Plain & Reinforced Concrete-1

CE-313

Lecture # 4

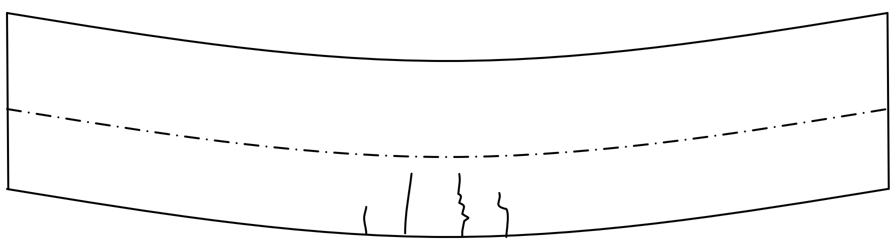
Flexural Analysis and Design of Beams

Plain & Reinforced Concrete-1

Different Types of Cracks

1. Pure Flexural Cracks

Flexural cracks start appearing at the section of maximum bending moment. These vertical cracks initiate from the tension face and move towards N.A.

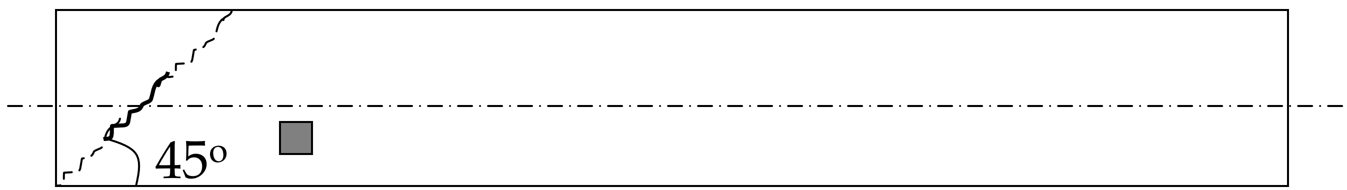


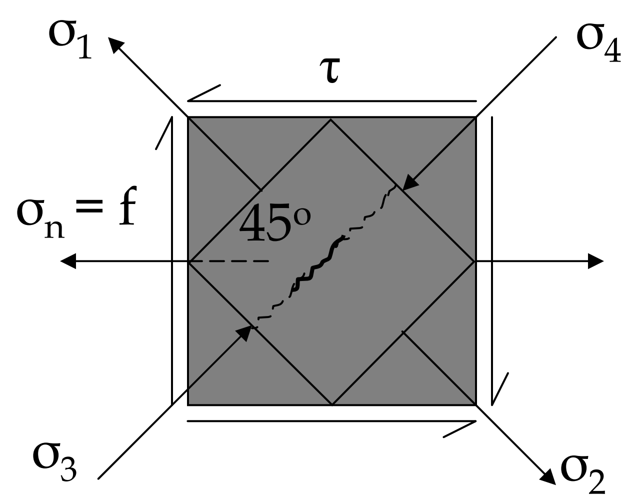
Plain & Reinforced Concrete-1

Different Types of Cracks (contd…)

2. Pure Shear/Web Shear Cracks

   These inclined cracks appear at the N.A due to shear stress and propagate in both direction





σ_1 and σ_2 are Major Principle Stresses

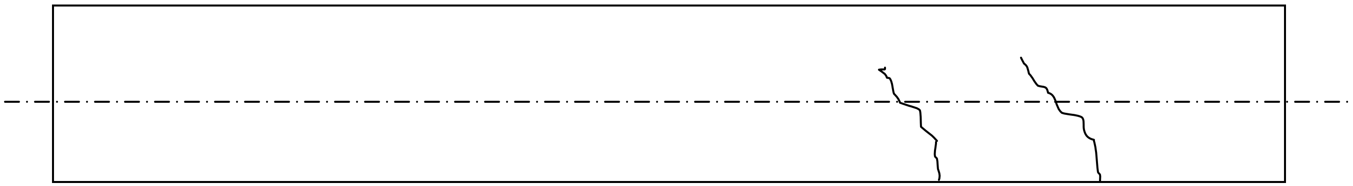
Plain & Reinforced Concrete-1

Pure Shear/Web Shear Cracks

Plain & Reinforced Concrete-1

Different Types of Cracks (contd…)

3. Flexural Shear Cracks
   In the regions of high shear due to diagonal tension, the inclined cracks develop as an extension of flexural cracks and are termed as Flexural Shear Cracks.



