Plain & Reinforced Concrete-1

CE-313

Lecture # 3

Flexural Analysis and Design of Beams

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Flexural Behavior of Beams Under Service Load

When loads are applied on the beam stresses are produced in concrete and steel reinforcement.

- If stress in steel bars is less than yield strength, steel is in elastic range.
- If stress in concrete is less than $0.6f_c'$ concrete is assumed to be with in elastic range.

Following are important points related to Elastic Range:

- Loads are un-factored
- Materials are in elastic range
- Allowable stress analysis and design is applicable.

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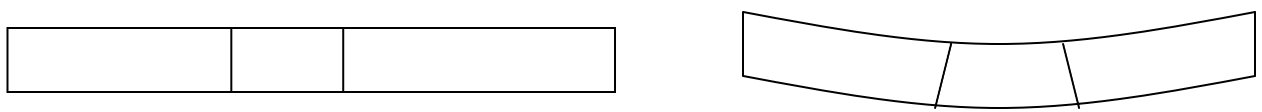
Assumption for the Study of Flexural Behavior

- Plane sections of the beam remain plane after bending.



- The material of the beam is homogeneous and obeys hooks law

Stress ∝ Strain

- Perfect bond exists between steel & concrete so whatever strain is produced in concrete same is produced in steel.
- All the applied loads up to to failure are in equilibrium with the internal forces developed in the material.
- At the strain of 0.003 concrete is crushed.

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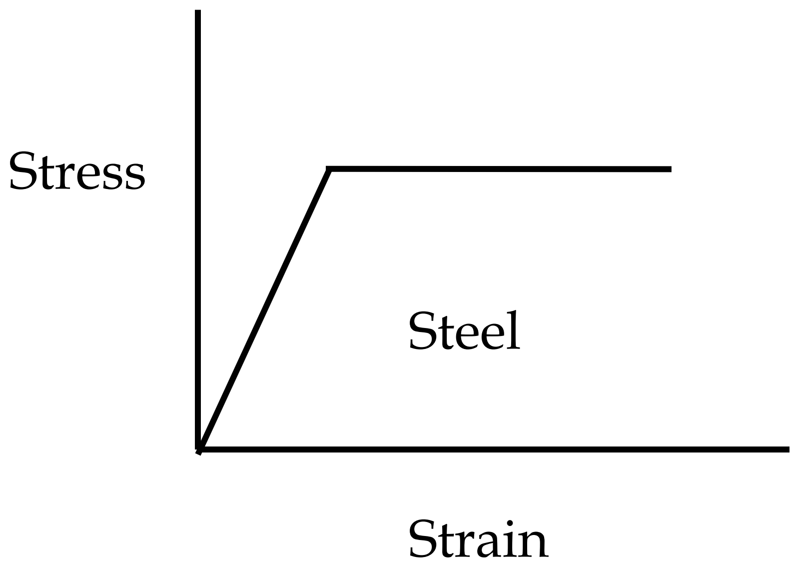
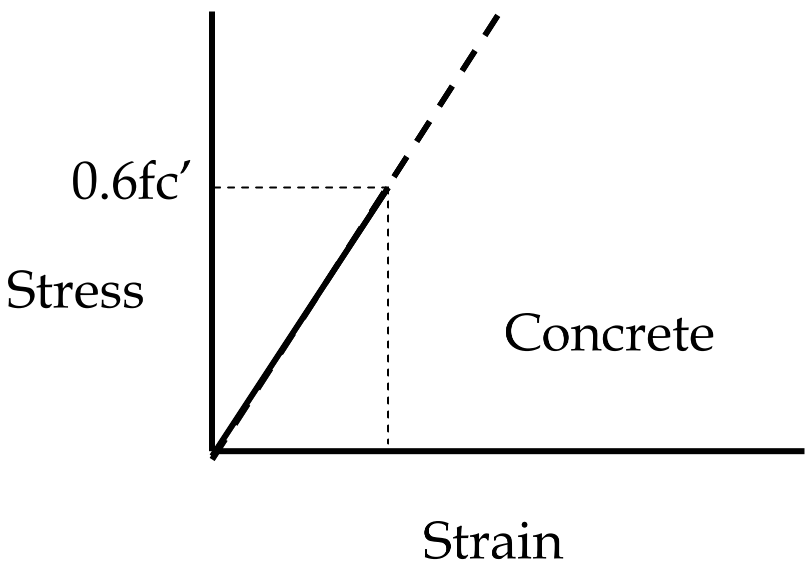
Assumption for the Study of Flexural Behavior (contd…)

- When cracks appear on the tension face of beam the concrete capacity to resist tension is considered zero.
- Stress and strain diagrams for steel and concrete are simplified.





