Plain and Reinforced Concrete II

Reference Books:

Concrete Structures by Prof. Dr. Zahid Ahmed Siddiqi

Reinforced concrete mechanics and design by James G. Macgregor

Design of concrete structures by Arthur H. Nilson David Darwin Charles W. Dolan

Reinforced Concrete by Edward G. Nawy

Code and Standard:

Building Code Requirements for Structural Concrete (ACI318-11)

American Society for Testing and Materials (ASTM)

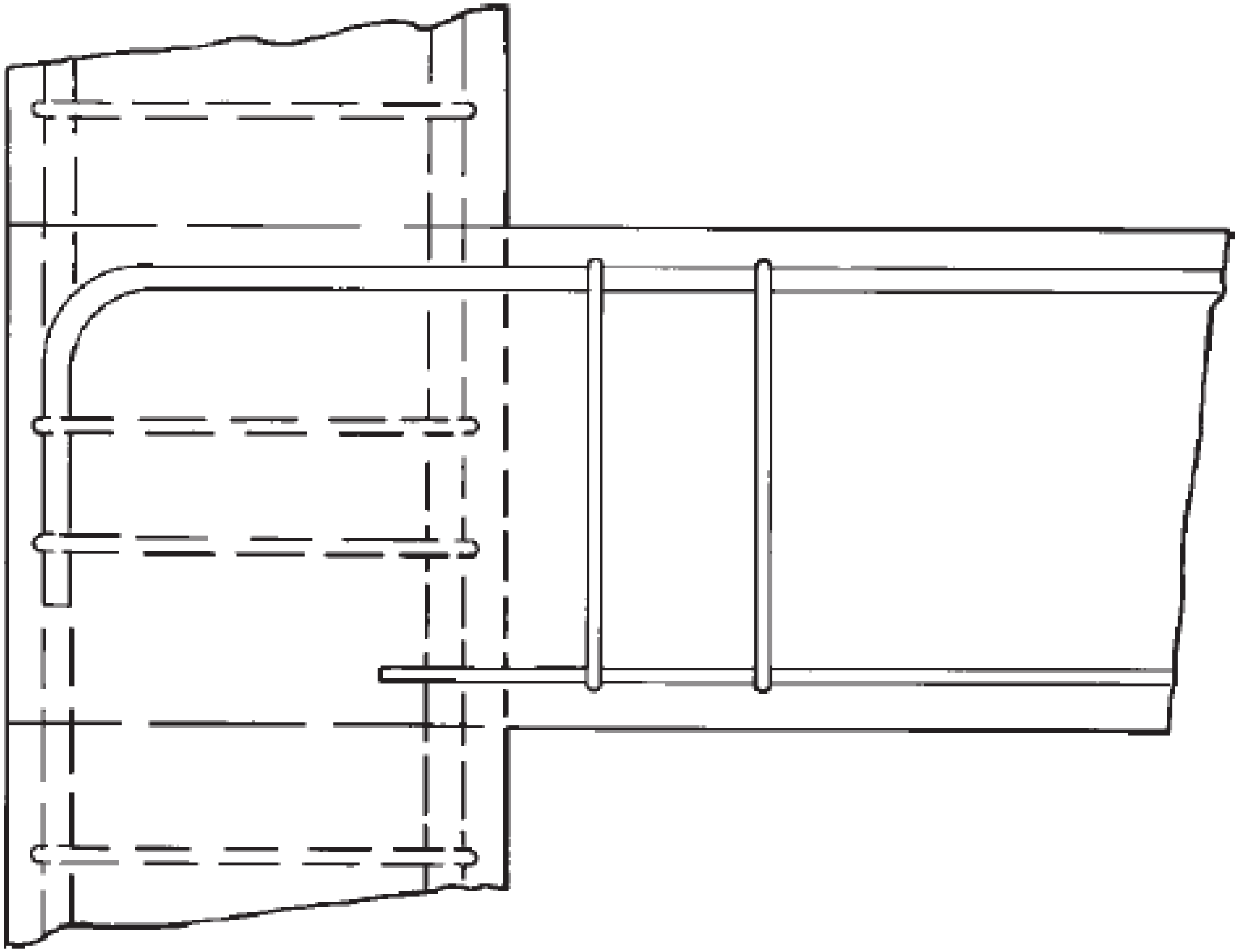
Bond and Development Length

Reference:

Reinforced concrete mechanics and design

by James G. Macgregor

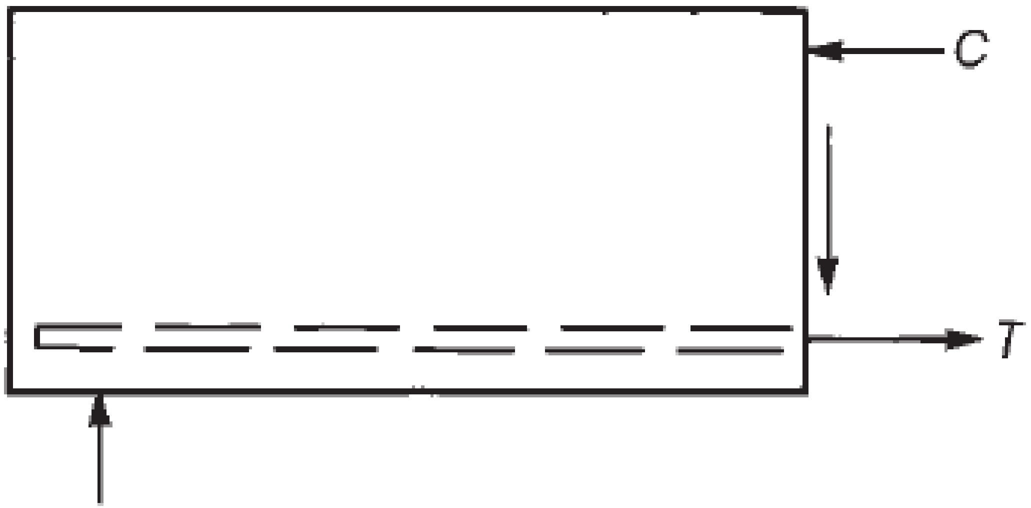
Bond and Development Length



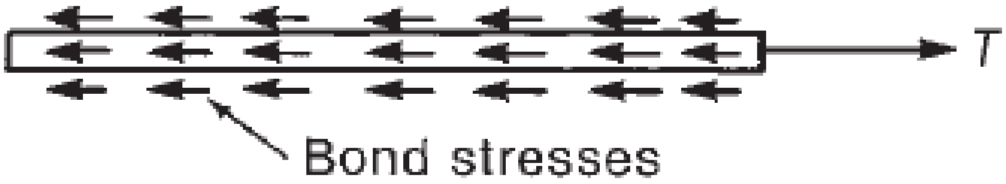
Bond and Development Length

- In a reinforced concrete beam, the flexural compressive forces are resisted by concrete, while the flexural tensile forces are provided by reinforcement.
- For this process to exist, there must be a force transfer, or *bond*, between the two materials.
- For the bar to be in equilibrium, bond stresses must exist.
- If these disappear, the bar will pull out of the concrete and the tensile force, *T*, will drop to zero, causing the beam to fail.

Bond and Development Length



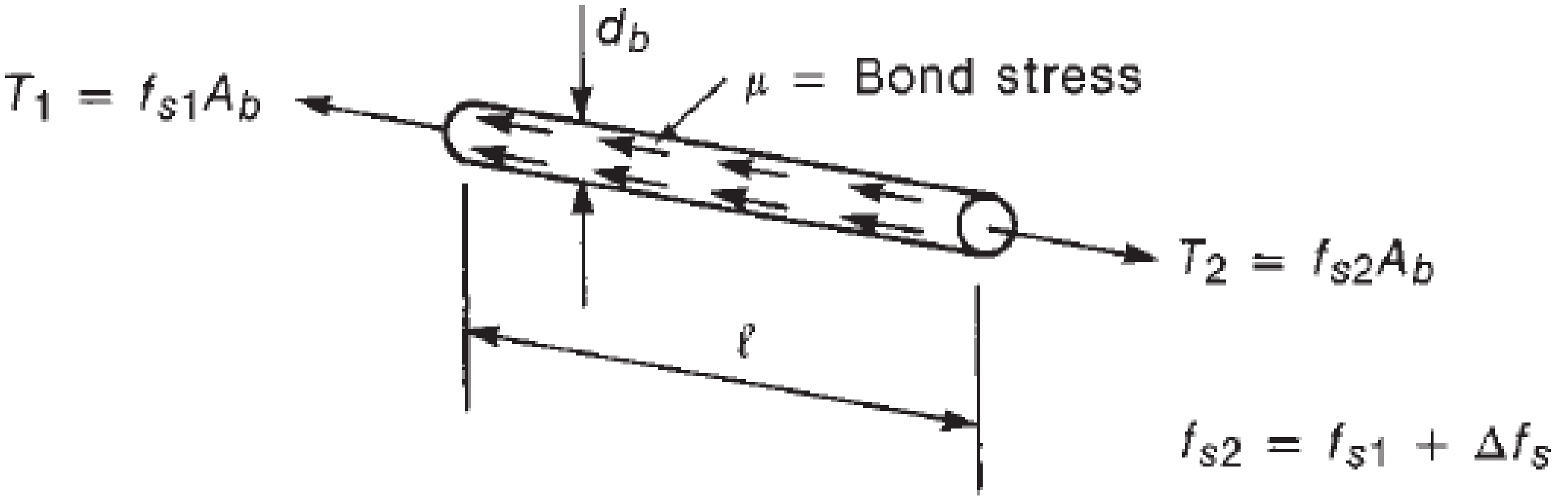
(a) Internal forces in beam.



(b) Forces on reinforcing bar.

Need for Bond Stresses

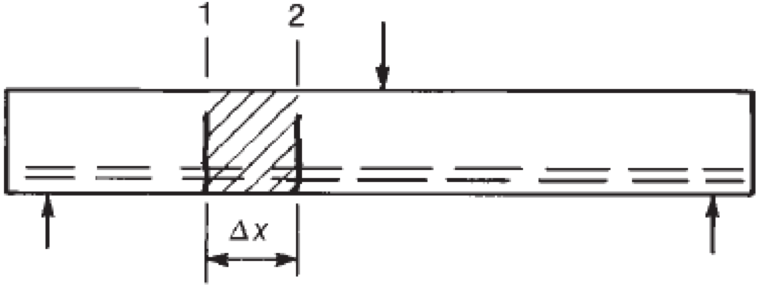
Relationship between change in bar stress and average bond stress



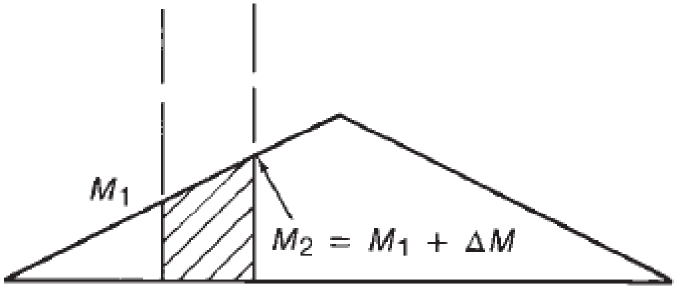
If “f_{s2}” is greater than “f_{s1}” bond stress, μ, must act on the surface of the bar to maintain equilibrium

$$(f_{s2} - f_{s1})\frac{\pi d_b{}^2}{4} = \mu_{avg}(\pi d_b)l$$

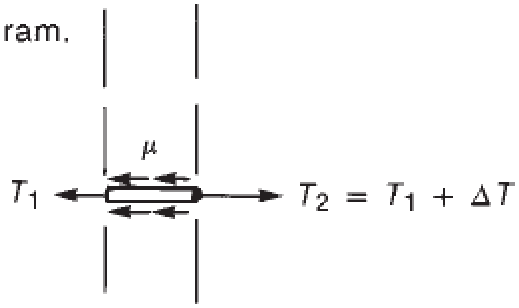
Bond and Development Length



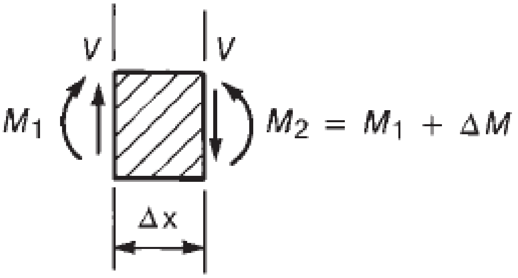
(a) Beam.



(b) Moment diagram.



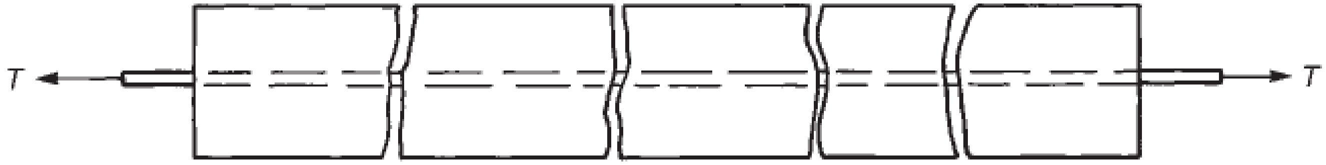
(c) Bar forces.



