Plain & Reinforced Concrete-1

CE-314

Lecture # 8

Flexural Analysis and Design of Beams

(Ultimate Strength Design of Beams)

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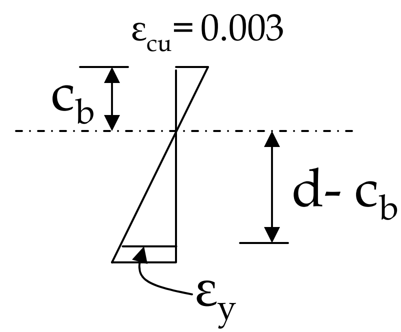
Balanced Steel Ratio, ρ_b

It is corresponding to that amount of steel which will cause yielding of steel at the same time when concrete crushes.

At ultimate stage:



Strain Diagram

$$\varepsilon_s = \varepsilon_y \quad \text{and} \quad f_s = f_y$$

$$\varepsilon_{cu} = 0.003$$

From the internal Force diagram

$$C_c = 0.85 f_c' \times b \times a_b$$

a_b = depth of equivalent rectangular stress block when balanced steel ratio is used.

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Balanced Steel Ratio, ρ_b (contd…)

$$T = A_s f_y = (\rho_b \times b \times d) \times f_y$$

For the longitudinal equilibrium

$$T = C_c$$

$$(\rho_b \times b \times d) \times f_y = 0.85 f_c' \times b \times a_b$$

$$\rho_b = 0.85 \frac{f_c'}{f_y} \frac{a_b}{d} \qquad (1)$$

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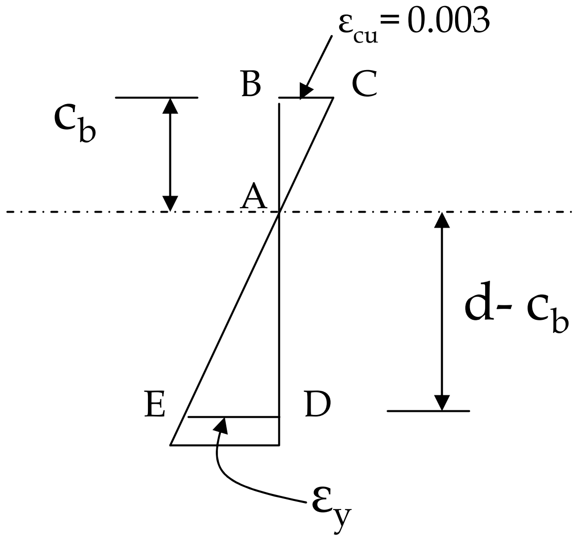
Balanced Steel Ratio, ρ_b (contd…)

From the strain diagram

Δ_s ABC & ADE



Strain Diagram

$$\frac{0.003}{c_b} = \frac{\varepsilon_y}{d - c_b}$$

$$c_b = \frac{0.003d}{0.003 + \varepsilon_y}$$

$$c_b = \frac{0.003}{0.003 + {f_y}/{E_s}} \times d \times \frac{E_s}{E_s}$$

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Balanced Steel Ratio, ρ_b (contd…)

$$c_b = \frac{0.003E_s}{0.003E_s + f_y} \times d$$

$$\boxed{c_b = \frac{600}{600 + f_y} \times d}$$

As we know $$a_b = \beta_1 \times C_b$$

$$a_b = \beta_1 \times \frac{600}{600 + f_y} \times d \qquad \text{——— (2)}$$

Put (2) in (1)

$$\boxed{\rho_b = 0.85\frac{f_c'}{f_y} \times \beta_1 \times \frac{600}{600 + f_y}}$$

Types of Cross Sections w.r.t. Flexural Behavior at Ultimate Load Level

1. Tension Controlled Section

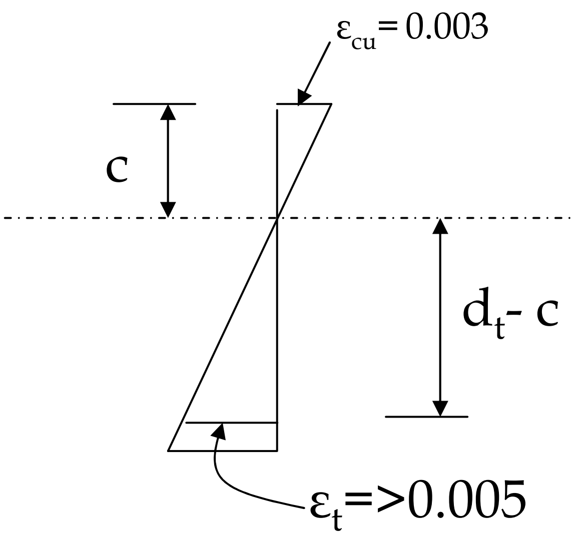
A section in which the net tensile strain in the extreme tension steel is greater than or equal to 0.005 when the corresponding concrete strain at the compression face is 0.003.