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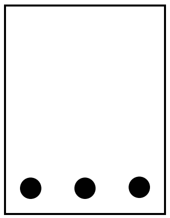
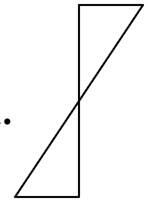
Flexural Behavior Beams

General Procedure for the Derivation of Formula

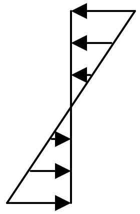
Step # 1 Draw the cross section of beam with reinforcement.



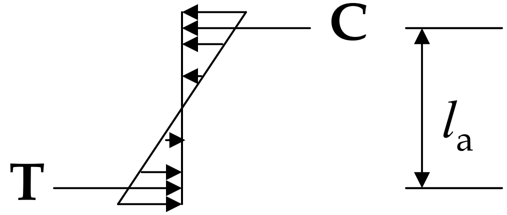
Step # 2 Draw the strain diagram for the cross section.



Step # 3 Draw the stress diagram.



Step # 4 Show location of internal resultant forces.



Step #5 Write down the equilibrium equations or use geometry of strain diagram to arrive at various results.

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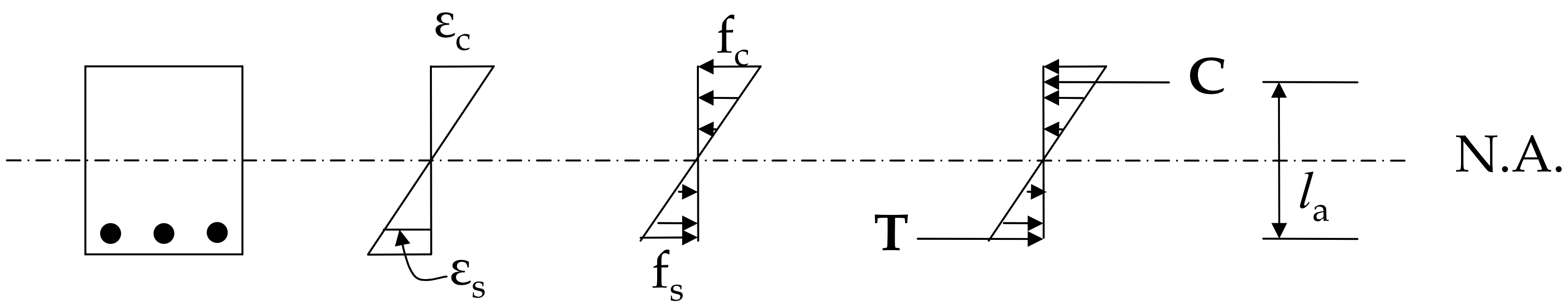
Flexural Behavior Beams (contd…)

