Plain & Reinforced Concrete-1

Sixth Term
Civil Engineering

CE-314

Lecture # 12

Flexural Analysis and Design of Beams

(Ultimate Strength Design of Beams)

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Design of Singly Reinforced Beam by Strength Method (for flexure only)

Data:

- Load, Span, SFD, BMD
- f_c', f_y, E_s
- Architectural depth, if any

Required:

- Dimensions, b & h
- Area of steel
- Detailing (bar bending schedule)

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Design of Singly Reinforced Beam by Strength Method (contd…)

Two Methods to Start the Procedure:

1. Select reasonable steel ratio between ρ_{min} and ρ_{max}. Then find b, h and A_s.
2. Select reasonable values of b, h and then calculate ρ and A_s.

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1. Using Trial Dimensions
   I. Calculate loads acting on the beam.
   II. Calculate total factored loads and plot SFD and BMD. Determine V_{umax} and M_{umax}.
   III. Select suitable value of beam width 'b'. Usually between L/20 to L/15. preferably a multiple of 75mm or 114 mm.
   IV. Calculate d_{min}.

$$d_{min} = \sqrt{\frac{M_u}{0.205 f_c' b}}$$

$h_{min} = d_{min} + 60$ mm for single layer of steel

$h_{min} = d_{min} + 75$mm for double layer of steel

} Round to upper 75 mm

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Design of Singly Reinforced Beam by Strength Method (contd…)

v. Decide the final depth.

$$h \geq h_{min}$$ For strength

$$h \geq h_{min}$$ For deflection

$$h \approx h_{a}$$ Architectural depth

$$h \approx h/12$$

Preferably "h" should be multiple of 75mm.

Recalculate "d" for the new value of "h"

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Design of Singly Reinforced Beam by Strength Method (contd…)

VI. Calculate "ρ" and "A_s".

Four methods

a) $$\rho = \omega\left(1 - \sqrt{1 - \frac{2.614R}{f_c'}}\right)$$

$$\omega = 0.85\,\frac{f_c'}{f_y} \qquad R = \frac{M_u}{bd^2}$$

b) Design Table

c) Design curves

d) Using trial Method

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Design of Singly Reinforced Beam by Strength Method (contd…)

VII. Check $A_s \geq A_{s\,min}$.

$$A_{s\,min} = \rho_{min}\,bd \qquad (\rho_{min} = 1.4/f_y \text{ to } f_c' \leq 31 \text{ MPa})$$

VIII. Carry out detailing

IX. Prepare detailed sketches/drawings.

X. Prepare bar bending schedule.

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1. Using Steel Ratio
   - I. Step I and II are same as in previous method.
   - III. Calculate ρ_{max} and ρ_{min} & select some suitable "ρ".
   - IV. Calculate bd^2 from the formula of moment

$$M_u = \phi_b M_n = 0.9\left(\rho b d^2\right) f_y \left(1 - \frac{\rho f_y}{1.7 f_c'}\right)$$

$$bd^2 = \frac{M_u}{0.9 \rho f_y \left(1 - \frac{\rho f_y}{1.7 f_c'}\right)}$$

   - V. Select such values of "b" and "d" that "bd^2" value is satisfied.
   - VI. Calculate A_s from the known steel ratio.
   - VII. Remaining steps are same as of previous method.

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Example:

Design the interior long beam B2 of an office building. Slab thickness = 160 mm, floor finishes = 75 mm of brick ballast and 50 mm of P.C.C. floor finishes. The beam B_2 supports a 228 mm thick wall of 3 m height. Use C-20 concrete and Grade 420 steel. US customary bars are to be used. Take b = 300 mm. Design for the following three options:

1. Depth obtained with maximum steel ratio, even if it violates the depth for deflection control
2. h = 835 mm
3. h = 910 mm, using trial method and prepare bar bending schedule.

