Plain & Reinforced Concrete-1

CE-314

Lecture # 7

Flexural Analysis and Design of Beams

(Ultimate Strength Design of Beams)

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Ultimate Strength Design of Beams

(Strength Design of Beams)

Strength design method is based on the philosophy of dividing F.O.S. in such a way that **bigger part** is applied on **loads** and **smaller part** is applied on **material strength.**

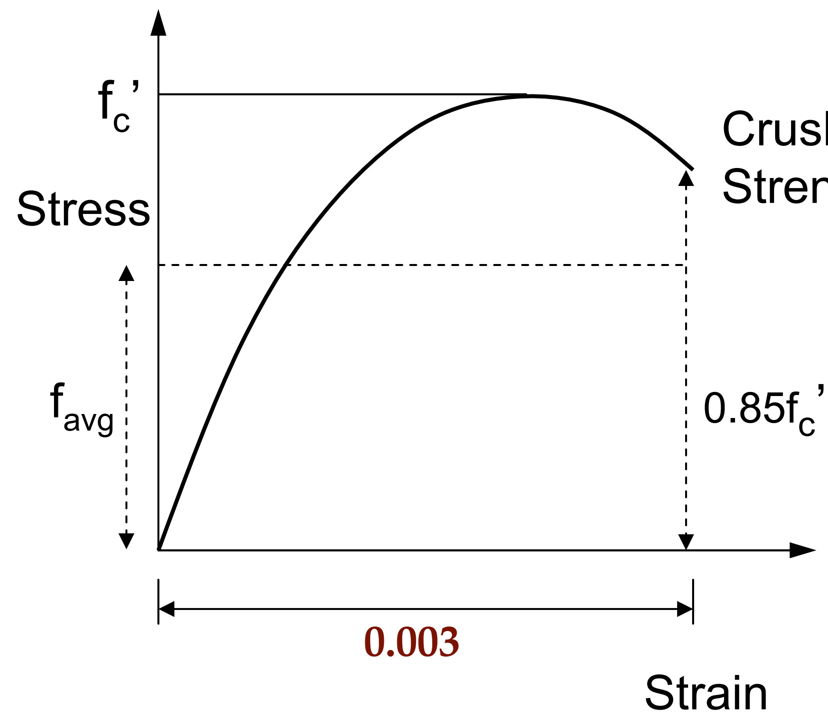
f_{avg} = Area under curve/0.003

