Plain & Reinforced Concrete-1

CE-313

Lecture # 5

Flexural Analysis and Design of Beams

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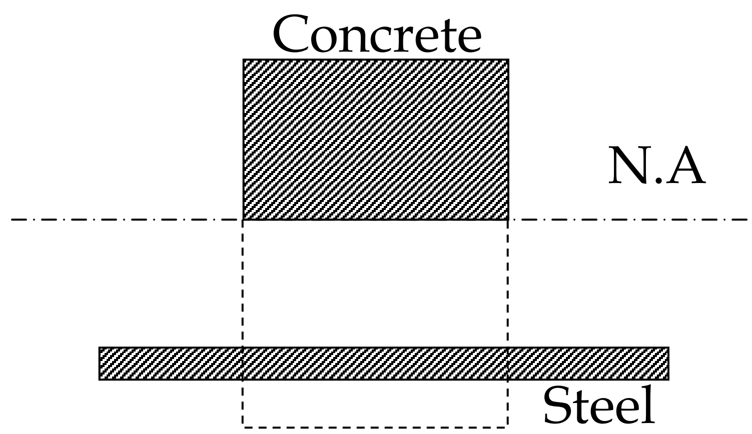
Cracked Transformed Section

When tensile stress cross f_r, crack appear on the tension face of beam.

- Tensile strength of concrete is neglected.
- All the concrete on the tension side of N.A. is neglected.



Only the Shaded area of cross-section is considered effective.

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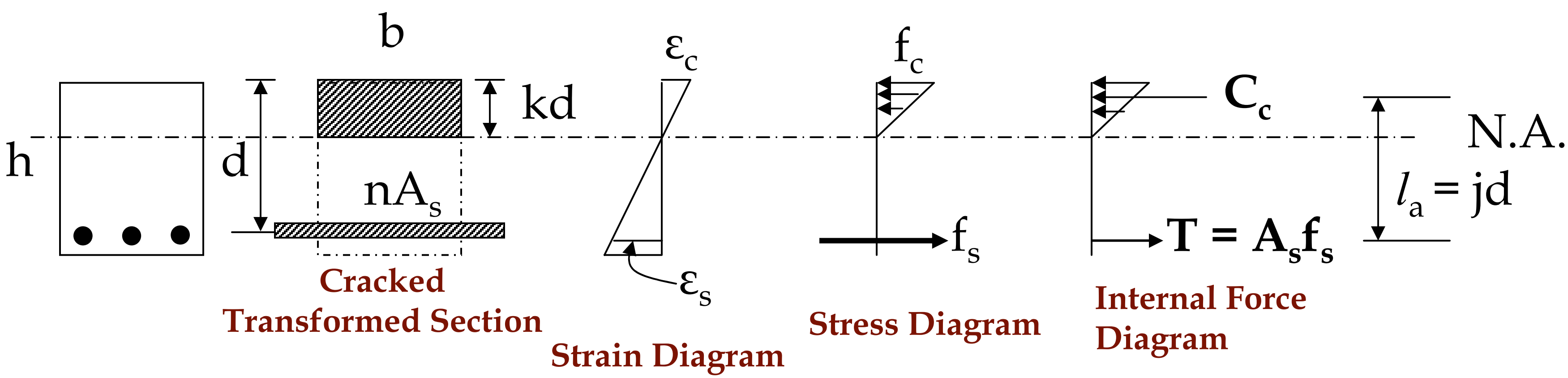
Cracked Transformed Section (contd…)



When the tension side is cracked the concrete becomes ineffective but the strains goes on increasing. The steel comes in action to take the tension.

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Cracked Transformed Section (contd…)

Calculations for "k", taking top face as reference

$$kd = [b \times kd \times kd/2 + nA_s d] / [b \times kd + nA_s]$$

$$b \times (kd)^2 + nA_s\, kd = b \times (kd)^2/2 + nA_s d$$

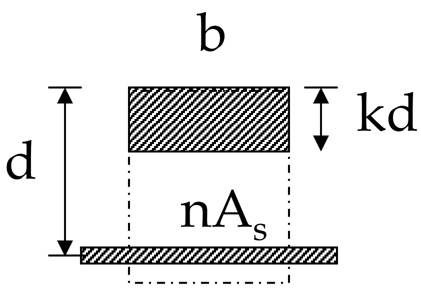
$$b \times (kd)^2/2 = nA_s d - nA_s\, kd$$

