UNIT 4

AZIMUTH DETERMINATION BY CELESTIAL OBSERVATION

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Introduction

The interaction of the earth with the sun and other stars is a three-dimensional happening best explained using a three-dimensional model. The relationship of the earth to the other celestial bodies is described in relation to the PZS or astronomical triangle and its six parts.

Azimuth determinations are generally made by observing either the sun or Polaris, although other stars may be used. The two methods for determining azimuth from the sun are the hour angle method and the altitude method. Accuracy requirements for the work, as well as accuracy of the data used in the calculations, will determine the method to be used.

Performance Expected on the Exams

Describe the interaction of the earth and the sun and other stars in relation to the PZS triangle.

Compute the astronomic azimuth of a line from a solar observation by either hour angle or altitude method.

Compute the astronomic azimuth of a line from a Polaris observation.

State the relative strengths and weaknesses of the hour angle and altitude methods of solar observation.

State the reasons for choosing a Polaris observation rather than a sun observation.

Key Terms

Celestial sphere
Astronomical triangle
Latitude
Longitude
Altitude
Declination
Greenwich hour angle
Local hour angle
Sidereal hour angle
Apparent time
Equation of time
Moment of time
True meridian time
Parallax
Refraction
Elongation
Culmination
Zenith
Polar distance
Vernal equinox
Autumnal equinox
Right ascension
Civil time
Elapsed time
Universal time
Atomic time
Sidereal time
Daylight saving time
The first point of Aries
Standard time
Atomic second

Video Presentation Outline

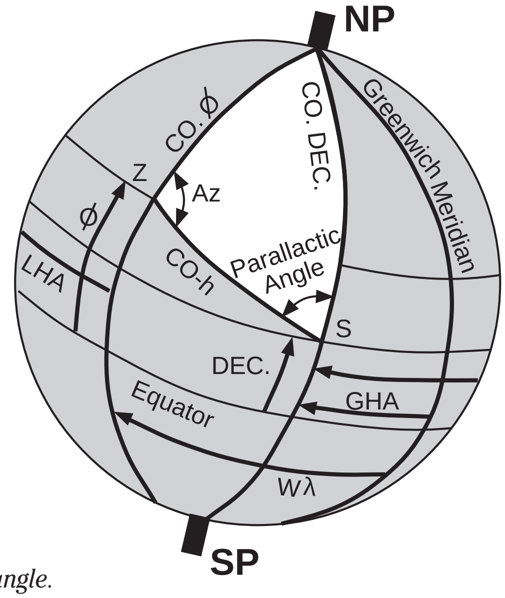


Figure 4-1. *The PZS triangle.*

The PZS Triangle

Co Ø	=	90° - LAT
CoDEC	=	90° - DEC
Co h	=	90° - h
NP	=	North Pole
Z	=	Zenith
S	=	Sun or star

- The three corners
- Time, latitude, longitude
- The six parts of the PZS triangle

Azimuth by Solar Observation

- Choosing between hour angle method and altitude method
- Hour angle method

$$\text{AZIMUTH} = \tan^{-1} \frac{\sin LHA}{\sin LAT \cos LHA - \cos LAT \tan DEC}$$

Where:

LHA = local hour angle of sun

Dec = declination of sun

LAT = latitude of observe

AZ = Azimuth of sun

To normalize AZ, add correction		
LHA	corr	corr
0-180	180(+AZ)	360(-AZ)
180-360	0(+AZ)	180(-AZ)

- Altitude method

$$Z = \cos^{-1} \frac{\sin DEC - \sin LAT \sin h}{\cos LAT \cos h}$$

Where:

AZ = Z (when sun is east of the local meridian)

AZ = 360 - Z (when sun is west of the local meridian)

