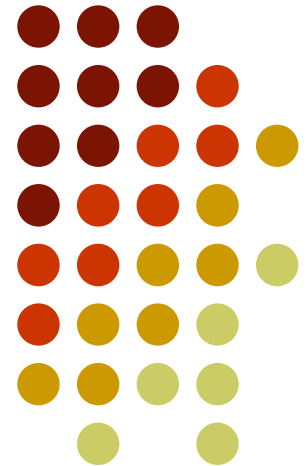


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CE-314

Lecture # 11

Flexural Analysis and Design of Beams (Ultimate Strength Design of Beams)

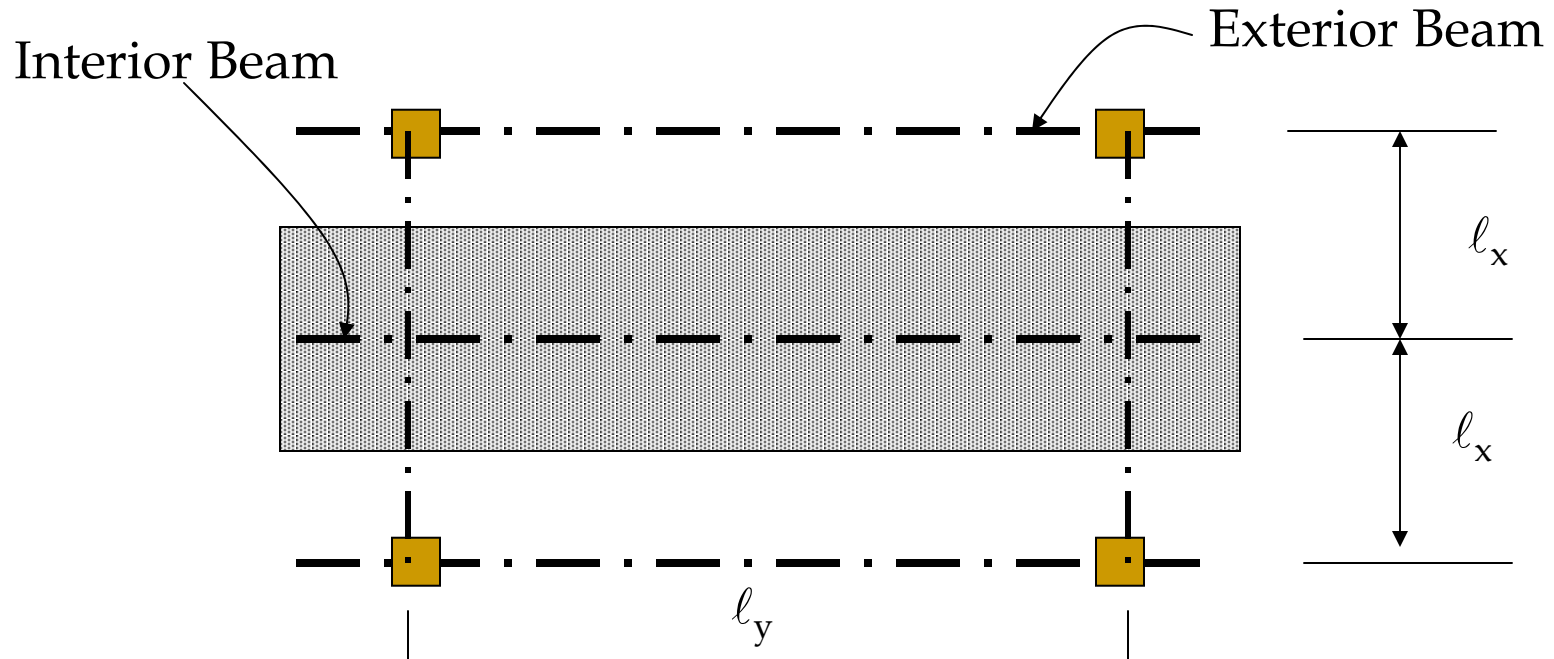


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Load Carried by the Beam

Beam Supporting One-way Slab ($l_y/l_x \geq 2$)