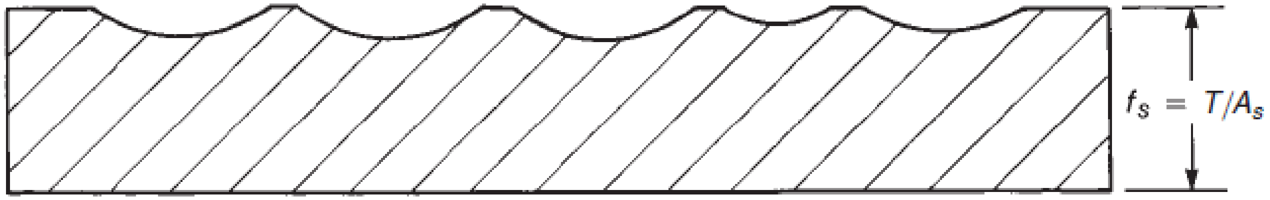
(d) Part of beam between sections 1 and 2.

Average flexural bond stress

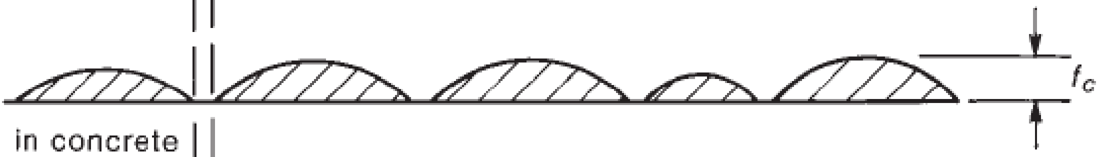
Bond Stresses in an Axially Loaded Prism



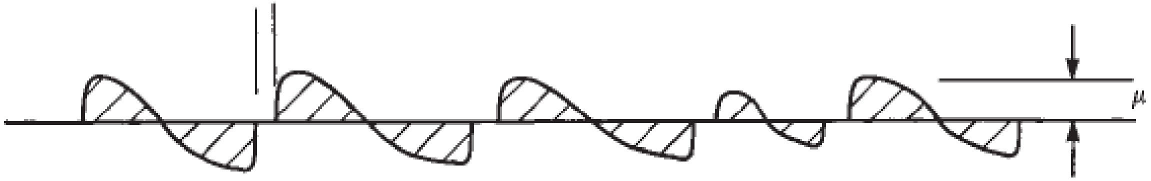
(a) Axially loaded prism.



(b) Variation in steel stress.



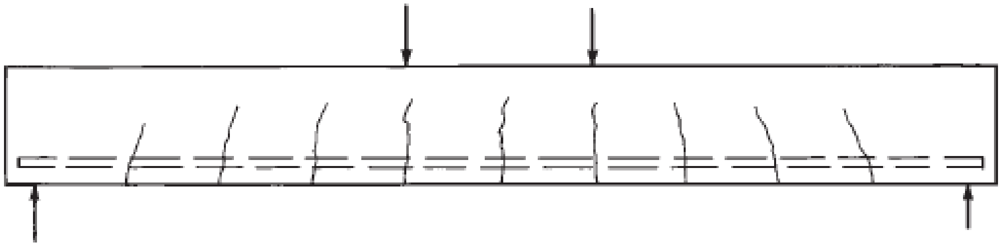
(c) Variation in concrete stress.



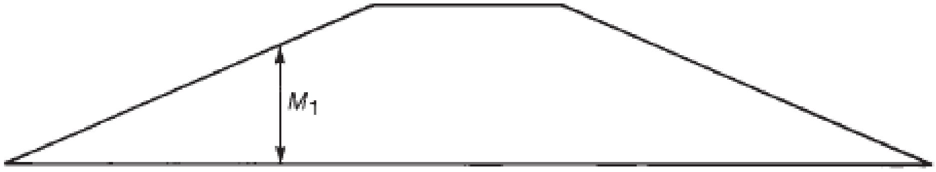
(d) Variation in bond stress—in-and-out bond stresses.

Steel, concrete, and bond stresses in a cracked prism

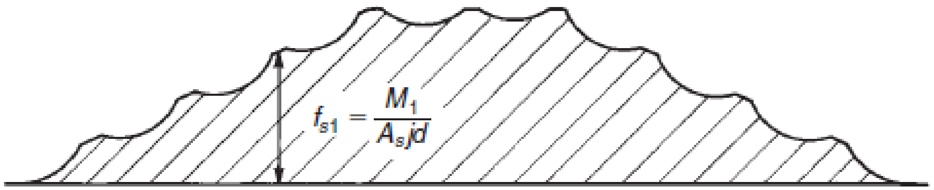
Steel, concrete and bond stresses in a cracked beam



(a) Cracked beam.

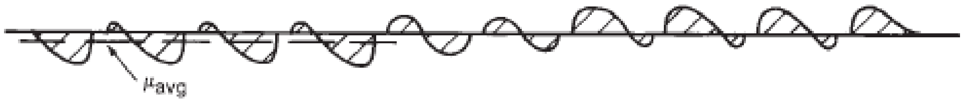


(b) Moment diagram.



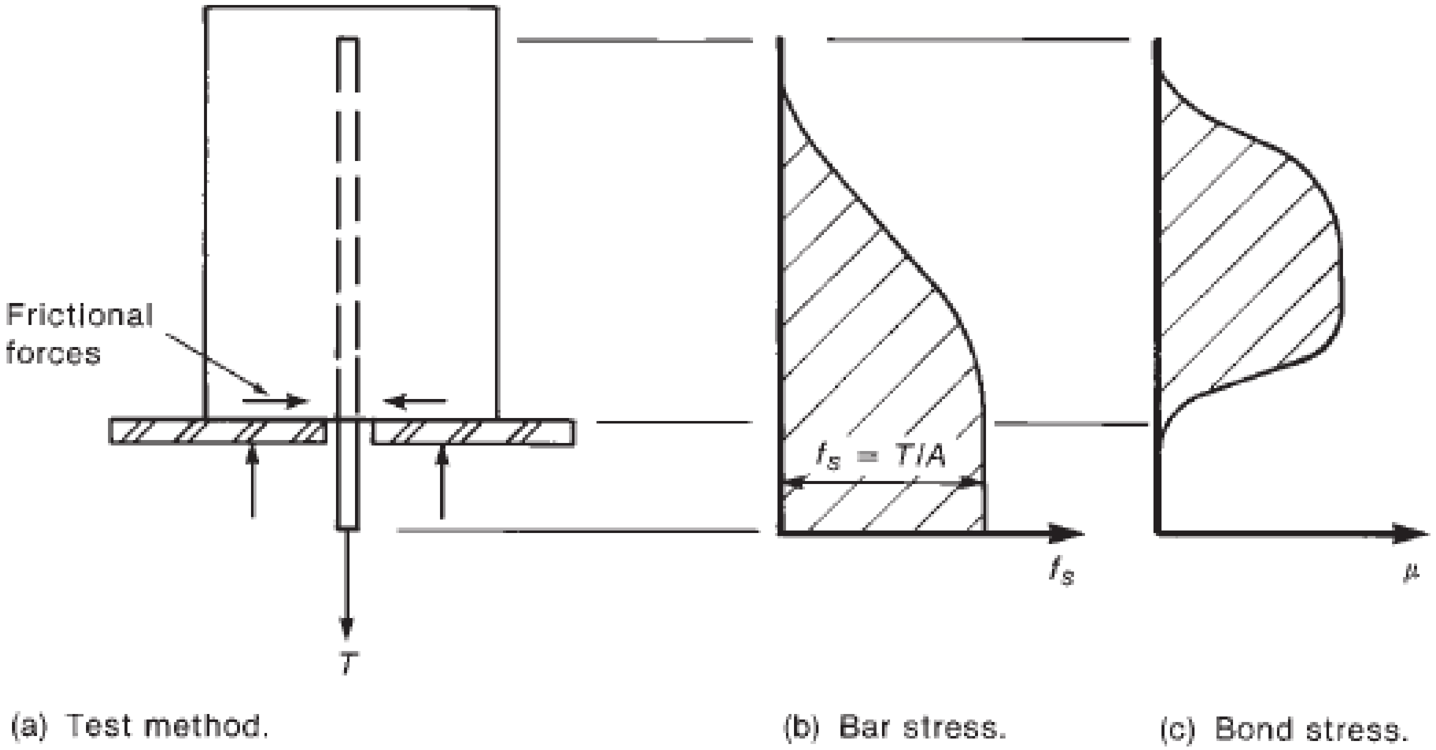
(c) Variation in steel stress.

(d) Tensile stress in concrete.



(e) In-and-out bond stresses.

Stress distribution in a pull-out test



(a) Test method.

(b) Bar stress.

(c) Bond stress.

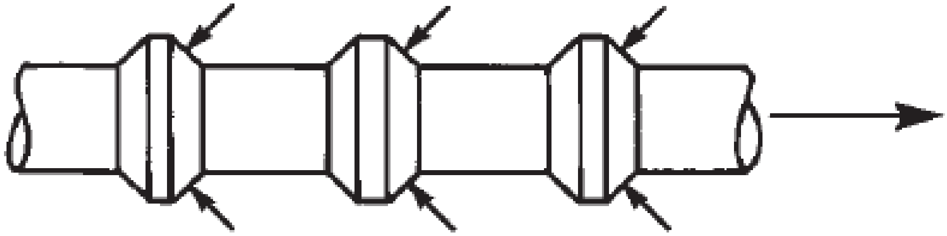
Mechanism of Bond Transfer

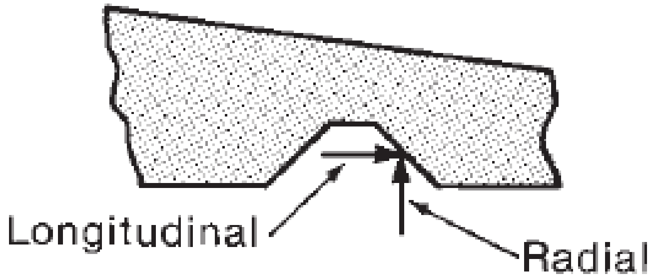
- A smooth bar embedded in concrete develops bond by adhesion between the concrete and the bar and by a small amount of friction.
- Both of these effects are quickly lost when the bar is loaded in tension, particularly because the diameter of the bar decreases slightly, due to Poisson's ratio.
- For this reason, smooth bars are generally not used as reinforcement.
- In cases where smooth bars must be embedded in concrete mechanical anchorage in the form of hooks are used.

Mechanism of Bond Transfer

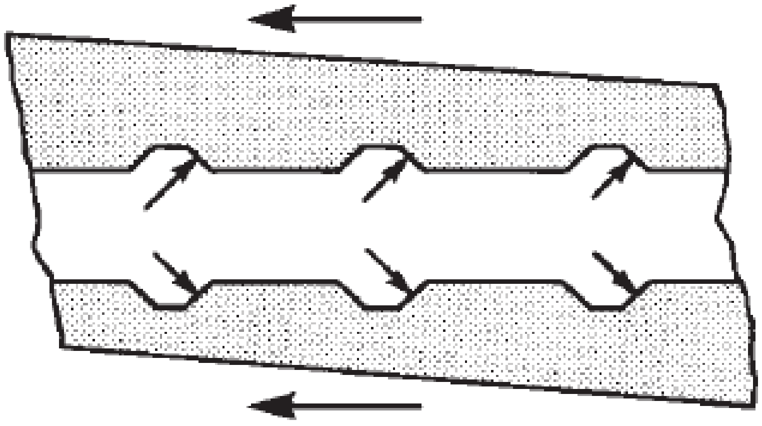
- Although adhesion and friction are present when a deformed bar is loaded for the first time, these bond-transfer mechanisms are quickly lost, leaving the bond to be transferred by bearing on the deformations of the bar.
- Equal and opposite bearing stresses act on the concrete.
- The forces on the concrete have both a longitudinal and a radial component.
- The latter causes circumferential tensile stresses in the concrete around the bar.
- Eventually, the concrete will split parallel to the bar, and the resulting crack will propagate out to the surface of the beam.
- The splitting cracks follow the reinforcing bars along the bottom or side surfaces of the beam.

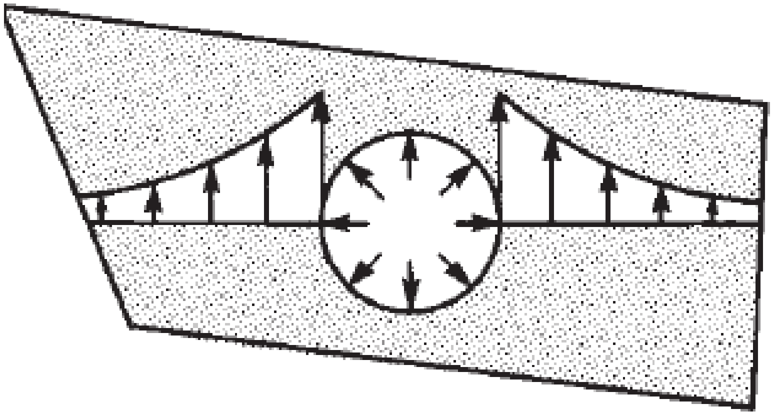
Bond-transfer mechanism


(a) Forces on bar.