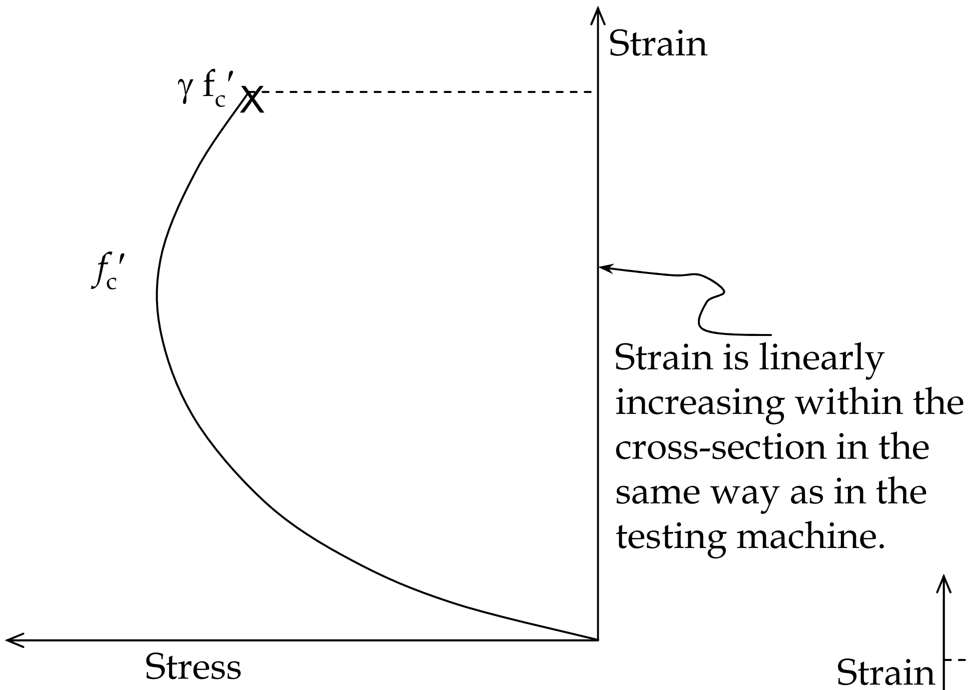
If $f_c' \le 28$ MPa

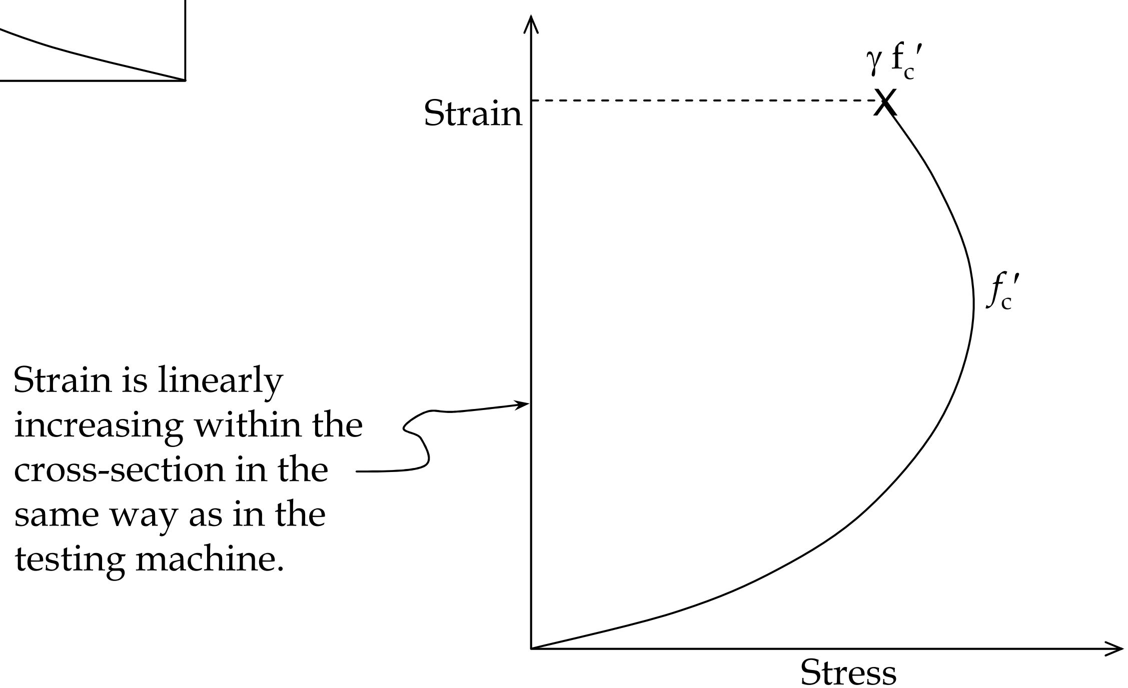
$$f_{avg} = 0.72\, f_c'$$

β_1 = Average Strength/Crushing Strength

$$\beta_1 = 0.72 f_c' \,/\, 0.85\, f_c' = 0.85$$




Strain
γ f_c′
f_c′
Stress
Strain is linearly increasing within the cross-section in the same way as in the testing machine.


γ f_c′
Strain
f_c′
Strain is linearly increasing within the cross-section in the same way as in the testing machine.
Stress

