Plain & Reinforced Concrete-1

CE-313

Lecture # 6

Flexural Analysis and Design of Beams

Example 2.5

Design a rectangular section for a simply supported beam of 5m span, subjected to a uniformly distributed load of 30 kN/m, using allowable stress design. Use C 30 concrete and Grade 420 steel.

Solution

$M = (30)(5)^2 / (8) = 93.75$ kN-m

For trial size of a beam, the total depth is normally taken closer to span/12 and width is taken approximately equal to depth / 3 for large beams to 2/3rd of depth for smaller beams. However, both must conform to the sizes of other components like columns and masonry walls.

- $h = L / 12 = 5 \times 1000 / 12 = 416.7$ mm
- Say the selected depth be $6 \times 75 = 450$ mm in terms of multiples of brick height.
- $b = 228$ mm (in terms of brick length)
- $d = h - 75 = 375$ mm
- $f_c' = 30$ MPa
- $f_y = 420$ MPa
- $f_c = 0.45 fc' = 13.5$ MPa
- $f_s = 168$ MPa
- $E_c = 4700\sqrt{f_c'} = 25{,}743$ MPa
- $n = E_s / E_c \cong 8$

k-Value For First Criterion

- $k = \dfrac{nf_c}{nf_c + f_s} = \dfrac{(8)(13.5)}{(8)(13.5) + (168)} = 0.391$
- $j = 1 - k/3 = 0.870$

k-Value For Second Criterion

- $$\rho_{\max} = 0.85 \times \frac{3}{8} \beta_1 \frac{f_c'}{f_y}$$
  $$= 0.85 \times \frac{3}{8} \times 0.85 \times \frac{30}{420} = 0.01935$$

- $\rho n = 0.1548$
- $k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$

  $= \sqrt{(0.1548)^2 + (2)(0.1548)} - (0.1548) = 0.423$
- $j = 0.86$
- This value is preferable and will be used in the present example. However, for practical designs, any one value should be calculated and used for a particular design.

Permissible Stresses

- $(f_s)_{permissible} = 0.4\ f_y = 168$ MPa
- $(f_c)_{permissible} = 0.45\ f_c' = 0.45 \times 30$

  $= 13.5$ MPa

Check For Minimum Effective Depth

The moment of resistance depending on the concrete strength is always evaluated to ensure under-reinforced behavior.

This will ensure that concrete will not crush in any case and the collapse is always by yielding of steel giving lot of warning.

$$M_r = M = \frac{f_c}{2} kj\, bd^2$$

$$= (13.5 / 2)(0.423)(0.86)\, bd^2$$

$$= 2.456\, bd^2$$ for moment in N-mm units

$$d_{min} = \sqrt{\frac{M \times 10^6}{2.456\, b}}$$ for moment in kN-m units

$$= \sqrt{\frac{93.75 \times 10^6}{2.456 \times 228}} = 409 \text{ mm}$$

Hence, the already selected depth is to be revised. Let the depth be slightly more than this minimum and preferably in multiples of the brick height (75 mm).

$d = 450$ mm

$h = d + 75 = 450 + 75 = 525$ mm

$b = 228$ (length of one brick)

Calculation Of Reinforcement

$$M_r = M = A_s f_s jd$$

$$A_s = \frac{M \times 10^6}{f_s \, j \, d}$$

$$= \frac{93.75 \times 10^6}{(168)(0.86)(450)} = 1442 \text{ mm}^2$$

Three # 25 may be used giving A_s = 1500 mm^2.

A maximum of half of the steel may be curtailed at ℓ_n / 20 distance from inner edge of the support (no curtailment according to the ACI Code), where ℓ_n is the clear distance between the supports.

Similarly, if bent-up bars are used, a maximum of half the bars may be bent up at a distance of ℓ_n / 7 from the inner edge of the support.