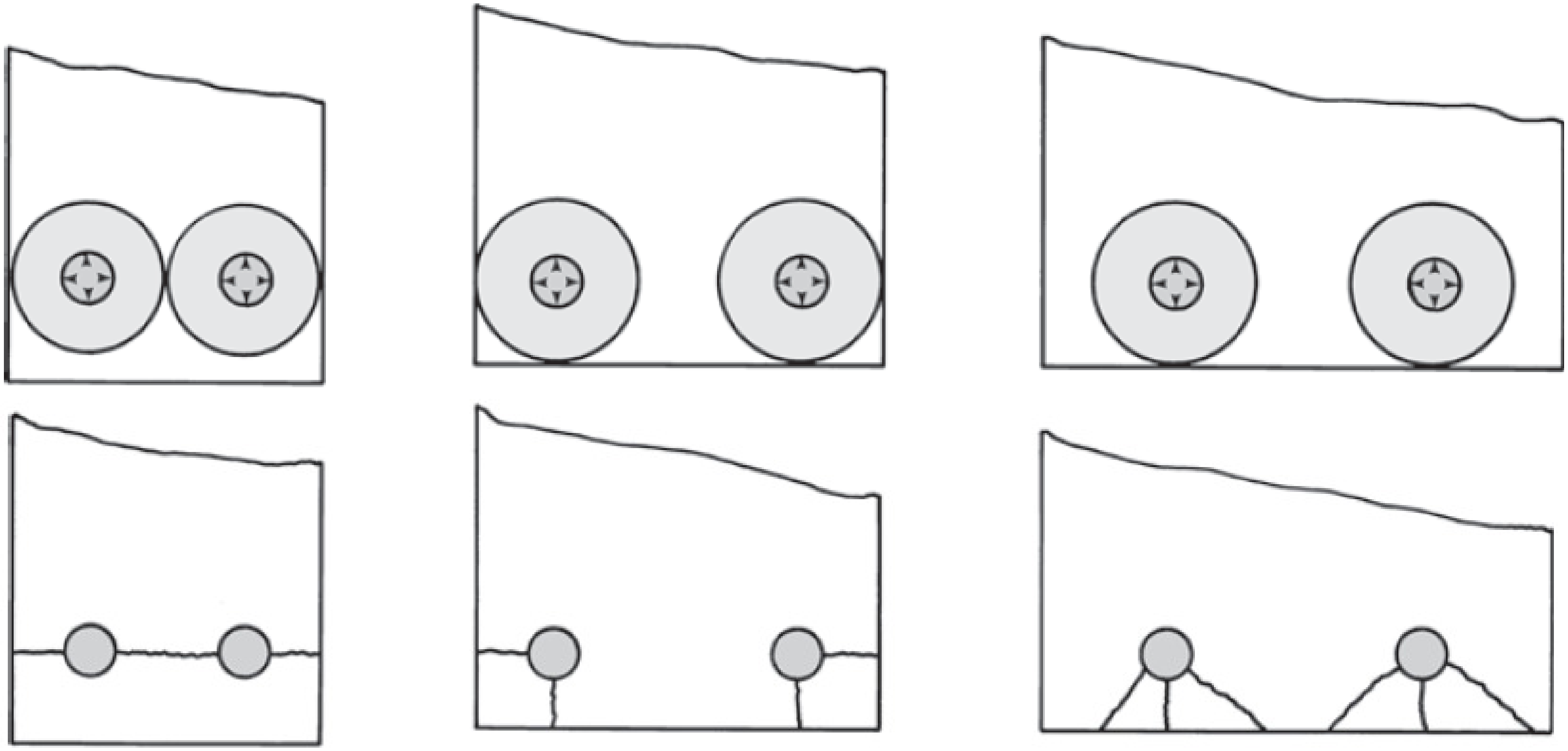
(c) Components of force on concrete.


(b) Forces on concrete.


(d) Radial forces on concrete and splitting stresses shown on a section through the bar.

Once these cracks develop, the bond transfer drops rapidly unless reinforcement is provided to restrain the opening of the splitting crack

Typical splitting-failure surfaces



(a) Side cover and half the bar spacing both less than bottom cover.

(b) Side cover = bottom cover, both less than half the bar spacing.

(c) Bottom cover less than side cover and half the bar spacing.

If the cover and bar spacings are large compared to the bar diameter, a *pull-out failure* can occur, where the bar and the annulus of concrete between successive deformations pull out along a cylindrical failure surface joining the tips of the deformations.

Development Length (l_d)

- ❖Because the actual bond stress varies along the length of a bar anchored in a zone of tension, the ACI Code uses the concept of l_d rather than bond stress.
- ❖l_d, is the shortest length of bar in which the bar stress can increase from zero to the yield strength (f_y).
- ❖If the distance from a point where the bar stress equals f_y to the end of the bar is less than the development length, the bar will pull out of the concrete.
- ❖The l_{ds} are different in tension and compression, because a bar loaded in tension is subject to in-and-out bond stresses and hence requires a considerably longer development length.
- ❖Also, for a bar in compression, bearing stresses at the end of the bar will transfer part of the compression force into the concrete.

Development Length (l_d)

The development length can be expressed in terms of the ultimate value of the average bond stress by setting (f_{s2} - f_{s1}) = f_y in equation.

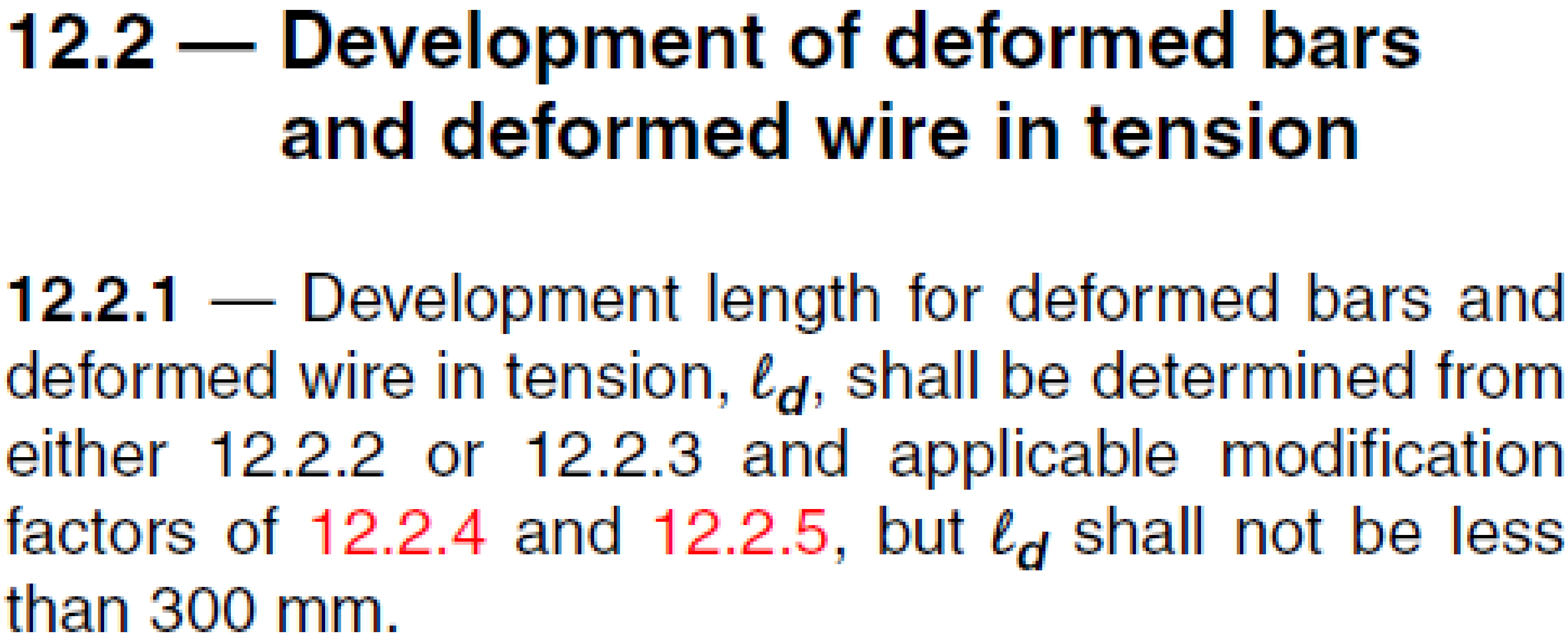
$$(f_{s2} - f_{s1})\frac{\pi d_b^{\ 2}}{4} = \mu_{avg}(\pi d_b)l$$

$$(f_y)\frac{\pi d_b^{\ 2}}{4} = \mu_{avg}(\pi d_b)l$$

$$\frac{f_y d_b}{4\mu_{avg}} = l_d$$

Here, μ_{avg} is the value of μ_{avg} at bond failure in a beam test.

Basic Development Equation ACI318-11



12.2 — Development of deformed bars and deformed wire in tension

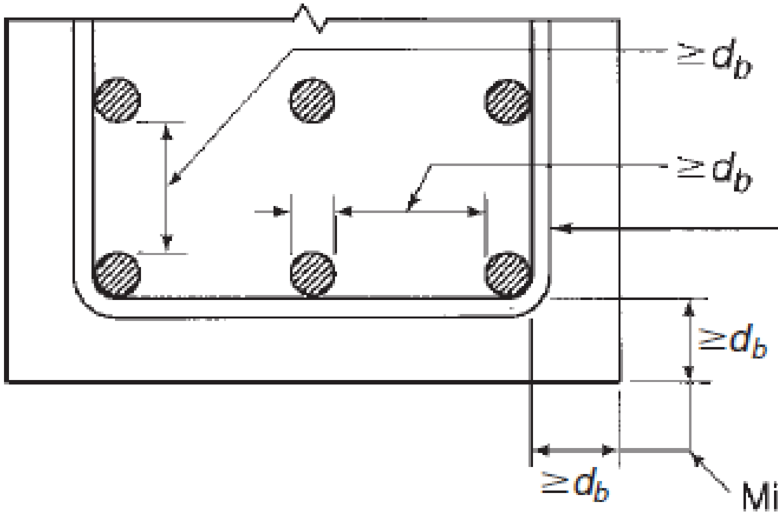
12.2.1 — Development length for deformed bars and deformed wire in tension, ℓ_d, shall be determined from either 12.2.2 or 12.2.3 and applicable modification factors of 12.2.4 and 12.2.5, but ℓ_d shall not be less than 300 mm.

Basic Development Equation ACI318-11

12.2.2 — For deformed bars or deformed wire, ℓ_d shall be as follows:

	Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Case 1 Case2	Clear spacing of bars or wires being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than d_b	$\left(\frac{f_y\psi_t\psi_e}{2.1\lambda\sqrt{f'_c}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e}{1.7\lambda\sqrt{f'_c}}\right)d_b$
	Other cases	$\left(\frac{f_y\psi_t\psi_e}{1.4\lambda\sqrt{f'_c}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e}{1.1\lambda\sqrt{f'_c}}\right)d_b$

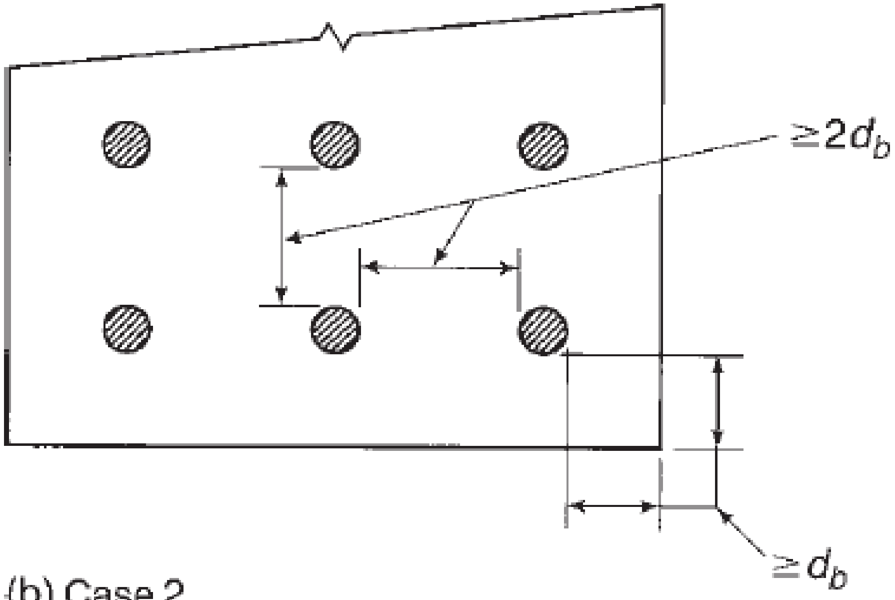
Explanation of Cases 1 and 2



Ties satisfying ACI Section 7.10.5 or stirrups satisfying ACI Code Sections 11.4.5 and 11.4.6.3 along development length

Minimum cover satisfying ACI Section 7.7.1

(a) Case 1.



(b) Case 2.

