CE 150 COMPUTER PROGRAMMING

BINARY NUMBER SYSTEM

by

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1

BASIC UNIT OF STORAGE

- A CIRCUIT in ON or OFF State
- Two numbers (or symbols) can be associated to these two states.
- The numbers are 1 and 0; the binary numbers
- The unit is termed as BIT (abbreviation of BInary digiT)

2

FUNDAMENTAL UNIT OF STORAGE

- 8 bits integrated to make a BYTE
- BYTE can store 256 different patterns consisting of 0s and 1s.
- 256 different numbers can be associated to these patterns.
- Storage exists of millions of Bytes

3

DATA TYPES

- Data may be classified into two broader classes:
  1. Numeric
  2. Non-Numeric
- Numbers may be whole or fractional
- Present discussion is limited to whole numbers only.

4

WHOLE NUMBERS

- WHOLE NUMBERS may be associated to bit patterns according to one of three conventions:

❖TRUE NOTATION

❖SIGNED NOTATION

❖EXCESS NOTATION

5

LOCAL VALUE

- Consider the decimal number 4035
- The local value of 5 is only <u>5</u> or (5x1 or $5x10^0$)
- The LV of 3 is <u>30</u> (3x10 or $3x\ 10^1$)

  The LV of 0 is <u>zero</u>
- The LV of 4 is <u>4000</u> (4x 1000 or $4x10^3$)
- In General LV at Nth position is 10^{N-1}

6

TRUE NOTATION

- Local Bit Values of pattern are added together to obtain the true whole number contained by the BYTE.

8	7	6	5	4	3	2	1	bit
128	64	32	16	8	4	2	1	LV
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	

- Zero (0) has a LV of zero at any bit position
- In General LV at Nth bit position is 2^{N-1}

7

EXAMPLE 1

- Consider the pattern 0 0 0 0 0 0 0 0 All the symbols are zero; each have local value zero and sum is 0. Thus the pattern 0 0 0 0 0 0 0 0 represents the decimal number 0 (ZERO) in true notation.

8

EXAMPLE 2

- 0 0 0 0 0 1 0 1

Gives the decimal number . .

$1x2^0+1x2^2+0$ ….=1 +4 = 5 or

$2^0+2^2=5$

- 1 0 0 1 0 0 0 0

= 2^4+2^7 = 16+128 = 144

9

EXAMPLE 3

- 1 1 1 1 1 1 1 1 is the largest whole number contained by BYTE in true notation.

Starting from right most bit (lowest bit)

= 1+2+4+8+16+32+64+128=255

The numbers 0 to 255 are associated to 256 patterns in a BYTE

10

EXCESS NOTATION

- This is one of the two ways to associate the Negative and Positive whole numbers to bit patterns.
- Consider three bit word for example:
- 000 001 010 011 . 100 101 110 111 are the 8 possible patterns. The pattern 100 equals to zero in this excess notation.

11

EXCESS NOTATION

- This number (100) is 4 in true notation.
- Thus 3 bit patterns would give Excess 4 numbers. The Excess is the true value of the pattern associated to the excess number zero (1 0 0 = 4)
- Excess 4 is denoted by E_4
- The Excess in E_4 is 4.

12

Examples in E_4

Binary	True	E4	Binary	True	E4
0 0 0	0	-4	1 0 0	4	0
0 0 1	1	-3	1 0 1	5	1
0 1 0	2	-2	1 1 0	6	2
0 1 1	3	-1	1 1 1	7	3

- E_4=True –4 .

13

EXCESS NOTATION

- Next Consider 4 bit patterns.
- 1 0 0 0 denotes ZERO in excess notation and its true value is 8.
- 4 bit patterns are thus denoted by E_8.
- Excess is 8 (true value of 1 0 0 0)
- E_8=True-8

14

EXAMPLES in 4 bits

- 1 0 0 1 (4 bits 1000 =8 so E_8)
- True Value = 1+8 = 9
- E_8 = 9 – 8 = 1
- Thus 1001 is 1 in Excess 8 notation.
- 0 0 0 1
- True value = 1
- E_8 = 1 - 8 = - 7

15

EXCESS in a BYTE

- In 8 bits, 1000 0000 is the pattern associated to the number zero (0). Its true value is 128. Therefore in a single byte Excess Notation is termed as $E_{128.}$
- The Excess is 128.
- E_{128}=True-128

16

EXAMPLES in E_{128}

- 0 1 0 0 1 1 1 1 (1 0 0 0 0 0 0 0) =128

True value = 1+2+4+8+64 = 79

The E_{128} value = 79 - 128 = - 49

- 0 0 0 0 0 0 0 0

True Value = 0

E_{128} = 0 -128= - 128 (the smallest possible number in a byte in excess notation)

17

EXAMPLES in E_{128}

- 1 1 1 1 1 1 1 1

True value = 255

E_{128} = 255 - 128 = 127. The largest possible number in 8 bits in Excess Notation.