$$b \times (kd)^2/2 = (1\text{-}k)\, nA_s d$$

$$bd^2/2 \times k^2 = (1\text{-}k)\, n\rho bd^2$$

$$k^2 = 2(1\text{-}k)\, n\rho$$

$$k^2 + 2n\rho\, k - 2n\rho = 0$$



Cracked Transformed Section

$$\rho = As / bd$$

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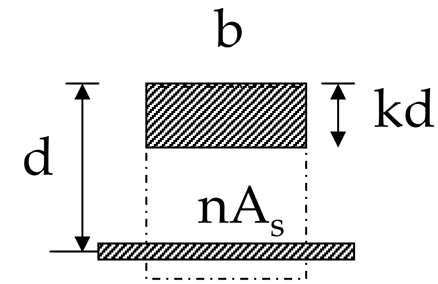
Cracked Transformed Section (contd…)

$$k^2 + 2n\rho\, k - 2n\rho = 0$$

$$k = \frac{-2n\rho \pm \sqrt{4(n\rho)^2 + 8n\rho}}{2}$$

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$$

Taking only positive value as distance can't be negative



Cracked Transformed Section

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Value of "j"

$$jd = d - kd/3$$

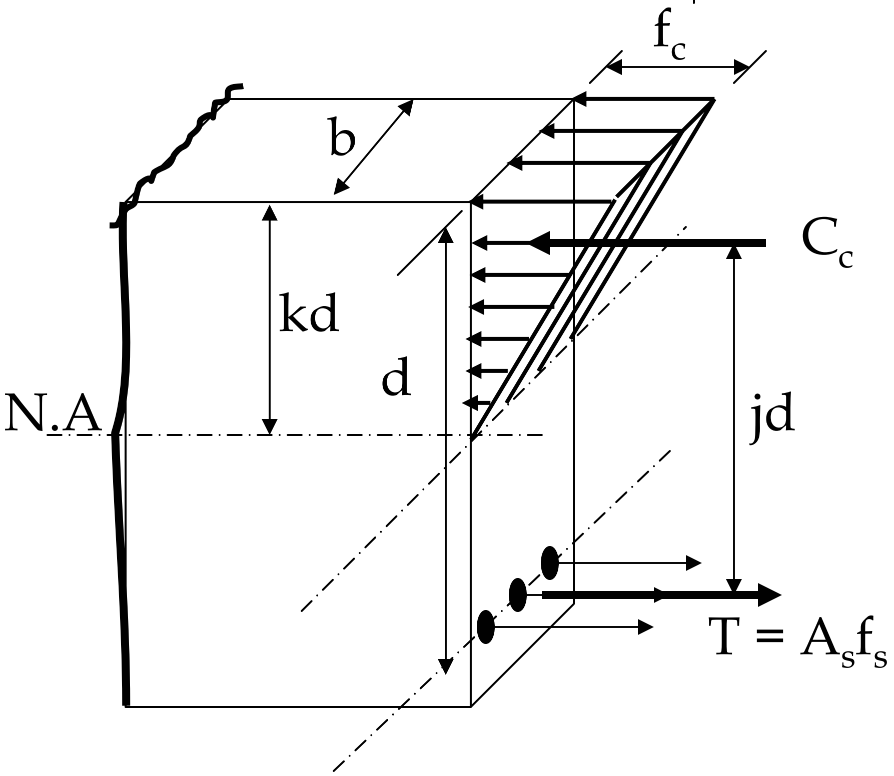
$$j = 1 - k/3$$

C_c = volume of stress diagram

$C_c = ½\, kd \times f_c \times b$

$$C_c = f_c/2 \times kd \times b$$

$$T = A_s f_s = n f_c A_s$$



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Resisting Moment Capacity

For longitudinal equilibrium

$$T = C_c$$

$$M_r = T \times jd = C_c \times jd$$

$$M_r = A_s f_s \times jd$$

And $$M_r = f_c/2 \times kdb \times jd$$

$$= \tfrac{1}{2} f_c \times kj \times bd^2 \text{ ------------- } 1$$

FOR MAXIMUM VALUE RESISTING MOMENT

f_s = Maximum allowable tensile stress in steel

= 0.5 f_y for grade 280 steel and 0.4 f_y for grade 420 steel

f_c = Maximum allowable concrete stress = 0.45 f_c'

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Minimum Reinforcement of Flexural Members

(ACI – 10.5.1)

$$A_{s\,min} = \frac{\sqrt{f_c'}}{4f_y} \times b_w d \geq 1.4/f_y \times b_w d$$

Critical if fc′ < 28 MPa

The minimum steel is always provided in structural members because when concrete is cracked then all load comes on steel, so there should be a minimum amount of steel to resist that load to avoid sudden failure.