Flexural Shear Cracks.

Plain & Reinforced Concrete-1

Tensile Strength of Concrete

There are considerable experimental difficulties in determining the true tensile strength of concrete. In direct tension test following are the difficulties:

1. When concrete is gripped by the machine it may be crushed due to the large stress concentration at the grip.
2. Concrete samples of different sizes and diameters show large variation in results.
3. If there are some voids in sample the test may show very low strength.
4. If there is some initial misalignment in fixing the sample, the results are not accurate.

Plain & Reinforced Concrete-1

Following are the few indirect methods through with tensile strength of concrete is estimated.

A. Split cylinder Test

This test is performed by loading a standard 150mmϕ x 300mm cylinder by a line load perpendicular to its longitudinal axis with cylinder placed horizontally on the testing machine platen.

The tensile strength can be defined as

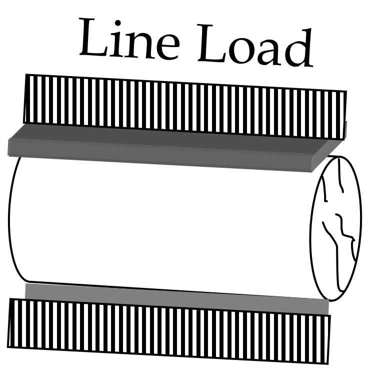
$$f_t = 2P / (\pi DL)$$

Where

P = Total value of load registered by machine

D = Diameter of concrete cylinder

L = Cylinder height



Plain & Reinforced Concrete-1

Tensile Strength of Concrete (contd…)

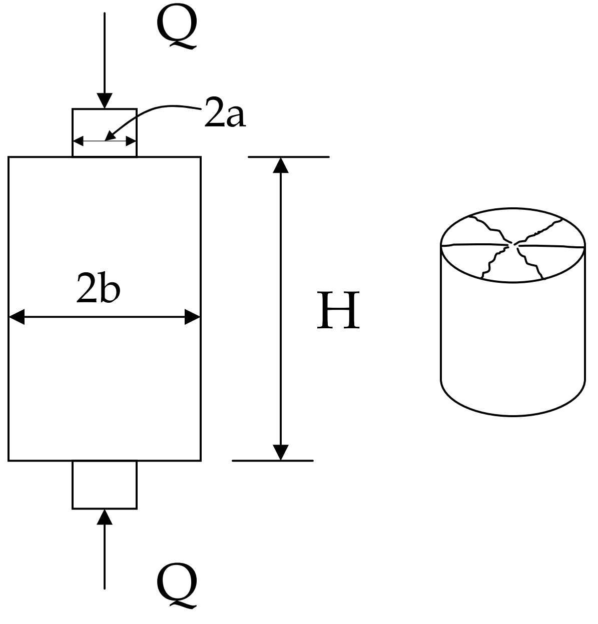
B. Double Punch Test

In this test a concrete cylinder is placed vertically between the loading platens of the machine and is compressed by two steel punches placed parallel to top and bottom end surfaces. The sample splits across many vertical diametrical planes radiating from central axis.

Tensile strength can be defined as

$$f_t = Q / [\pi (1.2bH-a^2)]$$

Q = Crushing Load



Plain & Reinforced Concrete-1

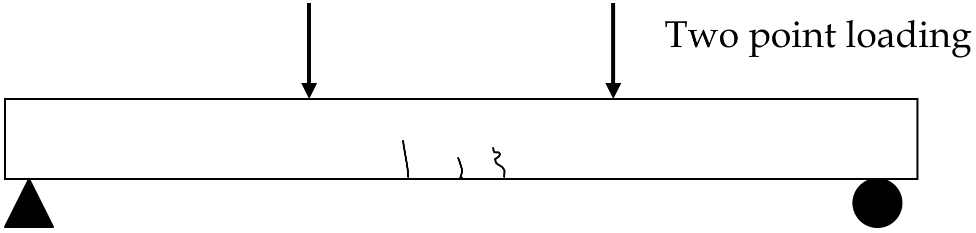
Tensile Strength of Concrete (contd…)

c. Modulus of Rupture Test

For many years, tensile strength has been measured in term of the modulus of rupture f_r, the computed flexural tensile stress at which a test beam of plain concrete fractures.

Because this nominal stress is computed on the assumption that concrete is an **elastic material**, and because this bending stress is localized at the outermost surface, it is larger than the strength of concrete in uniform axial tension.