Width of slab supported by interior beam = l_x

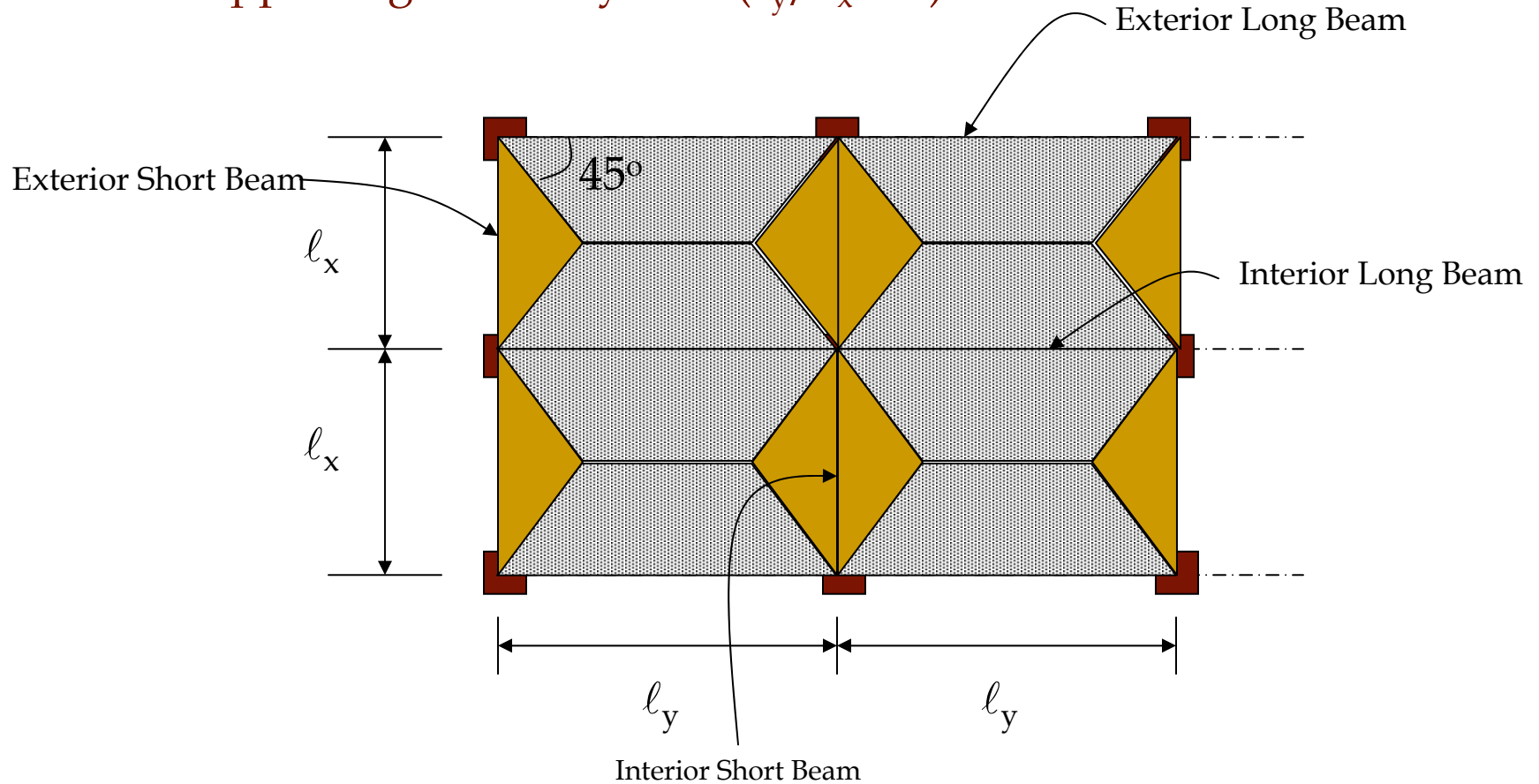
Width of slab supported by interior beam = $l_x/2 + \text{Cantilever width}$

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Load Carried by the Beam

Beam Supporting Two-way Slab ($l_y/l_x \leq 2$)



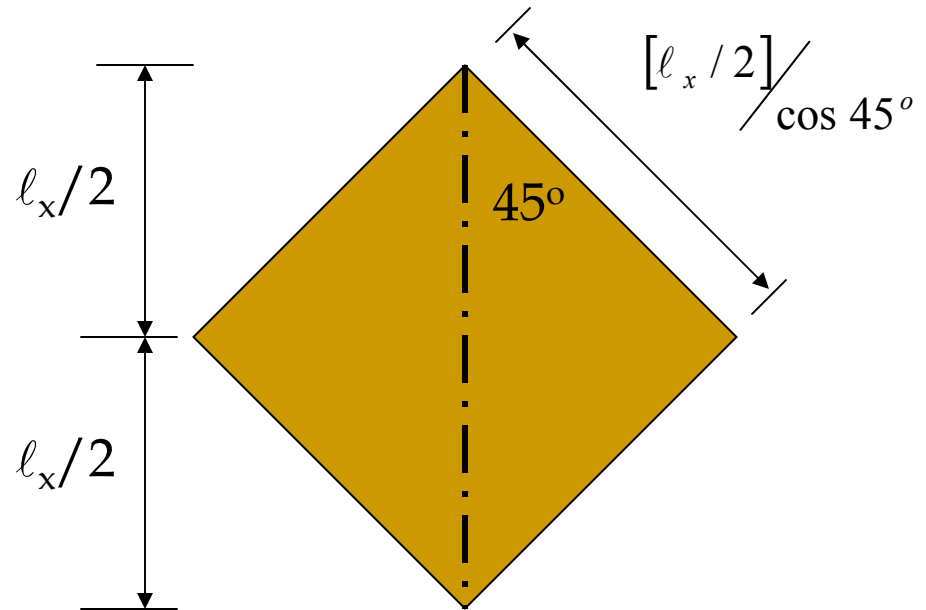
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Shorter Beams

For simplification this triangular load on both the sides is to be replaced by equivalent UDL, which gives same M_{\max} as for the actual triangular load.

$$\begin{aligned}\text{Area of Square} &= \left[\frac{[l_x/2]}{\cos 45^\circ} \right]^2 \\ &= \left[\frac{l_x}{2} \right]^2\end{aligned}$$