DEC = declination of sun

LAT = latitude of observer

h = vertical angle to the sun corrected for parallax and refraction

Example Problem

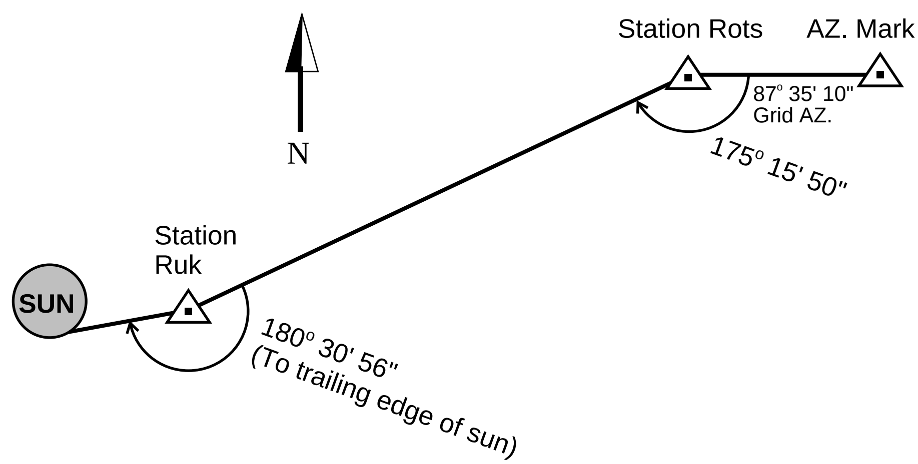
1989 LS Problem

(Portions of this problem have been deleted.)

California coordinates and basis of bearings for Station ROTS are shown on the sketch along with data for a solar observation at Station RUK.

Use the data provided and the diagram to reduce the solar observation either by the hour angle method or the altitude method. Show your work.

Determine the angle of closure at station RUK.



Solar Observation

Date	=	May 5, 1988
Time	=	5:23:35.0 PDST
		Watch is 0.5" fast
D.U.T.	=	- 0.3"
Vert. angle to center of sun	=	28° 05' 49" (Corrected for parallax and refraction)
LAT. RUK	=	36° 48' 57.0"
LONG. RUK	=	119° 46' 54.5"
Θ RUK	=	- 0° 27' 59"

Solar Ephemeris Table

	GHA	Declination	Semidiameter
May 4 W	180° 48' 22.7"	15° 58' 04.3"	15' 53.2"
May 5 Th	180° 49' 44.8"	16° 15' 19.0"	15' 53.0"
May 6 F	180° 50' 58.4"	16° 32' 17.5"	15' 52.7"
May 7 S	180° 52' 03.3"	16° 48' 59.7"	15' 52.5"

Azimuth by Observation of Polaris

The equation given for hour angle method for the sun can be used, or the following:

$$Z = \frac{p \sin LHA}{\cos h}$$

Where:

h = (true altitude of Polaris)

p = 90° - declination

Z is west of north when $0^\circ \leq LHA \leq 180^\circ$

Z is east of north when $180^\circ \leq LHA \leq 360^\circ$

AZ = Z (when Z is east of north)

AZ = 360 - Z (when Z is west of north)

Sample Test Questions

The following questions are from Problem B-3, 1988 LS:

Answer the following questions T (True) or F (False).
NOTE: False statements require a brief explanation.

1. Astronomic azimuth is based on true north.
2. Grid azimuth is based on true north.
3. Geodetic azimuth is astronomic azimuth - Θ + 2nd term.
4. To determine true north from observations on Polaris, the latitude of the observer must be known.
5. GHA = LHA - west longitude.
6. The best time to observe the sun to determine azimuth using the altitude method is 1/2 hour after sunrise or 1/2 hour before sunset.
7. Exact time is more important when using the hour angle method versus the altitude method.
8. The hour angle method requires both horizontal and vertical observations to determine the azimuth.
9. It is not necessary to know the latitude of the observer when using the hour angle method.
10. The best time to observe the sun to determine azimuth using the hour angle method is just after sunrise or just before sunset.
11. Standard time must be converted to local time to determine Greenwich time.
12. Local time is increased (in California) by eight hours to determine Greenwich time.
13. True solar time is local civil time minus the equation of time.
14. A level line at sea level is parallel with a level line at 8000 ft.
15. One sidereal day is longer than one solar day measured in civil time.