It is a measure of, but not identical with the real axial tensile strength.



Plain & Reinforced Concrete-1

There are some relationships which relate modulus of rupture, f_r, with compressive strength of concrete

$$f_r = 0.69 \sqrt{f_c'}$$

f_c' and f_r are in MPa. It also varies between 10 to 15% of f_c'.

ACI code gives a formula for f_r for strength calculations:

$$f_r = 0.5\lambda\sqrt{f_c'}$$

And for deflection control:

$$f_r = 0.62\sqrt{f_c'}$$

Where $\lambda = 1$ for normal weight concrete.

Mean split cylinder strength = $0.53 \sqrt{f_c'}$

Tensile strength ∝ square root of compressive strength.

True tensile strength varies between 8 to 15% of f_c'.

Few empirical formulae are developed for the tensile strength of concrete:

$$f_t = 0.25 \sqrt{f_c'} \text{ to } 0.42 \sqrt{f_c'}$$

Modular Ratio (n): "The ratio of modulus of elasticity of steel to modulus of elasticity of concrete is known as modular Ratio".

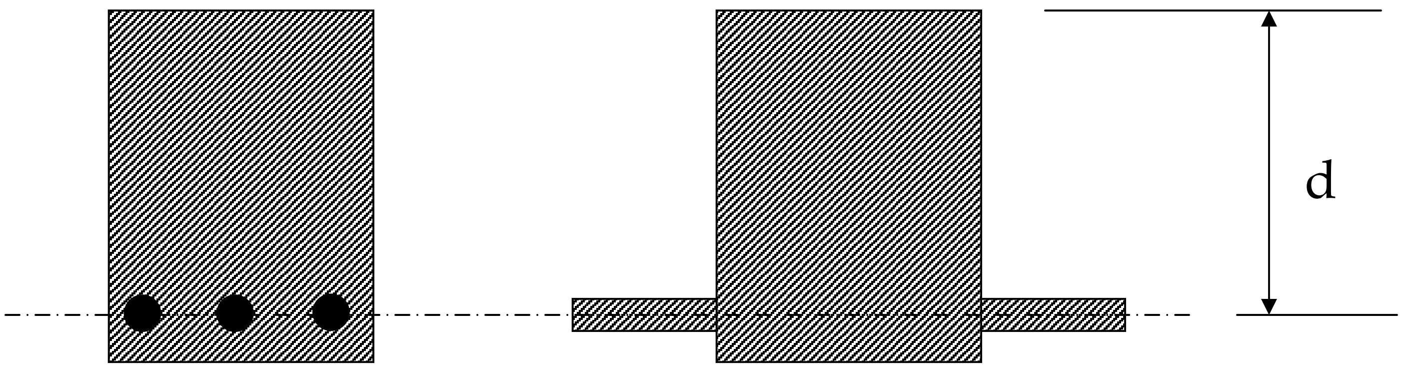
$$n = E_s / E_c$$

- Normally the value of n is 8 to 10
- It is a unit-less quantity

Plain & Reinforced Concrete-1

Transformed Section

Beam is a combination of concrete and steel. As a whole it is not a homogeneous material. In transformed section the steel area is replaced by an equivalent concrete area in order to calculate the section properties.



Transformed Section

- Width of the extended area is same as diameter of steel bar and its distance from compression face remains same.

Plain & Reinforced Concrete-1

Uncracked Transformed Section

When both steel and concrete are in elastic range and tensile stress at the tension face of concrete is less than tensile strength of concrete the section is un-cracked.

Within the elastic range, perfect bond (no slippage) exists between concrete and steel, so

$$\varepsilon_s = \varepsilon_c$$

$$f_s / E_s = f_c \;/\; E_c$$

$$f_s = (E_s / E_c)\, f_c$$

$$f_s = n\, f_c$$

Using this relationship, stress in steel can be calculated if stress in concrete and modular ratio are known.