1. When Both Steel and Concrete are in Elastic Range



Strain Diagram **Stress Diagram** **Resultant Force Diagram**

Both steel and concrete are resisting to applied action

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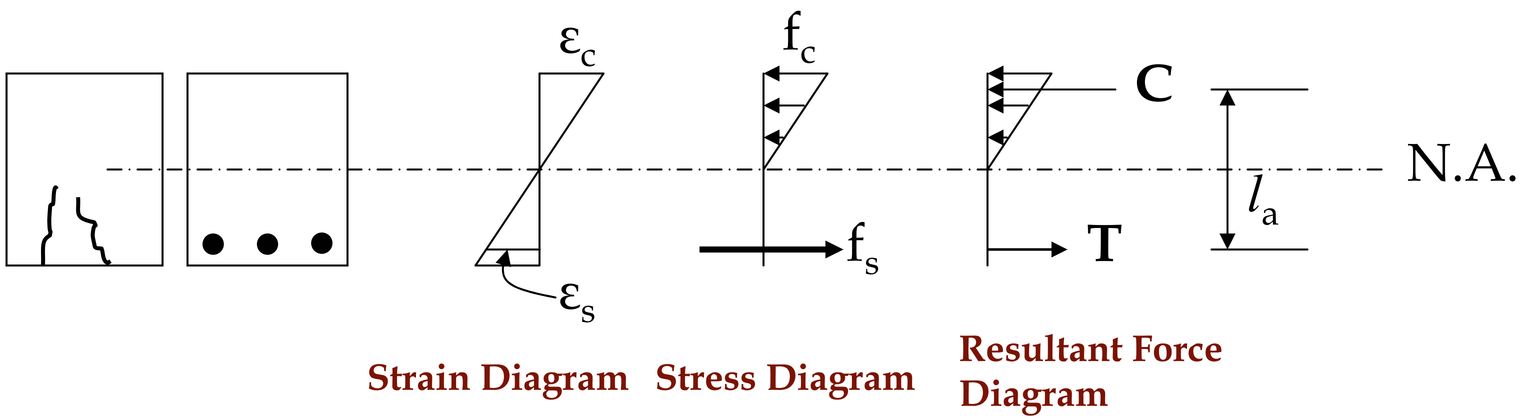
Flexural Behavior Beams (contd…)

2. When Cracks are Appeared on tension Side



When the tension side is cracked the concrete becomes ineffective but the strains goes on increasing. The steel comes in to action to take the tension.

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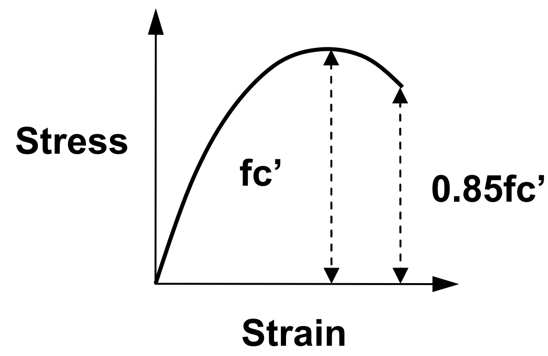
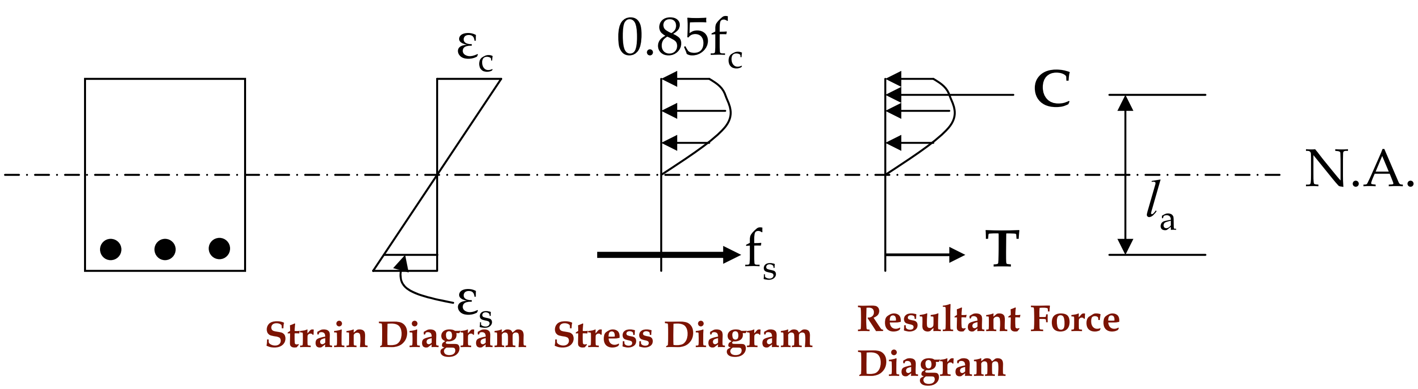
Flexural Behavior Beams (contd...)