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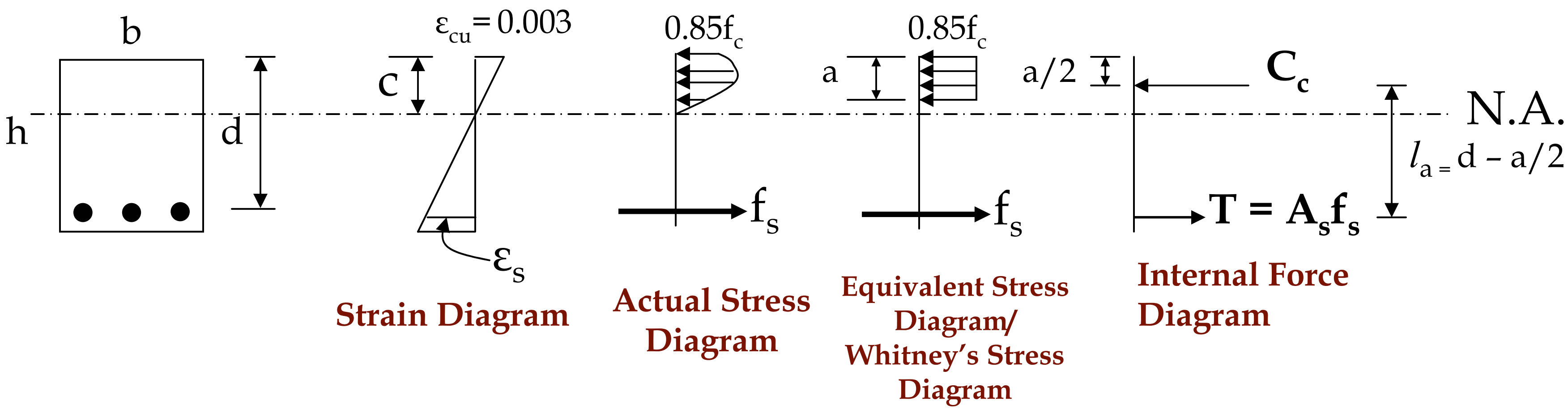
Ultimate Strength Design of Beams (contd…)



In ultimate strength design method the section is always taken as cracked.

c = Depth of N.A from the extreme compression face at ultimate stage

a = Depth of equivalent rectangular stress diagram.

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Ultimate Strength Design of Beams (contd…)

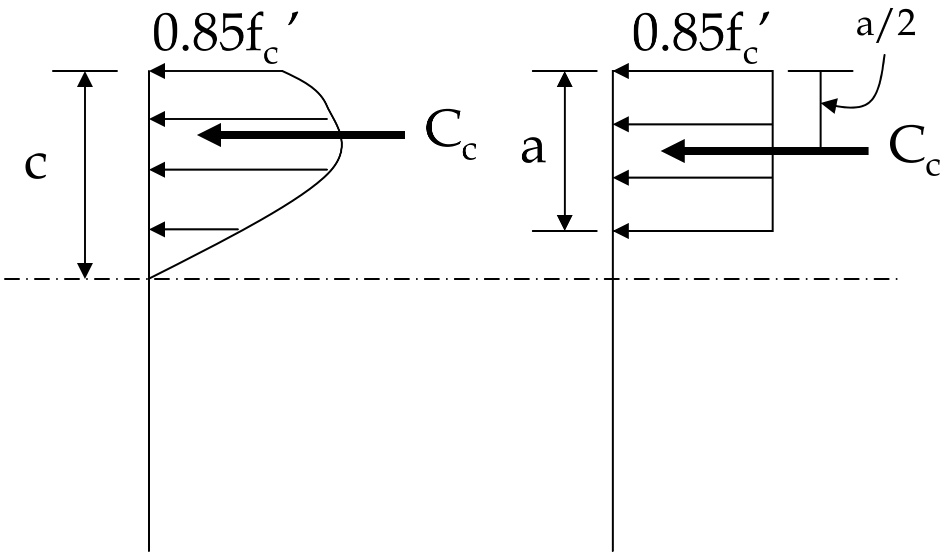
- **The resultant of concrete compressive force C_c, acts at the centriod of parabolic stress diagram.**
- **Equivalent stress diagram is made in such a way that it has the same area as that of actual stress diagram. Thus the magnitude of Cc and its position will remain unchanged.**

$$f_{av} \times b \times c = 0.85 f_c' \times b \times a$$

$$0.72 f_c' \times c = 0.85 f_c' \times a$$

$$a = \frac{0.72 f_c'}{0.85 f_c'} \times c$$

$$a = \beta_1 \times c$$



Actual Stress Diagram

Equivalent Stress Diagram/ Whitney's Stress Diagram

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Ultimate Strength Design of Beams (contd…)