Basic Development Equation ACI318-11

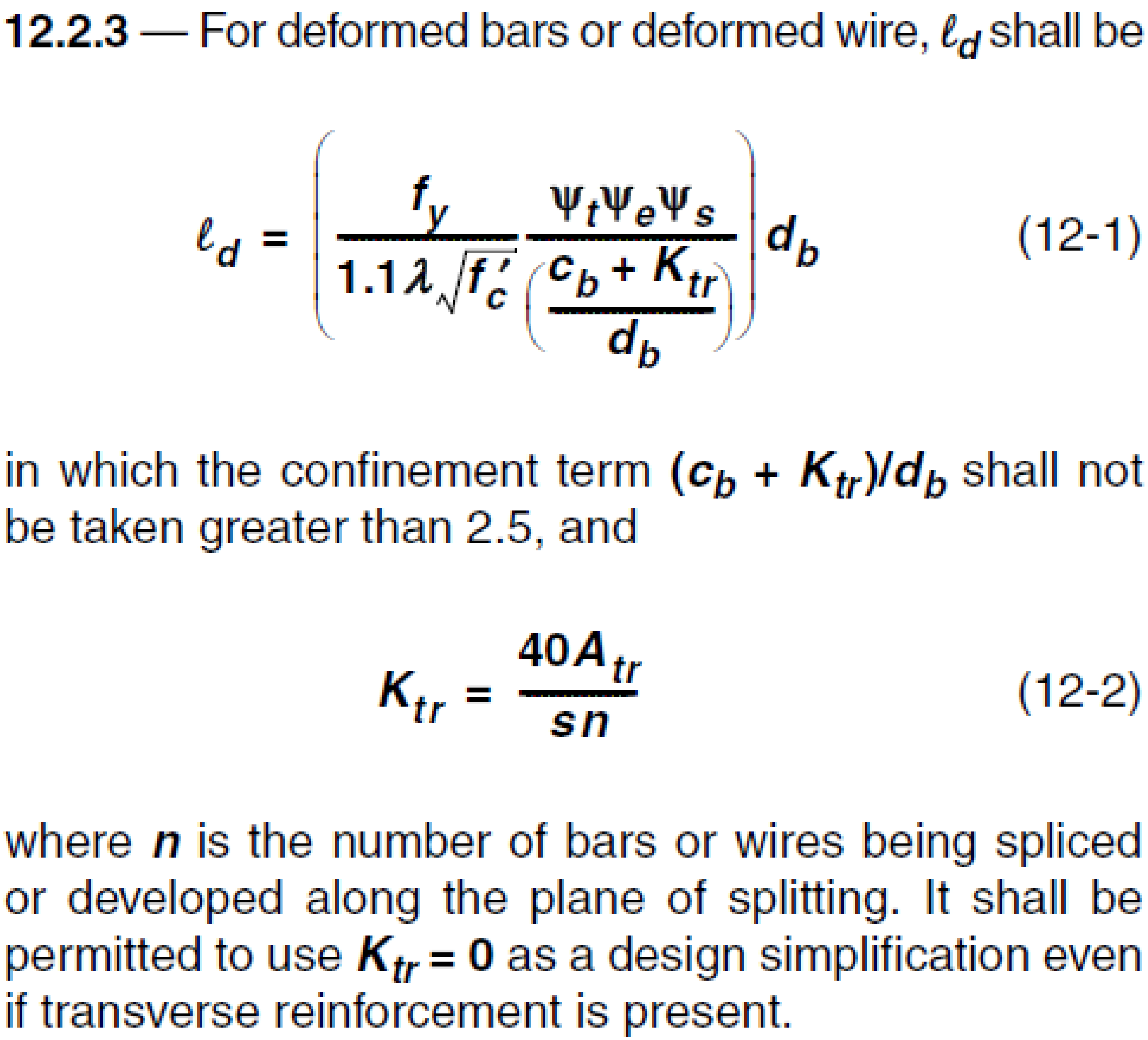
12.2.4 — The factors used in the expressions for development of deformed bars and deformed wires in tension in 12.2 are as follows:

(a) Where horizontal reinforcement is placed such that more than 300 mm of fresh concrete is cast below the development length or splice, $\psi_t = 1.3$. For other situations, $\psi_t = 1.0$.

(b) For epoxy-coated bars, zinc and epoxy dual-coated bars, or epoxy-coated wires with cover less than $3d_b$, or clear spacing less than $6d_b$, $\psi_e = 1.5$. For all other epoxy-coated bars, zinc and epoxy dual-coated bars, or epoxy-coated wires, $\psi_e = 1.2$. For uncoated and zinc-coated (galvanized) reinforcement, $\psi_e = 1.0$.

However, the product $\psi_t\psi_e$ need not be greater than 1.7.

Basic Development Equation ACI318-11



12.2.3 — For deformed bars or deformed wire, ℓ_d shall be

$$\ell_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s}{\left(\frac{c_b+K_{tr}}{d_b}\right)}\right)d_b \tag{12-1}$$

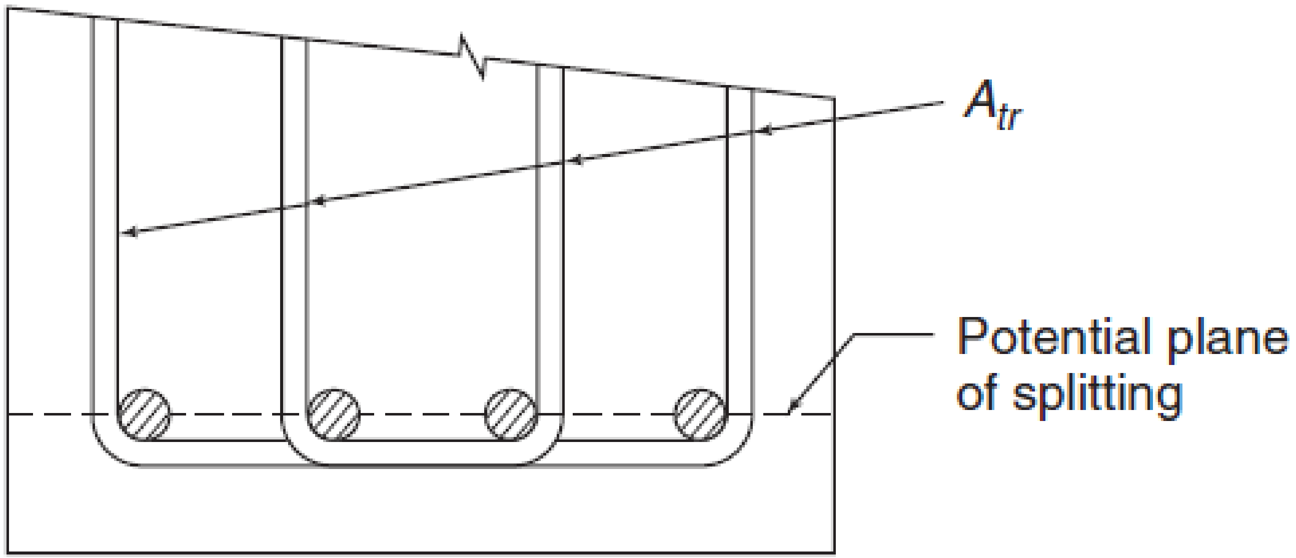
in which the confinement term $(c_b + K_{tr})/d_b$ shall not be taken greater than 2.5, and

$$K_{tr} = \frac{40A_{tr}}{sn} \tag{12-2}$$

where n is the number of bars or wires being spliced or developed along the plane of splitting. It shall be permitted to use $K_{tr} = 0$ as a design simplification even if transverse reinforcement is present.

Basic Development Equation ACI318-11

S = maximum center to center spacing of transverse reinforcement within l_d, mm

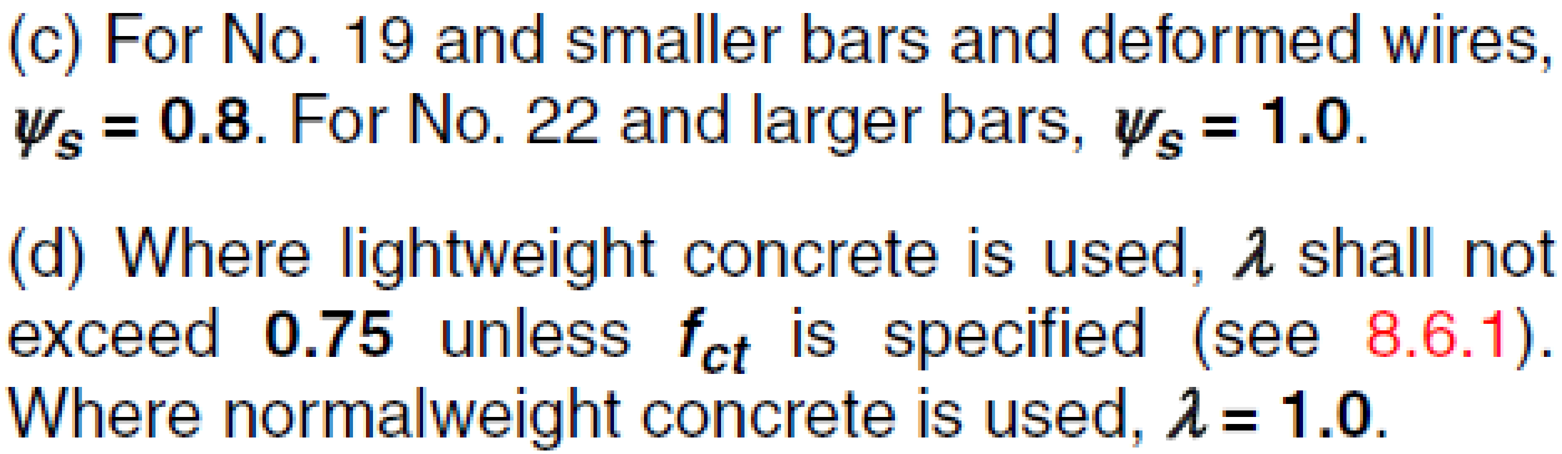


Bar Spacing Factor C_b

C_b is the smaller

- **The smallest distance measured from the surface of concrete to the center of the bar being developed, and**
- **One half of the center to center spacing of the bars or wires being developed**

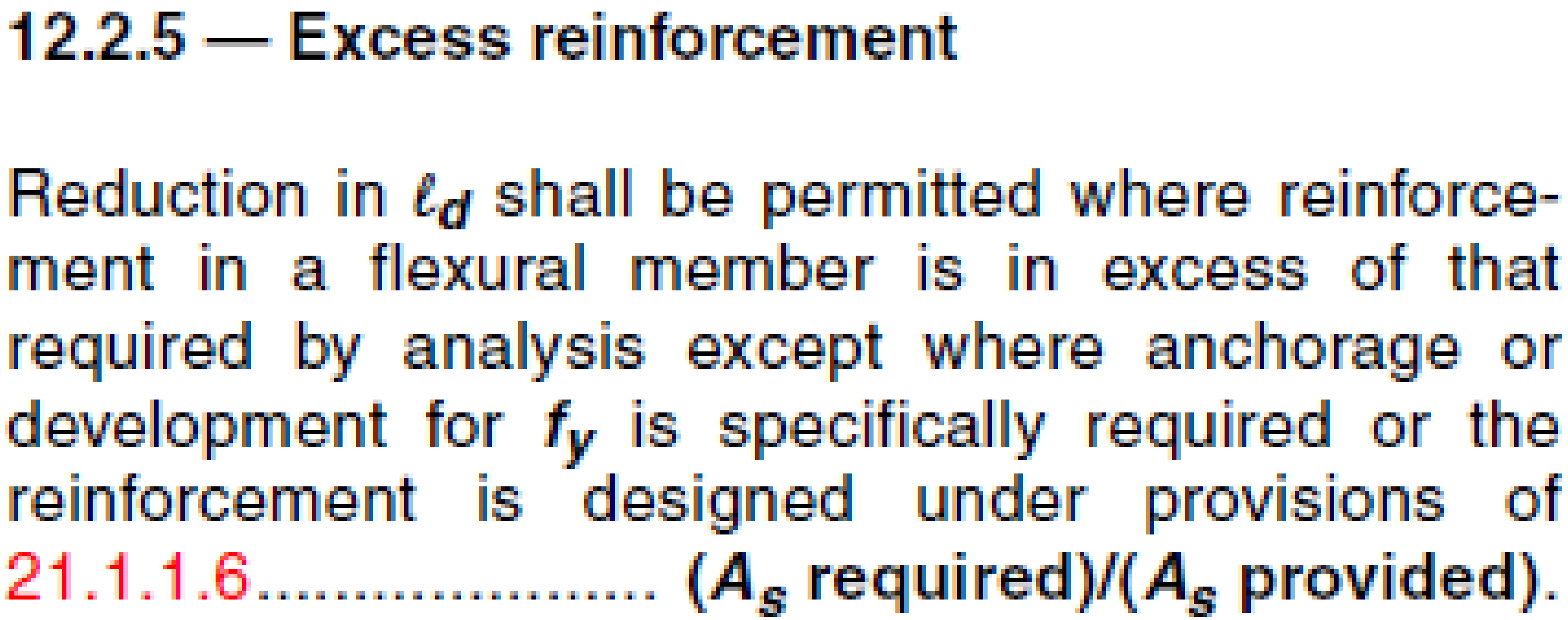
Basic Development Equation ACI318-11



(c) For No. 19 and smaller bars and deformed wires, $\psi_s = 0.8$. For No. 22 and larger bars, $\psi_s = 1.0$.

(d) Where lightweight concrete is used, λ shall not exceed **0.75** unless f_{ct} is specified (see 8.6.1). Where normalweight concrete is used, $\lambda = 1.0$.