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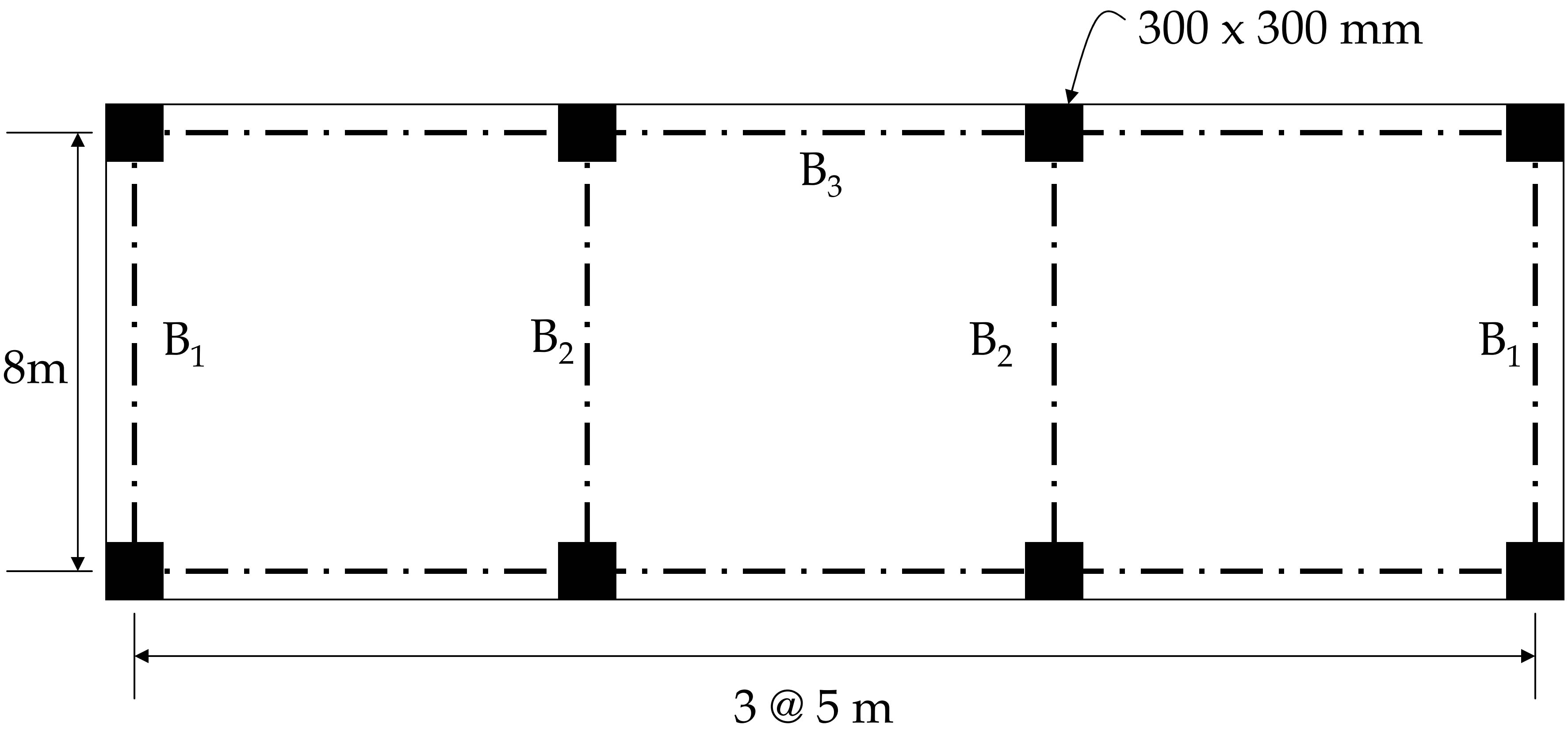
Example:



Beams are not monolith.

Solution:

Data

- L = 8 m
- Slab thickness = 160 mm
- Floor brick ballast = 75 mm
- Floor finish = 50 mm
- f_c' = 20 MPa
- f_y = 420 MPa
- b = 300 mm

Slab Load

R.C. slab: $\frac{160}{1000} \times 2400$ = 384 kgs/m^2

Brick ballast: $\frac{75}{1000} \times 1800$ = 135 kgs/m^2

P.C.C. + terrazzo: $\frac{50}{1000} \times 2300$ = 115 kgs/m^2

Total dead load: = 634 kgs/m^2

Live load: = 250 kgs/m^2

Total factored load= 1.2 D + 1.6 L

$= [1.2(634) + 1.6(250)] \times 9.81 / 1000$

$= 11.39$ kN/m^2

Brick Wall Load

Service dead load $= 1930 \times \frac{228}{1000} \times 3.0 \times \frac{9.81}{1000}$

$= 12.95$ kN/m

Factored dead load = 1.2 $\times$ 12.95 = 15.54 kN/m

Approximate Self Weight

Service dead load $= 2400 \times \frac{300}{1000} \times \frac{8}{12} \times \frac{9.81}{1000}$

$= 4.71$ kN/m

Factored dead load = 1.2 $\times$ 4.71 = 5.65 kN/m

Equivalent Width Of Slab Supported By Beam B_2

ℓ_y = 8 m : ℓ_x = 5 m :

$R = \ell_x / \ell_y = 5/8 = 0.625$

Equivalent slab width supported

$= (1 - R^2/3)\, \ell_x$

$= (1 - 0.625^2 / 3) \times 5 \times 1.1 = 4.78$ m

(10% extra for first interior beam)

Factored Slab Load Acting On Beam

Factored slab load on beam

= width of slab × slab load per unit area

= 4.78 × 11.39 = 54.49 kN/m

Total Factored Load

w_u = 54.49 + 15.54 + 5.65

= 75.68 kN/m

Total Factored Bending Moment

$$M_u = \frac{w_u \ell^2}{8} = \frac{75.68 \times 8^2}{8} = 605.44 \text{ kN-m}$$

Minimum Effective Depth For Singly Reinforced Section

$$d_{min} = \sqrt{\frac{M_u}{0.205 \times f_c' \times b}}$$

$$= \sqrt{\frac{605.44 \times 10^6}{0.205 \times 20 \times 300}} = 702 \text{ mm}$$

$$\rho_{min} = 1.4 / f_y = 1.4 / 420 = 0.00333$$

Depth For Deflection Control

Minimum depth of beam for deflection control

= L / 16 = 8000 / 16 = 500 mm

Case (i)

The minimum depth will be obtained by using the maximum permissible steel ratio. However, this will be equal to the already calculated d_{min}.