If there is a zero in the highest bit the number is –ve and positive otherwise.

18

GENERAL

- 8 7 6 5 4 3 2 1
- Highest Lowest
- The highest bit is also termed as the most significant bit as the lowest bit is called as the least significant bit.

19

SIGNED NOTATION

- Is another way of associating signed numbers to bit patterns.
- A ZERO (0) in the highest bit denotes non-negative number
- A ONE (1) in the highest bit denotes negative whole number.
- The local value of highest bit = $-(2)^{N-1}$

20

RULES for Signed Notation

- For ZERO in the last bit the TRUE value of the number is its SIGNED VALUE as well.
- For 1 in the last bit, add the local values of all other bits to $-(2)^{N-1}$ to obtain negative number.
- Signed Value = -ve value of last bit + Σ LV of all other bits

21

EXAMPLES in 3 bits

❖ 1 0 1
- Local Value of the Highest Bit = - 4
- Local Values of all other bits = 1
- The Signed Number = - 4 + 1 = - 3

❖ 0 1 1
- Zero in last bit indicates number is +ve; its true value is signed value:
- 0 + 2 + 1 = 3

22

EXAMPLES In 3 bits

❖ 0 1 1 is the largest +ve number
- Signed Value: 0 + 2 + 1 = 3

❖ 1 0 0 is the smallest number
- Signed Value: - 4
- Numbers range between – 4 and + 3

23

EXAMPLES in 4 bits

❖ 0 0 1 1 (is +ve Number)
- Signed Value = 3 = True Value

❖ 1 1 1 1 (is –ve Number)
- Signed Value: -8+(4+2+1) = -1
- 1 0 1 0
- Signed Value: - 8 + 2 = - 6

24

EXAMPLES in 8 bits

- 0 0 0 1 1 1 1 1
- Number is +ve and is = 16+8+4+2+1= 31
- 1 0 0 1 1 0 0 1
- = -128+(16+8+1) = - 103
- 1 1 1 1 1 1 1 1
- = -128+(64+32+16+8+4+2+1) = -1
- -1 in any number of bits is the pattern containing 1 in all bits.

25

WHOLE NUMBER DATA TYPES

- BYTE
- 1 Byte storage, True Notation
- 0 0 0 0 0 0 0 0 is the smallest number Value is 0
- 1 1 1 1 1 1 1 1 is the largest number Value is 255
- Range of BYTE numbers is (0 — 255)

26

WHOLE NUMBER DATA TYPES

- INTEGER
- 2 Byte storage, SIGNED Notation
- 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 is the smallest number and value is -32768
- 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 is the largest number and value is 32767
- Range of INTEGERS is (- 32768 — +32767)

27

WHOLE NUMBER DATA TYPES

- LONG INTEGER (LONG)
- 4 Byte storage in SIGNED Notation
- 1000 0000 0000 0000 0000 0000 0000 0000 is the smallest number and value is $-(2)^{31}$ = -2 14 74 83 648
- 0111 1111 1111 1111 1111 1111 1111 1111 is the largest number and value is $(2)^{31}-1$ = 2 14 74 83 647
- Range (-2 14 74 83 648 ----- +2 14 74 83 647)

28

Applications

TRUE NOTATION

- True notation is used to calculate the Values in Signed Notation as well as Excess Notation.
- It is the basic notation and applicable for all types of notations.

29

Applications

SIGNED NOTATION

- This notation is used to calculate the Values of Integers and Long Data Types.
- It is meaningful for whole numbers stored in 2B (16b) and 4B (32b) storage.

30

Applications

EXCESS NOTATION

- This notation is used to calculate the EXPONENT of real data type.
- It is meaningful for 8b and 11b only.
- These are used to store the exponent of Single Precision and Double Precision Real numbers. (To be discussed later on).

31

32