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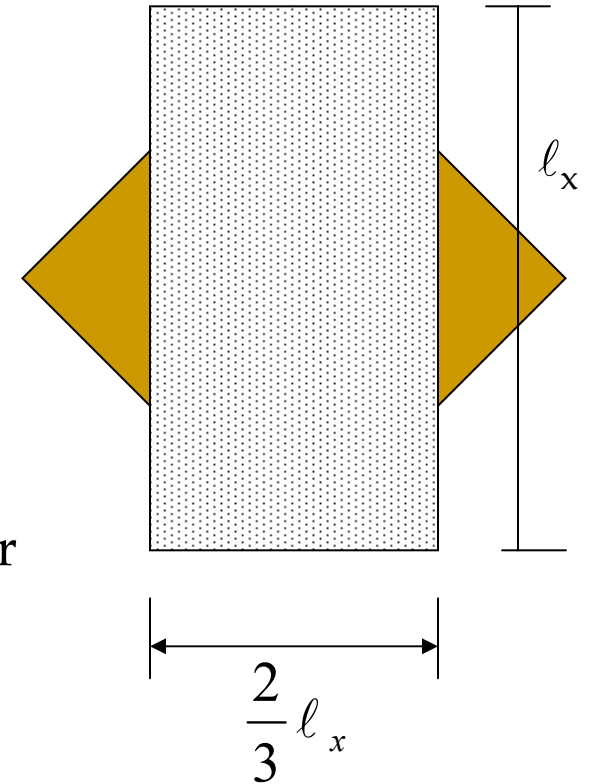


$$\text{Equivalent Rectangular Area} = \frac{4}{3} \times \frac{l_x^2}{2} = \frac{2}{3} l_x^2$$

Factor of 4/3 to convert this VDL into UDL.

$$\text{Equivalent width supported by interior short beam} = \frac{\frac{2}{3} l_x^2}{l_x} = \frac{2}{3} l_x$$

$$\text{Equivalent width supported by exterior short beam} = \frac{l_x}{3} + \text{Cantilever}$$

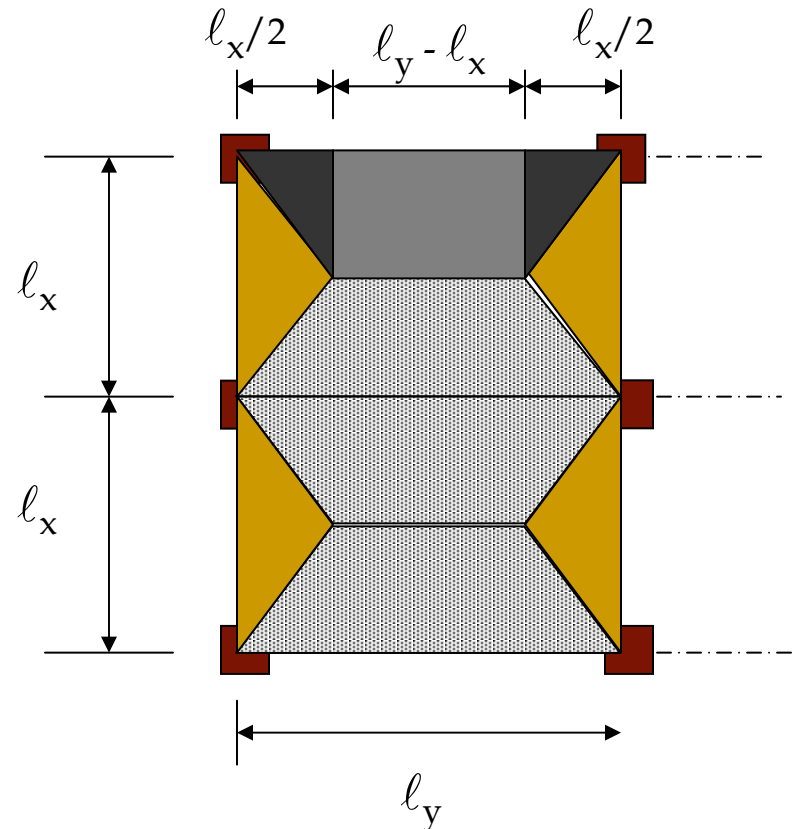


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Exterior Long Beam

$$\begin{aligned}\text{Supported Area} &= \frac{l_x}{2} \times (l_y - l_x) + \left(\frac{l_x}{2}\right)^2 \\ &= \frac{l_x l_y}{2} - \frac{l_x^2}{2} + \frac{l_x^2}{4} \\ &= \frac{l_x l_y}{2} - \frac{l_x^2}{4} \\ &= \frac{1}{2} \left(l_x l_y - \frac{l_x^2}{2} \right)\end{aligned}$$



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Exterior Long Beam

$$F = \frac{1 - R^2 / 3}{1 - R / 2}$$

where $R = \frac{l_x}{l_y}$

For Square panel

$$R = 1 \text{ and } F = 4/3$$

Factor F converts trapezoidal load into equivalent UDL for maximum B.M. at center of simply supported beam.