Basic Development Equation ACI318-11



12.2.5 — Excess reinforcement

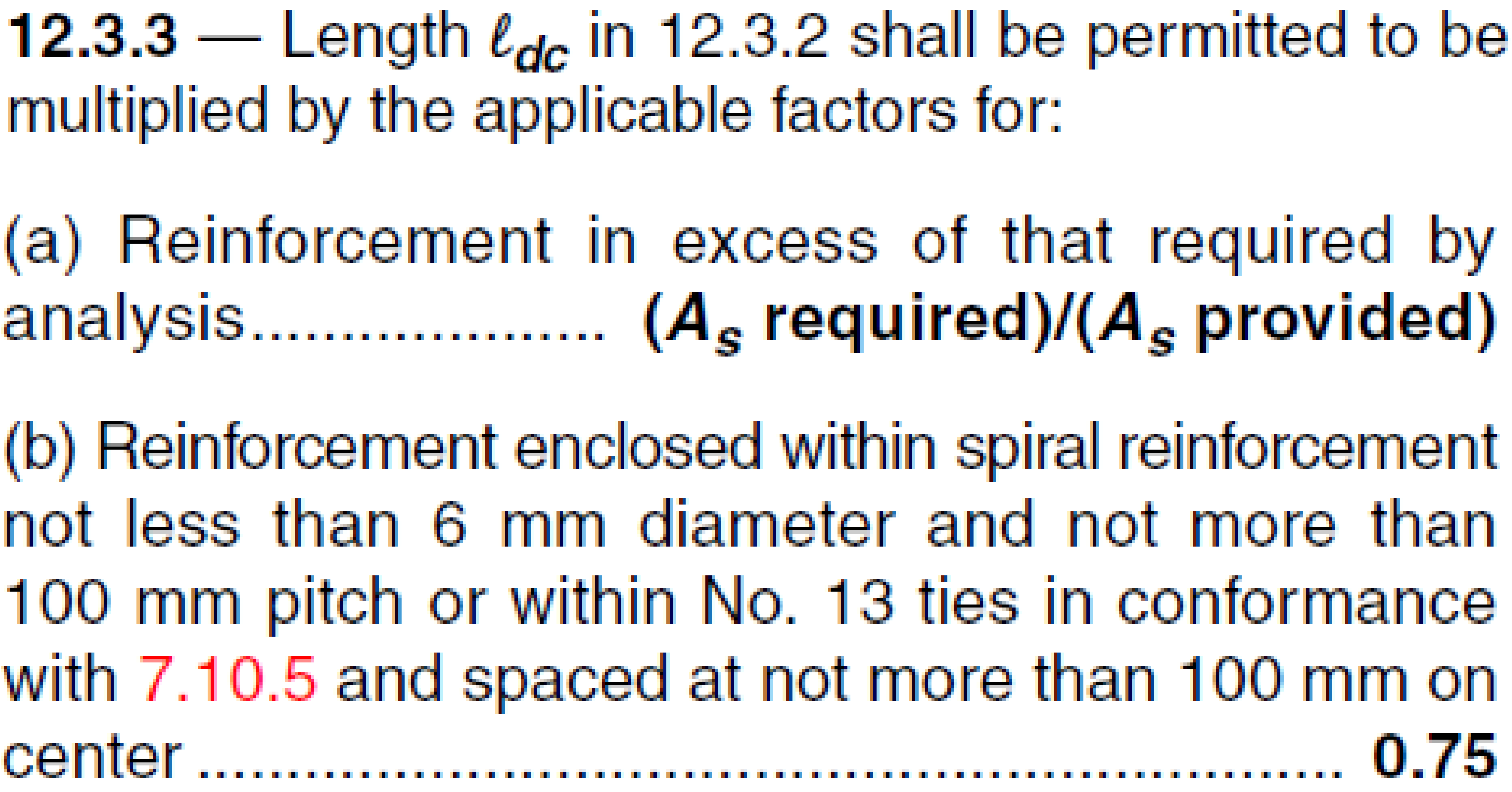
Reduction in ℓ_d shall be permitted where reinforcement in a flexural member is in excess of that required by analysis except where anchorage or development for f_y is specifically required or the reinforcement is designed under provisions of 21.1.1.6...................... $(A_s \text{ required})/(A_s \text{ provided})$.

Development of deformed bars and deformed wire in compression

12.3.1 — Development length for deformed bars and deformed wire in compression, ℓ_{dc}, shall be determined from 12.3.2 and applicable modification factors of 12.3.3, but ℓ_{dc} shall not be less than 200 mm.

12.3.2 — For deformed bars and deformed wire, ℓ_{dc} shall be taken as the larger of $(0.24 f_y / \lambda \sqrt{f_c'})d_b$ and $(0.043 f_y)d_b$, with λ as given in 12.2.4(d) and the constant 0.0003 carries the unit of mm^2/N.

Development of deformed bars and deformed wire in compression



12.3.3 — Length ℓ_{dc} in 12.3.2 shall be permitted to be multiplied by the applicable factors for:

(a) Reinforcement in excess of that required by analysis..................... **(A_s required)/(A_s provided)**

(b) Reinforcement enclosed within spiral reinforcement not less than 6 mm diameter and not more than 100 mm pitch or within No. 13 ties in conformance with 7.10.5 and spaced at not more than 100 mm on center **0.75**

Development of standard hooks in tension

12.5.1 — Development length for deformed bars in tension terminating in a standard hook (see 7.1), ℓ_{dh}, shall be determined from 12.5.2 and the applicable modification factors of 12.5.3, but ℓ_{dh} shall not be less than the larger of $8d_b$ and 150 mm.

12.5.2 — For deformed bars, ℓ_{dh} shall be $(0.24\psi_e f_y / \lambda\sqrt{f_c'})d_b$ with ψ_e taken as 1.2 for epoxy-coated reinforcement, and λ taken as 0.75 for lightweight concrete. For other cases, ψ_e and λ shall be taken as 1.0.

Development of standard hooks in tension

12.5.3 — Length ℓ_{dh} in 12.5.2 shall be permitted to be multiplied by the following applicable factors:

(a) For No. 36 bar and smaller hooks with side cover (normal to plane of hook) not less than 65 mm. and for 90-degree hook with cover on bar extension beyond hook not less than 50 mm....................... **0.7**

(b) For 90-degree hooks of No. 36 and smaller bars that are either enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than $3d_b$ along ℓ_{dh}; or enclosed within ties or stirrups parallel to the bar being developed, spaced not greater than $3d_b$ along the length of the tail extension of the hook plus bend **0.8**

Development of standard hooks in tension

(c) For 180-degree hooks of No. 36 and smaller bars that are enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than **$3d_b$** along ℓ_{dh} **0.8**

(d) Where anchorage or development for f_y is not specifically required, reinforcement in excess of that required by analysis .... **(A_s required)/(A_s provided)**

12.5.5 — Hooks shall not be considered effective in developing bars in compression.

Development of standard hooks in tension

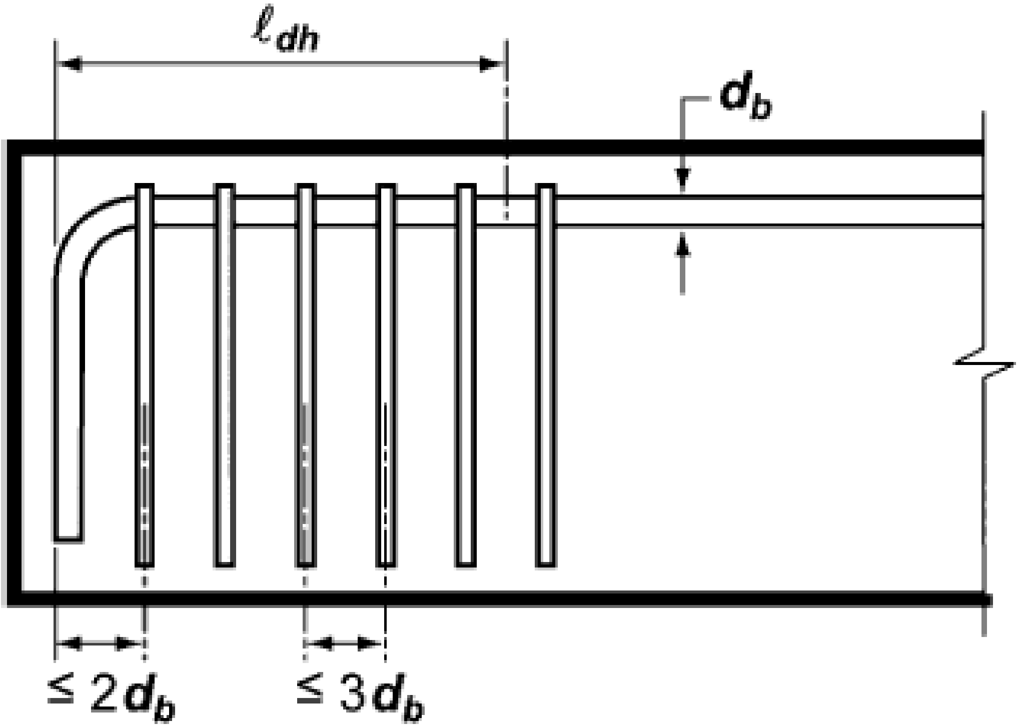


Fig. R12.5.3(a)—Ties or stirrups placed perpendicular to the bar being developed, spaced along the development length ℓ_{dh}.

Development of standard hooks in tension

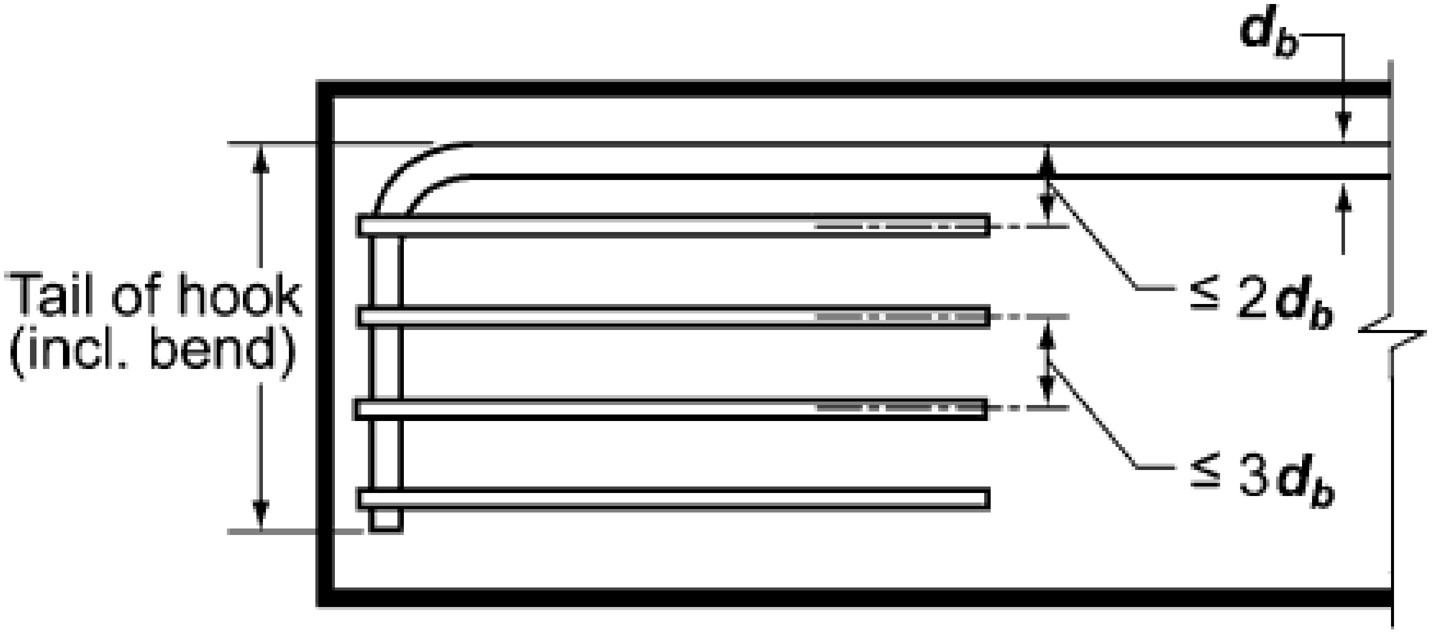


Fig. R12.5.3(b)—Ties or stirrups placed parallel to the bar being developed, spaced along the length of the tail extension of the hook plus bend.