Which of the following is the best answer:

16. At what time will the effects of a small error in the determination of observer's latitude be minimized when making azimuth observations on Polaris?
    A. 12:00 midnight
    B. when Polaris is at elongation
    C. when Polaris is at culmination
    D. when the LHA is 90 degrees
    E. when the GHA is 90 degrees

17. At what time will the effects of a small error in the determination of the time of observation be minimized when making azimuth observations on Polaris?
    A. 12:00 midnight
    B. when Polaris is at elongation
    C. when Polaris is at culmination
    D. when LHA is 180 degrees
    E. when GHA is 180 degrees

The following questions are from Problem B-3, 1991 LS:

There are two methods by which azimuth can be determined by observation of the sun. Answer the following questions concerning these methods.

18. Name the two methods that can be used to determine azimuth by observations of the sun.

19. Which method is more accurate? Explain your answer.

20. The following two questions concern the method that uses vertical angle observations:
    A. How would inconsistencies of the angular (vertical and horizontal) observations be detected?
    B. How would calculations for the effect of the semidiameter of the sun be eliminated?

21. For each method, indicate whether parallax and refraction are taken into account. Explain your answer.

22. When using the method that uses only the horizontal angle, what is the single most important area where errors, excluding time and angular measurements, would most likely occur?

23. What is an appropriate source for accurate time determination?

24. When using the method that uses only the horizontal angle, if observations are made on the trailing limb of the sun, how does that affect angular calculations?

25. For each method, describe how the time of day of the observations affects azimuth determination.

26. For each method, describe how averaging the observations for calculation purposes would affect the final azimuth determination.

Use the solar ephemeris table from the sample problem in the video presentation outline to solve problems 27 and 28.

27. What is the local hour angle for a solar observation at 4:01:37.2 p.m. PDST on May 4, 1988, at a longitude of 120° 39' 15" west?

28. What is the sun's declination at 3:36:44 p.m. PDST May 6, 1988?

The following questions are from Problem B-4, 1992 LS:

Problem Statement: The following questions are independent and require a demonstration of your knowledge in using the sun or a star to determine a bearing basis for a survey.

Problem Requirements: Answer the questions with a brief sentence and show your calculations if any are required.

29. An observation on the sun is taken on Thursday, April 2, at 5:05:30.2 p.m. Pacific Standard Time (120 degrees west longitude). Afterward, the observer turns to the radio time station and determines that the observation clock is 2.6 seconds fast and also notes three double ticks between the ninth and fifteenth second after the minute tone.
    A. What is the Coordinated Universal Time (UCT) time and date?
    B. What is the correct UT1 time after applying the DUT correction?

30. Precise time is not available. Name a stellar body and time to observe to obtain an astronomic bearing within 10 seconds of true north in North America.

31. The following observations are recorded. Determine which one is inconsistent with the remaining observations.

	Time	Clockwise Horizontal Angle
(a)	9:05:01	40° 10' 15"
(b)	9:06:20	40° 12' 05"
(c)	9:07:10	40° 13' 17"
(d)	9:08:40	40° 14' 18"

32. The star Sirius is observed for a basis of bearing using the altitude method. The measured vertical angle is +45° 10' 10". The elevation is 1000 ft above sea level and the temperature is 60° F. What is the correction for parallax to be applied to the vertical angle?

33. A horizontal clockwise angle of 135° 30' 30" is measured to the trailing edge of the sun. The vertical angle to the center is +38° 25' above the horizon. The sun's semidiameter is 16' 45". What is the correct horizontal angle to the center of the sun?

34. At a latitude of 40° 42' north, a Polaris observation is taken from which a traverse is run due west 6000 ft. A second Polaris observation is then taken. Compute the correction of meridian convergence for the second observation.