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Equivalent width

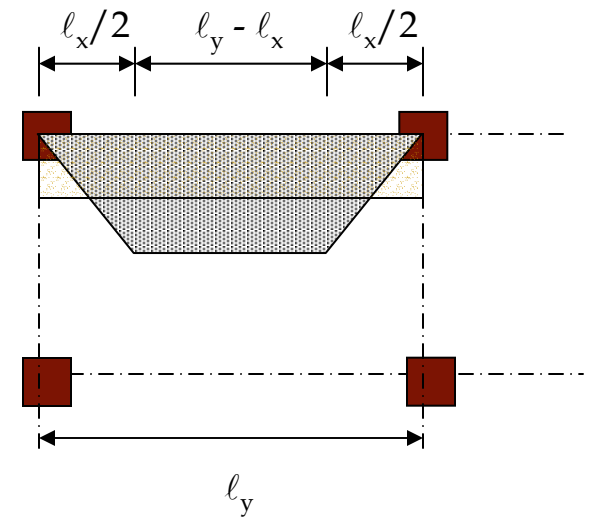
$$= \frac{(\text{Area..Supported}) \times F}{\text{Span...Length}}$$

$$= \left[\frac{1}{2} \left(l_x l_y - \frac{l_x^2}{2} \right) \times \frac{1 - R^2/3}{1 - R/2} \right] \times \frac{1}{l_y}$$

$$= \left[\frac{l_x}{2} \left(1 - \frac{l_x/l_y}{2} \right) \times \frac{1 - R^2/3}{1 - R/2} \right]$$

$$= \left[\frac{l_x}{2} (1 - R^2/3) \right] + \text{Cantilever (if present)}$$

Equivalent width



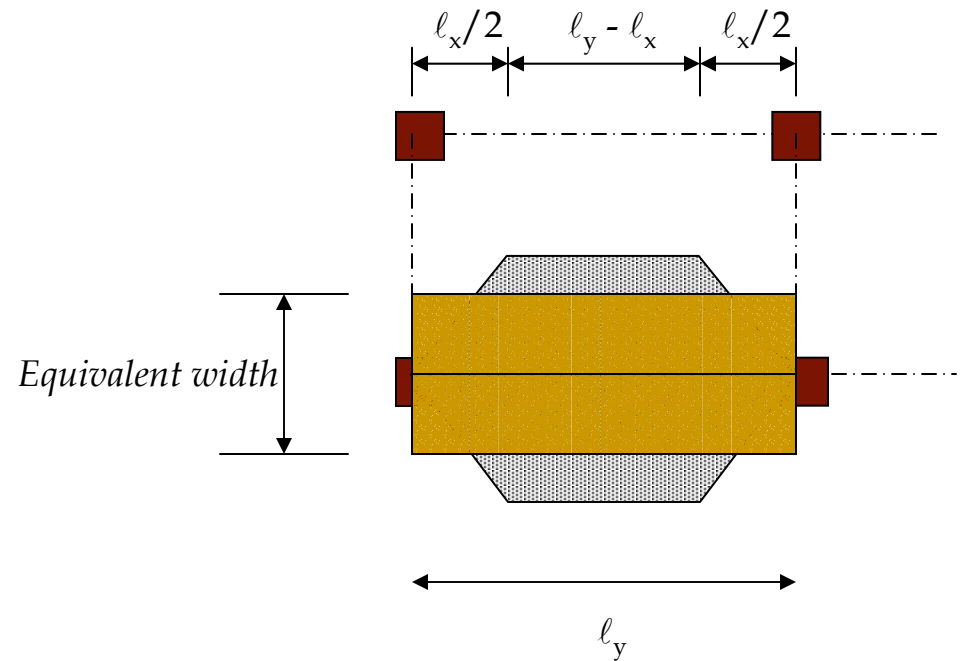
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Interior Long Beam

Equivalent width

$$= l_x \left(1 - R^2/3 \right)$$



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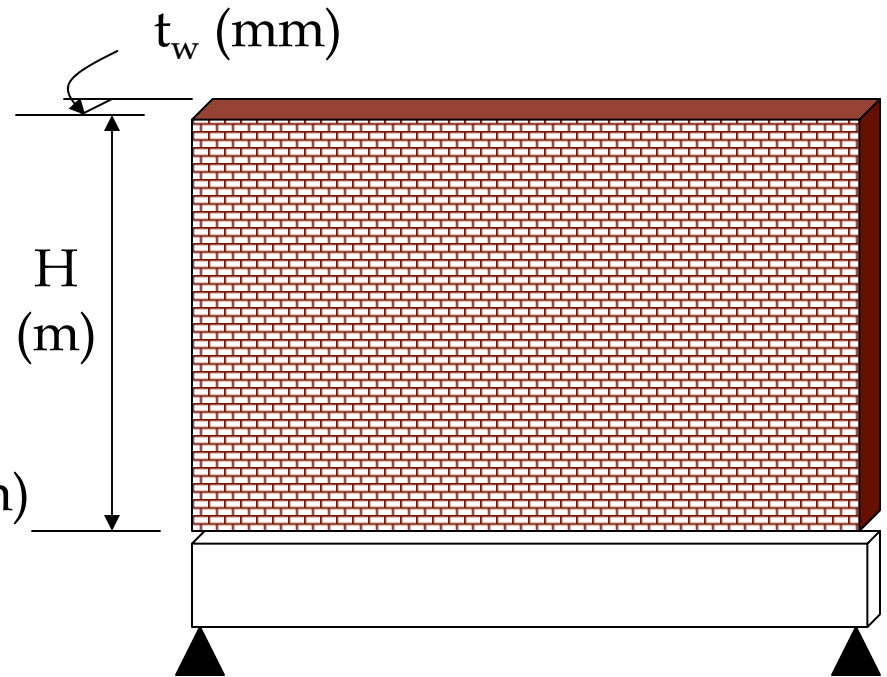