For a design to be safe $M < M_r$

For an economical design $M = M_r$

$$M = A_s f_s \times jd$$

$$A_s = M / (f_s \times jd)$$

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Minimum Depth of Section

To satisfy a cross section against concrete failure

$$M = M_r$$

$$M = C_c \text{ x } jd$$

$$M = (f_c/2 \text{ x } kd \text{ x } b) \text{ x } jd$$

$$d_{\min} = \sqrt{\frac{2M}{f_c\, kj\, b}}$$ where $f_c = 0.45 f_c'$

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Determination of "k" value for design

$$k = \sqrt{\left(n^2\rho^2 + 2n\rho\right)} - n\rho$$

Value of k can not be determined as ρ is not know. There are two different approaches to establish the value of k.

1. Simultaneous Occurring of Maximum Permissible Steel and Concrete Stresses.

2. Assuming some suitable steel ratio

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Determination of "k" value (contd…)

1- Simultaneous Occurring of Maximum Permissible Steel and Concrete Stresses.

Consider Δ ABC & Δ ADE

$$\varepsilon_s / (d-kd) = \varepsilon_c / (kd)$$

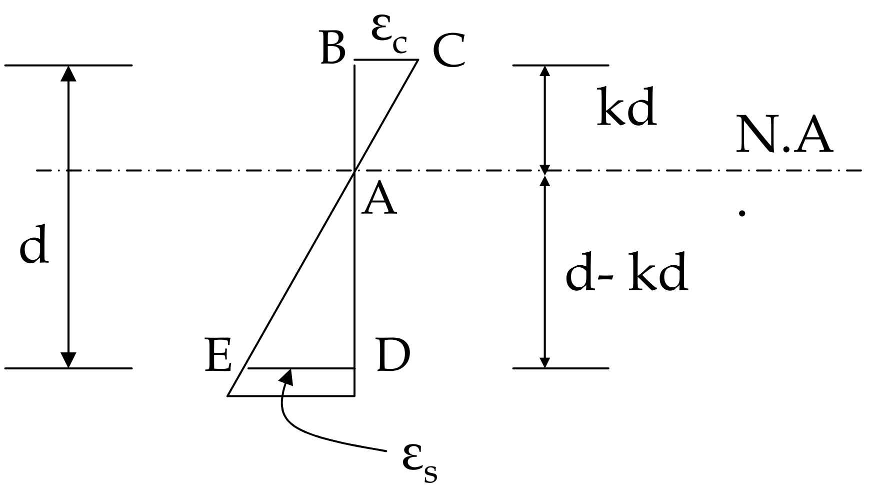
$$(f_s / E_s) / (d-kd) = (f_c / E_c) / (kd)$$

$$f_s \times kd = (E_s / E_c) \times f_c \times (d-kd)$$

$$f_s \times k = n \times f_c \times (1-k)$$

$$f_s \times k + nf_c k = nf_c$$

$$k = nf_c / (f_s + nf_c)$$



Strain Diagram

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Determination of "k" value (contd…)

2- Assuming some suitable steel ratio

In this approach, some suitable value of steel ratio **(less than or equal to ρ_{max})** is selected at the start of calculations and used for the determination of "k"

$$\rho_{max} = 0.85 \times 3/8 \times \frac{f_c'}{f_y}\frac{600}{600+f_y}$$

Calculate ρ_{max} and select some value less than this, then,

$$k = \sqrt{\left(n^2\rho^2 + 2n\rho\right)} - n\rho$$

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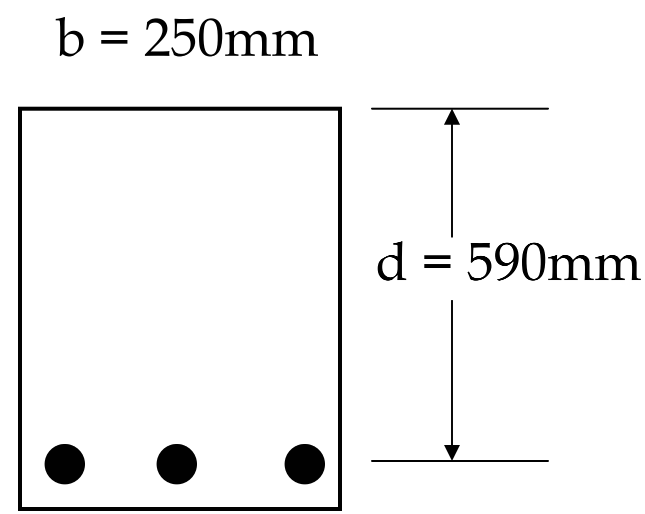
Example 2.3 (un-cracked transformed section)

A rectangular beam of size 250mm x 650mm with effective depth of 590mm is reinforced with 3 # 25 US customary bars. Concrete has specified compressive strength of 28MPa. Yield strength of steel is 420 MPa. Check these stresses against the maximum permissible ACI stresses caused by the bending moment of 120kN-m. Also calculate moment of inertia of the cracked transformed section.