3. When Compression Stresses Cross Elastic Range





It is clear that the stress diagram is in fact obtained by rotating the stress strain diagram of concrete.

Strains keeps on changing linearly in all three cases.

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Flexural Behavior Beams (contd…)

Final Equation for Calculating Moment Capacity

$$M_r = T \times l_a = C \times l_a$$

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Flexural/Bending Stress Formula

$$f = \pm My/I$$ **(Valid in Elastic Range Only)**

$$f = \pm M/(I/y)$$

$$f = \pm M/S$$

f = Flexural Stress

S = Elastic Section Modulus

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Shear Stress Formula

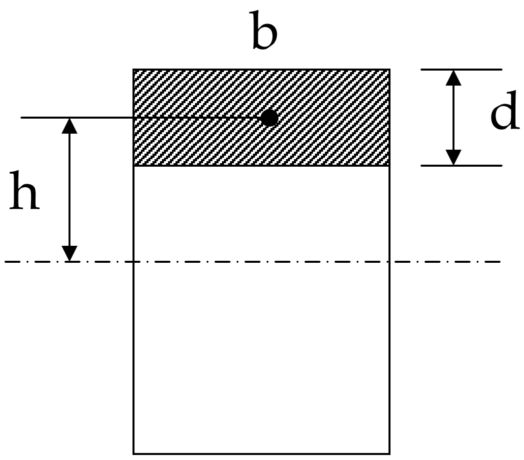
$$\tau = VAy/(I\,b)$$

(Valid in Elastic Range Only)

$$\tau = VQ/(I\,b)$$

τ = Shear Stress

Q = First moment of area



First Moment of Shaded Area, Q = (b x d) h

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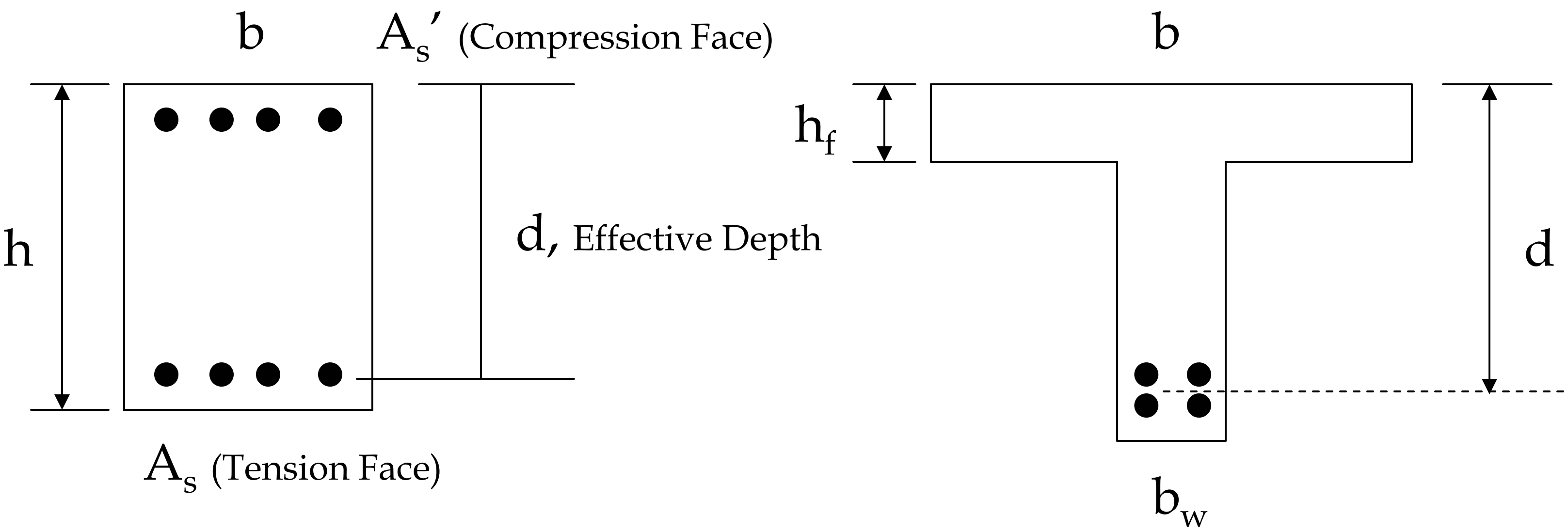
Notation



