B}{N_2^2} \therefore N_2 = N_1\sqrt{\frac{H_B}{H_C}} = 1000 \times \sqrt{\frac{16}{21{\cdot}9}}$$
$$= 855\,\text{rev/min}$$

By using equation (1) and (2) to scale down the original values of H and q the pump characteristic at 855 rev/min could be plotted as shown in Fig. 14.4.

Problems

1 The external diameter of the impeller of a centrifugal pump is 250 mm and the internal diameter is 150 mm, the corresponding width of the impeller at this latter point being 15 mm. The vane angle at outlet makes an angle of 45 deg backwards to the tangent of the impeller circle. If the radial velocity of flow is constant, the discharge being $2{\cdot}7\,\text{m}^3/\text{min}$ when the speed is 1100 rev/min calculate (a) the impeller angle at inlet; (b) the angle of the guide vanes in the diffuser ring; (c) the pressure rise through the pump assuming a diffuser ring efficiency of 60 per cent and neglecting frictional losses.

Answer 143° 39′, 38° 27′, 11·8 m

2 A centrifugal pump when running at 1500 rev/min is to deliver $90\,\text{dm}^3/\text{s}$ against a head of 24 m. The flow at entry is radial, and the radial velocity of flow is to be constant through the impeller at 3·6 m/s. The diffuser vanes may be assumed to convert 50 per cent of the kinetic head at exit from the impeller into pressure head. The outer diameter is to be twice the inner and the width of the impeller at exit is to be 12 per cent of the diameter. Neglecting impeller losses and the influence of blade

thickness, determine the diameter and widths at inlet and outlet and the impeller and guide vane angles.

Answer 129 mm, 258 mm, 62 mm, 31 mm, 19° 33′, 28° 55′, 14° 38′

3 A centrifugal pump lifts water against a head of 36 m, the manometric efficiency being 80 per cent. The suction and delivery pipes are both of 150 mm bore, the impeller is of 375 mm diam and 25 mm wide at the outlet: its exit blade angle is 25 deg and the specified rotational speed is 1320 rev/min. If the total loss by friction in the pipeline at this speed is estimated at 9 m, calculate the probable rate of discharge at this speed.

Answer 0·06 m^3/s

4 The impeller of a centrifugal pump is 325 mm diam and 19 mm wide at outlet. The blade angle at outlet is 35 deg, wheel speed 1600 rev/min, suction lift 1·5 m, and estimated loss of head on the suction side 2·1 m. The static lift from the pump centre is 39 m and the delivery pipe losses 9·6 m. If the manometric efficiency of the pump is 76 per cent and the overall efficiency 68 per cent find the discharge in dm^3/s and the power needed if both the suction and delivery pipes are 125 mm diam.

Answer 35·5 dm^3/s, 20·74 kW

5 A centrifugal pump impeller has an external diameter of 300 mm and discharge area of 0·11 m^2. The blades are bent backwards so that the direction of the relative velocity at the discharge surface makes an angle of 145 deg with the tangent to this surface drawn in the direction of impeller rotation. The diameters of the suction and delivery pipes are 300 mm and 225 mm respectively.

Gauges at points on the suction and delivery pipes close to the pump and each 1·5 m above the level in the supply sump showed heads of 3·6 m below and 18·6 m above atmospheric pressure when the pump was delivering 0·2 m^3/s of water at 1200 rev/min. It required 71 kW to drive the pump.

Find (*a*) the overall efficiency; (*b*) the manometric or hydraulic efficiency, assuming that water enters the impeller without shock or whirl; (*c*) the loss of head in the suction pipe.

Answer 61·3 per cent, 71·3 per cent, 1·7 m

6 A centrifugal pump has to discharge 225 dm^3/s of water and develop a head of 22·5 m when the impeller rotates at 1500 rev/min. Determine (*a*) the impeller diameter and (*b*) the blade angle at the outlet edge of the impeller.

Assume that the manometric efficiency is 75 per cent; the loss of head in the pump due to fluid resistance is $0{\cdot}033v^2$ m, where v m/s is the absolute velocity with which the water is discharged from the impeller; the area of the impeller outlet surface is $1{\cdot}2D^2 m^2$ where D is the impeller diameter in m; and water enters the impeller without whirl.