$$\rho_{max} = 0.375 \times 0.85\, \beta_1 \frac{f_c'}{f_y}$$

$$= 0.375 \times 0.85 \times 0.85 \times \frac{20}{420}$$

$$= 0.0129$$

$$M_u = \phi_b \rho b d^2 f_y \left(1 - \frac{1}{1.7}\frac{\rho f_y}{f_c'}\right)$$

$$605.44 \times 10^6 = 0.9 \times 0.0129 \times 300 \times d^2 \times 420 \times \left(1 - \frac{1}{1.7}\frac{0.0129 \times 420}{20}\right)$$

$$d^2 = 492{,}328$$

$$d = 702 \text{ mm}$$

$$A_s = \rho\, b\, d = (0.0129)(300)(702)$$

$$= 2717 \text{ mm}^2$$

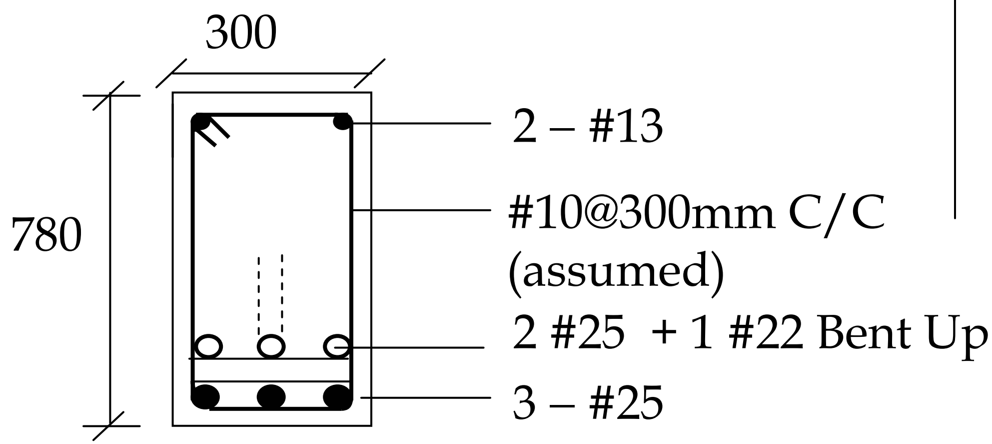
6 No.25 provide 3060 mm^2 (12.6 % higher)

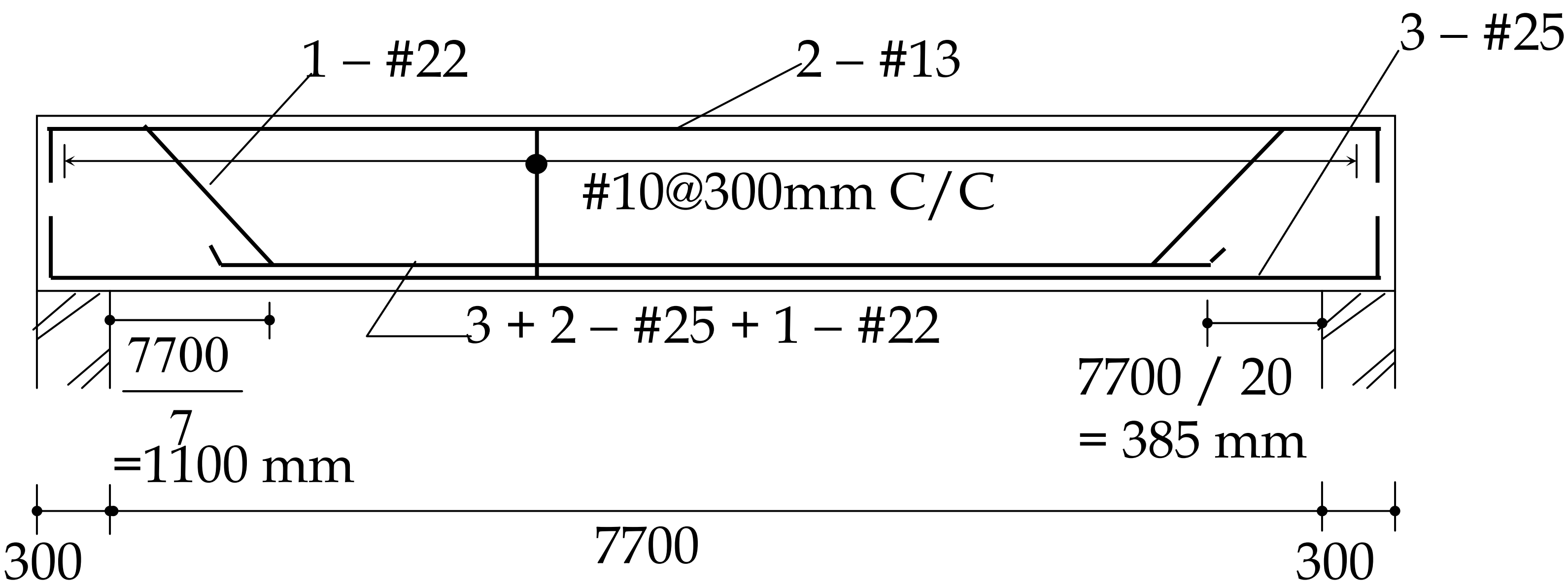
5 No.25 + 1 No.22 provide 2937 mm^2

(8.1 % higher)

Clear Cover = 40 mm

a) Mid-span Cross Section





b) Longitudinal Section

- When using bent-up reinforcement, the bars may be bent at a distance of ℓ_n / 7 from a simply supported end, if at least 50% of the total steel is continued beyond the bent-up point. For curtailed bars, this distance for simply supported end may be taken equal to ℓ_n / 20 (no curtailment according to the ACI Code).