Development of standard hooks in tension

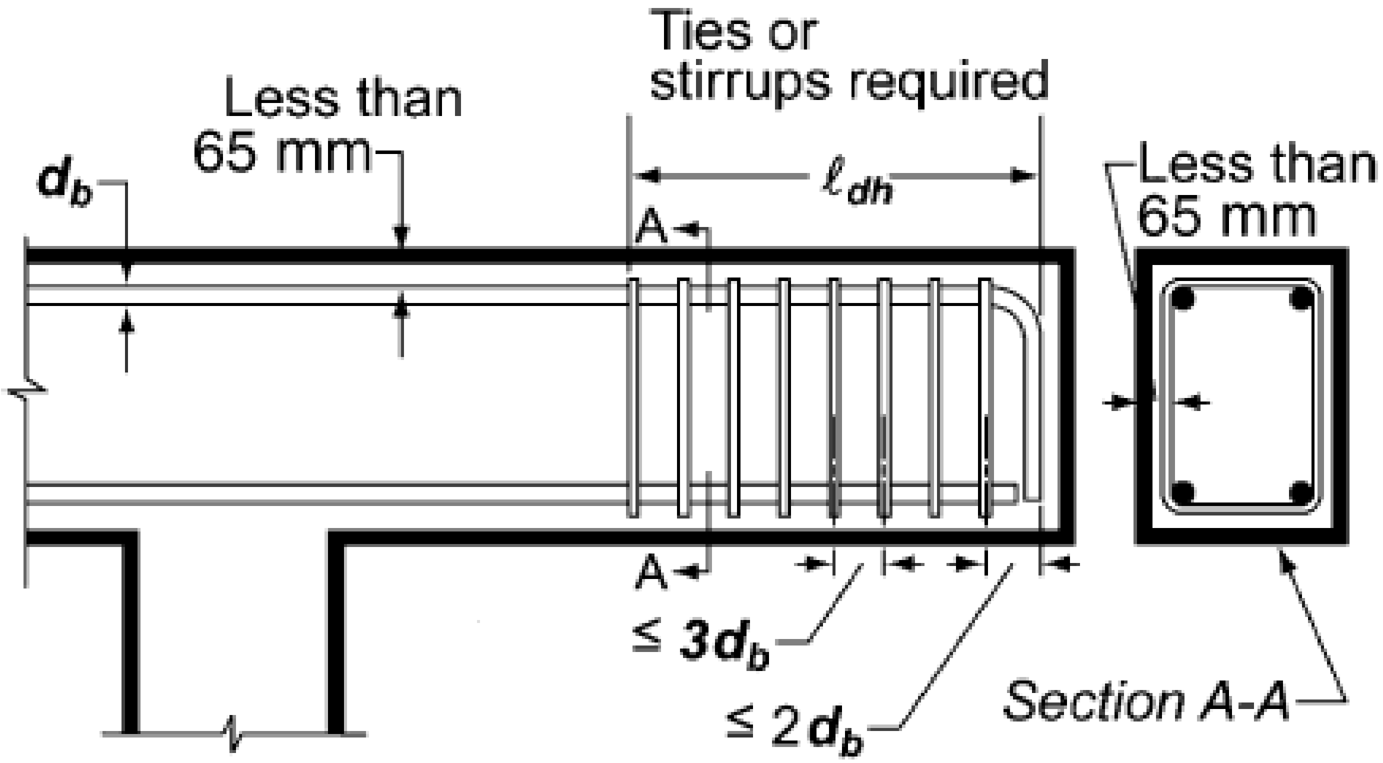
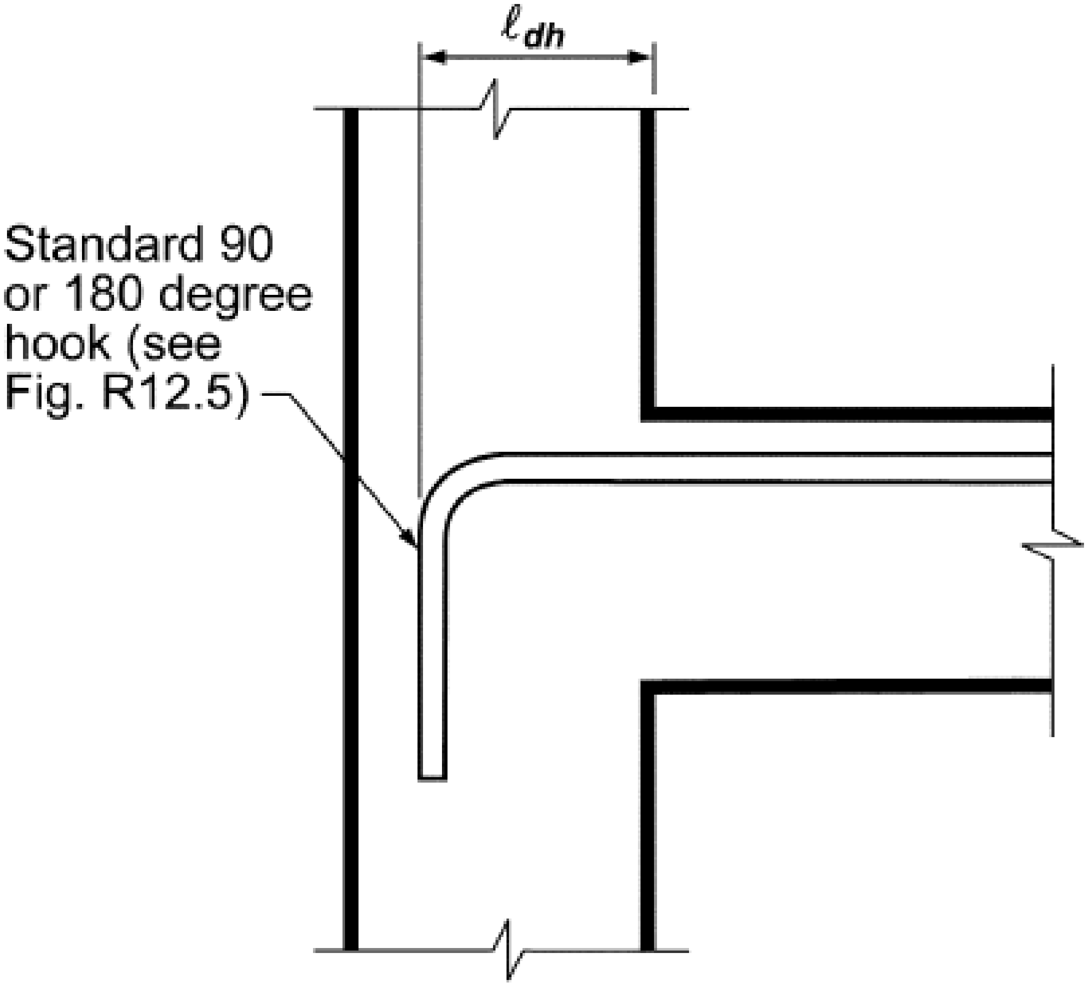


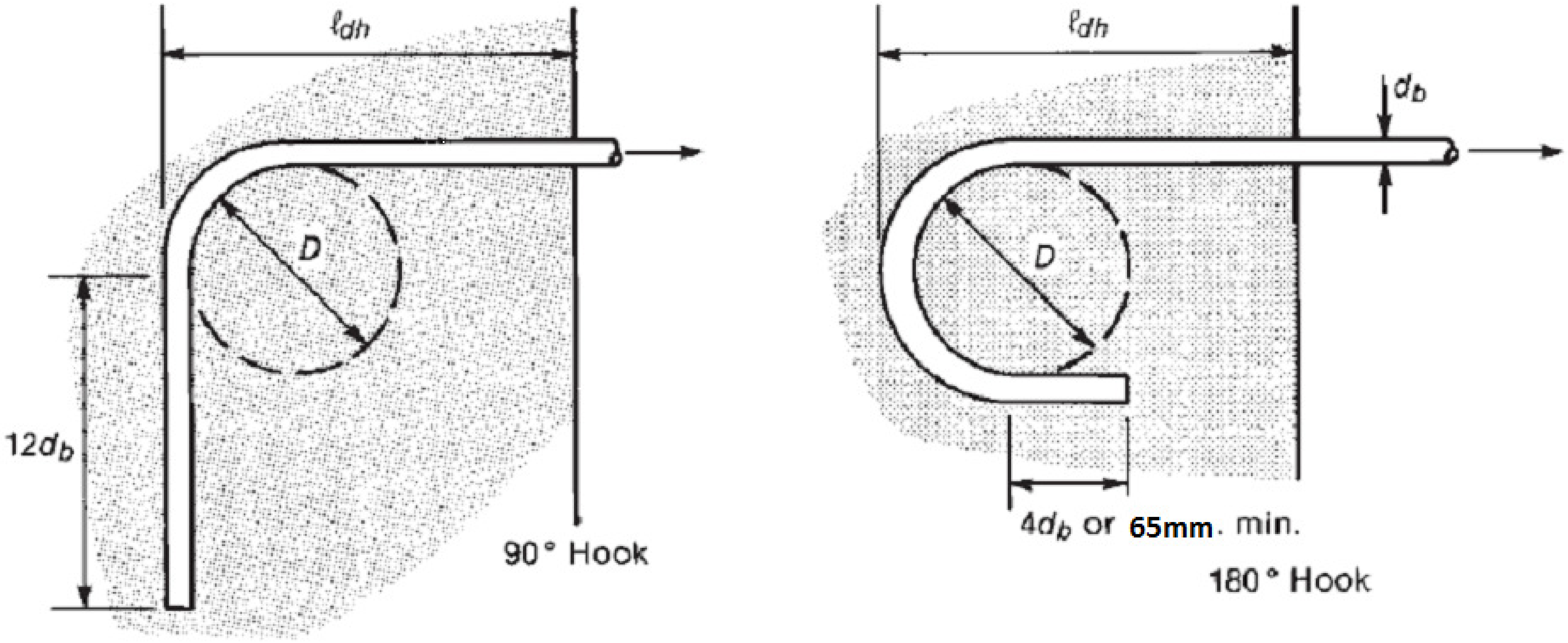
Fig. R12.5.4—Concrete cover per 12.5.4.

Standard Hooks



(a) Anchorage into exterior column

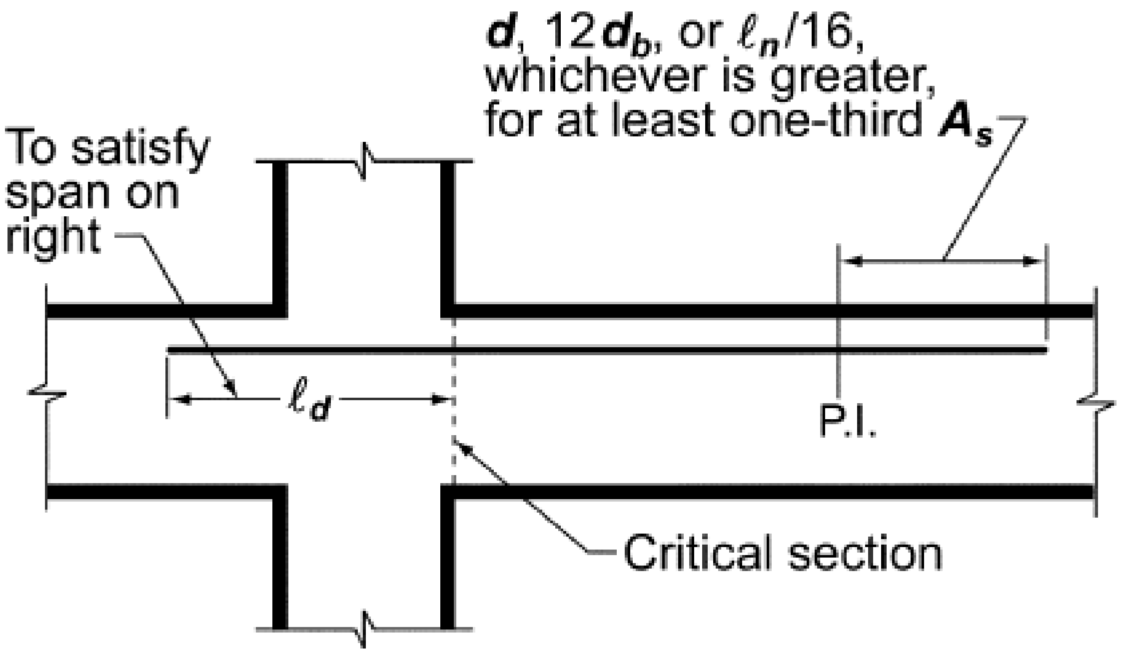
Standard Hooks



No. 10 through No. 25 $D = 6d_b$
No. 29, 32 and 36 $D = 8d_b$
No. 43 and No. 57 $D = 10d_b$