Answer 0·253 m, 30°

7 A centrifugal pump is required to discharge 0·56 m^3/s of water and develop a head of 12 m when the impeller rotates at

750 rev/min. The manometric efficiency is to be 80 per cent, the loss of head in the pump due to friction being assumed to be $0{\cdot}0276V^2$ m of water, where V is the velocity with which water leaves the impeller. Water enters the impeller without shock or whirl and the velocity of flow is constant at 2·7 m/s. Obtain (*a*) the impeller diameter and outlet area, and (*b*) the blade angle at the outlet edge of the impeller. Explain briefly why the direction of the actual velocity at discharge from the impeller differs usually from the direction given by the outlet velocity diagram.
Answer 0·364 m, 0·207 m^2, 34°

8 A centrifugal pump with an impeller of 190 mm diam gives at maximum efficiency a discharge of 3·9 m^3/min of fresh water at 1800 rev/min against a head of 4·2 m. What should be the speed of rotation of a similar impeller of 380 mm diam to give 54·5 m^3/min of sea water of density 1025 kg/m^3 and what pressure would it then generate?
Answer 3150 rev/min, 515 kN/m^3

9 A scale model, one-fifth full size, is to be tested in order to predict the performance of a large centrifugal pump working against a head H. Show that, provided the viscosity of the fluid has no appreciable effect on the performance of the pump, the test may be carried out under any convenient head.

What head would be required for the test if viscosity is taken into account, (*a*) when the pump and model both use water, (*b*) when the kinematic viscosity of the fluid dealt with by the pump is 5 times that used by the model, and what would be the ratio of the corresponding speeds of rotation in each case? Establish the formulae required.
Answer (*a*) $H_m = 25H$, $N_m = 25N$; (*b*) $H_m = H$, $N_m = 5N$

10 A centrifugal fan has to deliver 4·25 m^3/s when running at 750 rev/min. The diameter of the impeller at inlet is 525 mm and at outlet is 750 mm. It may be assumed that the air enters radially at a speed of 15 m/s. The vanes are set backwards at outlet at 70° to the tangent and the width at outlet is 100 mm. The volute casing gives a 30 per cent recovery of the outlet velocity head. The losses in the impeller may be taken as equivalent to 25 per cent of the outlet velocity head. The specific volume of air is 0·8 m^3/kg and the blade thickness effects may be neglected. Determine the manometric efficiency, the power required and the pressure at discharge.
Answer 57·9 per cent, 2·08 kW, 39·2 m of air

11 A centrifugal pump impeller is of 250 mm external diameter and the water passage is 32 mm wide at exit. The circumference is reduced by 12 per cent on account of vane thickness. The impeller vanes are inclined at 140 deg to the forward tangent at exit. Manometric efficiency = 83 per cent, rev/min = 1000, Q = 2·86 m^3/min. Calculate the conversion efficiency of the diffuser ring. Assume no losses in the impeller.
Answer 56 per cent

12 The impeller of a centrifugal pump has an external diameter of 250 mm and an effective outlet area of 170 cm². The blades are bent back so that the angle at the outlet edge is 148 deg to the tangent drawn to the direction of impeller rotation. The diameters of the suction and delivery openings are 150 mm and 125 mm respectively.

When running at 1450 rev/min and delivering 28 dm³/s of water, the pressure heads at the suction and delivery openings were found to be respectively 4·5 m *below* and 13·5 m *above* atmospheric pressure, the points at which these pressure heads were measured being at the same level. The motor driving the pump supplied 8 kW. Water enters the impeller without shock or whirl.

Assuming that the true outlet whirl component = 0·7 of the ideal one, obtain: (*a*) the overall efficiency; (*b*) the manometric efficiency based on the true whirl component.