- Spacing between bars $= \dfrac{300 - 2\times 40 - 2\times 10 - 3\times 25}{2}$

$= 62.5$ mm (OK)

Case (ii)

- h = 835 mm
- $d \approx 835 - 75 = 760$ mm $> d_{min}$
- M_u = 605.44 kN-m

Method 1

$$\omega = \frac{0.85 f_c'}{f_y} = \frac{0.85 \times 20}{420} = 0.0405$$

$$R = \frac{M_u}{bd^2} = \frac{605.44 \times 10^6}{300 \times 760^2} = 3.494$$

- $\rho = \omega\left(1-\sqrt{1-\frac{2.614\,R}{f_c'}}\right)$

$= 0.0405\left(1-\sqrt{1-\frac{2.614\times 3.494}{20}}\right)$

$= 0.01065 > \rho_{min}$ (OK)

- $A_s = \rho\, b\, d$

$= 0.01065 \times 300 \times 760 = 2428\ \text{mm}^2$

4 - #22 + 2 - #25 will be sufficient

($2568\ \text{mm}^2$) (OK)

Method 2

- $R = 3.494$ MPa as before.
- From the table corresponding to $f_c' = 20$ MPa and $f_y = 420$ MPa,

  $\rho = 0.0108$ (by approximate interpolation)
- Almost the same reinforcement as in the first method.

Method 3

- $R = 3.266$ MPa as before.
- Using design curves for $f_c' = 20$ MPa and $f_y = 420$ MPa, corresponding to $R = 3.494$ MPa, the following steel ratio is obtained:

  $\rho \cong 0.0107$
- Almost the same reinforcement as in the first method.

Method 4

Trial 1:

Assume $a = d / 3 = 760/3 = 253$ mm

$$A_s = \frac{M_u}{0.9 f_y \left(d - \frac{a}{2}\right)} = \frac{605.44 \times 10^6}{0.9 \times 420 \left(760 - \frac{253}{2}\right)}$$

$= 2528 \text{ mm}^2$

Trial 2:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2528 \times 420}{0.85 \times 20 \times 300} = 208 \text{ mm}$$

$$A_s = \frac{M_u}{0.9 f_y \left(d - \frac{a}{2}\right)} = \frac{605.44 \times 10^6}{0.9 \times 420 \left(760 - \frac{208}{2}\right)} = 2442 \text{ mm}^2$$

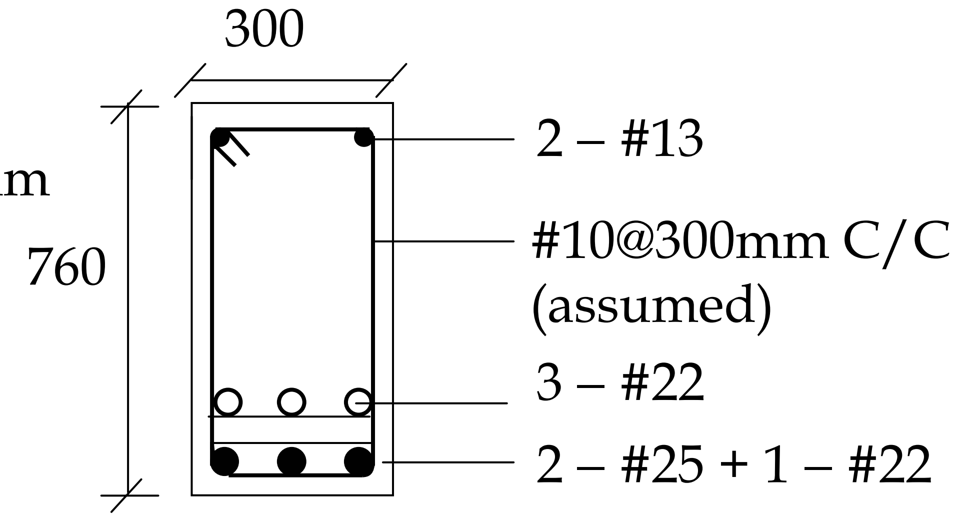
Trial 3:

$$a = \frac{2442 \times 420}{0.85 \times 20 \times 300} = 201 \text{ mm}$$

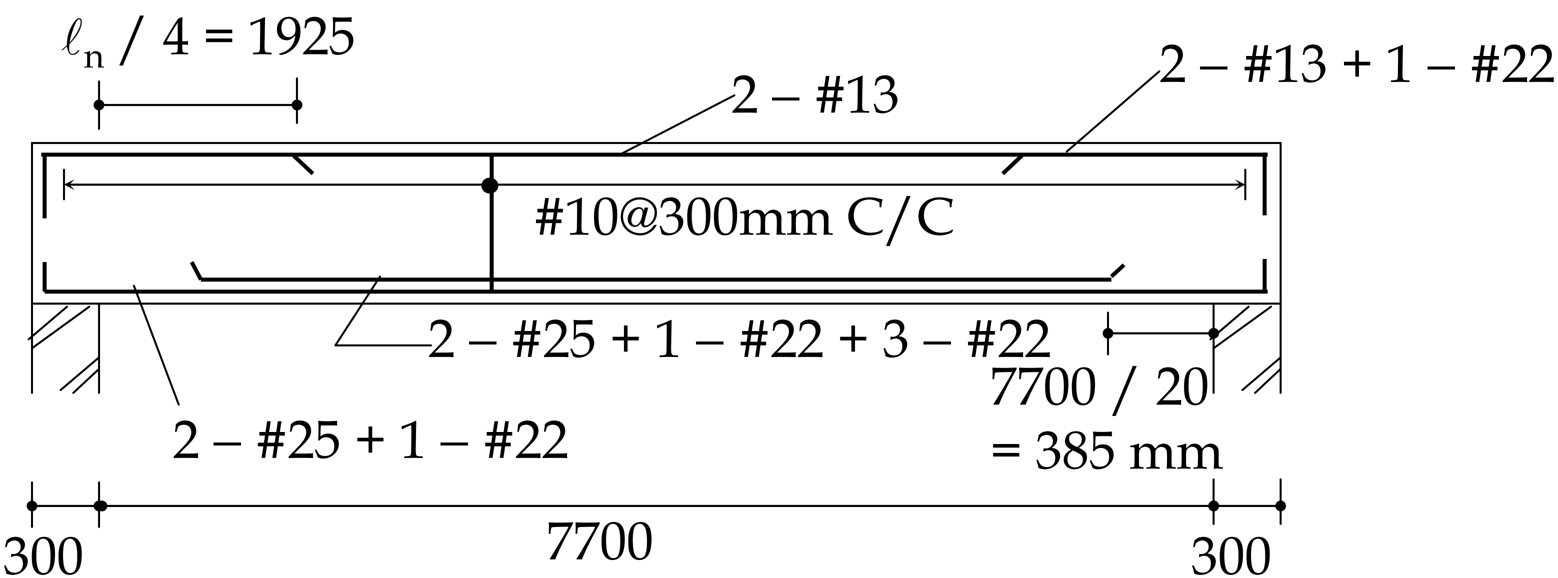
$$A_s = \frac{605.44 \times 10^6}{0.9 \times 420 \left(760 - \frac{201}{2}\right)} = 2429 \text{ mm}^2$$

(sufficiently close to the previous answer)

Clear Cover = 40 mm



a) Mid-span
Cross Section



b) Longitudinal Section

Case (iii)

h = 910 mm

d = 910 – 75 = 835 mm

Trial 1: Assume a = d / 3 = 835/3

= 278 mm

$$A_s = \frac{M_u}{0.9 f_y \left(d - \frac{a}{2} \right)} = \frac{605.44 \times 10^6}{0.9 \times 420 \left(835 - \frac{278}{2} \right)}$$

= 2301 mm^2

Trial 2: $a = \dfrac{A_s f_y}{0.85 f_c' b}$

$$= \frac{2301 \times 420}{0.85 \times 20 \times 300} = 190 \text{ mm}$$

$$A_s = \frac{605.44 \times 10^6}{0.9 \times 420 \left(835 - \frac{190}{2}\right)} = 2164 \text{ mm}^2$$