Consider a beam having steel area A_s . In order to obtain a transformed section, the area of steel (A_s) is replaced by an equivalent area of concrete so that equal force is developed in both.

$$A_g = A_c + A_s$$

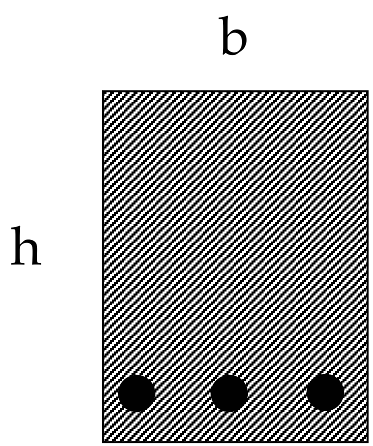
Total tensile force, T

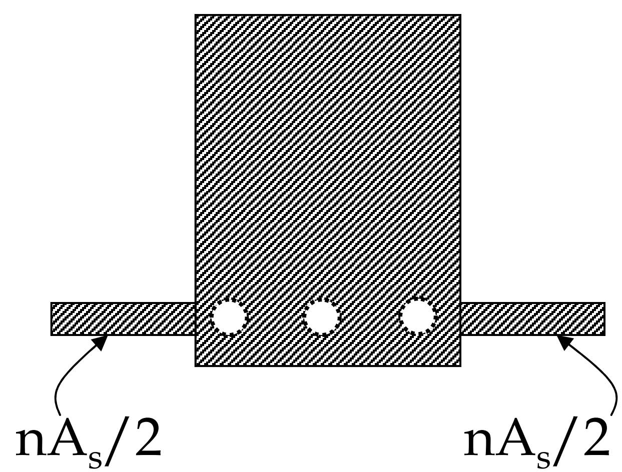
$$T = f_c A_c + f_s A_s$$

$$= f_c (A_g - A_s) + n f_c A_s$$

$$= f_c (Ag - A_s + n A_s)$$

$$= f_c \{A_g + (n - 1) A_s\}$$



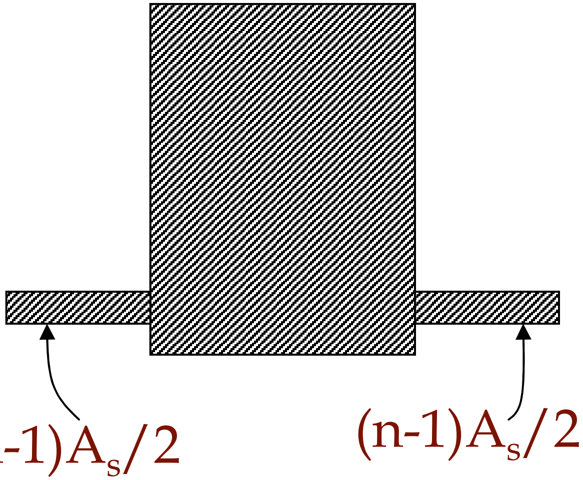


Plain & Reinforced Concrete-1

The equivalent area $nA_s/2$ is shown on either side but steel inside the beam is removed which creates a space that is filled by area of concrete, thus the equivalent area on either side becomes

$$nA_s/2 - A_s/2$$

$$(n-1)A_s / 2$$



Transformed Section

- Once the transformed section has been formed the sectional properties **(A, Location of N.A., I, S etc)** are calculated in usual manner

Total Area of transformed section = $b \times h + (n-1)A_s = A_g + (n-1)A_s$

Example 2.1

A rectangular beam of size 250 $\times$ 650 mm, with effective depth equal to 590 mm, is reinforced with three No. 25 US customary bars. C 28 concrete and Grade 420 steel are to be used. Determine the stresses at the top, bottom and level of reinforcement caused by a bending moment of 50 kN-m. The member is within its elastic range.

- A_s = 3×510 = $1530\ mm^2$
- E_s = 200,000 MPa
- f_c' = 28 MPa
- E_c = 470024,870 MPa
- n = $Es / Ec \cong 8$