$$\frac{\varepsilon_t}{0.003} = \frac{d_t - c}{c}$$

$$\varepsilon_t = 0.003\frac{d_t - c}{c} \geq 0.005$$

$$0.003\frac{d_t}{c} \geq 0.005 + 0.003$$

$$c \leq \frac{3}{8} d_t$$ and $$a \leq \beta_1 \frac{3}{8} d_t$$



Strain Diagram

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2. Compression Controlled Section **(over-reinforced section)**

The section in which net steel strain in the extreme tension steel at failure is lesser than ε_y when corresponding concrete strain is 0.003. This means that failure occurs by crushing of concrete.

- Capacity of steel remain unutilized.
- It gives brittle failure without warning.

$$a > a_b \qquad c > c_b$$

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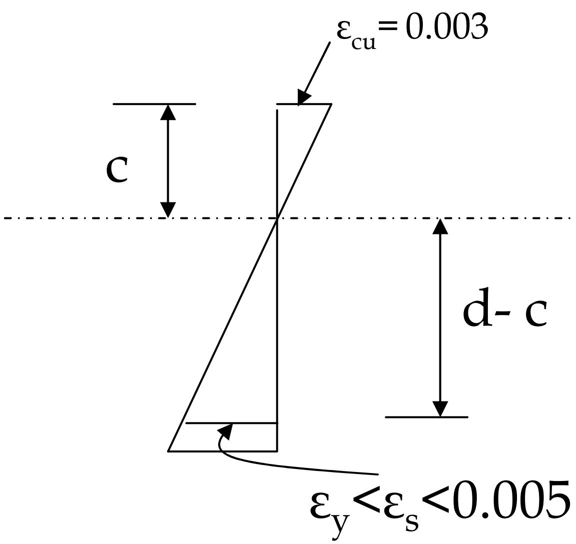
Types of Cross Sections w.r.t. Flexure at Ultimate Load Level (contd…)

3. Transition Section

The section in which net tensile strain in the extreme tension steel at ultimate conditions is greater than ε_y but less than 0.005 when corresponding concrete strain is 0.003.

$$a = \beta_1 \frac{3}{8} d_t \quad to \quad a_b = \beta_1 \frac{600}{600 + f_y} d_t$$



Strain Diagram

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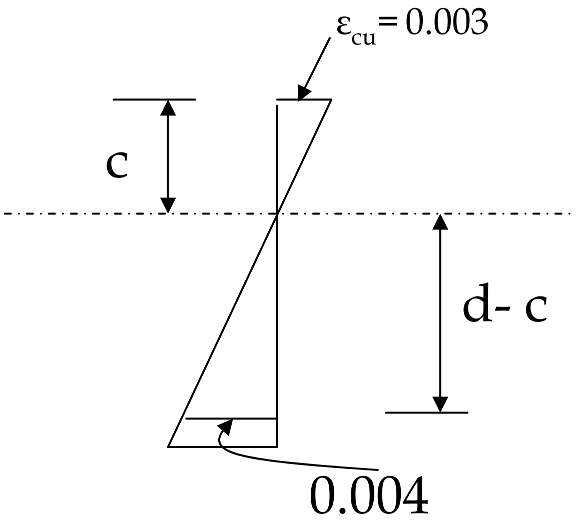
Transition Section Requirements For Design

To ensure under-reinforced behavior, ACI code establishes a minimum net tensile strain of 0.004 at the ultimate stage.

$$\frac{0.003}{c} = \frac{0.004}{d - c}$$

$$c \le \frac{3}{7} d \qquad a \le \beta_1 \frac{3}{7} d$$



Strain Diagram

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Types of Cross Sections w.r.t. Flexure at Ultimate Load Level (contd…)

Both the "Tension Controlled Sections" and "Transition Sections" are "Under-Reinforced Section"

In Under-Reinforced Sections steel starts yielding before the crushing of concrete and:

$$\rho < \rho_b$$

It is always desirable that the section is under-reinforced otherwise the failure will initiate by the crushing of concrete. As concrete is a brittle material so this type of failure will be sudden which is NOT DESIREABLE.