36. A solar observation is taken at 120° 15' 45" west longitude. Determine the Local Hour Angle of the sun at 12:00:00 UT1 on January 1, 1990, given:

Date	GHA (Sun)
January 1, 1990	179° 10' 39.5"
January 2, 1990	179° 03' 33.8"

37. Given the following calculated azimuths of a line, determine the answers to a and b below.
   A. The standard deviation of the set
   B. The 90% error of the mean

Azimuths

1. 40° 21' 10"
2. 40° 21' 12"
3. 40° 21' 05"
4. 40° 20' 55"
5. 40° 20' 58"

Answer Key

1. T

2. Astro Az = true Az

3. Geod AZ = Grid Az + θ - 2nd term

4. $Z = P \dfrac{\text{Sin LHA}}{\text{cos h}}$

5. GHA = LHA + west longitude

6. Altitude above 20° and two hours before or after apparent local noon.

7. T

8. No vertical angle needed.

9. $AZ = \tan^{-1} \dfrac{\text{sin LHA}}{\text{sin LAT cos LHA - cos LAT tan DEC}}$

10. T

11. Standard time is converted to Greenwich time by adding eight (1 hour for every 15° of west longitude).

12. Local time must be converted to standard and then to Greenwich time.

13. TST = CT + ET

14. Level surfaces at higher altitudes are more elliptical than at lower altitudes.

15. One sidereal day = 23h 56m in civil time.

16. C. At culmination the bearing is 0 at all LAT.

17. B. Polaris has no visual movement right or left at elongation for ± 3 min.

18. Altitude method and hour angle method.

19. Hour angle method—because time can be very accurately determined, and inaccuracies in measuring the vertical angle and determining refraction make the altitude method less reliable.

20. A. By plotting the horizontal/vertical angles vs. time;
    B. Sighting the left and right edge of the sun or Roeloff's Prism.

21. A. Hour angle method—No. There is no vertical angle.
    B. Altitude method—Yes. It is applied only to the vertical angle.

22. In the determination of the latitude and longitude of the point. Examples: Scaling on a U.S.G.S. quad sheet or leveling the instrument.

23. Time cube method—monitoring WWV.

24. The method does not take account of the semidiameter of the sun.

25. Hour angle method—because a vertical angle is not being measured, the time of day of observation, except as an extreme, does not affect the determination of the azimuth. The lower the elevation, the less the effect of misleveling will be.

    Altitude method—best time is 8 a.m. - 10 a.m. and 2 p.m. - 4 p.m. Vertical angle not changing enough from 10 a.m. - 2 p.m. and refraction correction becomes very large and uncertain at low altitudes.

26. Altitude method—averaging of adjusted data would not have any measurable effect on the final azimuth.

    Hour angle method—averaging will give errors due to the fact that the sun travels in an apparent curved path and the semidiameter correction would be affected by the change in altitude.

27.

Time 5-4-88	4:01:37.2
PM Correction	+ 12:00:00
Zone Correction	+ 7:00:00
UT1 5-4-88	23:01:37.2

$$\begin{aligned} GHA &= GHA\ 0^h + (GHA\ 24^h - GHA\ 0^h + 360)\frac{UT1}{24} \\ &= 180^\circ\ 48'\ 22.7'' + (180^\circ\ 49'\ 44.8'' - 180^\circ\ 48'\ 22.7'' + 360)\frac{23{:}01{:}37.2}{24} \\ &= 526^\circ\ 13'\ 59.5'' \end{aligned}$$

Normalize GHA by subtracting 360°

$$\begin{aligned} LHA &= GHA - \text{west longitude} \\ &= 166^\circ\ 13'\ 59.5'' - 120^\circ\ 39'\ 15'' \\ LHA &= 45^\circ\ 34'\ 44.5'' \end{aligned}$$

28.