f_c = concrete stress at any load level at any distance form the N.A

f_c'= 28 days cylinder strength

ε_c = Strain in concrete any load level

ε_{cu} = Ultimate concrete strain, 0.003

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Notation (contd…)

f_y = Yield strength of concrete

f_s = Steel stress at a particular load level

ε_s = strain in steel at a particular level, $\varepsilon_s = f_s/E_s$

ε_y = Yield strain in steel

E_s = Modulus of elasticity of steel

E_c = Modulus of elasticity of concrete

ρ (rho) = Steel Ratio, $\rho = A_s/A_{ec} = A_s/(b \times d)$

T = Resultant tensile force

C = Resultant compressive force

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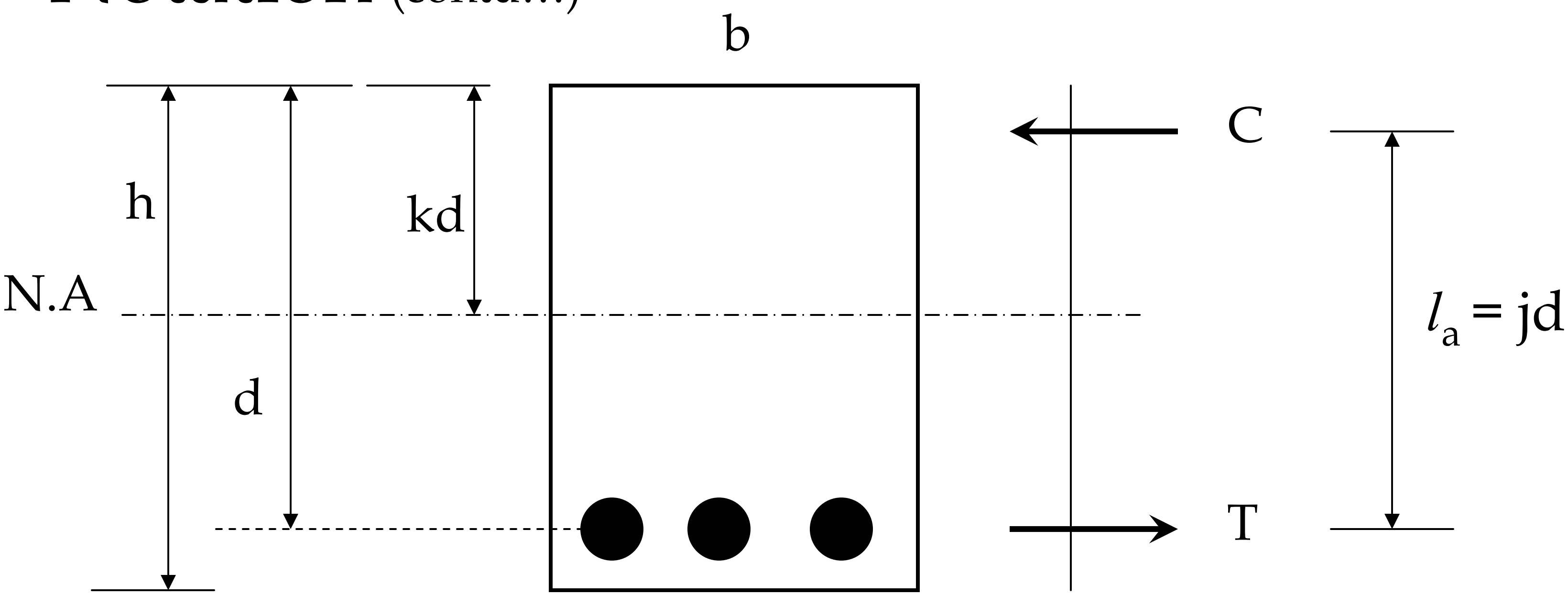
Notation (contd…)



jd = Lever arm $\quad j = l_a / d$ (valid for elastic range)

kd = Depth of N.A. from compression face, $\quad k = c/d$

j and k are always less than 1.

Concluded