- $\ell_n = 5000 - 228 = 4772$ mm
- $\ell_n / 20 \approx 250$ mm
- Length of straight main bars

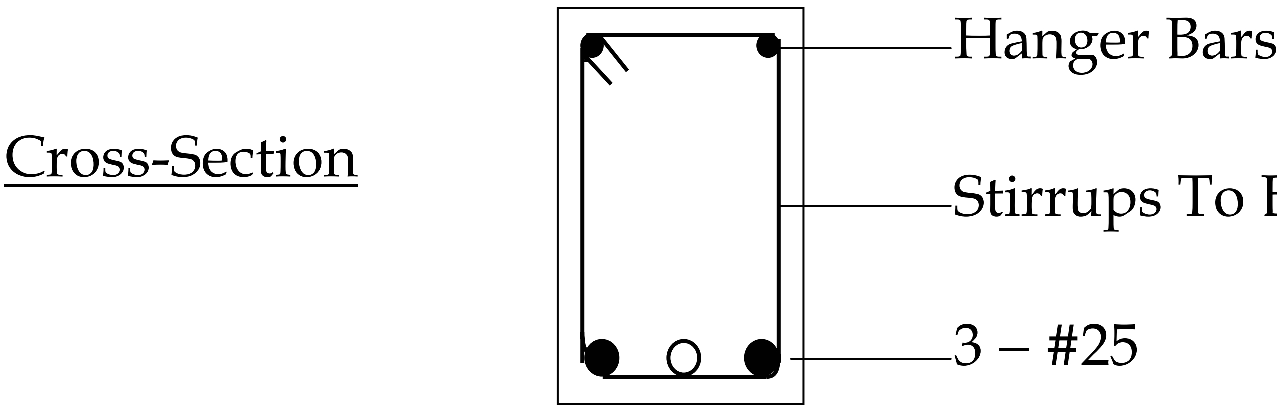
  $\approx 5000 + 228 = 5228$ mm
- Length of curtailed bar

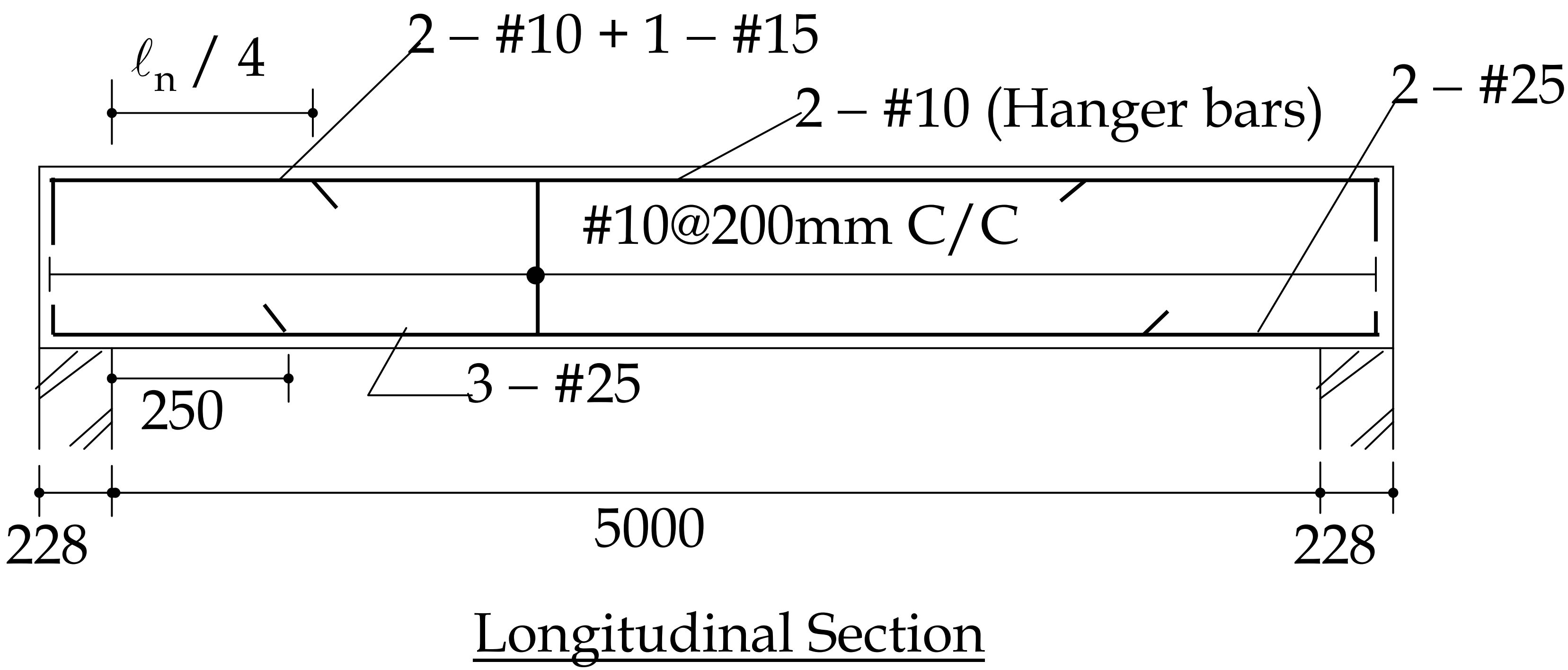
  $= 5000 - 2 \times 250 = 4500$ mm
- Weight of tension steel