Time 5-6-88	3:36:44	PMPDST
PM Correction	12:00:00	
Zone Correction	7:00:00	
UT1 5-6-88	22:36:44	

$$\text{Dec} = \text{Dec } 0^h + (\text{Dec } 24^h - \text{Dec}0^h)\frac{\text{UT1}}{24}$$

$$= 16° \ 32' \ 17.5'' + (16° \ 48' \ 59.7'' - 163° \ 2' \ 17.5'')\frac{22:36:44}{24}$$

$$\text{Dec} = 16° \ 48' \ 01.7'' + \text{correction}$$

Correction for linear interp. $= (0.0000395)\ (\text{Dec}0^h)\ \text{Sin}\ (7.5\ \text{UT1})$

29. Time 5:05:30.2 P.M. PST April 2
    PM +12
    Zone +8
    Watch -2.6
    25:05:27.6

    A. UCT 1:05:27.6 April 3
       DUT -0.3
    B. UT1 1:05:27.3 April 3

    1. Polaris at elongation (the star has no noticeable movement for approximately 10 minutes of time at elongation).

    2.

	Time	D	Angle	D
(a)	9:05:01		40° 10' 15"	
		79"		110"
(b)	9:06:20		40° 12' 05"	
		50"		72"
(c)	9:07:10		40° 13' 17	
		90"		61"
(d)	9:08:40		40° 14' 18"	

The (d) observation is inconsistent with the others because the ratio of time to angle of (c) to (d) does not match the ratio of (a) to (b) or (b) to (c). The ratios of (a) to (b) and (b) to (c) are nearly the same.

    3. There is no parallax correction for stars. They are light years away.

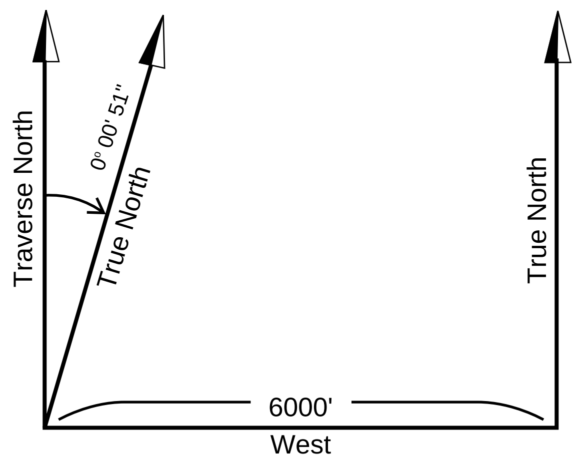
4. The semi-diameter (SD) has to be divided by the cosine of the vertical angle before it can be added to the trailing edge of the sun.

$$\frac{SD}{\cos h} = 0° \, 21' \, 22.7''$$

$$135° \, 30' \, 30'' + 0° \, 21' \, 22.7'' = 135° \, 51' \, 52.7''$$

5. There are approximately 6080 feet in 1 minute of longitude at the equator.

6080' x cos LAT = 4609



There are approximately 4609 feet in 1 minute of longitude at LAT 40° 42".

$$\text{DIF LONG} = 60'' \, \frac{6000}{4609}$$

$$= 78.1''$$

$$\text{Convergence} = \text{DIF LONG} \times \sin \text{LAT}$$

$$= 50.9''$$

6.

$$\text{GHA} = \text{GHA } 0^h + (\text{GHA } 24^h - \text{GHA } 0^h + 360) \, \frac{\text{UT1}}{24}$$

$$= 179° \, 10' \, 39.5'' + (179° \, 03' \, 33.8'' - 179° \, 10' \, 39.5'' + 360) \, \frac{12}{24}$$

$$\text{GHA} = 359° \, 07' \, 06.6''$$

$$\text{LHA} = \text{GHA} - \text{west longitude}$$

$$\text{LHA} = 238° \, 51' \, 21.6''$$

7.

	DIFF	$DIFF^2$
40° 21' 10'	+06"	36"
40° 21' 12"	+08"	64"
40° 21' 05"	+01"	01"
40° 20' 55"	-09"	81"
40° 20' 58"	-06"	36"

Average = 40° 21" 04" $\sum DIFF^2 = 218''$

a. $s_s = \sqrt{\dfrac{\sum DIFF^2}{(n-1)}} = \pm 7.38''$

b. $E_{90} = 1.6449 \sqrt{\dfrac{\sum DIFF^2}{n(n-1)}} = \pm 5.43$

References

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Mackie, J. B., *The Elements of Astronomy for Surveyors,* Charles Griffin House, England, 1985.