Wall Load (if present) on Beam

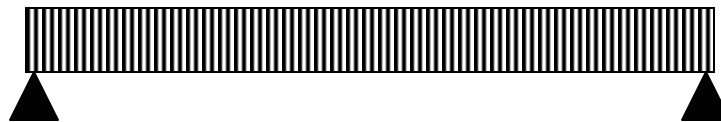
UDL on beam

$$= \left(1m \times H \times \frac{t_w}{1000} \right) \times 1930 \times \frac{9.81}{1000}$$

$$= 0.019 \times t_w \times H \text{ (kN/m)}$$



$$0.019 \times t_w \times H$$



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Wall Load on the Lintel

Equivalent UDL on lintel if height of slab above lintel is greater than $0.866L$

$$UDL = 0.11 \times t_w \times L \quad \text{kN/m}$$

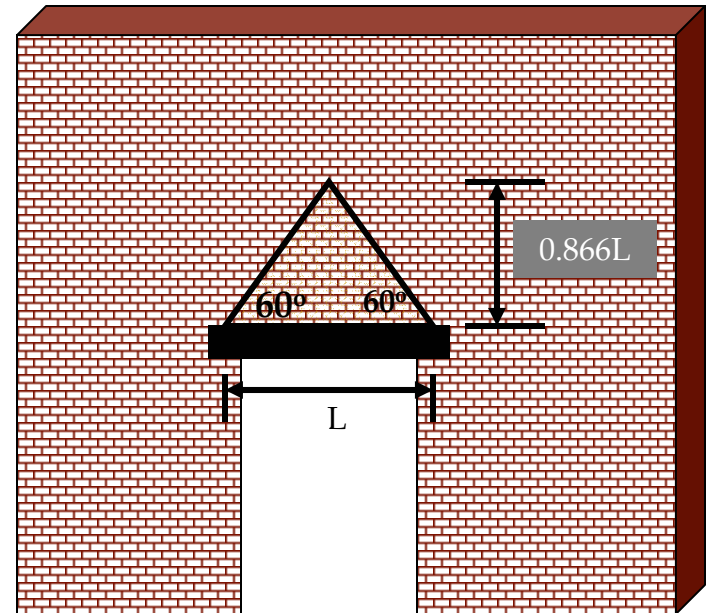
t_w = wall thickness in "mm"

L = Opening size in "m"

If the height of slab above lintel is less than $0.866L$

Total Wall Load + Load from slab in case of load bearing wall

$$\begin{aligned} UDL &= (\text{Equivalent width of slab supported}) \times (\text{Slab load per unit area}) \\ &= m \times \text{kN/m}^2 = \text{kN/m} \end{aligned}$$



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Slab Load per Unit Area

Top Roof

Slab Thickness = 125 mm

Earth Filling = 100 mm

Brick Tiles = 38 mm

Dead Load

$$\text{Self wt. of R.C. slab} = \frac{125}{1000} \times 2400 = 300 \text{ kg / m}^2$$

$$\text{Earth Filling} = \frac{100}{1000} \times 1800 = 180 \text{ kg / m}^2$$

$$\text{Brick Tiles} = \frac{38}{1000} \times 1930 = 74 \text{ kg / m}^2$$

$$\text{Total Dead Load, } W_d = 554 \text{ kg/m}^2$$

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Top Roof

Live Load

$$W_L = 200 \text{ kg/m}^2$$

Total Factored Load, W_u

$$W_u = 1.2 W_d + 1.6 W_L$$

$$W_u = (1.2 \times 554 + 1.6 \times 200) \times \frac{9.81}{1000}$$

$$W_u = 9.66 \text{ kN / m}^2$$

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Intermediate Floor

Slab Thickness	= 150 mm
Screed (brick ballast + 25% sand)	= 75 mm
P.C.C.	= 40 mm
Terrazzo Floor	= 20 mm

Dead Load

$$\text{Self wt. of R.C. slab} = \frac{150}{1000} \times 2400 = 360 \text{ kg / m}^2$$

$$\text{Screed} = \frac{75}{1000} \times 1800 = 135 \text{ kg / m}^2$$

$$\text{Terrazzo + P.C.C} = \frac{(20 + 40)}{1000} \times 2300 = 138 \text{ kg / m}^2$$

$$\text{Total Dead Load, } W_d = \underline{\underline{633 \text{ kg/m}^2}}$$

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Intermediate Floor

Live Load

$$\text{Occupancy Live Load} = 250 \text{ kg/m}^2$$

$$\text{Moveable Partition Load} = 150 \text{ kg/m}^2$$

$$W_L = 250 + 150 = 400 \text{ kg/m}^2$$

Total Factored Load, W_u

$$w_u = 1.2w_d + 1.6w_L$$

$$w_u = (1.2 \times 633 + 1.6 \times 400) \times \frac{9.81}{1000}$$

$$w_u = 13.73 \text{ kN / m}^2$$

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Self Weight of Beam

Service Self Wight of Beam = $b \times h \times 1\text{m} \times 2400$

$$= \frac{L}{12} \times \frac{L}{18} \times 1\text{m} \times 2400 = 11.11L^2 \quad \text{kg/m}$$

Factored Self Wight of Beam

$$= 11.11L^2 \times 1.2 \times \frac{9.81}{1000} = 0.131L^2 \quad \text{kN/m}$$

Self weight of beam is required to be calculated in at the stage of analysis, when the beam sizes are not yet decided, so approximate self weight is computed using above formula.