- Additional steel area = $(n - 1)\ A_s$

  = $10,710\ mm^2$

$$\bar{y} = \frac{(250)(650)(325) + (10{,}710)(590)}{173{,}210}$$

$$= \mathbf{341\ mm}$$

$$I = \frac{(250)(650)^3}{12} + (250)(650)(16)^2 + (10{,}710)(249)^2$$

$$= 642{,}700 \times 10^4\ \text{mm}^4$$

$$(f_c)_{\text{top}} = \frac{M\,y}{I}$$

$$= \frac{50 \times 10^6 \times 341}{642{,}700 \times 10^4}$$

$$= \mathbf{2.65\ MPa\ (compressive)}$$

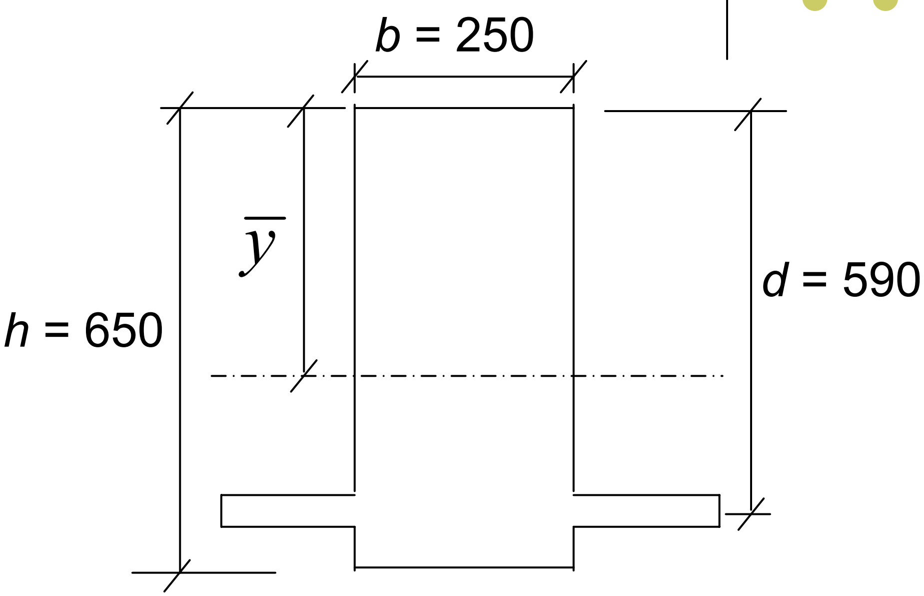


Fig. 2.7. Transformed Area for Example 2.1.

$$(f_c)_{bot} = \frac{M\,y}{I} = \frac{50\times10^6\times309}{642{,}700\times10^4}$$

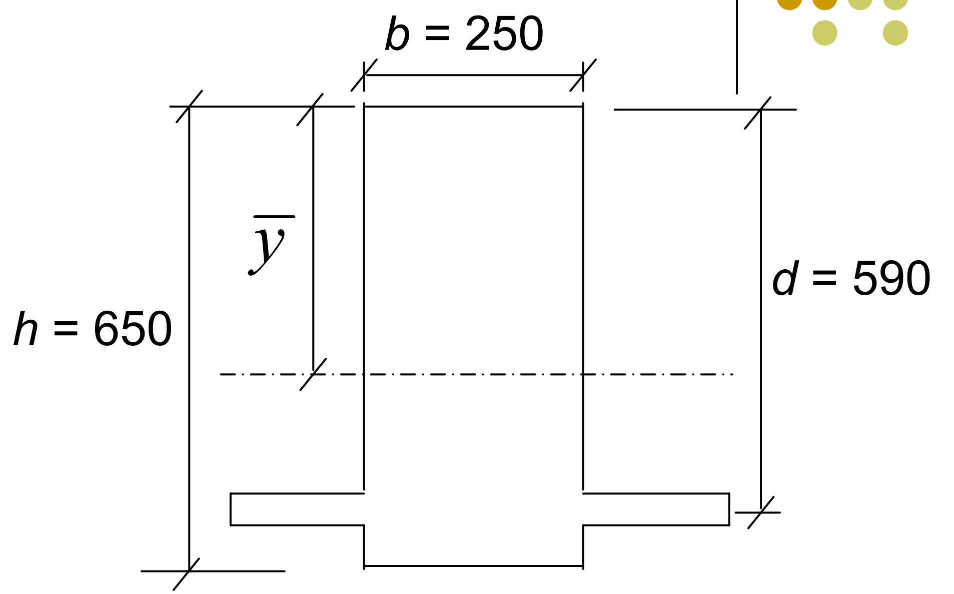
$$= \mathbf{2.40\ MPa\ (tensile)}$$

$$f_s = n\,\frac{M\,y}{I}$$

$$= 8\times\frac{50\times10^6\times(590-341)}{642{,}700\times10^4}$$

$$= \mathbf{15.50\ MPa}$$

$$f_r \text{ for strength} = 0.5\sqrt{f_c'} = \mathbf{2.65\ MPa}$$



The bottom concrete stress in tension is lesser than its modulus of rupture indicating that the section is really uncracked. Further, the stresses are much lesser than f_c'/ 2 and f_y / 2 showing elastic behavior. *Hence, the assumption of uncracked transformed section is justified.*

Concluded