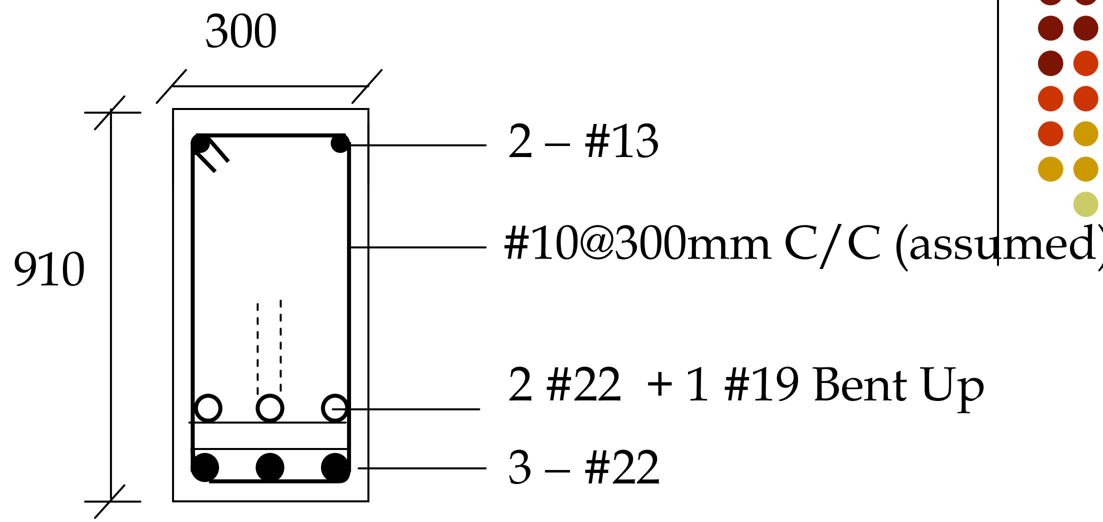
Trial 3: $a = 178$ mm

$A_s = 2147 \text{ mm}^2$

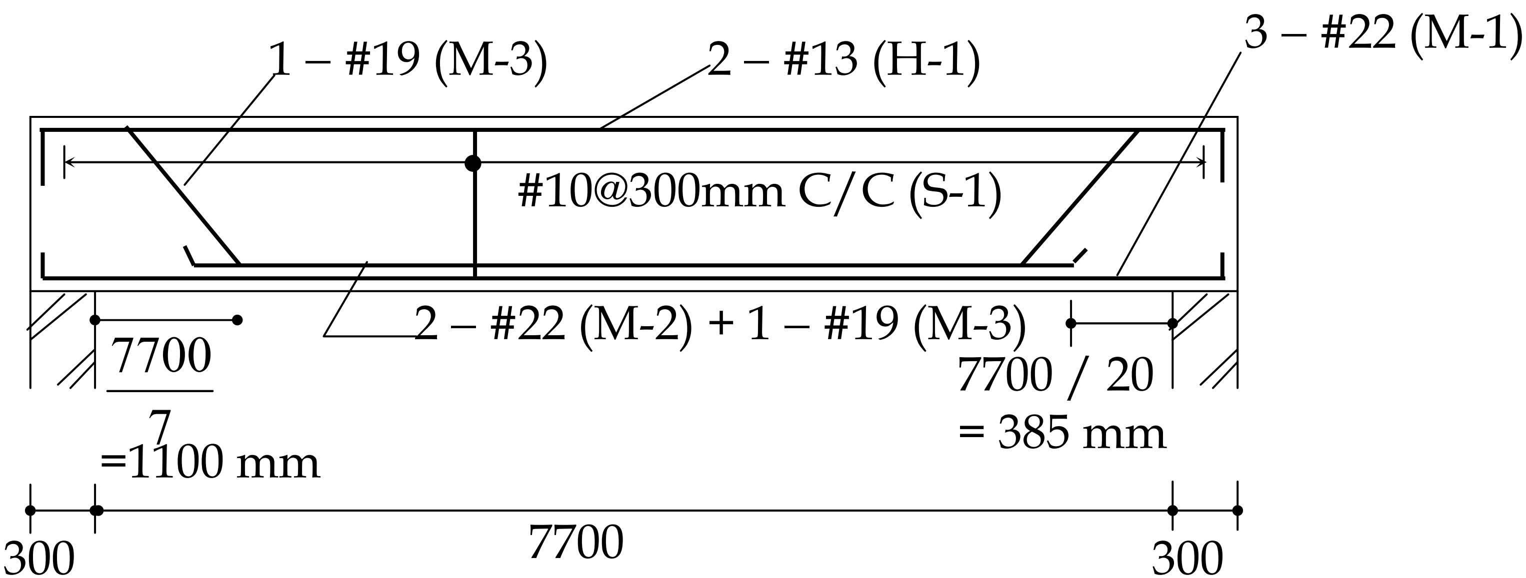
(sufficiently close to the previous trial)

The area of steel is provided by 5 #22 + 1 #19 give 2219 mm².

Clear Cover = 40 mm

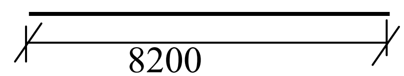
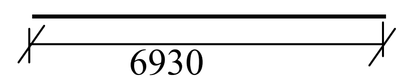
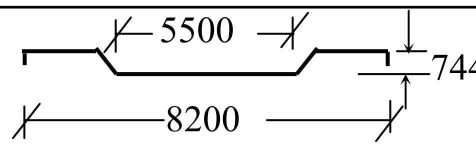
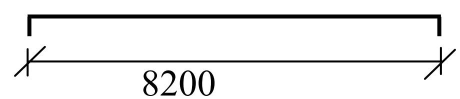
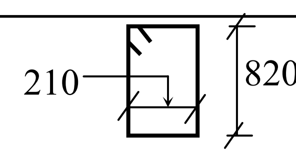


a) Mid-span Cross Section



b) Longitudinal Section

Table 3.5. Bar Bending Schedule. Steel Grade: 420

S. No.	**Bar Designation**	**No. Of Bars**	**Len. Of Bar (m)**	**Dia. Of Bar**	**Weight Of Bars**				**Shape Of Bar**
					No. 10	**No. 13**	**No. 19**	**No. 22**	
1	M-1	3	8.200	#22				74.84	
2	M-2	2	6.930	#22				42.16	
4	M-3	1	9.500	#19			21.23		
5	H-1	2	8.434	#13		16.77			
6	S-1	27	2.420	#10	36.59				
				$\sum =$	38.5	17.6	22.3	122.9	

Total Steel Required ≅ 202 kgs

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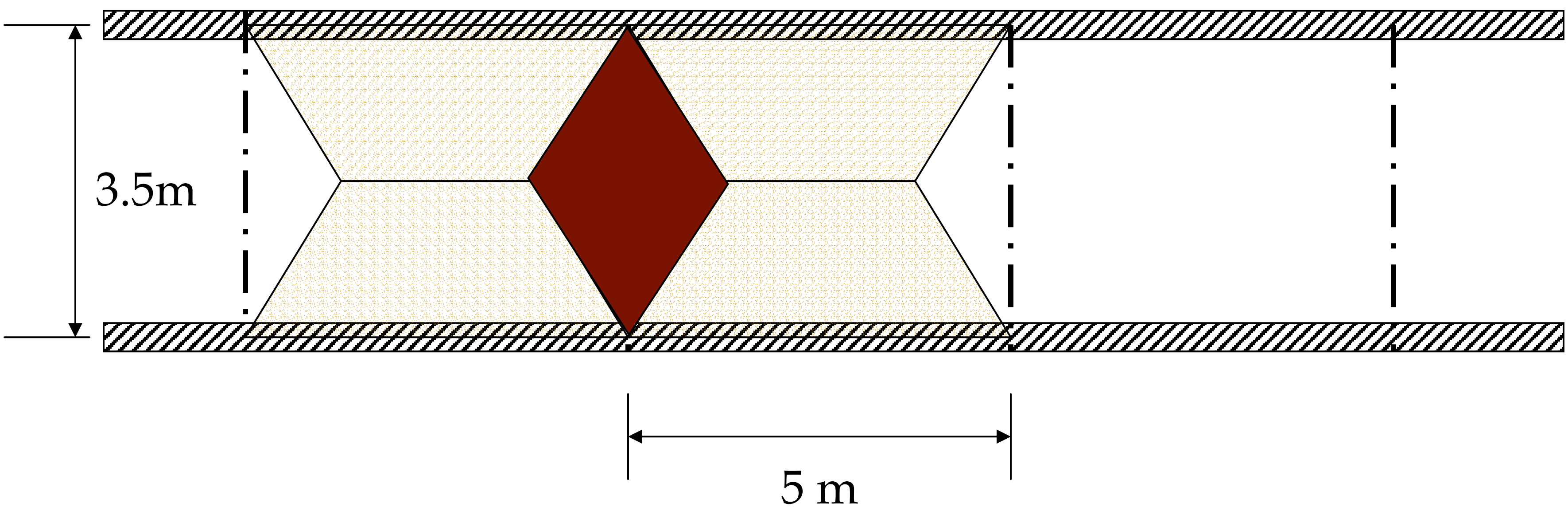
Example:

Design a singly reinforced rectangular flexural member to be used as interior simply supported short beam. Slab panel is 5m x 3.5m. Factored slab load is 10 kN/m². $f_c' = 17.25$ MPa. $f_y = 300$ MPa, b = 228mm, architectural depth = 375 mm.



Data

- L = 3.5 m
- Factored slab load = 10.00 kN/m^2
- f_c' = 15 MPa
- f_y = 300 MPa
- b = 228 mm
- h_a = 375 mm

Approximate Self Weight

- Service dead load $= 2400 \times \frac{228}{1000} \times \frac{3.5}{12} \times \frac{9.81}{1000}$
  $= 1.57$ kN/m
- Factored dead load $= 1.2 \times 1.57$
  $= 1.88$ kN/m

Equivalent Width Of Slab Supported By Beam B1

$L_y = 5$ m : $L_x = 3.5$ m

Equivalent slab width supported $= 2/3\ L_x$

$= 2/3 \times 3.5 = 2.34$ m

Factored Slab Load Acting On Beam

- Factored slab load on beam
  $= \text{width of slab} \times \text{slab load per unit area}$
  $= 2.34 \times 10.00 = 23.4$ kN/m

Total Factored Load

- $w_u = 23.4 + 1.88 = 25.3$ kN/m

Total Factored Bending Moment

- $M_u = \dfrac{w_u \ell^2}{8} = \dfrac{25.3 \times 3.5^2}{8} = 38.74$ kN-m

Minimum Effective Depth For Singly Reinforced Section

- $h_{min} = d_{min} + 60$ (assuming one layer of steel)

$$= \sqrt{\frac{M_u}{0.205 f_c' \times b}} + 60$$

$$= \sqrt{\frac{39.28.74 \times 10^6}{0.205 \times 17.25 \times 228}} + 60 = 279 \text{ mm}$$

Depth For Deflection Control

- Minimum depth of beam for deflection control (h_{min}) $= L / 20$

$= 3500 / 20 = 175$ mm

Maximum Architectural Depth

- $h_{a,max}$ = 375 mm

Most General Depth

- h = L / 12 = 3500 /12 = 292 mm
- The depth may be selected in multiples of the brick height, if possible.

Selected Depth

- h = 300 mm
- d = h – 60 = 240 mm

Minimum Steel Ratio

$$\rho_{min} = 1.4 / f_y = 1.4 / 300 = 0.00467$$

Maximum Steel Ratio

$$\rho_{max} = 0.375 \times 0.85\, \beta_1 \frac{f_c'}{f_y}$$

$$= 0.375 \times 0.85 \times 0.85 \times \frac{17.25}{300} = 0.0156$$

Calculation Of Steel

Trial 1: Assume $a = d / 3 = 240/3 = 80$ mm

$$A_s = \frac{M_u}{0.9 f_y \left(d - \frac{a}{2}\right)}$$

$$= \frac{38.74 \times 10^6}{0.9 \times 300 \left(240 - \frac{80}{2}\right)} = 717 \text{ mm}^2$$

Trial 2: $a = \frac{A_s f_y}{0.85 f_c' b}$

$$= \frac{77 \times 300}{0.85 \times 17.25 \times 228} = 64 \text{ mm}$$

$$A_s = \frac{38.74 \times 10^6}{0.9 \times 300 \left(240 - \frac{64}{2}\right)} = 690 \text{ mm}^2$$

Trial 3: $a = \dfrac{690 \times 300}{0.85 \times 17.25 \times 228} = 62$ mm

$$A_s = \frac{38.74 \times 10^6}{0.9 \times 300 \left(240 - \frac{62}{2}\right)} = 687 \text{ mm}^2$$

(sufficiently close to the previous answer)