(a) Standard hooks—ACI Code Sections 7.1 and 7.2.1

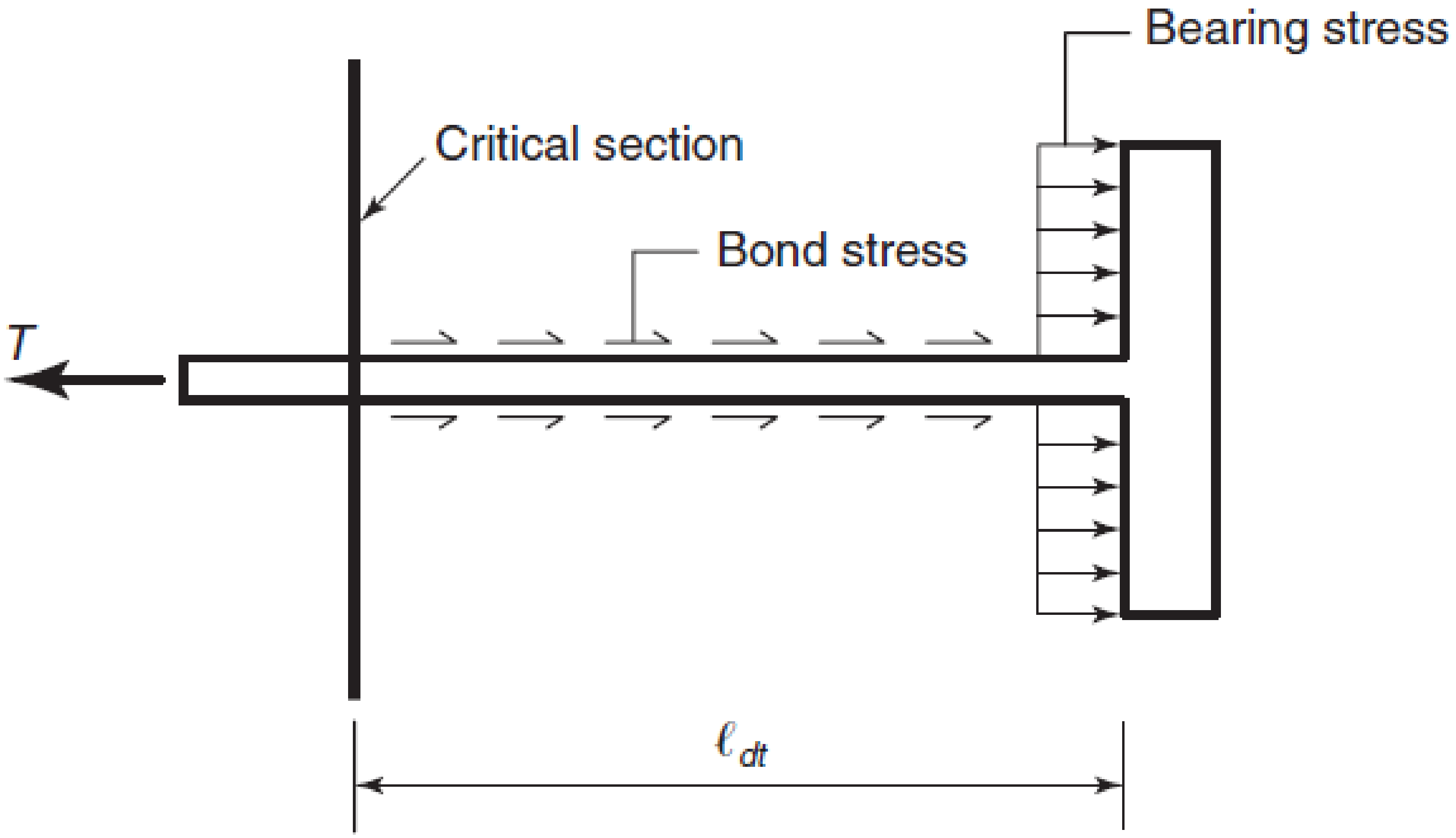
Development of Negative Moment Reinforcement

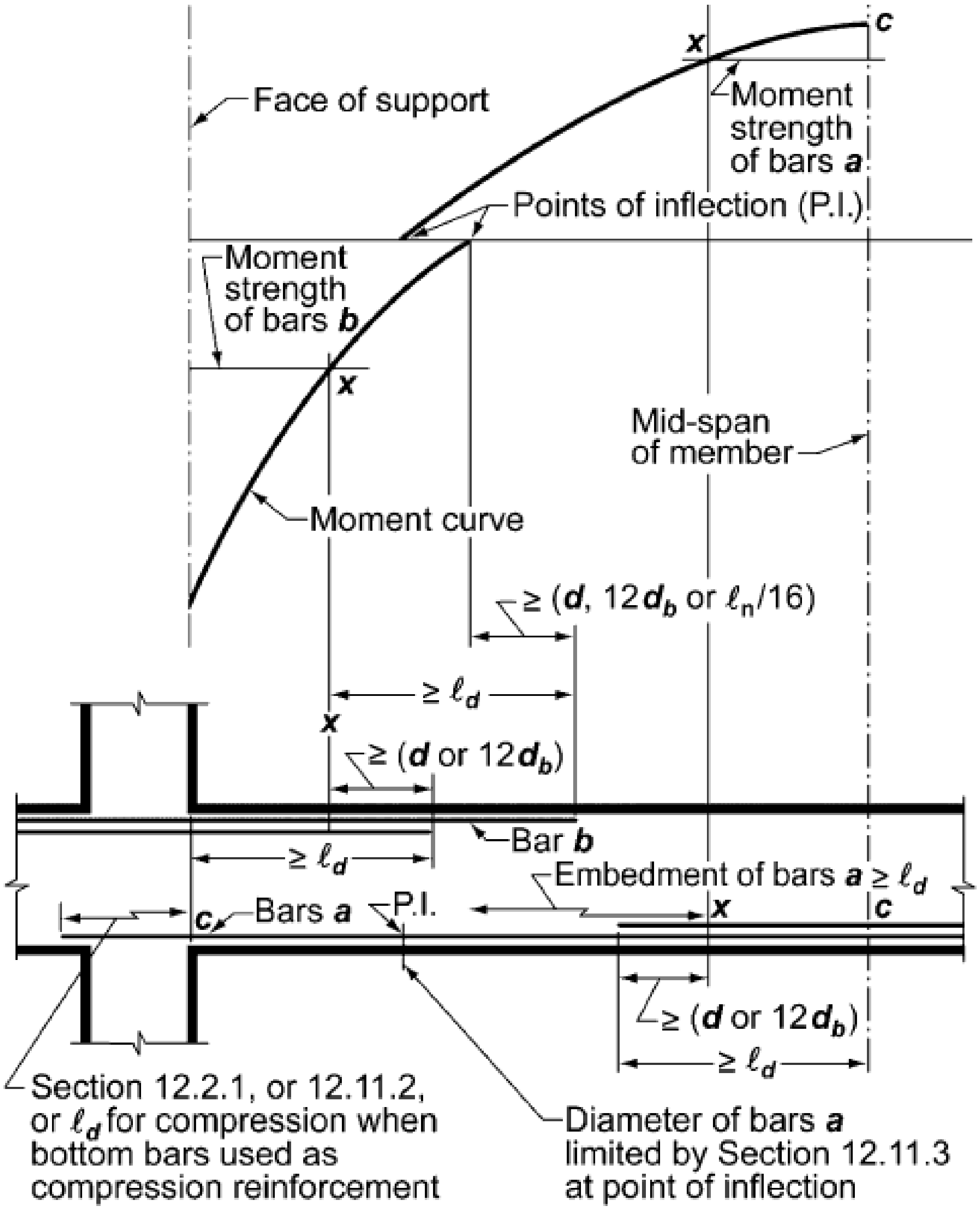


Note: Usually such anchorage becomes part of the adjacent beam reinforcement.

(b) Anchorage into adjacent beam

Development length of headed bars in tension




x
c
Face of support
Moment strength of bars a
Points of inflection (P.I.)
Moment strength of bars b
x
Mid-span of member
Moment curve
≥ (d, 12d_b or ℓ_n/16)
≥ ℓ_d
x
≥ (d or 12d_b)
Bar b
≥ ℓ_d
Embedment of bars a ≥ ℓ_d
c
Bars a
P.I.
x
c
≥ (d or 12d_b)
≥ ℓ_d
Section 12.2.1, or 12.11.2, or ℓ_d for compression when bottom bars used as compression reinforcement
Diameter of bars a limited by Section 12.11.3 at point of inflection

Splices of Reinforcement

Splices should, if possible, be located away from points of maximum tensile stress.

Frequently, reinforcement in beams and columns must be spliced.

There are four types of splices:

- Lapped splices,
- Mechanical splices,
- Welded splices, and
- End-bearing splices

All four types of splices are permitted, as limited in ACI Code Sections 12.14, 12.15, and 12.16

Tension Lap Splices

Failure of a tension lap splice without stirrups enclosing the splice. (Photograph courtesy of J. G. MacGregor.)

Tension Lap Splices

- In a lapped splice, the force in one bar is transferred to the concrete, which transfers it to the adjacent bar.
- The transfer of forces out of the bar into the concrete causes radially outward pressures on the concrete these pressures, in turn, cause splitting cracks along the bars.
- The splitting cracks generally initiate at the ends of the splice, where the splitting pressures tend to be larger than at the middle.
- Large transverse cracks occur at the discontinuities at the ends of the spliced bars.
- Transverse reinforcement in the splice region delays the opening of the splitting cracks and hence improves the splice capacity.

Tension Lap Splices

12.15 — Splices of deformed bars and deformed wire in tension

12.15.1 — Minimum length of lap for tension lap splices shall be as required for Class A or B splice, but not less than 300 mm, where:

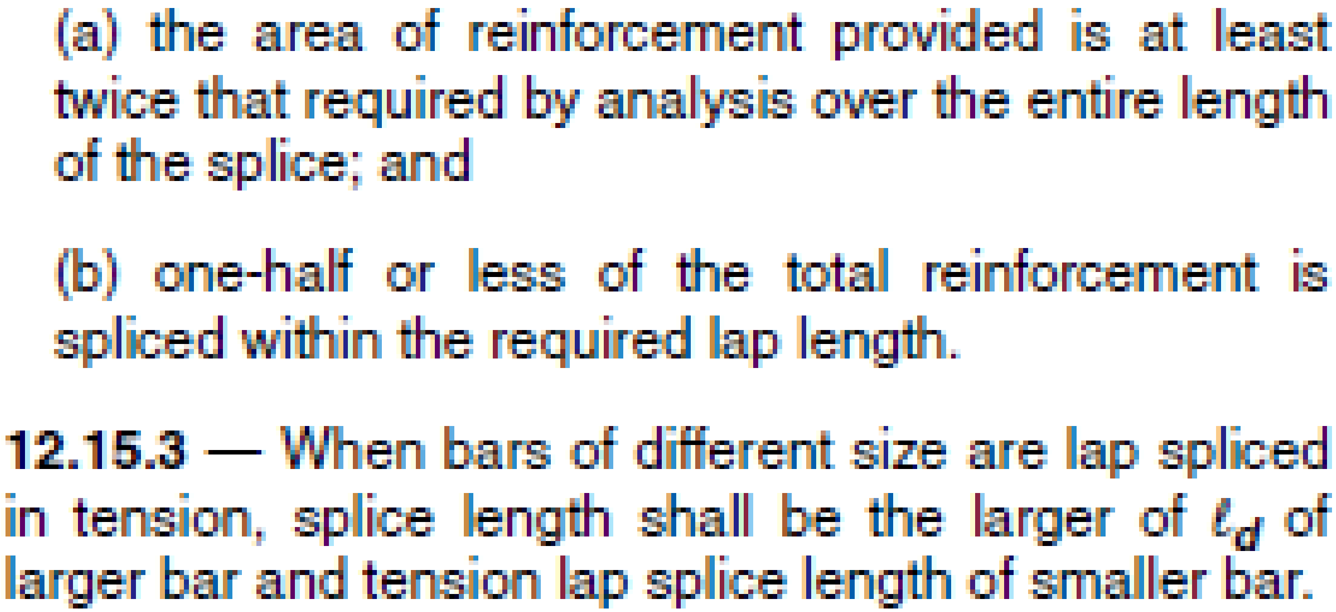
Class A splice.. **$1.0\ell_d$**

Class B splice.. **$1.3\ell_d$**

where ℓ_d is calculated in accordance with 12.2 to develop f_y, but without the 300 mm minimum of 12.2.1 and without the modification factor of 12.2.5.

12.15.2 — Lap splices of deformed bars and deformed wire in tension shall be Class B splices except that Class A splices are allowed when:

Tension Lap Splices



(a) the area of reinforcement provided is at least twice that required by analysis over the entire length of the splice; and

(b) one-half or less of the total reinforcement is spliced within the required lap length.

12.15.3 — When bars of different size are lap spliced in tension, splice length shall be the larger of ℓ_d of larger bar and tension lap splice length of smaller bar.

