Plain & Reinforced Concrete-1

CE-314

Lecture # 10

Flexural Analysis and Design of Beams

(Ultimate Strength Design of Beams)

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Capacity Analysis of Singly Reinforced Rectangular Beam by Strength Design method

Data

1. Dimensions, b, h, d and L (span)
2. f_c', f_y, E_c, E_s
3. A_s

Required

1. $\phi_b M_n$
2. Load Carrying Capacity

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Solution

Step # 1Calculte the depth of N.A assuming the section to be under-reinforced.

$$f_s = f_y \quad \text{and} \quad \varepsilon_s \geq \varepsilon_y$$

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad \text{and} \quad c = \frac{a}{\beta_1}$$

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Solution

Step # 2 Calculate ε_s and check the assumption of step# 1

$$\varepsilon_s = \varepsilon_t = 0.003\frac{d-c}{c}$$ For extreme point

If $\varepsilon_s \geq \varepsilon_y$, the assumption is correct

If $\varepsilon_s \leq \varepsilon_y$, the section is under-reinforced. So "a" is to be calculated again by the formula of over reinforced section

$$a = \frac{A_s \times 600\left(\frac{\beta_1 d - a}{a}\right)}{0.85 f_c' b}$$

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Capacity Analysis of Singly Reinforced Rectangular Beam by Strength Design method (contd…)

Solution

Step # 3 Decide ϕ factor

For $\varepsilon_s \geq 0.005$, $\phi = 0.9$ (Tension controlled section)

For $\varepsilon_s \leq \varepsilon_y$, $\phi = 0.65$ (Compression controlled section)

For $\varepsilon_y \leq \varepsilon_s \leq 0.005$, Interpolate value of ϕ (Transition Section)

Step # 4 Calculate $\phi_b M_n$

$$\phi_b M_n = \phi_b A_s f_y \left(d - \frac{a}{2} \right)$$

For under-reinforced Section

$$\phi_b M_n = \phi_b \times 0.85 f_c' ba \left(d - \frac{a}{2} \right)$$

For over-reinforced Section

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Capacity Analysis of Singly Reinforced Rectangular Beam by Strength Design method (contd…)

Alternate Method

Step # 1 to step # 3 are for deciding whether the section is tension over reinforced or under-reinforced. Alternatively it can be done in the following manner.

1. Calculate ρ and ρ_{max} if $\rho < \rho_{max}$ section is under-reinforced.
2. Calculate d_{min}, if $d \geq d_{min}$, section is tension controlled

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Example

A singly reinforced rectangular beam has a width of 228 mm and effective depth of 450 mm. Use C-18 concrete and Grade 420 steel. Calculate flexural capacity for the following three cases.

1. 2 # 25 bars (SI size)
2. 3 # 25 + 2 # 15 (SI)
3. Capacity for balanced steel

Solution

- b = 228 mm
- d = 450 mm
- f_c' = 18 MPa
- f_y = 420 MPa
- E_s = 200,000 MPa
- $\phi_b M_n$ = ?

  Case (i) A_s = 1000 mm^2

  Case (ii) A_s = 1500 + 400 = 1900 mm^2

  Case (iii) balanced failure

Case (i)

- $a = \dfrac{A_s f_y}{0.85 f_c' b} = \dfrac{1000 \times 420}{0.85 \times 18 \times 228} = 120.4$ mm

- $\varepsilon_s = 0.003 \dfrac{\beta_1 d - a}{a} = 0.003 \dfrac{0.85 \times 450 - 120.4}{120.4}$

  $= 0.00653$

  $> \varepsilon_y = 420 / 200{,}000 = 0.0021$

  $\varepsilon_s > \varepsilon_y \Rightarrow$ tension failure, $\phi_b = 0.90$

- $\phi_b M_n = \phi_b A_s f_y (d - a/2)$

  $= (0.9)(1000)(420)(450 - 120.4 / 2) / 10^6$

  $= 147.3$ kN-m

Case (ii)

- First assuming the under-reinforced behavior, we get,

1. $a = \dfrac{A_s f_y}{0.85 f_c' b} = \dfrac{1900 \times 420}{0.85 \times 18 \times 228} = 228.8$ mm

2. $\varepsilon_s = 0.003 \dfrac{\beta_1 d - a}{a}$

$= 0.003 \dfrac{0.85 \times 450 - 228.8}{228.8}$

$= 0.00202$

$< \varepsilon_y = 420 / 200{,}000 = 0.0021$

Revise '*a*' by considering the expression for the compression-controlled section.