A reinforcement of 2 #20 + 1 #15 provides the required area of steel

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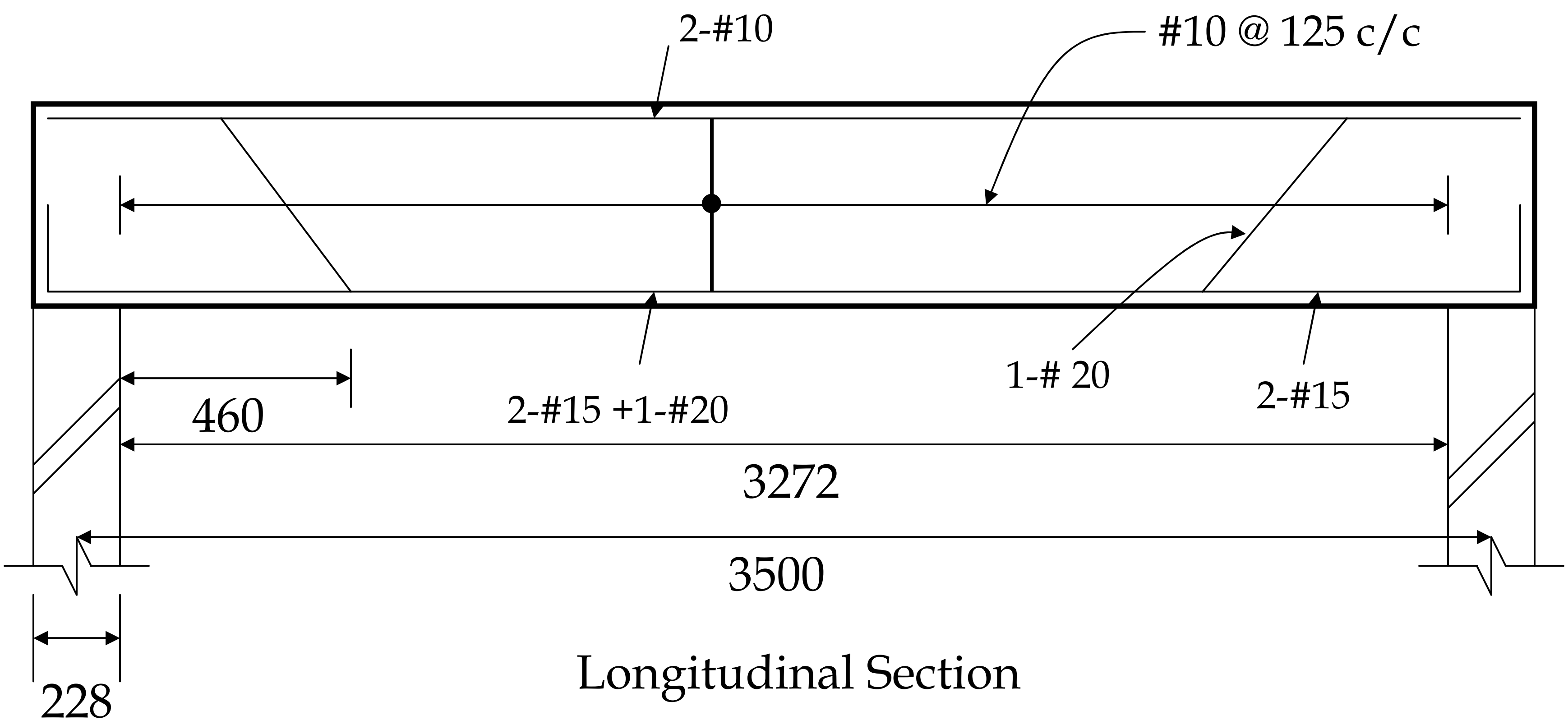
Detailing



Longitudinal Section

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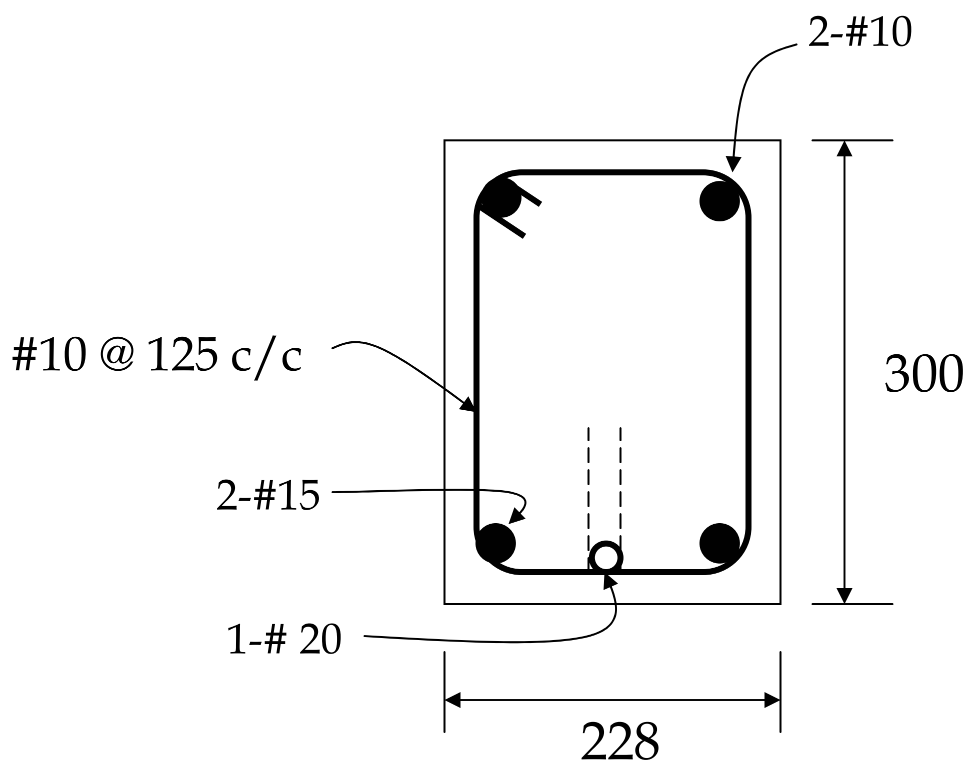
Detailing



Mid-span Cross Section

Assignment
Problems of Chapter No. 3