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Strength Reduction Factor **(Resistance Factor)**, ϕ

- Tension Controlled Section, ϕ = 0.9
- Compression Controlled Section

Member with lateral ties, ϕ = 0.65

Members with spiral reinforcement, ϕ = 0.75

- Transition Section

For transition section ϕ is permitted to be linearly interpolated between <u>0.65 or 0.75</u> to <u>0.9</u>.

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Strength Reduction Factor (Resistance Factor), Φ

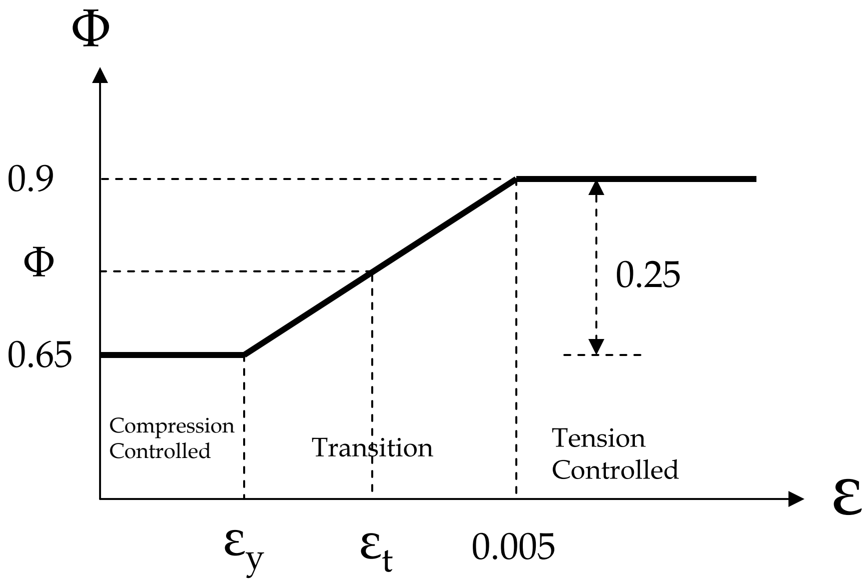
- Transition Section (contd…)

For members with ties

$$\phi = 0.65 + \frac{0.25}{0.005 - \varepsilon_y}\left(\varepsilon_t - \varepsilon_y\right)$$

For members with spirals

$$\phi = 0.75 + \frac{0.15}{0.005 - \varepsilon_y}\left(\varepsilon_t - \varepsilon_y\right)$$



ε_t = strain in extreme tension steel when concrete crushes.

$$\varepsilon_t = 0.003\left(\frac{d_t}{c} - 1\right)$$

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Maximum Steel Ratio, ρ_{max}

For

$$T = C$$

$$A_s \times f_y = 0.85 f_c' \times b \times a$$

$$\rho \times bd \times f_y = 0.85 f_c' \times b \times a$$

$$\rho = 0.85 \frac{f_c'}{f_y} \times \frac{a}{d}$$

For tension controlled section $\varepsilon_s = 0.005$ $a = \beta_1 \frac{3}{8} d$

So

$$\rho_{max} = 0.85 \frac{f_c'}{f_y} \times \left(\frac{3}{8} \beta_1 \right)$$

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Maximum Steel Ratio, ρ_{max} (contd…)

For transition section

$$\varepsilon_s = 0.004 \qquad a = \beta_1 \frac{3}{7} d$$

$$\rho_{max} = 0.85 \frac{f_c'}{f_y} \times \left(\frac{3}{7} \beta_1 \right)$$

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Minimum Reinforcement of Flexural Members

(ACI – 10.5)

$$\rho_{min} = \frac{0.25\sqrt{f_c'}}{f_y} \geq \frac{1.4}{f_y}$$

- The minimum steel is always provided in structural members because when concrete is cracked then all load comes on steel, so there should be a minimum amount of steel to resist that load to avoid sudden failure.
- The minimum steel ratio of slabs is less than this value. This is because of less chances of sudden failure in slabs due to two-way action.
- This formula is not used for slabs.

Concluded