- $\rho = A_s / bd = \dfrac{1900}{228 \times 450} = 0.0185$

$$\left(\frac{0.85 f_c'}{0.003 E_s \rho}\right) a^2 + a\,d - \beta_1 d^2 = 0$$

$$\left(\frac{0.85 \times 18}{600 \times 0.0185}\right) a^2 + 450\,a - 0.85 \times 450^2 = 0$$

$$1.378\,a^2 + 450\,a - 172{,}125 = 0$$

- $a = \dfrac{-450 + \sqrt{450^2 + 4 \times 1.378 \times 172{,}125}}{2 \times 1.378}$ = 226 mm

3. The section is compression-controlled $\Rightarrow$ $\phi_b = 0.65$

4. $\phi_b M_n = \phi_b\, 0.85\, f_c{}'\, b\, a\ (d - a / 2)$

   $= (0.65)(0.85)(18)(228)(226)(450 - 226 / 2) / 10^6$

   $= 172.7$ kN-m

Case (iii)

1. $\rho_b = 0.85\,\beta_1 \dfrac{f'_c}{f_y} \dfrac{600}{f_y + 600}$

$$= 0.85 \times 0.85 \frac{18}{420} \frac{600}{1020} = 0.01821$$

2. Let $A_s = \rho_b\, b\, d$

$$= 0.01821 \times 228 \times 450 = 1869 \text{ mm}^2$$

3. $a_b = \dfrac{A_s f_y}{0.85 f'_c b}$

$$= \frac{(1869)(420)}{(0.85)(18)(228)} = 225 \text{ mm}$$

4. $\phi_b M_n = \phi_b A_s f_y (d - a/2)$

$= (0.65)(1869)(420)(450 - 225 / 2) / 10^6$

$= 172.2$ kN-m

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Selection of Steel Bars for Beams

1. When different diameters are selected the maximum difference can be a gap of one size.
2. Minimum number of bars must be at least two, one in each corner.
3. Always place the steel symmetrically.
4. Preferably steel may be placed in a single layer but it is allowed to use 2 to 3 layers.
5. Selected sizes should be easily available in market.
6. A lesser number of bars is usually preferable because of ease of cutting, placing of steel and pouring of concrete.

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Selection of Steel Bars for Beams (contd…)

7. ACI Code Requirements

There must be a minimum clearance between bars (only exception is bundled bars).

- Concrete must be able to flow through the reinforcement.
- Bond strength between concrete and steel must be fully developed.

Minimum spacing must be lesser of the following

- Nominal diameter of bars
- 25mm in beams & 40mm in columns
- 1.33 times the maximum size of aggregate used.

We can also give an additional margin of 5 mm.

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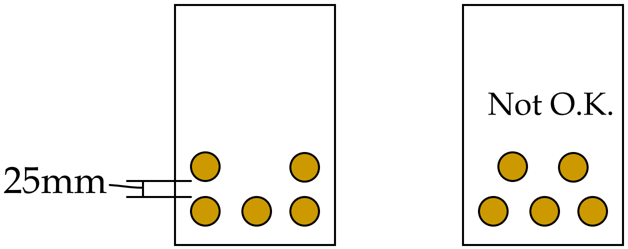
Selection of Steel Bars for Beams (contd…)

8. A minimum clear gap of 25 mm is to be provided between different layers of steel.
9. The spacing between bars must not exceed a maximum value for crack control, usually applicable for slabs.

What is Detailing?



- Deciding diameter of bars
- Deciding no. of bars
- Deciding location of bent-up and curtailment of bars
- making sketches of reinforcements.

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Concrete Cover to Reinforcement

Measured as clear thickness outside the outer most steel bar.

Purpose

- To prevent corrosion of steel
- To improve the bond strength
- To improve the fire rating of the building
- It reduces the wear of steel and attack of chemicals specially in factories.

Concrete Cover to Reinforcement (contd…)

ACI Code Minimum Clear Cover Requirements

1. Concrete permanently exposed to earth — 75 mm
2. Concrete occasionally exposed to earth
   - # 19 to # 57 bars — 50 mm
   - # 16 and smaller bars — 40 mm
3. Sheltered Concrete
   - Slabs and walls — 20 mm
   - Beams and columns — 38 mm

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Number of Bars in a Single Layer (for beams)

$$N_b = 0.02b_w - 1.4$$

Rounded to lower whole number

b_w = width of web of beam

For I-shaped beam, width of bottom flange should be used in place of b_w .

Concluded