- After calculation of steel requirement for bending moment, shear force and torque at important sections of a beam, detailing for reinforcement is carried out.
- Detailing means selection of the bar sizes, number of bars, bar spacing, bar positions and bar cut-offs and showing the results on a neat sketch.
- A bar bending schedule is then prepared which helps the cutting, bending and placing of bars.

- The number, length, diameter, steel grade and shape of each bar are separately written.
- Each type of bar is given a separate designation on the drawings and the corresponding values are entered in the bar-bending schedule.
- This table gives the total amount of steel required for a particular project. The columns of a typical bar bending schedule are given in Table 3.3.



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Bar Bending Schedule

Serial #	Bar Designation	Number of Bars	Length of one Bar	Dia of bar	Weight of Steel Required					Shape of Bar
					#10	#13	#15	#19	#25	
1	M-1			#25					✓	
2	S-1			#10	✓					
3	H-1			#15			✓			

Σ

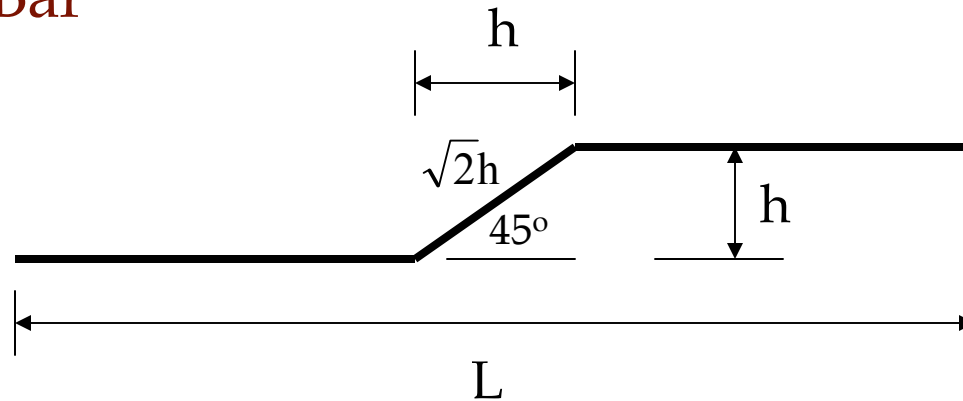
Total weight of steel = 1.05Σ , 5% increase, for wastage during cutting and bending

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Bar Bending Schedule (contd...)

Bent-up Bar



Additional Length = $0.414 h$

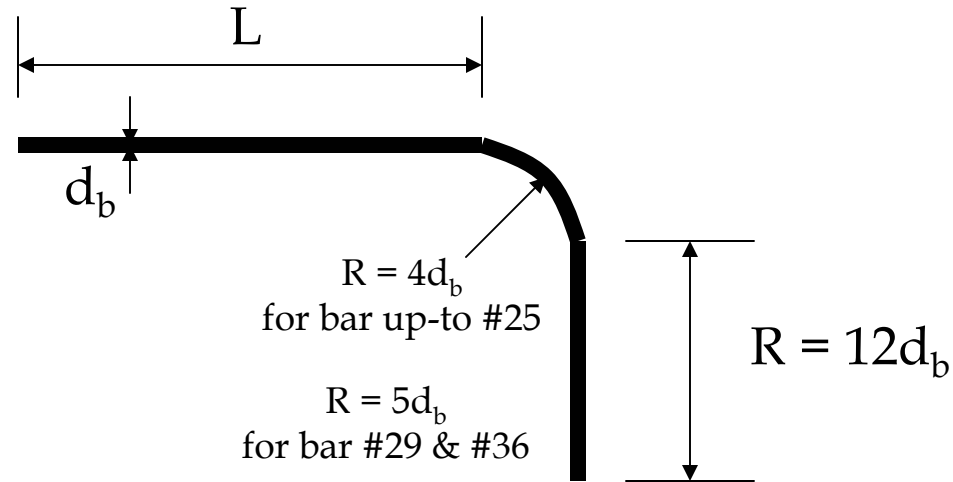
Total Length = $L + 0.414 h$

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Bar Bending Schedule (contd...)

90°-Standard Hooks (ACI)