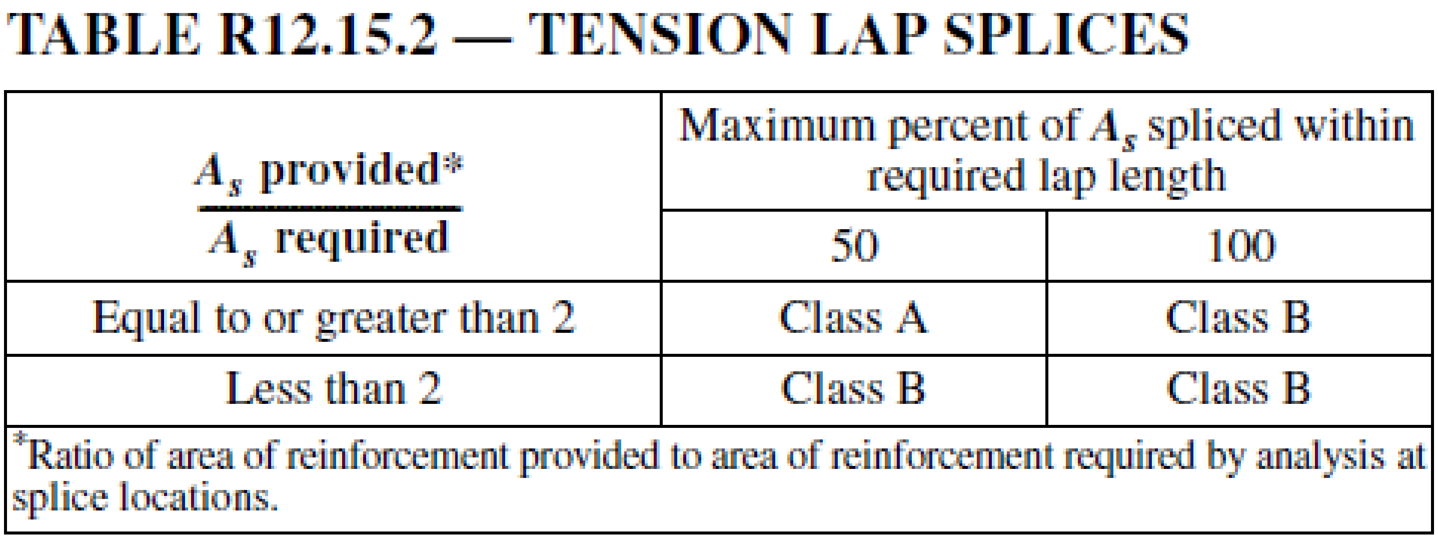
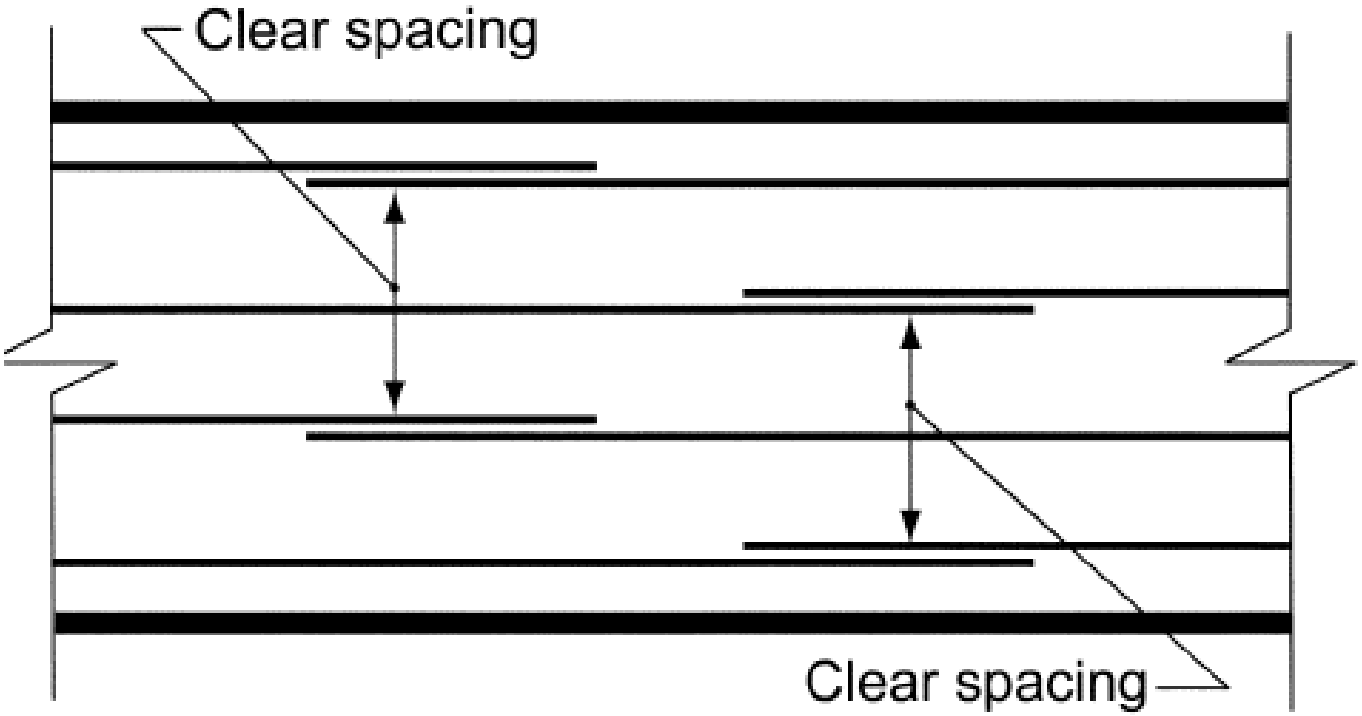


TABLE R12.15.2 — TENSION LAP SPLICES

$\dfrac{A_s \text{ provided*}}{A_s \text{ required}}$	Maximum percent of A_s spliced within required lap length	
	50	100
Equal to or greater than 2	Class A	Class B
Less than 2	Class B	Class B

*Ratio of area of reinforcement provided to area of reinforcement required by analysis at splice locations.

Tension Lap Splices



(b) Staggered splices

Tension Development Length of Deformed Bars

The user may easily construct simple, useful expressions. For example, in all structures with normalweight concrete ($\lambda = 1.0$), uncoated reinforcement ($\psi_e = 1.0$), No. 22 or larger bottom bars ($\psi_t = 1.0$) with $f_c' = 28$ MPa and Grade 420 reinforcement, the equations reduce to

$$\ell_d = \frac{(420)(1.0)(1.0)}{1.7(1.0)\sqrt{28}} d_b = 47 d_b$$

or

$$\ell_d = \frac{(420)(1.0)(1.0)}{1.1(1.0)\sqrt{28}} d_b = 72 d_b$$

Thus, as long as minimum cover of d_b is provided along with a minimum clear spacing of $2d_b$, or a minimum clear cover of d_b and a minimum clear spacing of d_b are provided along with minimum ties or stirrups, then $\ell_d = 47 d_b$. The penalty for spacing bars closer or providing less cover is the requirement that $\ell_d = 72 d_b$.

12.16 Splices of deformed bars in compression

12.16.1 — Compression lap splice length shall be

$0.071 f_y d_b$ for f_y of 420MPa or less, or

$(0.13 f_y - 24) d_b$ for f_y greater than 420MPa,

but not less than 300 mm.

For f_c' less than 21 MPa, length of lap shall be increased by one-third.

Splices of deformed bars in compression

12.16.2 When bars of different size are lap spliced in compression, splice length shall be the larger of l_{dc} of larger bar and compression lap splice length of smaller bar.

Lap splices of No. 43 and No. 57 bars to No. 36 and smaller bars shall be permitted.

Splice requirements for columns

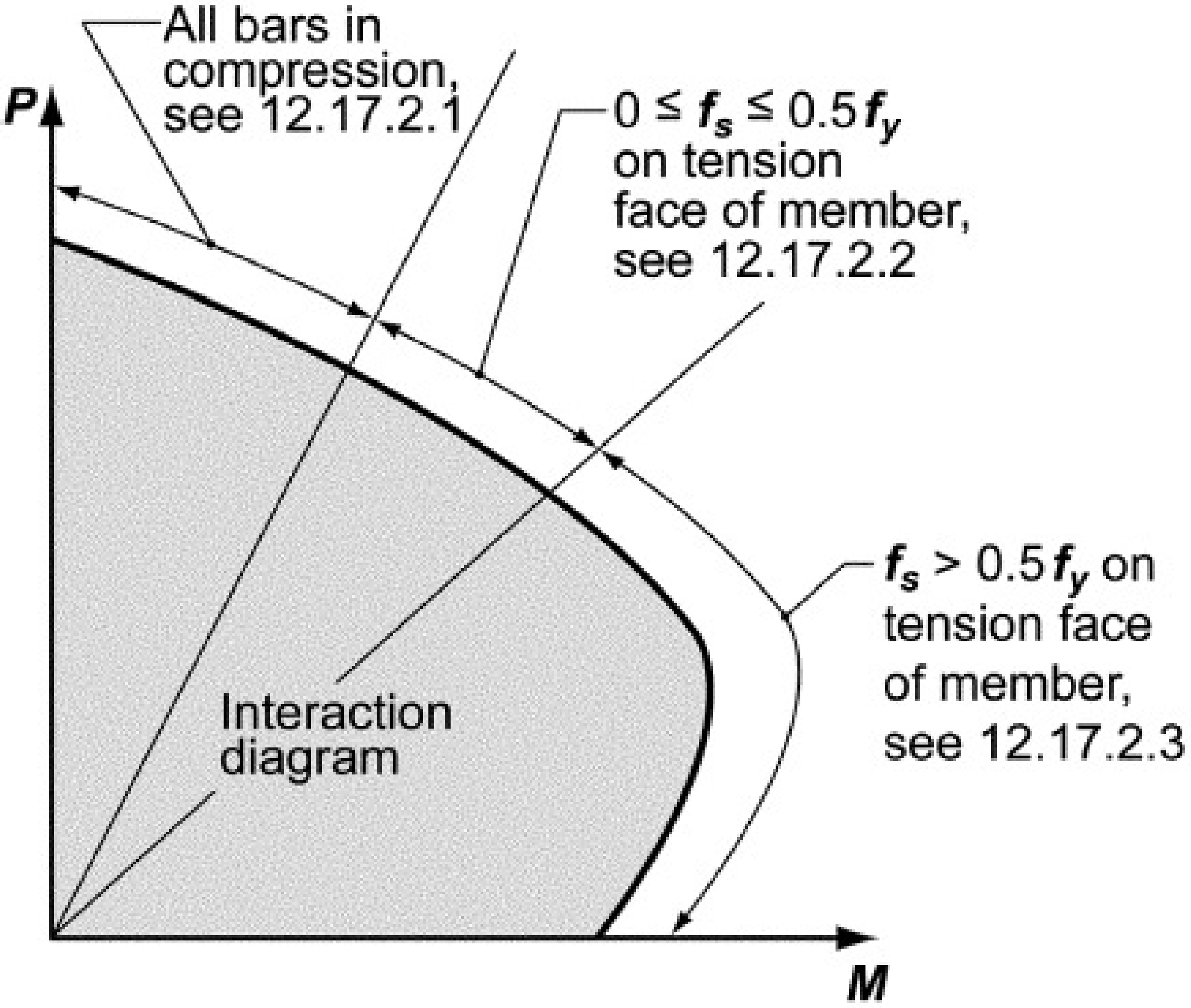


Fig. R12.17—Special splice requirements for columns.

Splice requirements for columns

12.17.2.1 — Where the bar stress due to factored loads is compressive, lap splices shall conform to 12.16.1, 12.16.2, and, where applicable, to 12.17.2.4 or 12.17.2.5.

12.17.2.2 — Where the bar stress due to factored loads is tensile and does not exceed 0.5*fy* in tension, lap splices shall be Class B tension lap splices if more than one-half of the bars are spliced at any section, or Class A tension lap splices if half or fewer of the bars are spliced at any section and alternate lap splices are staggered by *ld*.

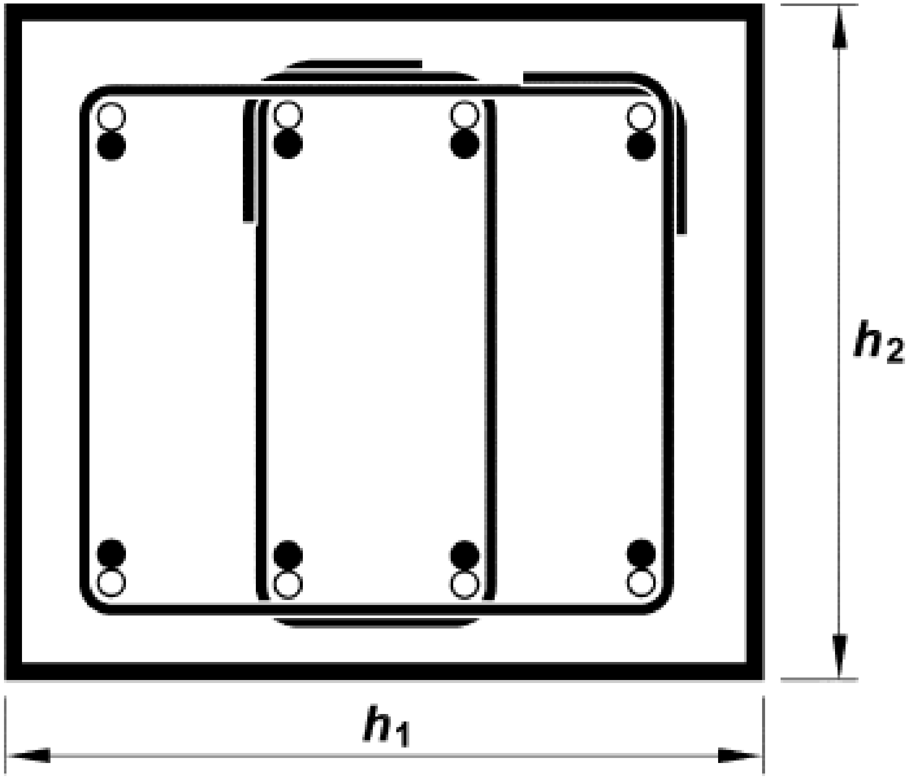
12.17.2.3 — Where the bar stress due to factored loads is greater than 0.5*fy* in tension, lap splices shall be Class B tension lap splices

Splice requirements for columns

12.17.2.4 — In tied reinforced compression members, where ties throughout the lap splice length have an effective area not less than 0.0015*hs* in both directions, lap splice length shall be permitted to be multiplied by 0.83, but lap length shall not be less than 300 mm. Tie legs perpendicular to dimension *h* shall be used in determining effective area.

12.17.2.5 — In spirally reinforced compression members, lap splice length of bars within a spiral shall be permitted to be multiplied by 0.75, but lap length shall not be less than 300 mm.

Splice requirements for columns



Direction 1: $4A_b \geq 0.0015h_1s$

Direction 2: $2A_b \geq 0.0015h_2s$

where A_b is the area of the tie

Fig. R.12.17.2—Example application of 12.17.2.4.