Factor β_1

$$\beta_1 = 0.85 \quad \text{for } f_c' \leq 28 \text{ MPa}$$

Value of β_1 decreases by 0.05 for every 7 MPa increase in strength with a minimum of 0.65

$$\beta_1 = 1.05 - 0.00714 f_c' \quad \geq 0.65$$

$$\leq 0.85$$

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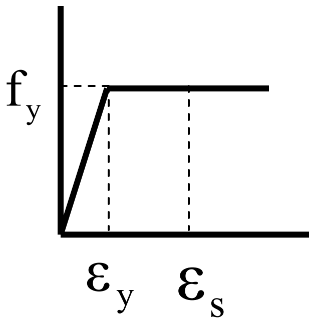
Determination of N.A. Location at Ultimate Condition

CASE-I: Tension Steel is Yielding at Ultimate Condition

$$\varepsilon_s \geq \varepsilon_y \quad \text{and} \quad f_s = f_y$$



CASE-II: Tension Steel is Not Yielding at Ultimate Condition

$$\varepsilon_s < \varepsilon_y \quad \text{or} \quad f_s < f_y$$

For 280 grade steel

$$\varepsilon_y = \frac{f_y}{E} = \frac{280}{200{,}000} = 0.0014$$

For 420 grade steel

$$\varepsilon_y = \frac{f_y}{E} = \frac{420}{200{,}000} = 0.0021$$

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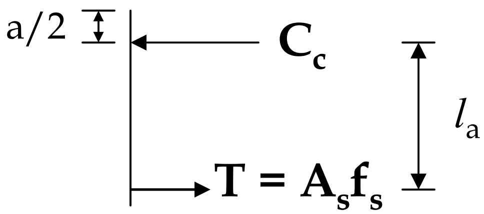
CASE-I: Tension Steel is Yielding at Ultimate Condition

$$T = A_s \times f_s = A_s \times f_y$$

$$C_c = 0.85 f_c' \times b \times a$$

$$l_a = d - \frac{a}{2}$$



Internal Force Diagram

For longitudinal Equilibrium

$$T = C_c$$

$$A_s \times f_y = 0.85 f_c' \times b \times a$$

$$a = \frac{A_s \times f_y}{0.85 f_c' \times b}$$

and

$$c = \frac{a}{\beta_1}$$

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CASE-I: Tension Steel is Yielding at Ultimate Condition (contd…)

Nominal Moment Capacity, M_n depending on steel = T x l_a

$$M_n = A_s \times f_y \times \left(d - \frac{a}{2}\right)$$

Design Moment Capacity

$$\phi_b M_n = \phi_b A_s \times f_y \times \left(d - \frac{a}{2}\right)$$

Nominal Moment Capacity, M_n based on concrete strength = C_c x l_a

$$M_n = 0.85 f_c' \times b \times a \times \left(d - \frac{a}{2}\right)$$

$$\phi_b M_n = \phi_b 0.85 f_c' \times b \times a \times \left(d - \frac{a}{2}\right)$$

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Minimum Depth for Deflection Control

$$\Delta \propto \frac{1}{I} \qquad \Delta \propto \frac{1}{(\text{Depth})^3}$$

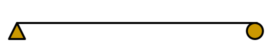
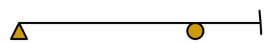
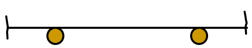
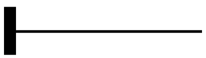
For UDL

$$\Delta \propto wL^4$$

$$\Delta \propto (wL)L^3$$

The required minimum depth is more for 420-grade steel than 280-grade steel because, at the ultimate load, the steel will be yielding. The yield strain is higher for higher-grade steel meaning more rotation of the critical sections associated with more deflections. The depth is increased in such cases to reduce the rotation angle for the high yield strain at the critical sections.

Minimum Depth for Deflection Control (Contd…)

Steel Grade	Simply Supported	One End Continuous	Both End Continuous	Cantilever
				
280 or 300	L/20	L/23	L/26	L/10
420	L/16	L/18.5	L/21	L/8
520	L/14	L/16	L/18.4	L/7

Concluded