$$\text{Total Length} = L + 18d_b$$

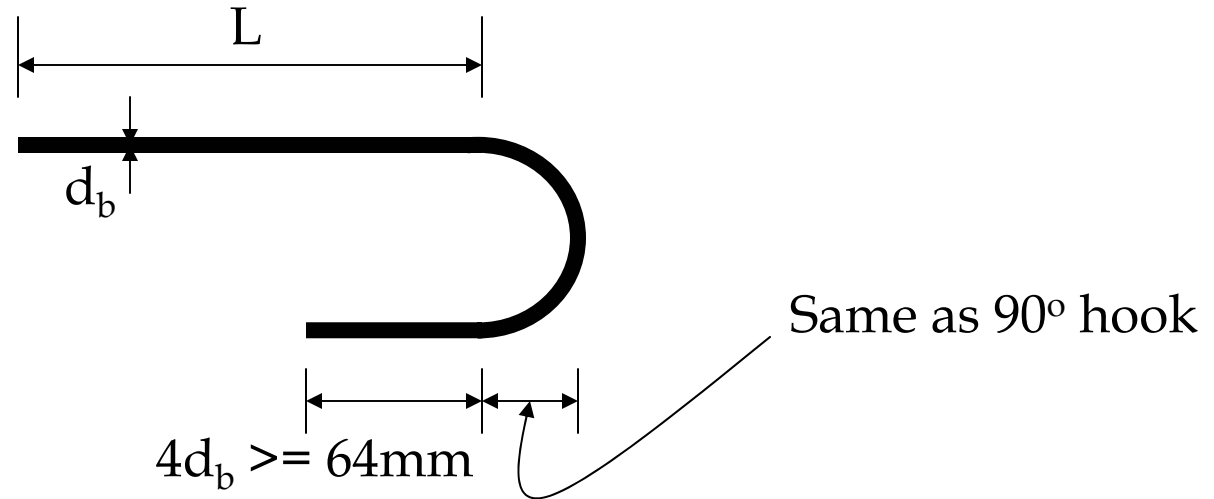
For $R = 4d_b$

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Bar Bending Schedule (contd...)

180°-Standard Hooks (ACI)