  $= (2 \times 5.228 + 4.500) \times 1.05 \times 3.925$

  $= 62$ kg

  (including 5% wastage)


Cross-Section
Hanger Bars
Stirrups To Be Designed Later
3 – #25


2 – #10 + 1 – #15
ℓn / 4
2 – #10 (Hanger bars)
2 – #25
#10@200mm C/C
250
3 – #25
228
5000
228
Longitudinal Section

Check for ACI Minimum and Maximum Steel

- $A_{s,min} = \dfrac{1.4}{f_y} b\,d$

  $= \dfrac{1.4}{420}(228)(450) = 342\ \text{mm}^2$

- The provided steel is more than this amount, hence the condition is satisfied.

- $\rho = 1500 / (228)(450) = 0.0146$

  $< \rho max = 0.01935$ *OK*

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Reduced Moment of Inertia Due to Cracking

Elastic deflection can be expressed in general form

$$\Delta = f\,(\text{loads, spans, supports}) \;/\; (\,EI\,)$$

Deflection of a cracked section can be expressed as

$$\Delta = f\,(\text{loads, spans, supports}) \;/\; (\,EI_e)$$

As the number of cracks and their width increases the M.O.I of cross section reduces and deflection increases.

I_e = Effective moment of inertia of cracked section

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$$I_e = [\ M_{cr}/M_a\ ]^3\ I_g + [\ 1 - \{M_{cr}/M_a\}^3\ ]I_{cr} \leq I_g$$

Where

M_{cr} = Cracking moment $\qquad M_{cr} = f_r\ I_g/y_t$

M_a = Maximum moment in the member at a load level where we want to calculate deflection

I_g = Gross moment of inertia of un-cracked transformed section

I_{cr} = Moment of inertia of cracked transformed section

f_r = Modulus of rupture

y_t = Extreme tension fiber distance from N.A.

Effective moment of inertia calculated by the above expression can be used to calculate the immediate deflection of beams after cracking

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Long Term Deflection

Long term deflection is caused by creep and shrinkage.

$$\Delta_t = \lambda_\Delta \Delta_i$$

where

Δ_t =Long term deflection

Δ_i = Instantaneous Deflection

λ_Δ = Multiplier for additional deflection due to long term effect

$$\text{Total Deflection} = \Delta_i + \Delta_t$$

$$\text{Total Deflection} = \Delta + \lambda \Delta$$

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$$\lambda = \xi / (1 + 50 \rho')$$

ρ' = compression steel ratio

= area of compression steel / (b x d)

= A_s' / (b x d)

ξ = Time dependent factor for sustained loads

Elapsed Time	ξ
5 years or more	2.0
12 months	1.4
6 months	1.2
3 months	1.0

Concluded