Answer (*a*) 61·4 per cent; (*b*) 83·4 per cent

13 A centrifugal pump delivers 11·8 cubic metres of water per minute at 1200 rev/min with a manometric efficiency of 75 per cent. The impeller is 300 mm diam with a width at exit of 75 mm. The blades occupy 12 per cent of the periphery and are swept backwards making an angle of 40 deg with the tangent at the outer periphery. Calculate the fraction k of the kinetic energy of discharge from the impeller which is subsequently recovered in the casing assuming no loss of head in the impeller. What is the manometric efficiency if $k = 0$?

Make simple sketches to show two conventional methods used to regain the kinetic energy of discharge from the impeller.

Answer 0·402, 58 per cent

14 Derive the expression for the specific speed of a centrifugal pump in terms of N the speed of impeller rotation, Q the quantity discharged and H the head developed.

A multi-stage centrifugal pump having 6 stages with 225 mm diam impellers develops a head of 120 m when running at 1500 rev/min and discharging 5·45 cubic metres of water per minute.

Four geometrically similar stages having 300 mm diam impellers are used to build a multi-stage pump which is to run at 1000 rev/min. Assuming that each stage in both pumps operates under dynamically similar conditions, obtain (*a*) the quantity in m³/min that will be discharged by this pump and (*b*) the head that it will develop.

Answer (*a*) 8·61 m³/min, (*b*) 63·2 m

15 A centrifugal pump produced the following performance data when running at 1500 rev/min on a test run.

Flow m³/s	0·075	0·150	0·200	0·250	0·300
Total head m	70	68	64	58	49
Input power kW	97	127	147	163	170

The pump is required to deliver water from a sump to a reservoir whose level is 60 m above that of the sump. Suction and delivery

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pipes of 300 mm diam will have a combined length of 120 m (f = 0·006), 12 m of which is on the suction side, and the pump inlet is 3 m above the water level in the supply sump. What will be the efficiency and the discharge of the pump at the test speed? What would be the most economical speed to operate the pump and what suction head would occur at the pump inlet under these optimum speed conditions?

Answer 85·3 per cent, 0·2 m^3/s; 1620 rev/min, 4·3 m

16 Define the term "specific speed" of a centrifugal pump and deduce an expression for it in terms of the head H, the discharge Q and the speed N.

A multi-stage centrifugal pump is required to lift 1·8 m^3/min of water from a mine, the total head including friction being 750 m. If the speed of the pump is 2900 rev/min, find the least number of stages if the specific speed per stage is not to be less than 150 in SI units.

Answer 10 stages

15

Reciprocating pumps

The reciprocating mechanism consists of a piston or displacer moving in a cylinder which liquid enters or leaves through suitable valves. The piston is given a reciprocating motion by means of a connecting rod and crank.

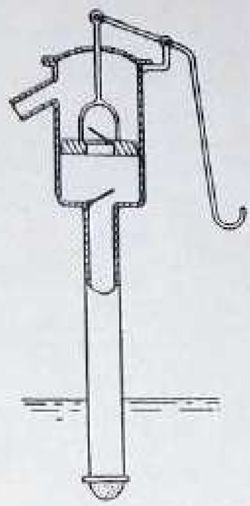

Figure 15.1

Suction pumps (Fig. 15.1) are used solely to raise water to the pump cylinder level. On the suction stroke the movement of the piston forms a partial vacuum in the cylinder and the atmospheric pressure forces the liquid in the sump into the cylinder. Theoretically the lift cannot exceed the head of liquid equivalent to atmospheric pressure, which in the case of water is 10·4 m, but if the pressure falls below the vapour pressure the liquid will boil in the cylinder and the pump cease to function. The available lift in the case of water is thus limited to about 0·8 m at ordinary temperatures.

Force pumps (Fig. 15.2) are similar to suction pumps but on the delivery stroke the liquid is forced into a delivery pipe and can be raised to any desired height above the pump centre-line. The same limitations on the lift from sump to pump cylinder apply as for the suction pump.

Single-acting pumps make one delivery per revolution of the crank for each cylinder (Fig. 15.2).

Double-acting pumps (Fig. 15.3) make two deliveries per revolution of the crank for each cylinder.

A *multi-cylinder* pump has two or more cylinders. Three cylinders