$f_c' = 28$ MPa
$f_y = 420$ MPa
$M = 120$ kN-m

(positive moment)

$f_{top} = ?$ $f_{bottom} = ?$



Solution

Example 2.1 shows that if uncracked section is considered for the moment given in this example, the stresses will become twice that of the Example 2.1 exceeding the modulus of rupture.

Cracked section must therefore be considered for this example.

- $n = 8$ as in Example 2.1
- $\rho = \dfrac{1530}{(250)(590)} = 0.01037$
- $\rho n = 0.08298$
- $k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$

  $= \sqrt{(0.08298)^2 + 2\,(0.08298)} - (0.08298) = 0.333$
- $j = 1 - k/3 = 0.889$

- $f_s = \dfrac{M}{A_s\, jd}$

  $= \dfrac{120 \times 10^6}{(1530)(0.889)(590)} = 149.5$ MPa

$$f_c = \frac{2M}{kj\,bd^2} = \frac{(2)(120 \times 10^6)}{(0.333)(0.889)(250)(590)^2}$$
$$= 9.32 \text{ MPa}$$

- $(f_s)_{permissible} = 0.4\, f_y = 168$ MPa
- $(f_c)_{permissible} = 0.45\, f_c' = 0.45 \times 28$
  $= 12.6$ MPa
- Hence both the steel and concrete stresses are within the ACI limits.
- f_r for strength $= 0.5\sqrt{f_c'} = 2.65$ MPa
- As the concrete stress at the bottom of beam exceeds modulus of rupture, the cracked transformed section is actually to be considered.

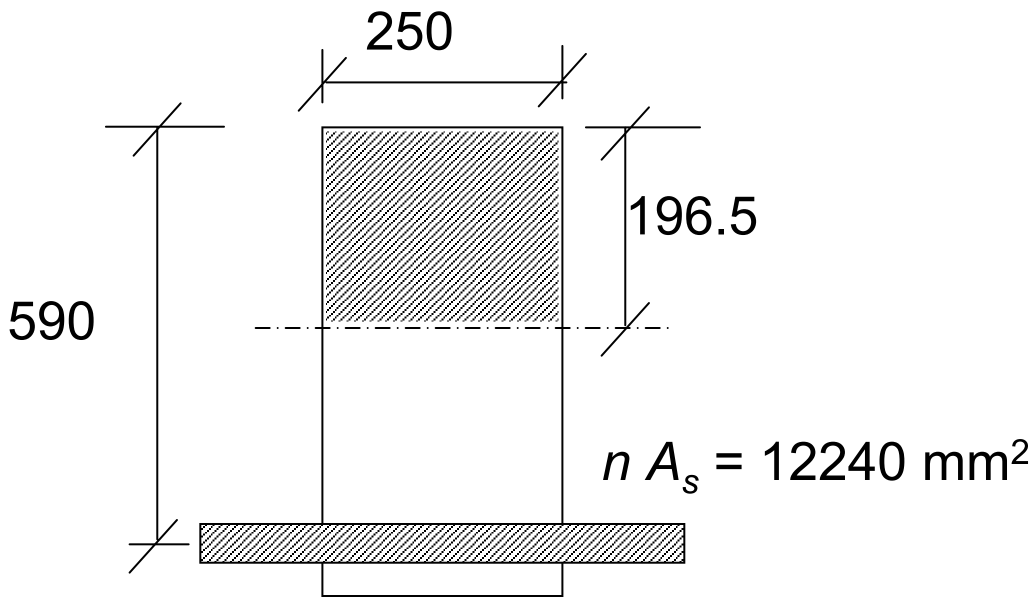


Fig. 2.12. Cracked Transformed Section For Example 2.3.

$$I = \frac{250 \times 196.5^3}{12} + 250 \times 196.5 \times 98.25^2$$

$$+ 12240 \times 393.50^2$$

$$= 252,800 \times 10^4 \text{ mm}^4$$

Concluded