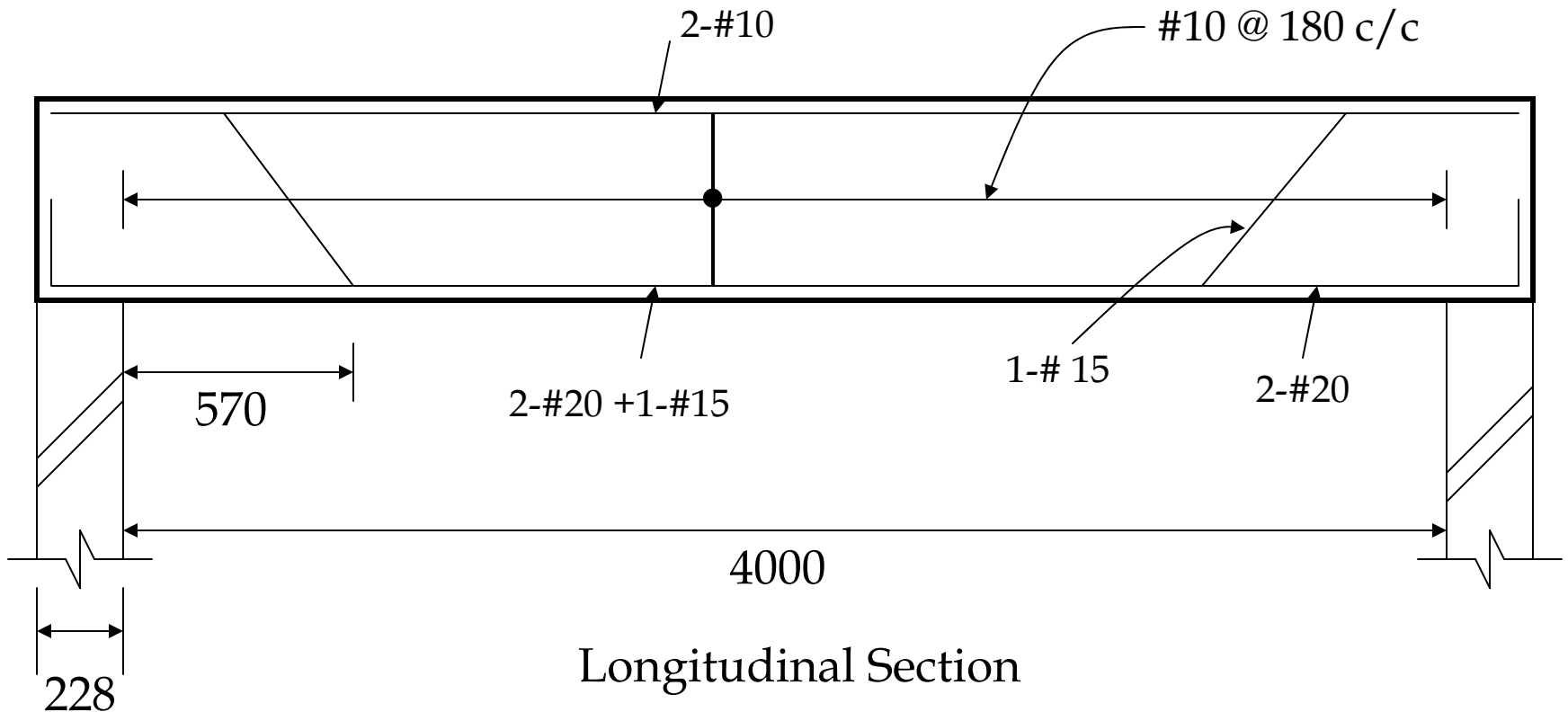
$$\text{Total Length} = L + 20d_b$$

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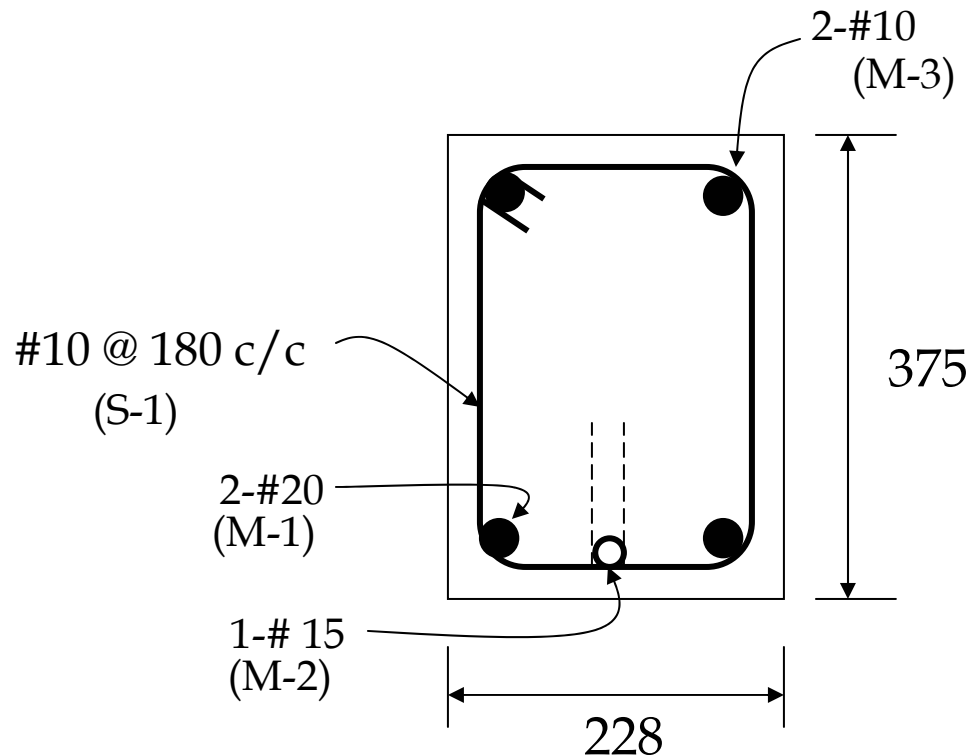


Bar Bending Schedule (contd...)

Example: Prepare bar bending schedule for the given beam. Clear cover = 40 mm and SI steel bars are used.



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Cross Section

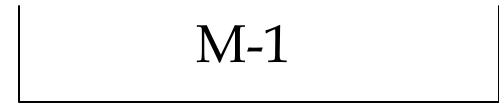
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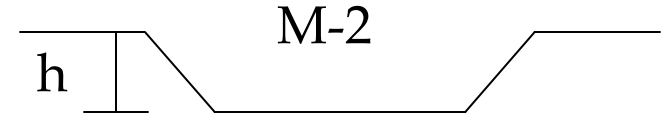
Bar Bending Schedule (contd...)

Example:

$$\begin{aligned} M-1 &= 4000 + 2 \times 228 - 2 \times 60 + 2 \times (18 \times 20) \\ &= 5056 \end{aligned}$$



$$h = 375 - 2 \times 40 - 2 \times 10 - 2 (15/2) = 260$$



$$\begin{aligned} M-2 &= 4000 + 2 \times 228 - 2 \times 40 + (0.414 \times 260) \times 2 \\ &= 5092 \end{aligned}$$

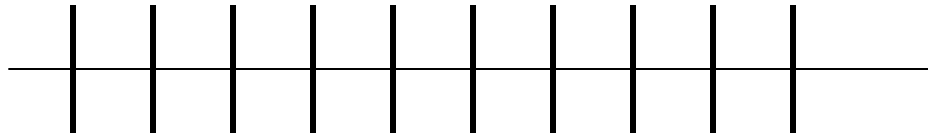
$$\begin{aligned} M-1 &= \text{same as M-1} \\ &= 5056 \end{aligned}$$



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Shear stirrups



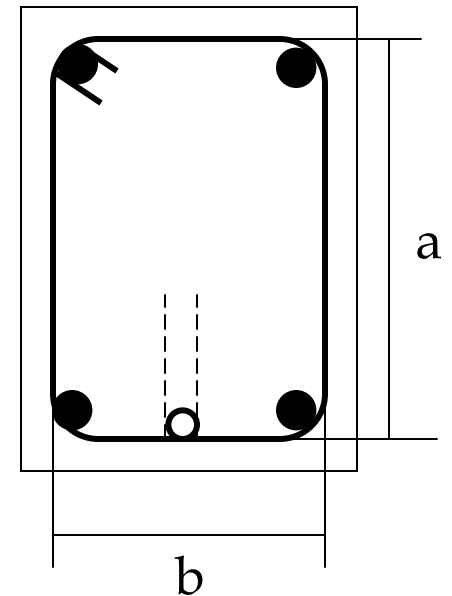
$$\text{Number of Bars} = 4436 / 180 = 25$$

Round-up

$$a = 375 - 2 \times 40 - 10 = 285 \text{ mm}$$

$$b = 228 - 2 \times 40 - 10 = 138 \text{ mm}$$

$$\text{Total length of S-1} = 2 (138 + 285 + 18 \times 10) = 1206 \text{ mm}$$



Plain & Reinforced Concrete-1



Bar Bending Schedule

Serial #	Bar Designation	Number of Bars	Length of one Bar (m)	Dia of bar	Weight of Steel Bars			Shape of Bar
					#10	#15	#20	
1	M-1	2	5.056	20			23.9	
2	M-2	1	5.092	15		10.0		
3	M-3	2	5.056	10	8.0			
4	S-1	25	1.206	10	23.7			
1.05 Σ					33.3	10.5	25.1	

Total Weight = 69 kg



Concluded